Image Alignment

16-385 Computer Vision (Kris Kitani)

Carnegie Mellon University

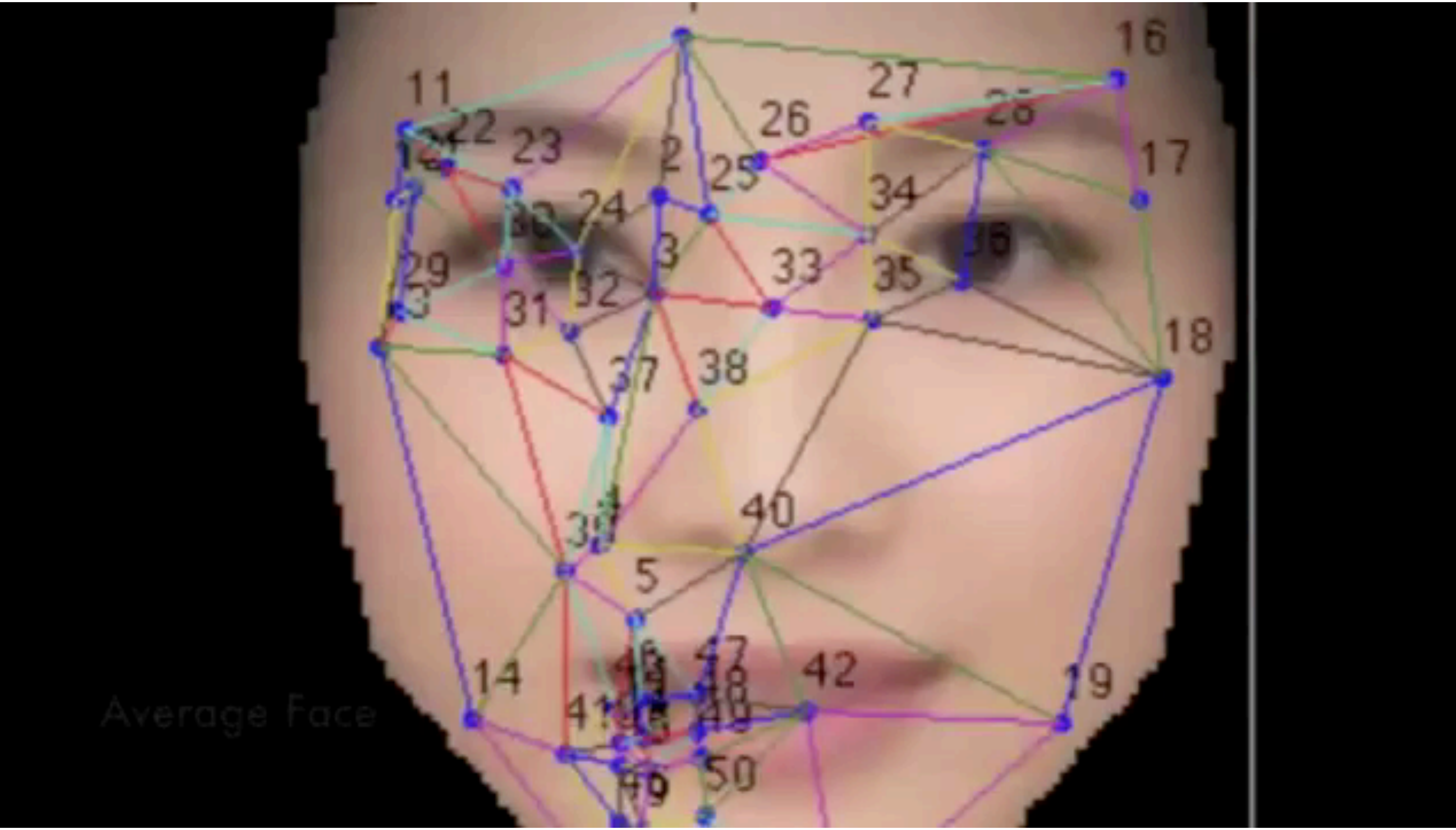
Average Face

Image Alignment

(start with an initial solution, match the image and template)

Image Alignment Objective Function

$$\sum_{\boldsymbol{x}} \left[I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p})) - T(\boldsymbol{x}) \right]^2$$

Given an initial solution…several possible formulations

Additive Alignment

$$\sum_{\boldsymbol{x}} \left[I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p} + \Delta\boldsymbol{p})) - T(\boldsymbol{x}) \right]^2$$

incremental perturbation of parameters

Image Alignment Objective Function

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$$\sum_{\boldsymbol{x}} \left[I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p}+\Delta\boldsymbol{p})) - T(\boldsymbol{x}) \right]^2$$

