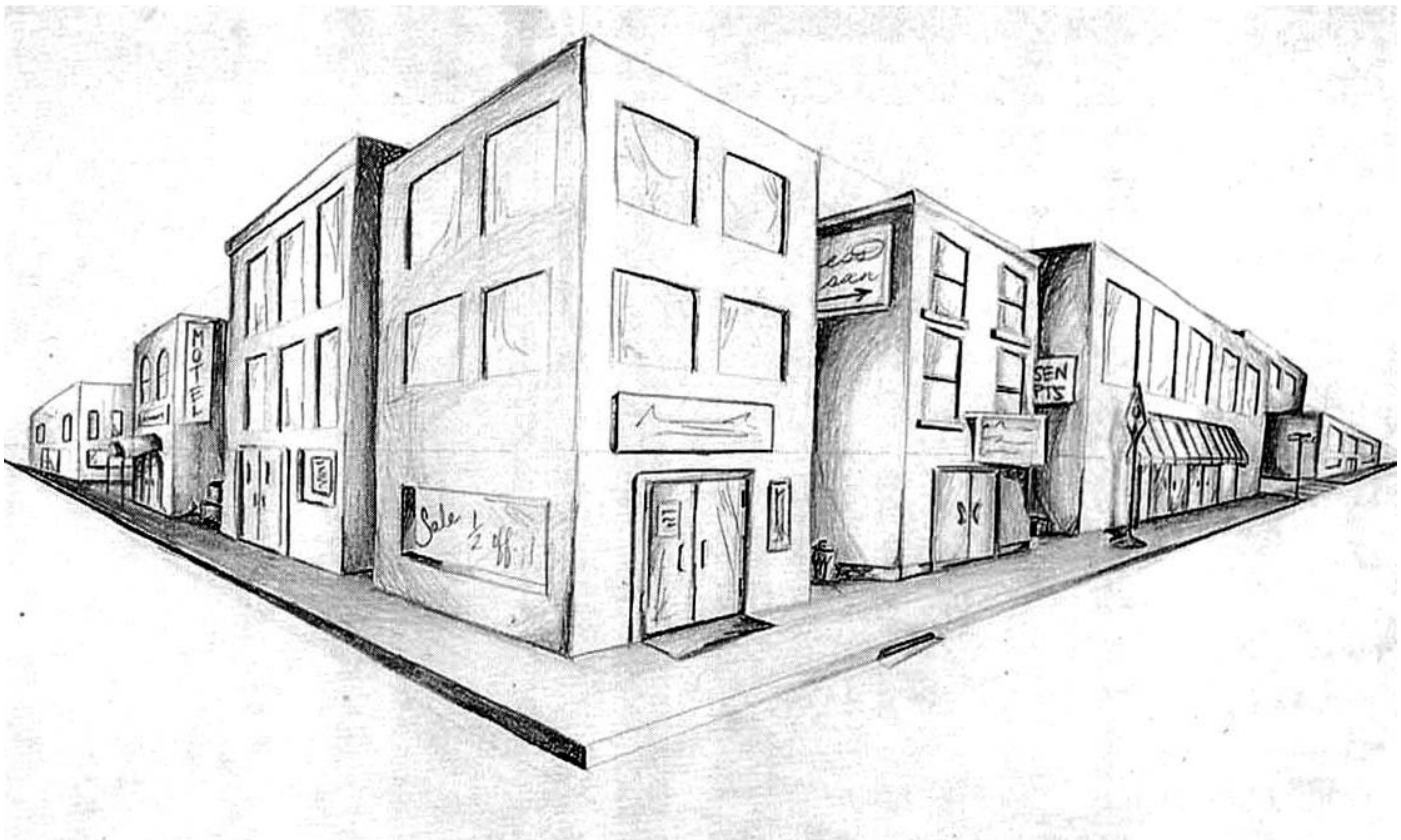
Detecting corners



http://www.cs.cmu.edu/~16385/

16-385 Computer Vision
Spring 2020, Lecture 5

Course announcements

- Homework 1 posted on course website.
  - Due on February 5th at 23:59.
  - This homework is in Matlab.
  - How many of you have looked at/started/finished homework 1?
- First theory quiz on course website and due on February 3rd, at 23:59.
- From here on, all office hours will be at Smith Hall 200.
  - Conference room next to the second floor restrooms.