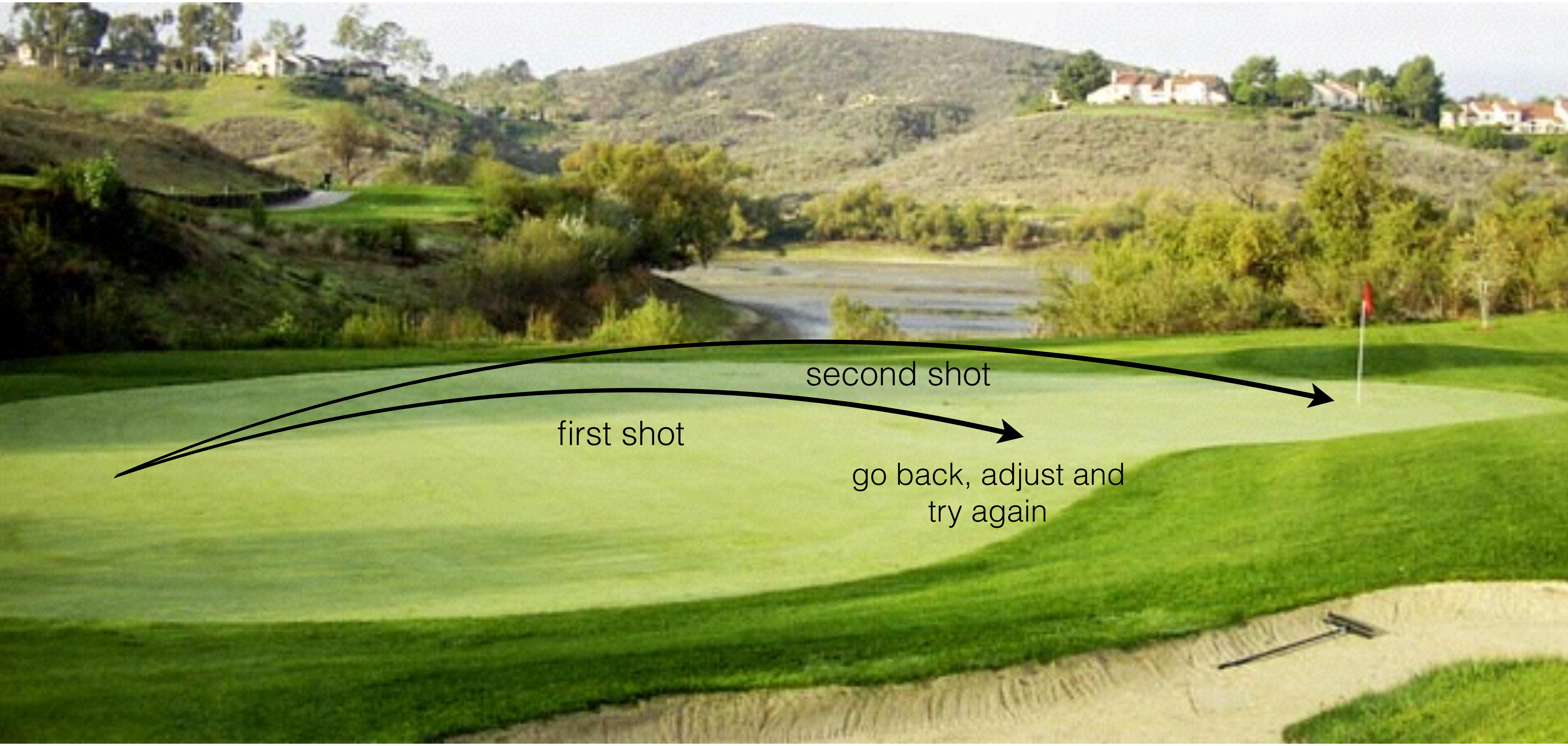
incremental perturbation of <u>parameters</u>

Compositional Alignment

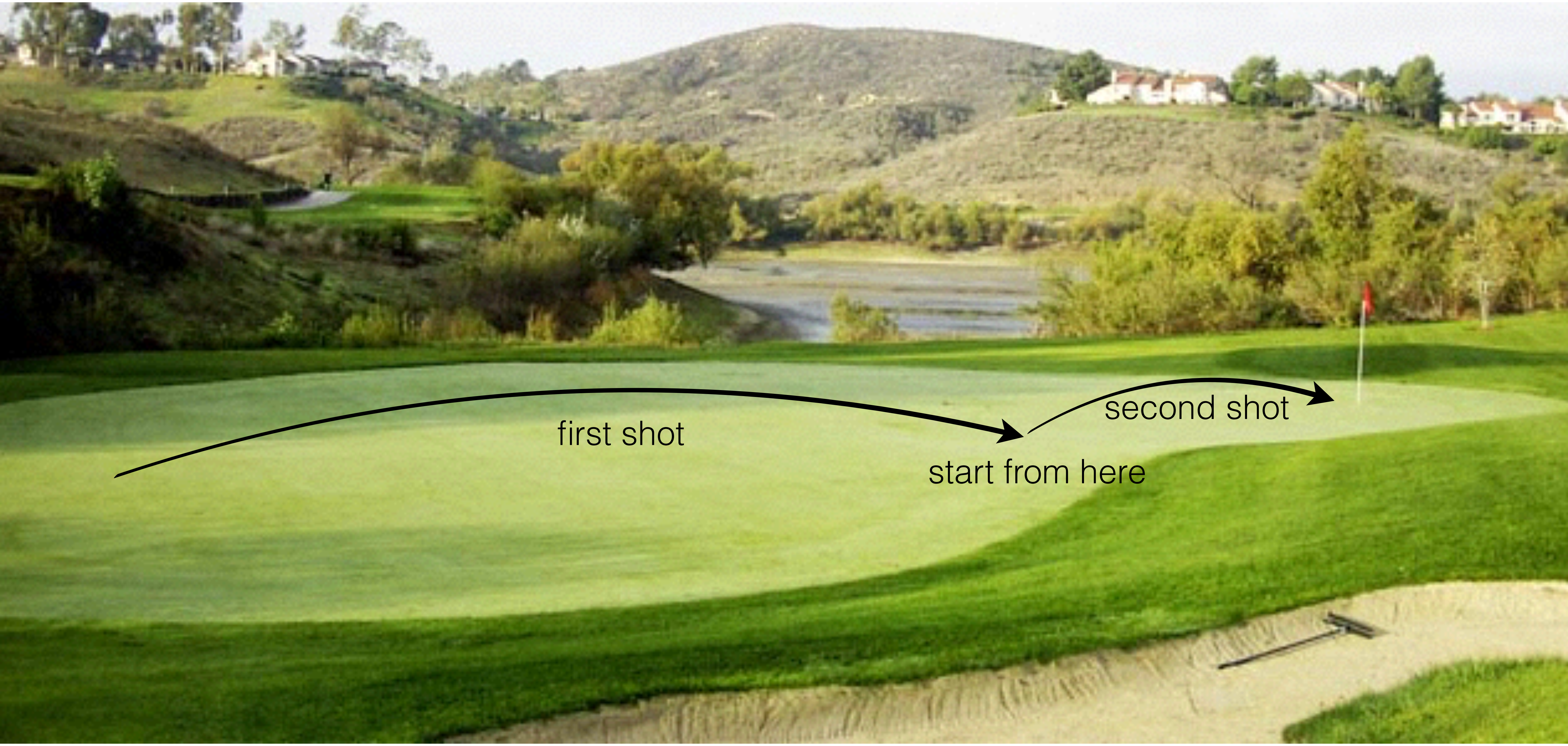
$$\sum_{\boldsymbol{x}} \left[I(\mathbf{W}(\ \mathbf{W}(\boldsymbol{x};\Delta\boldsymbol{p});\boldsymbol{p}\ ) - T(\boldsymbol{x}) \right]^2$$

incremental <u>warps</u> of image

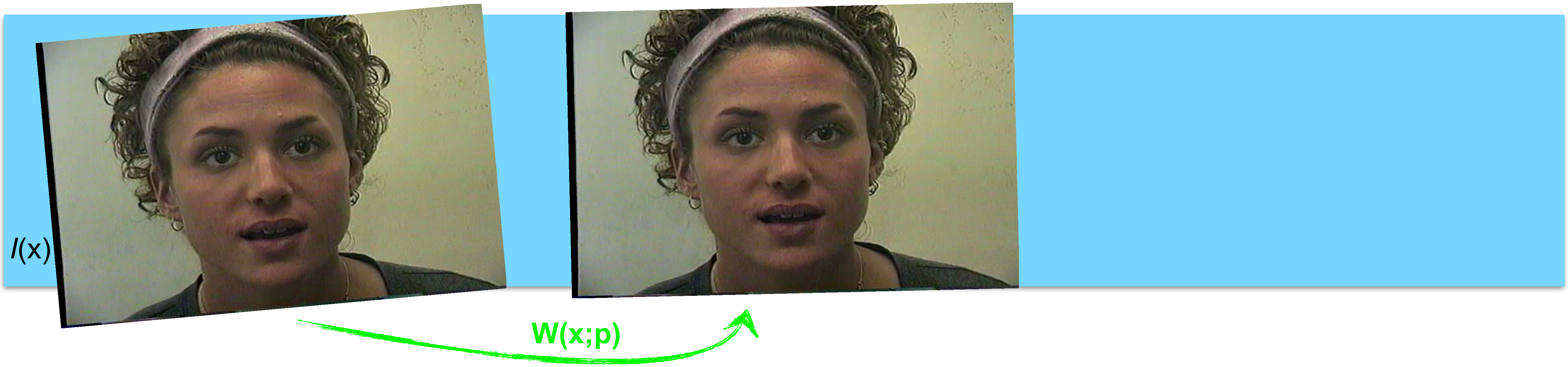
Additive strategy



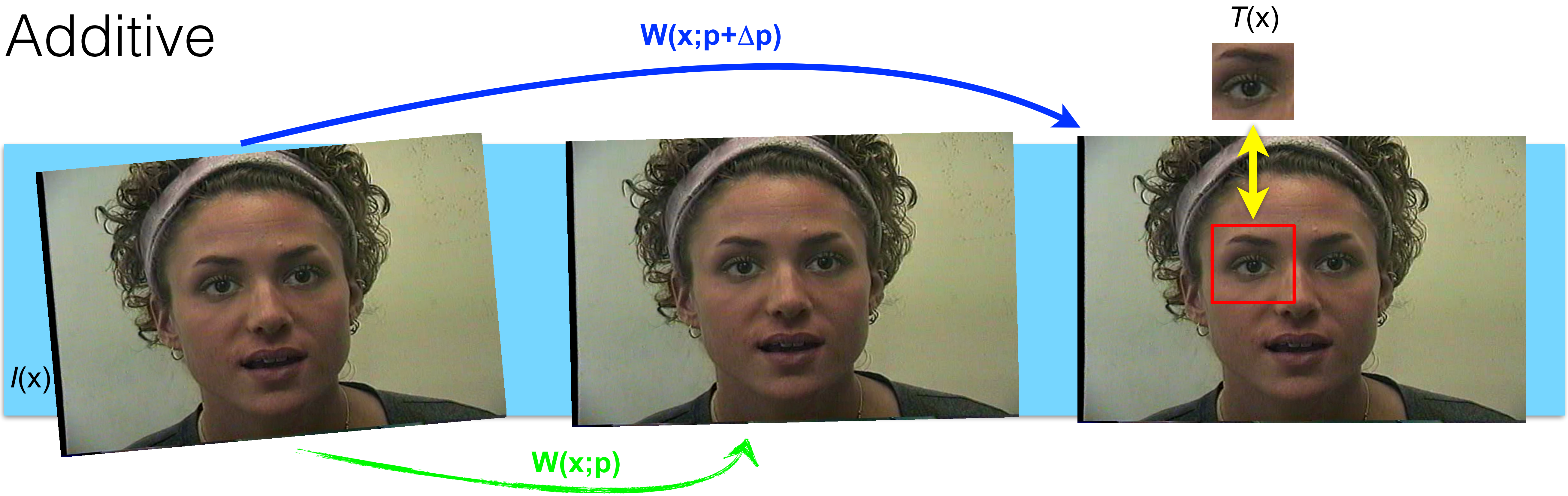
Compositional strategy



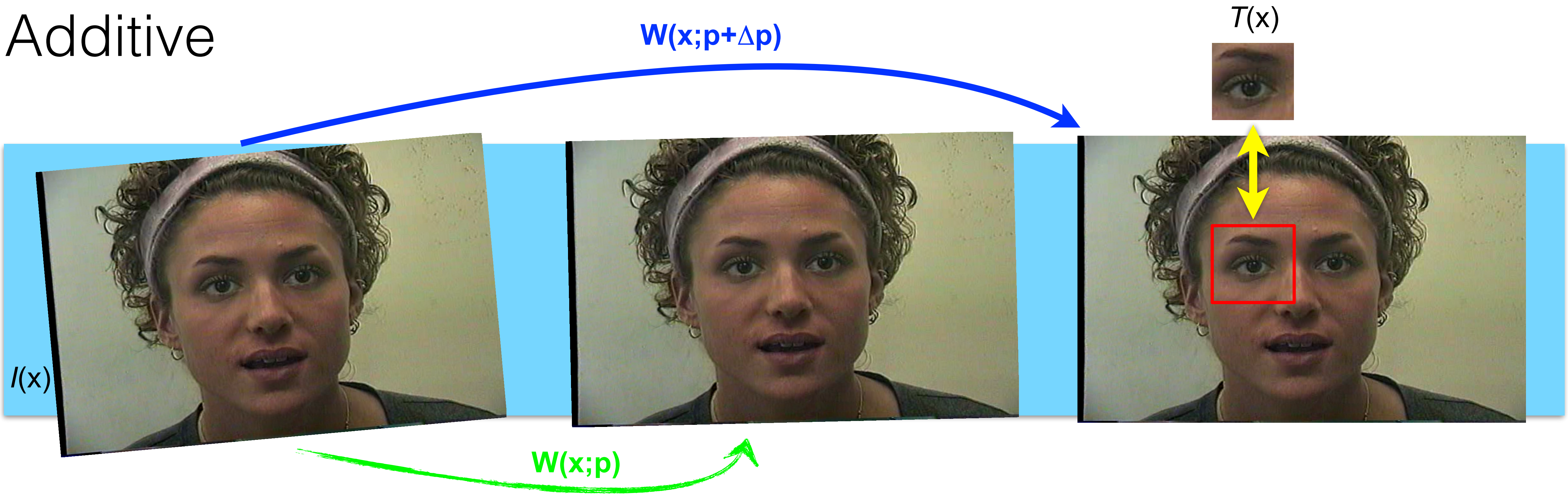
Additive



Additive

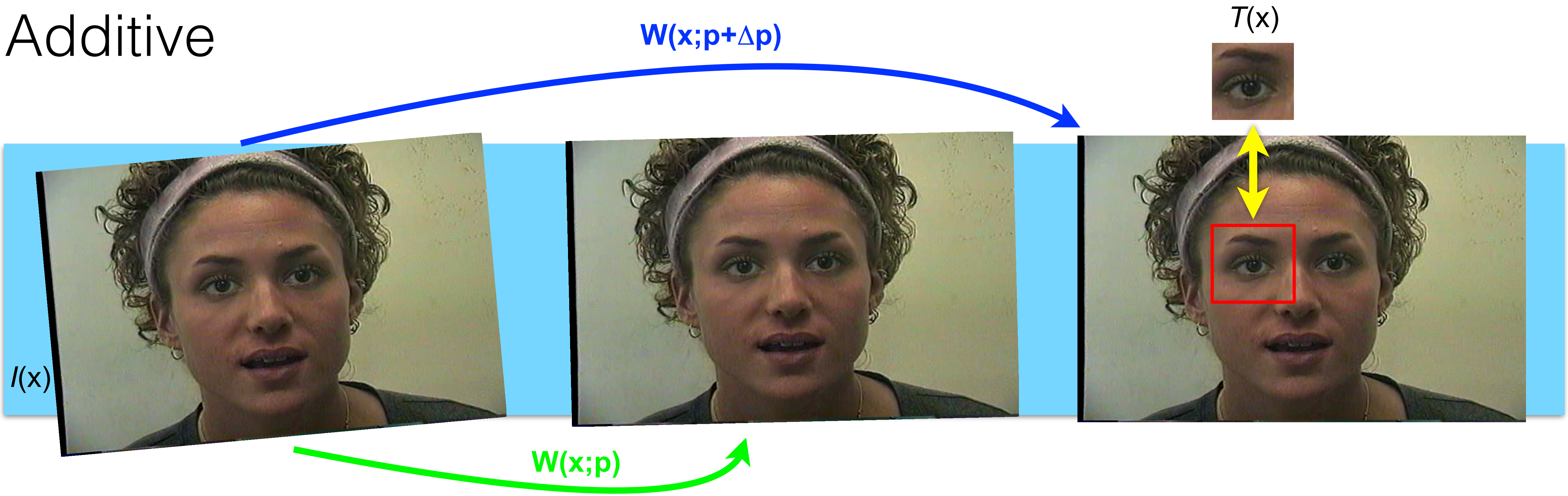


Additive

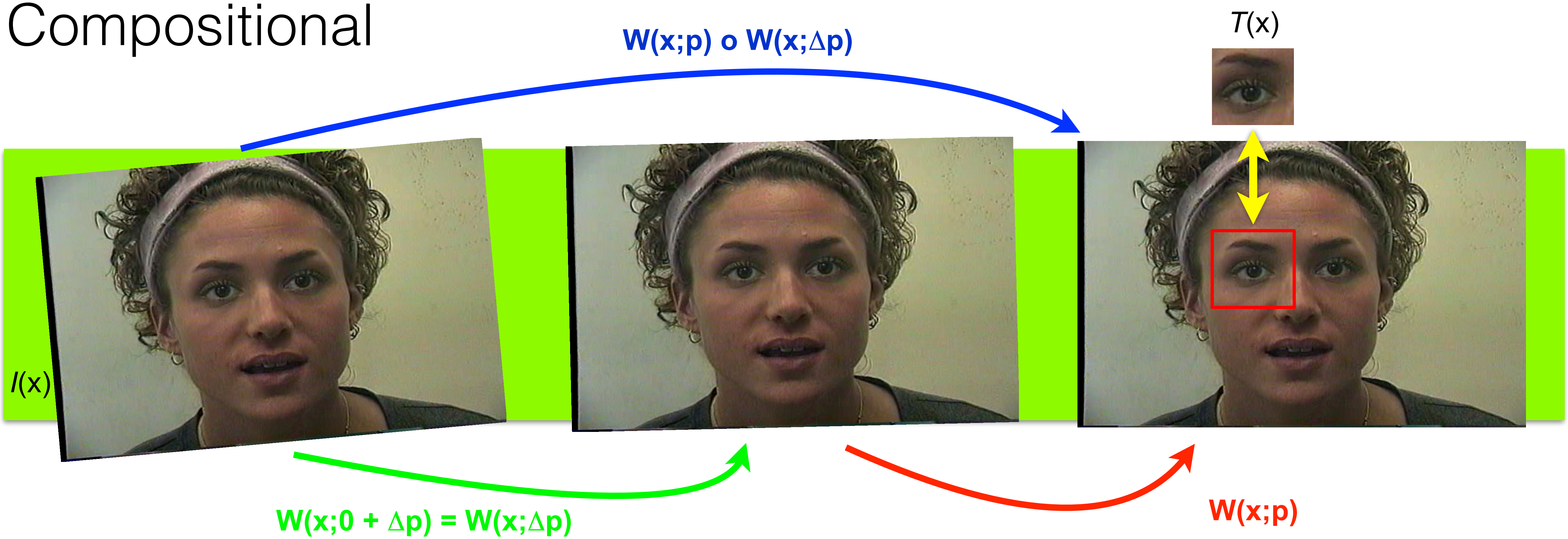


Compositional

Additive



Compositional



Compositional Alignment

Original objective function (SSD)

$$\min_{\boldsymbol{p}} \sum_{\boldsymbol{x}} \left[I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p})) - T(\boldsymbol{x}) \right]^2$$

Assuming an initial solution **p** and a compositional warp increment

$$\sum_{\boldsymbol{x}} \left[I(\mathbf{W}(\ \mathbf{W}(\boldsymbol{x};\Delta\boldsymbol{p});\boldsymbol{p}\ ) - T(\boldsymbol{x}) \right]^2$$

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Another way to write the composition

$$\mathbf{W}(\boldsymbol{x};\boldsymbol{p}) \circ \mathbf{W}(\boldsymbol{x};\Delta\boldsymbol{p}) \equiv \mathbf{W}(\ \mathbf{W}(\boldsymbol{x};\Delta\boldsymbol{p});\boldsymbol{p}\ )$$

Identity warp

$$\mathbf{W}(\boldsymbol{x};\mathbf{0})$$

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Identity warp

$$\mathbf{W}(\boldsymbol{x};\mathbf{0})$$

Skipping over the derivation…the new update rule is

$$\mathbf{W}(\boldsymbol{x};\boldsymbol{p}) \leftarrow \mathbf{W}(\boldsymbol{x};\boldsymbol{p}) \circ \mathbf{W}(\boldsymbol{x};\Delta\boldsymbol{p})$$

So what’s so great about this compositional form?