Overview of today's lecture

Leftover from Lecture 4:

- More on Hough lines.
- Hough circles.

New in lecture 5:

- Why detect corners?
- Visualizing quadratics.
- Harris corner detector.
- Multi-scale detection.
- Multi-scale blob detection.

Slide credits

Most of these slides were adapted from:

- Kris Kitani (15-463, Fall 2016).

Some slides were inspired or taken from:

- Fredo Durand (MIT).
- James Hays (Georgia Tech).

Why detect corners?

Why detect corners?

Image alignment (homography, fundamental matrix)

3D reconstruction

Motion tracking

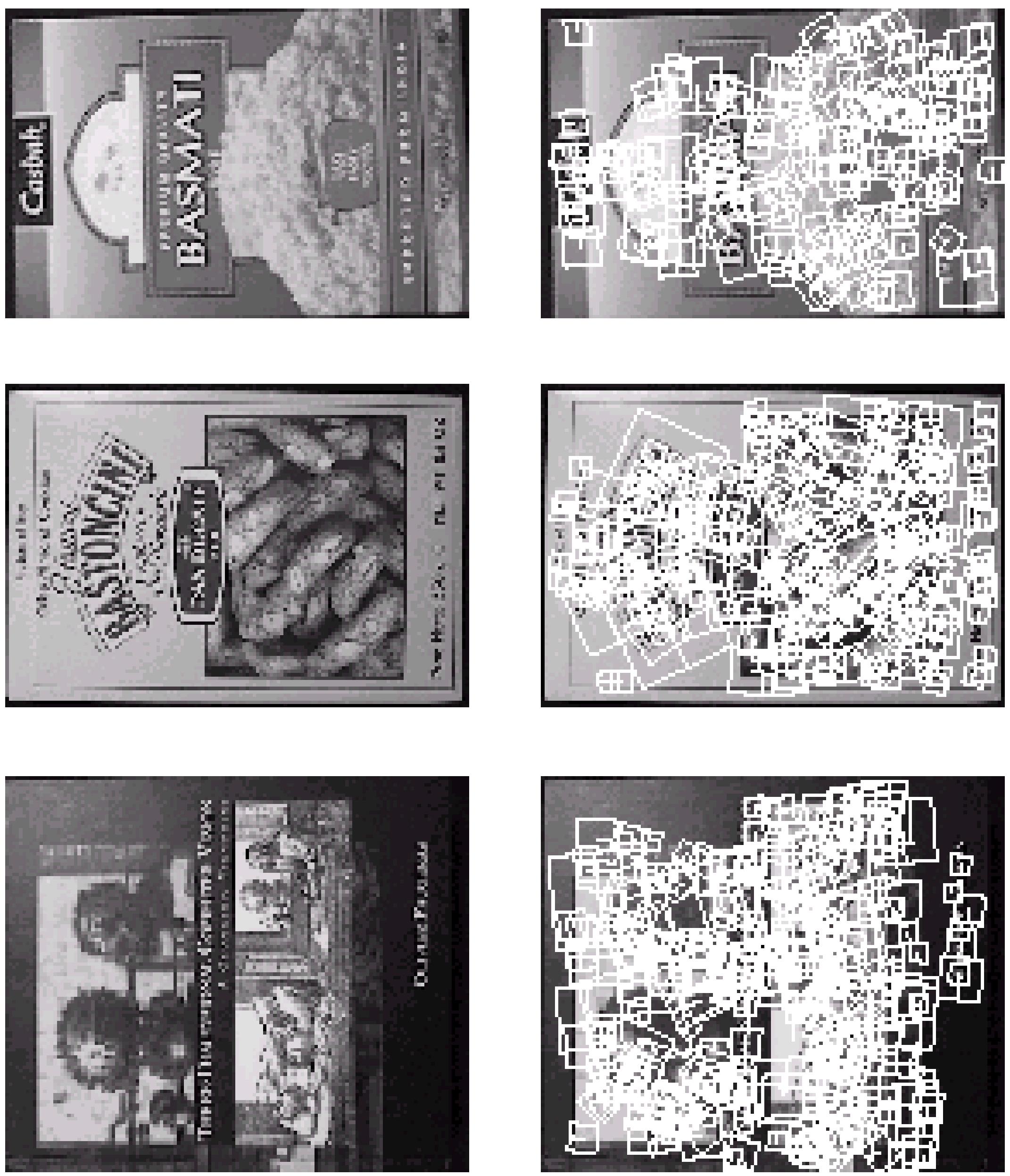
Object recognition

Indexing and database retrieval

Robot navigation