Additive Alignment

$$\sum_{\boldsymbol{x}} \left[I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p}+\Delta\boldsymbol{p})) - T(\boldsymbol{x}) \right]^2$$

Compositional Alignment

$$\sum_{\boldsymbol{x}} \left[I(\mathbf{W}(\ \mathbf{W}(\boldsymbol{x};\Delta\boldsymbol{p});\boldsymbol{p}\ ) - T(\boldsymbol{x}) \right]^2$$

linearized form

$$\sum_{\boldsymbol{x}} \left[I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p})) + \nabla I(\boldsymbol{x}') \frac{\partial \mathbf{W}}{\partial \boldsymbol{p}} \Delta\boldsymbol{p} - T(\boldsymbol{x}) \right]^2$$

linearized form

$$\sum_{\boldsymbol{x}} \left[I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p})) + \nabla I(\boldsymbol{x}') \frac{\partial \mathbf{W}(\boldsymbol{x};\mathbf{0})}{\partial \boldsymbol{p}} \Delta\boldsymbol{p} - T(\boldsymbol{x}) \right]^2$$

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Jacobian of W(x;p)

Compositional Alignment

$$\sum_{\boldsymbol{x}} \left[I(\mathbf{W}(\ \mathbf{W}(\boldsymbol{x};\Delta\boldsymbol{p});\boldsymbol{p}\ ) - T(\boldsymbol{x}) \right]^2$$

linearized form

$$\sum_{\boldsymbol{x}} \left[I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p})) + \nabla I(\boldsymbol{x}') \frac{\partial \mathbf{W}(\boldsymbol{x};\mathbf{0})}{\partial \boldsymbol{p}} \Delta\boldsymbol{p} - T(\boldsymbol{x}) \right]^2$$

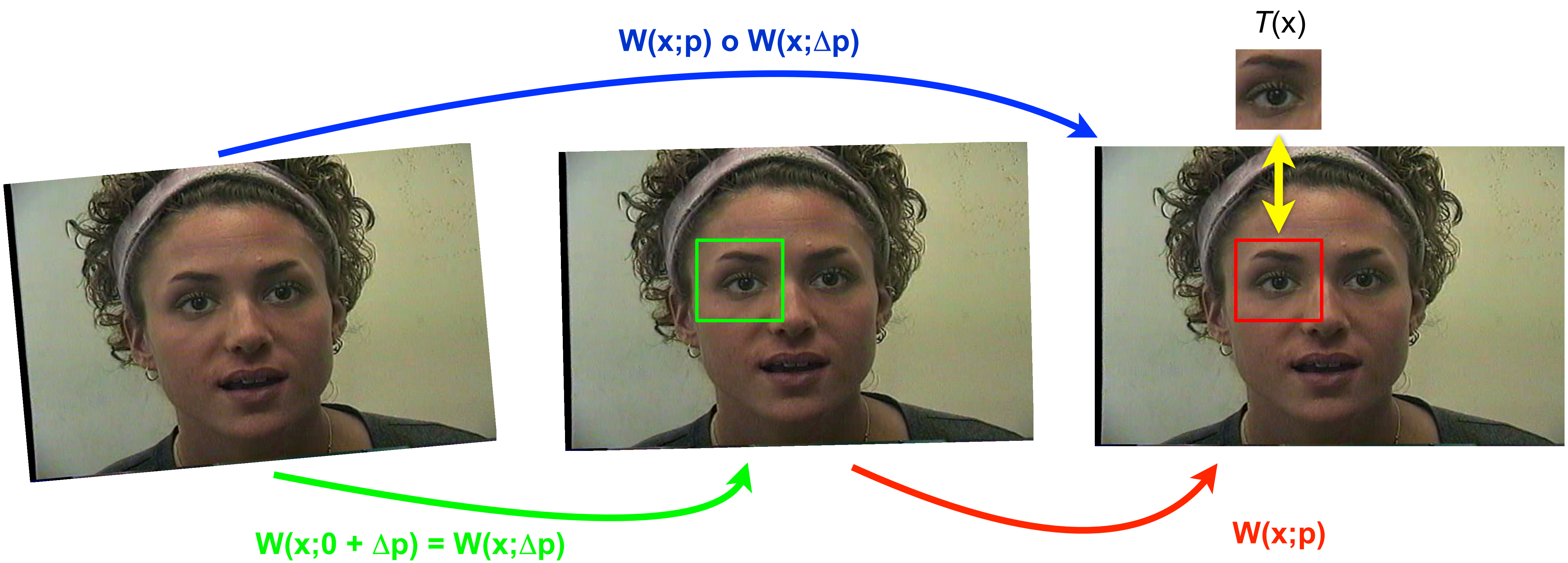
Jacobian of W(x;0)

The Jacobian is constant.
Jacobian can be precomputed!

Compositional Image Alignment

Minimize

$$\sum_{\mathbf{x}} [\, I(\mathbf{W}(\mathbf{W}(\mathbf{x};\Delta\mathbf{p});\mathbf{p})) - T(\mathbf{x})\,]^2 \approx \sum_{\mathbf{x}} \left[I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I(\mathbf{W}) \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x}) \right]^2$$



Jacobian is simple and can be precomputed

Lucas Kanade (Additive alignment)

1. Warp image $I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p}))$
2. Compute error image $[T(\boldsymbol{x}) - I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p}))]^2$
3. Compute gradient $\nabla I(\boldsymbol{x}')$
4. Evaluate Jacobian $\frac{\partial \mathbf{W}}{\partial \boldsymbol{p}}$
5. Compute Hessian H
6. Compute $\Delta \boldsymbol{p}$
7. Update parameters $\boldsymbol{p} \leftarrow \boldsymbol{p} + \Delta \boldsymbol{p}$

Shum-Szeliski (Compositional alignment)

1. Warp image $I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p}))$
2. Compute error image $[T(\boldsymbol{x}) - I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p}))]^2$
3. Compute gradient $\nabla I(\boldsymbol{x}')$
4. Evaluate Jacobian $\frac{\partial \mathbf{W}(\boldsymbol{x};\mathbf{0})}{\partial \boldsymbol{p}}$
5. Compute Hessian H
6. Compute $\Delta \boldsymbol{p}$
7. Update parameters $\mathbf{W}(\boldsymbol{x};\boldsymbol{p}) \leftarrow \mathbf{W}(\boldsymbol{x};\boldsymbol{p}) \circ \mathbf{W}(\boldsymbol{x};\Delta\boldsymbol{p})$

Any other speed up techniques?

Why not compute warp updates on the template?

Additive Alignment

$$\sum_{\boldsymbol{x}} [I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p}+\Delta\boldsymbol{p})) - T(\boldsymbol{x})]^2$$

Compositional Alignment

$$\sum_{\boldsymbol{x}} [I(\mathbf{W}(\ \mathbf{W}(\boldsymbol{x};\Delta\boldsymbol{p});\boldsymbol{p}\ ) - T(\boldsymbol{x})]^2$$

Why not compute warp updates on the template?

Additive Alignment

$$\sum_{\boldsymbol{x}} [I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p}+\Delta\boldsymbol{p})) - T(\boldsymbol{x})]^2$$

Compositional Alignment

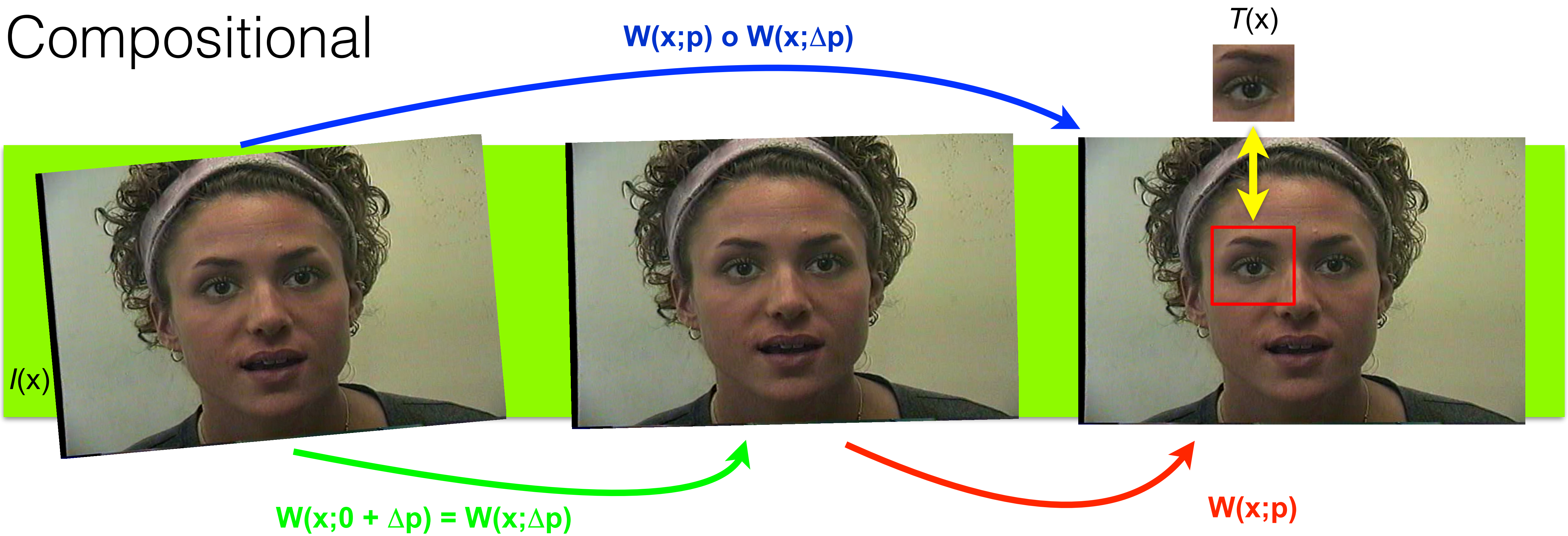
$$\sum_{\boldsymbol{x}} [I(\mathbf{W}(\ \mathbf{W}(\boldsymbol{x};\Delta\boldsymbol{p});\boldsymbol{p}\ ) - T(\boldsymbol{x})]^2$$

What happens if you let the template be warped too?