Planar object instance recognition

Database of planar objects

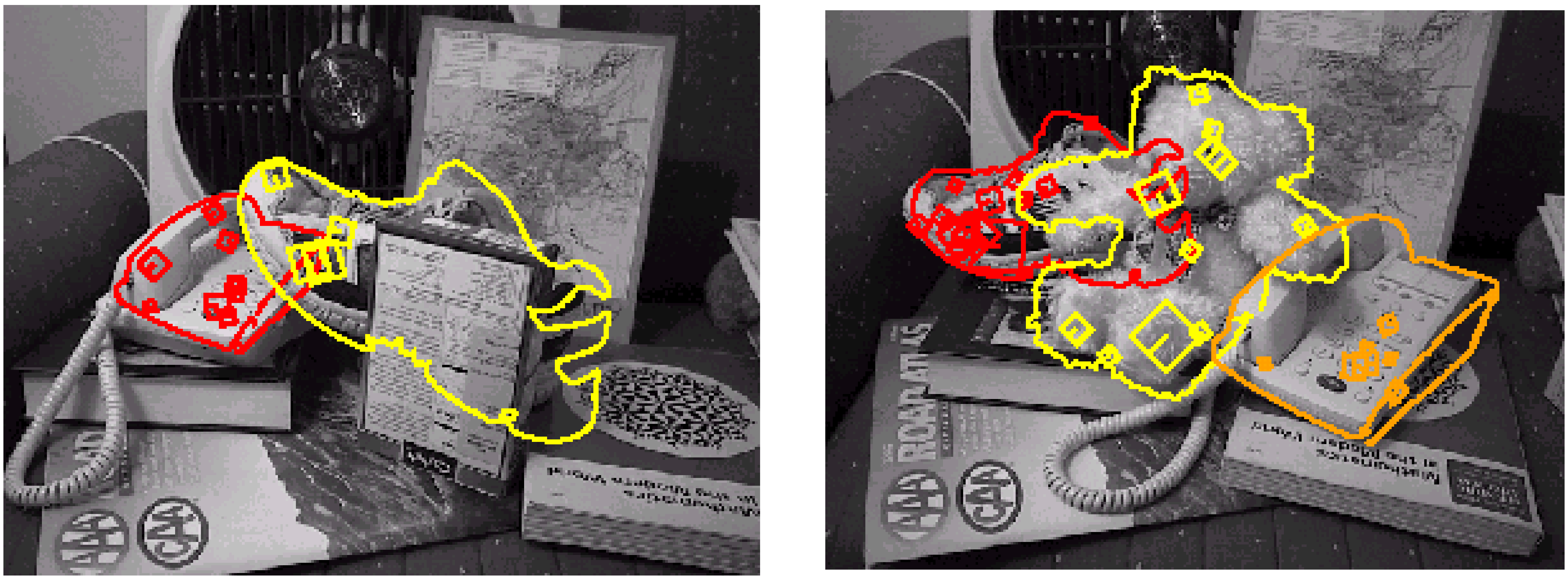


Instance recognition

3D object recognition

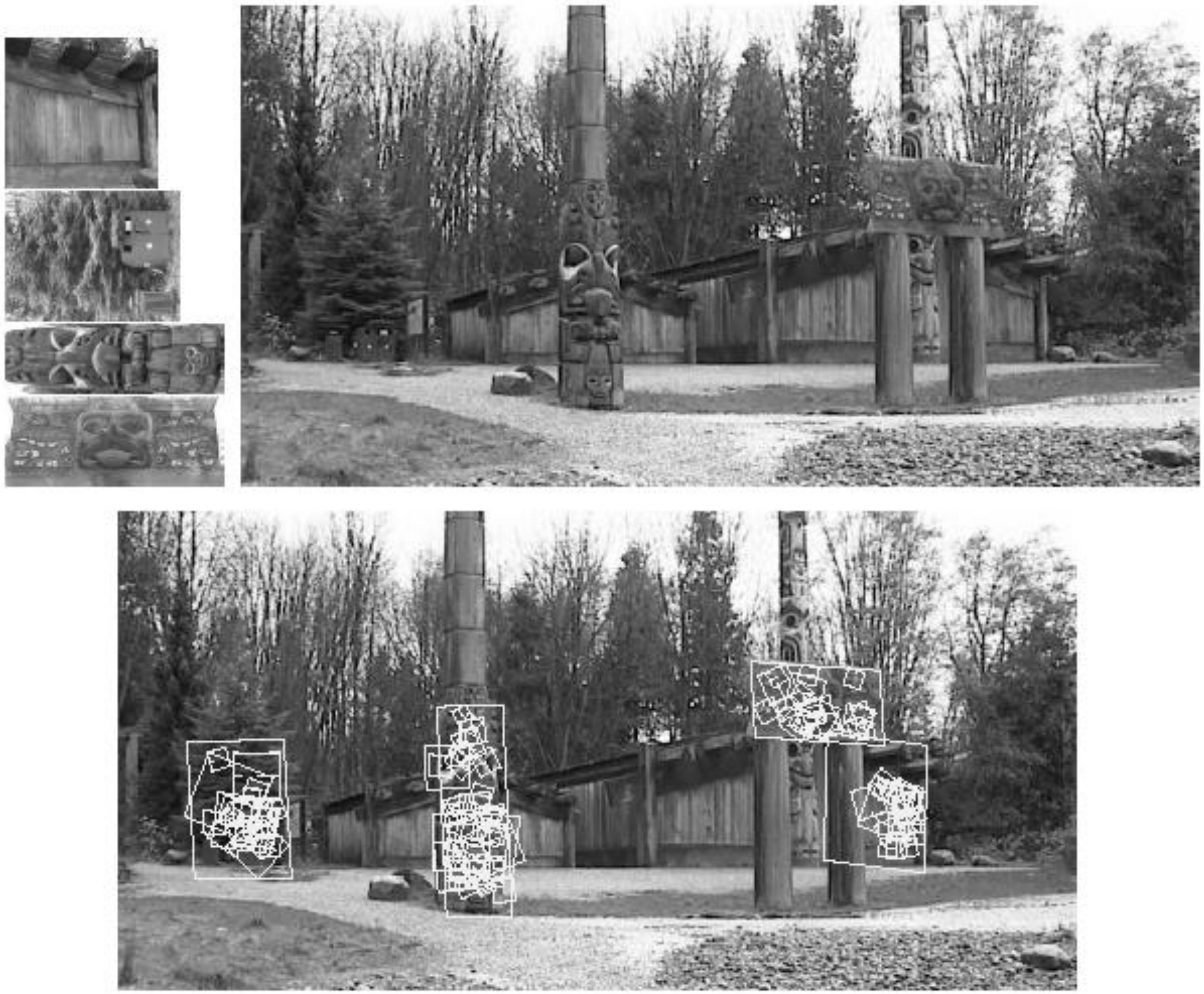
Database of 3D objects