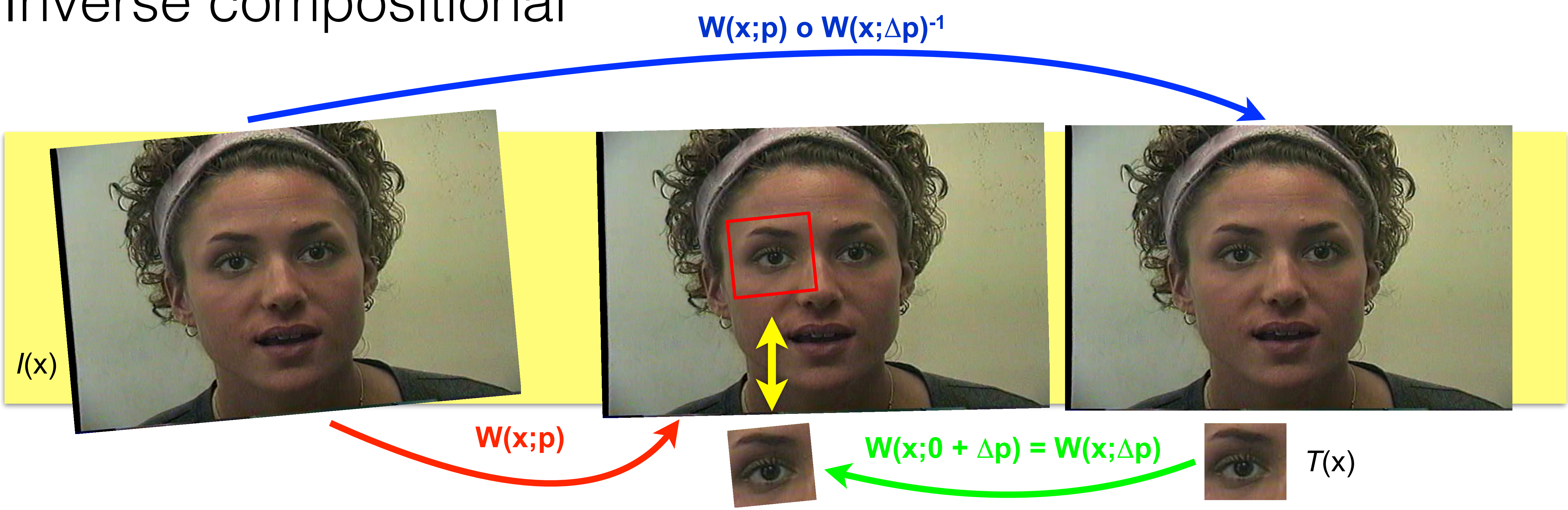
Inverse Compositional Alignment

$$\sum_{\boldsymbol{x}} [T(\mathbf{W}(\boldsymbol{x};\Delta\boldsymbol{p}) - I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p}))]^2$$

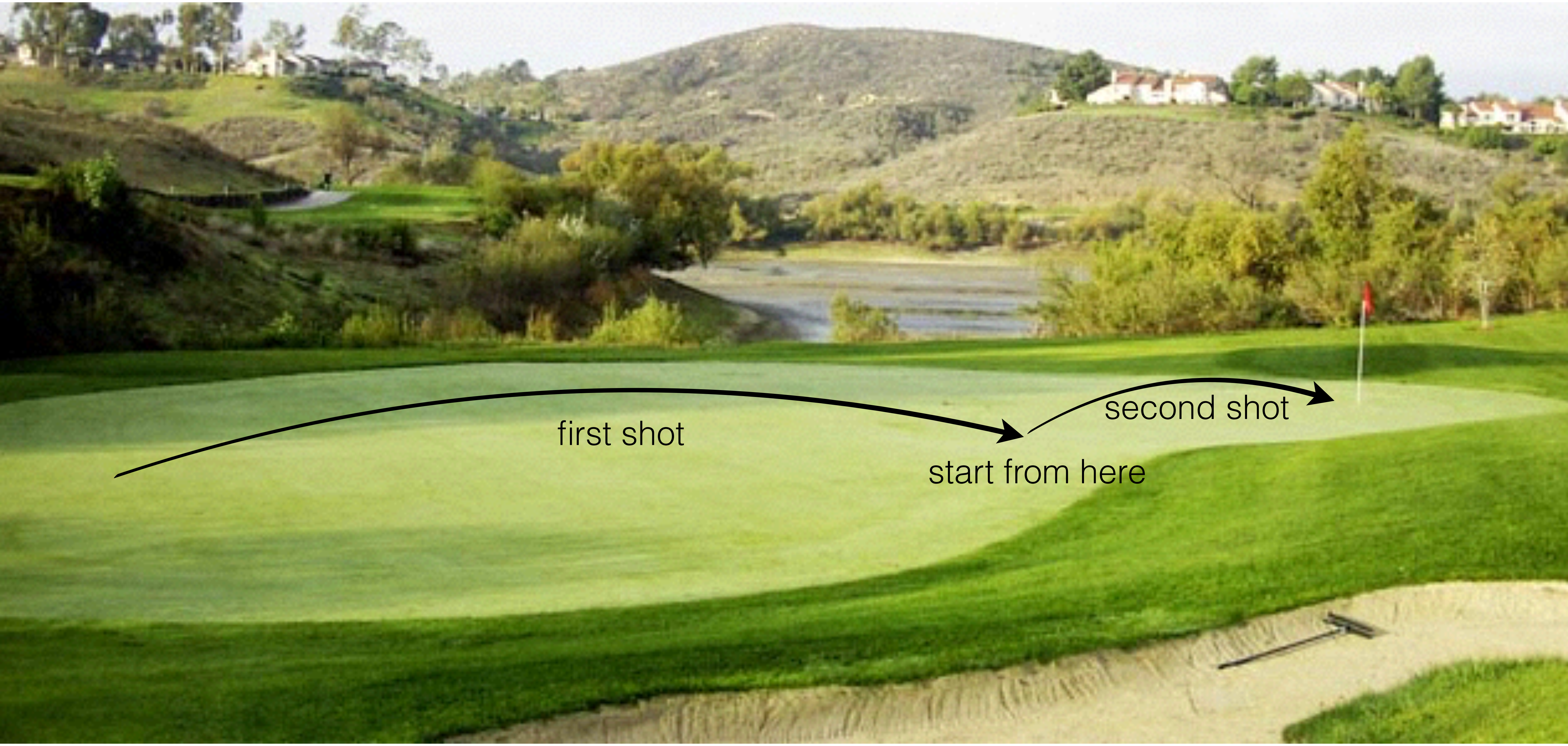
Compositional



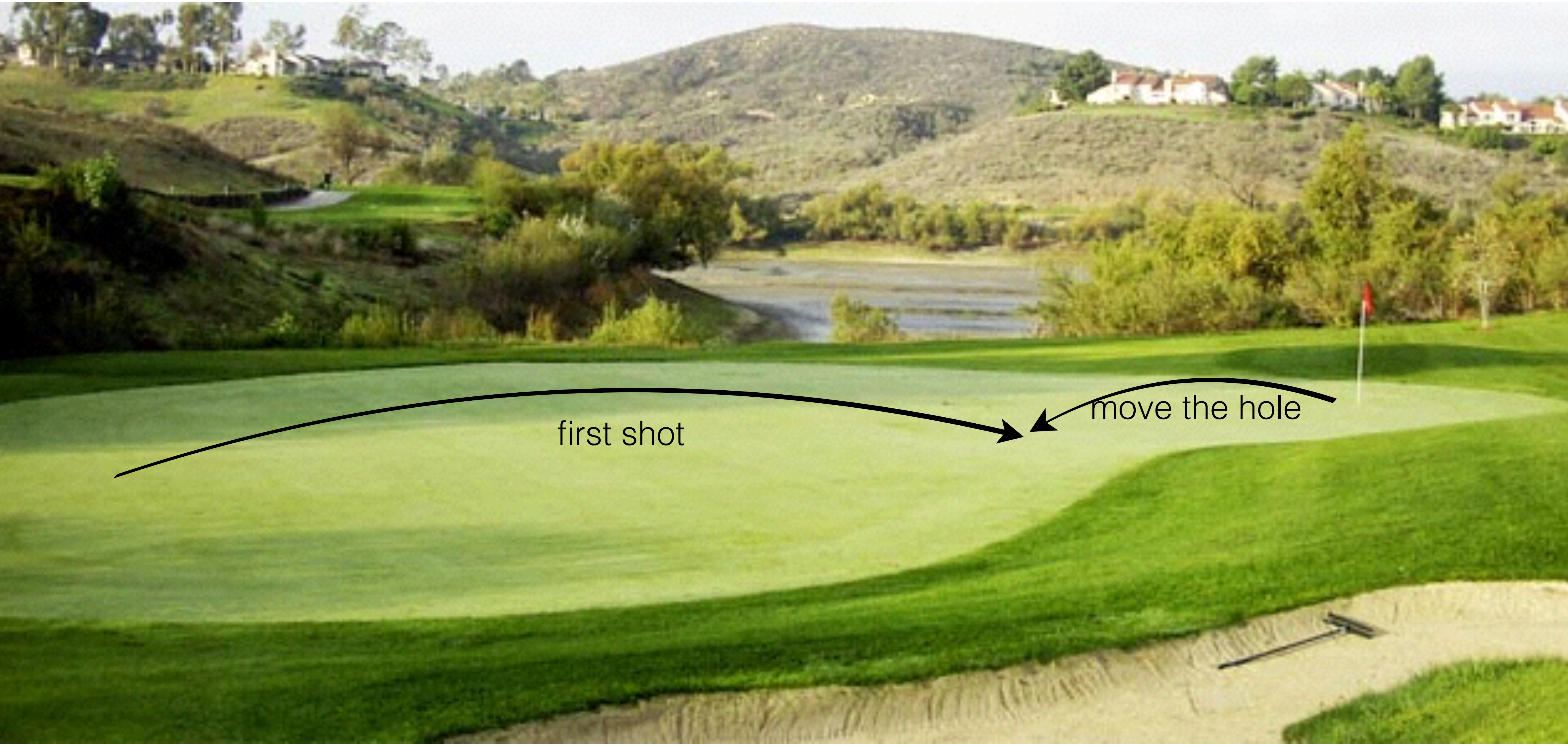
Inverse compositional



Compositional strategy



Inverse Compositional strategy



So what’s so great about this inverse compositional form?

Inverse Compositional Alignment

Minimize

$$\sum_{\boldsymbol{x}} [T(\mathbf{W}(\boldsymbol{x}; \Delta\boldsymbol{p}) - I(\mathbf{W}(\boldsymbol{x}; \boldsymbol{p}))]^2 \approx \sum \boldsymbol{x} \left[T(\mathbf{W}(\boldsymbol{x}; \mathbf{0})) + \nabla T \frac{\partial \mathbf{W}}{\partial \boldsymbol{p}} \Delta\boldsymbol{p} - I(\mathbf{W}(\boldsymbol{x}; \boldsymbol{p})\right]^2$$

Solution

$$H = \sum_{\boldsymbol{x}} \left[\nabla T \frac{\partial \mathbf{W}}{\partial \boldsymbol{p}}\right]^\top \left[\nabla T \frac{\partial \mathbf{W}}{\partial \boldsymbol{p}}\right]$$

can be precomputed from template!

$$\Delta\boldsymbol{p} = \sum_{\boldsymbol{x}} H^{-1} \left[\nabla T \frac{\partial \mathbf{W}}{\partial \boldsymbol{p}}\right]^\top [T(\boldsymbol{x}) - I(\mathbf{W}(\boldsymbol{x}; \boldsymbol{p}))]$$

Update

$$\mathbf{W}(\boldsymbol{x}; \boldsymbol{p}) \leftarrow \mathbf{W}(\boldsymbol{x}; \boldsymbol{p}) \circ \mathbf{W}(\boldsymbol{x}; \Delta\boldsymbol{p})^{-1}$$

Properties of inverse compositional alignment

Jacobian can be precomputed

It is constant - evaluated at W(x;0)

Gradient of template can be precomputed

It is constant

Hessian can be precomputed

$$H = \sum_{\boldsymbol{x}} \left[\nabla T \frac{\partial \mathbf{W}}{\partial \boldsymbol{p}} \right]^{\top} \left[\nabla T \frac{\partial \mathbf{W}}{\partial \boldsymbol{p}} \right]$$

$$\Delta \boldsymbol{p} = \sum_{\boldsymbol{x}} H^{-1} \left[\nabla T \frac{\partial \mathbf{W}}{\partial \boldsymbol{p}} \right]^{\top} [T(\boldsymbol{x}) - I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p}))]$$

(main term that needs to be computed)

Warp must be invertible

Lucas Kanade (Additive alignment)

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Baker-Matthews (Inverse Compositional alignment)

1. Warp image $I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p}))$

2. Compute error image $[T(\boldsymbol{x}) - I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p}))]$

3. Compute gradient $\nabla T(\mathbf{W})$

4. Evaluate Jacobian $\frac{\partial \mathbf{W}}{\partial \boldsymbol{p}}$

5. Compute Hessian H $$H = \sum_{\boldsymbol{x}} \left[\nabla T \frac{\partial \mathbf{W}}{\partial \boldsymbol{p}}\right]^{\top} \left[\nabla T \frac{\partial \mathbf{W}}{\partial \boldsymbol{p}}\right]$$

6. Compute $\Delta \boldsymbol{p}$ $$\Delta \boldsymbol{p} = \sum_{\boldsymbol{x}} H^{-1} \left[\nabla T \frac{\partial \mathbf{W}}{\partial \boldsymbol{p}}\right]^{\top} [T(\boldsymbol{x}) - I(\mathbf{W}(\boldsymbol{x};\boldsymbol{p}))]$$

7. Update parameters $\mathbf{W}(\boldsymbol{x};\boldsymbol{p}) \leftarrow \mathbf{W}(\boldsymbol{x};\boldsymbol{p}) \circ \mathbf{W}(\boldsymbol{x};\Delta\boldsymbol{p})^{-1}$

Algorithm	Efficient	Authors
Forwards Additive	No	Lucas, Kanade
Forwards compositional	No	Shum, Szeliski
Inverse Additive	Yes	Hager, Belhumeur
Inverse Compositional	Yes	Baker, Matthews