3D objects recognition



Recognition under occlusion

Location Recognition



Robot Localization

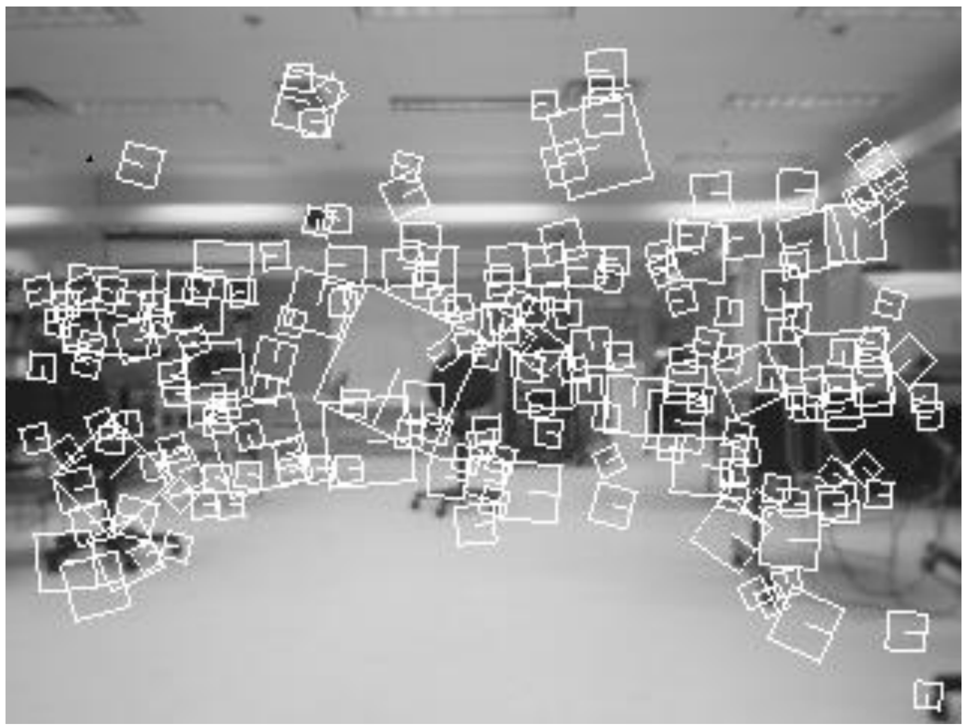
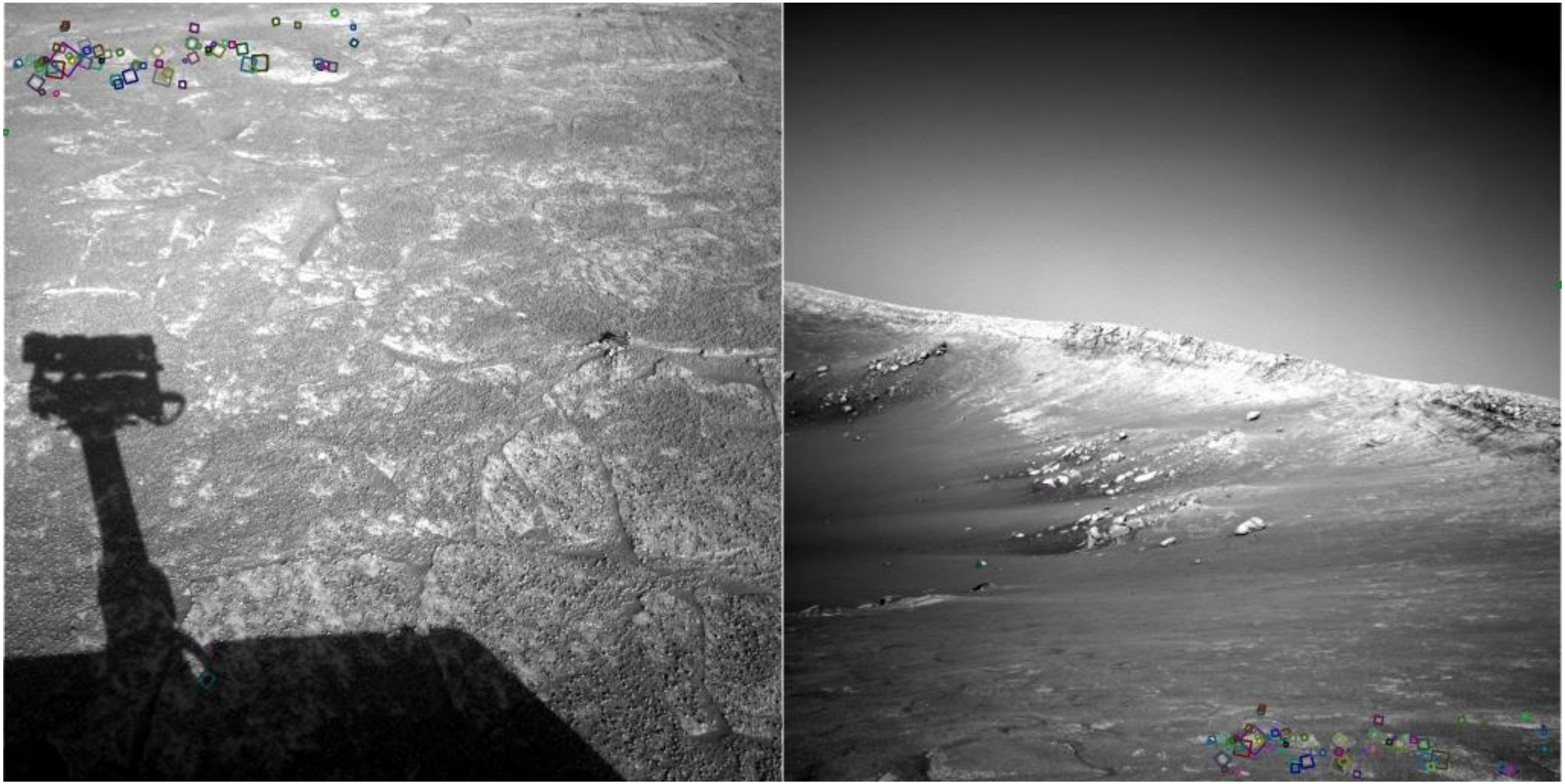
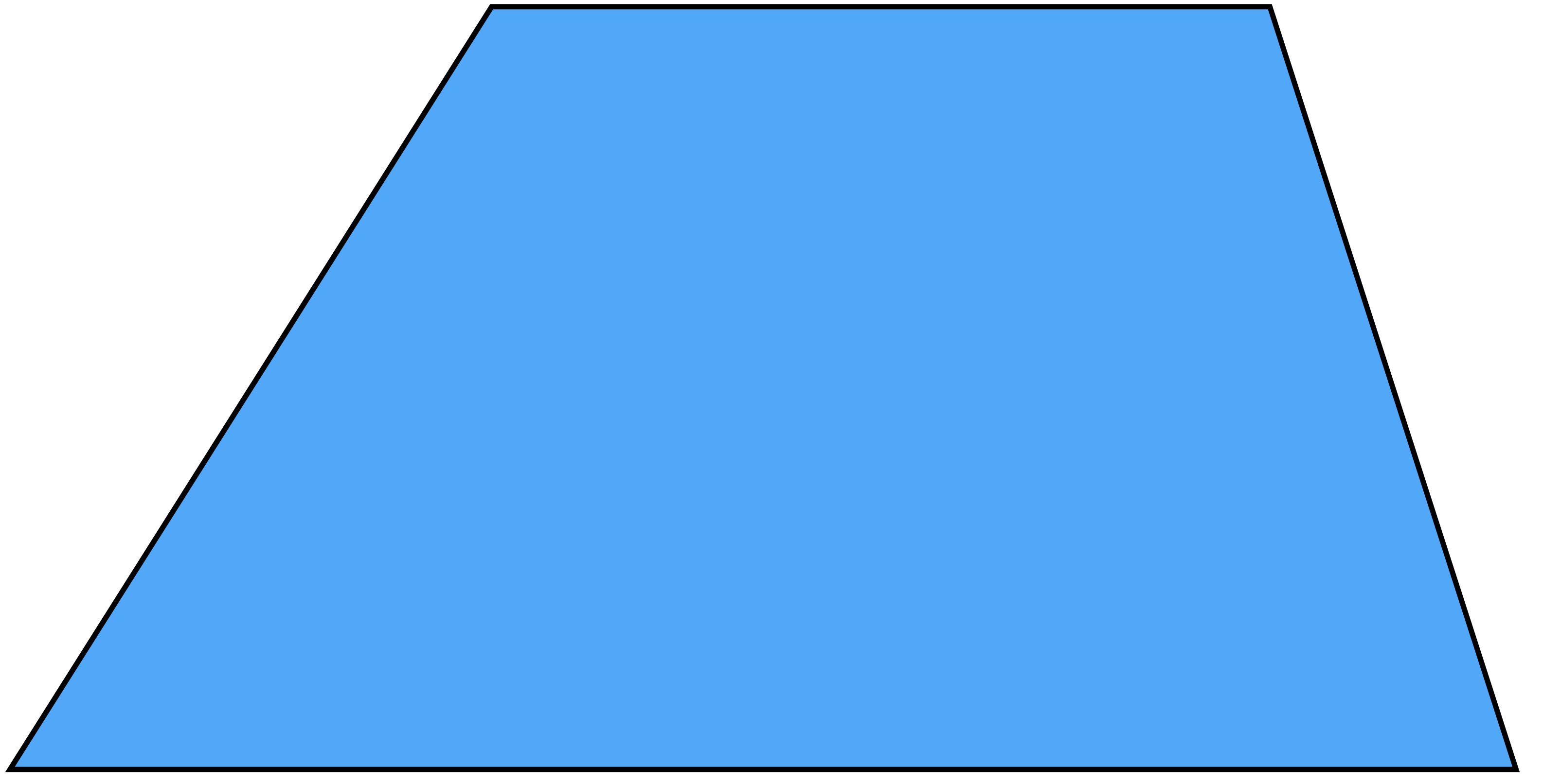


Image matching

NASA Mars Rover images

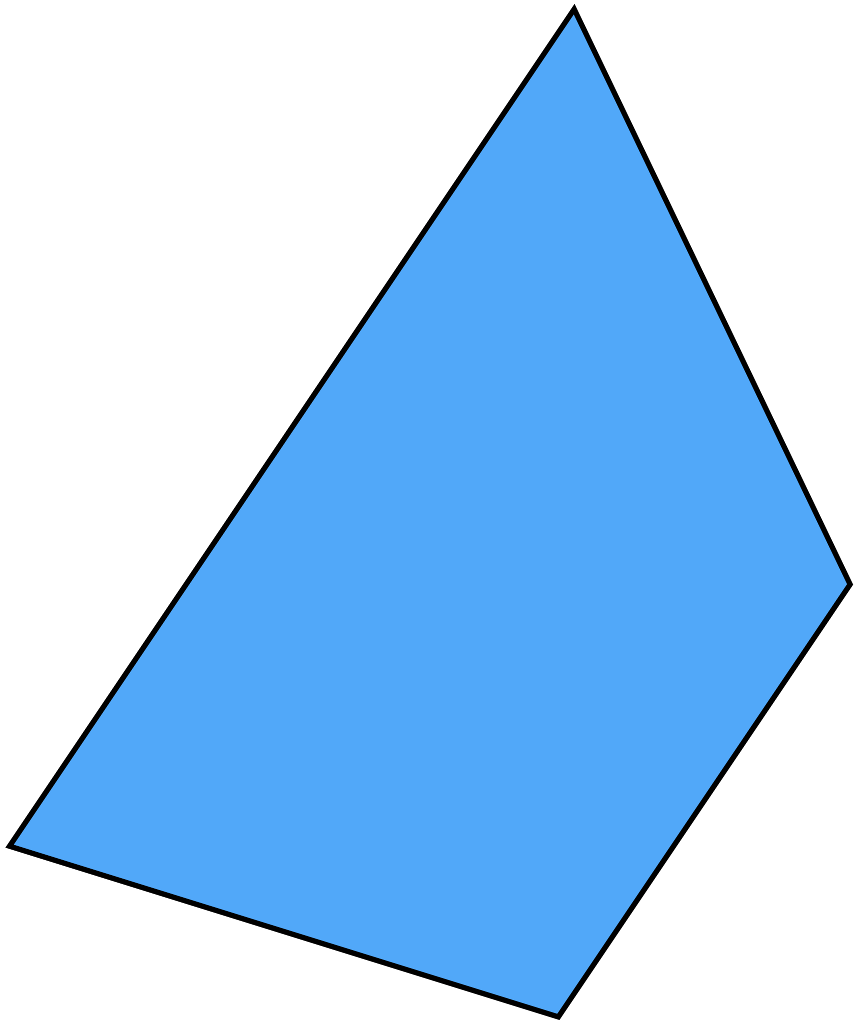
Where are the corresponding points?





Pick a point in the image.
Find it again in the next image.

What type of feature would you select?



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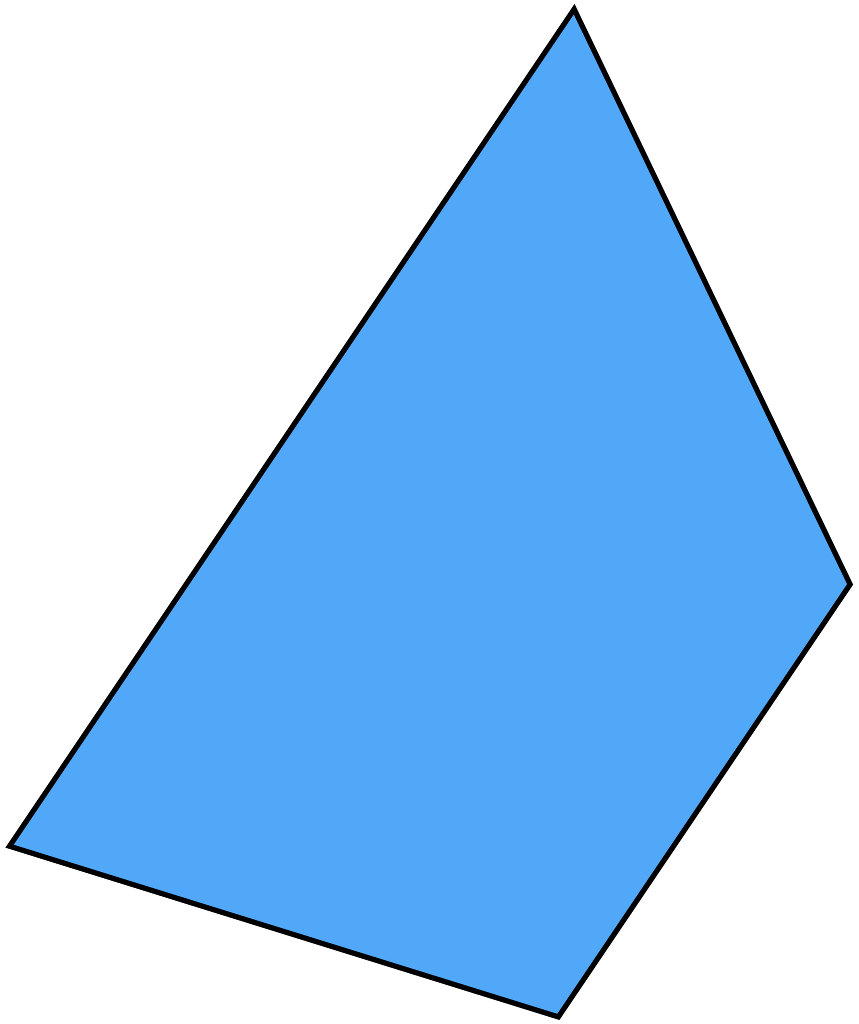
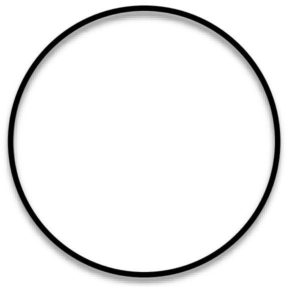


Pick a point in the image.
Find it again in the next image.

What type of feature would you select?

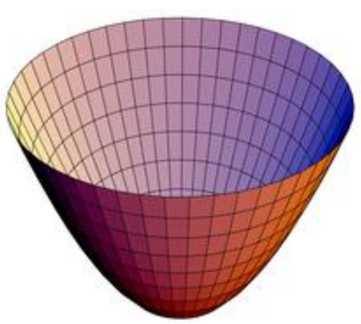
a corner

Visualizing quadratics



Equation of a circle

$$1 = x^2 + y^2$$



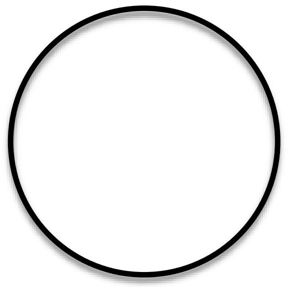
Equation of a ‘bowl’ (paraboloid)

$$f(x, y) = x^2 + y^2$$

If you slice the bowl at

$$f(x, y) = 1$$

what do you get?



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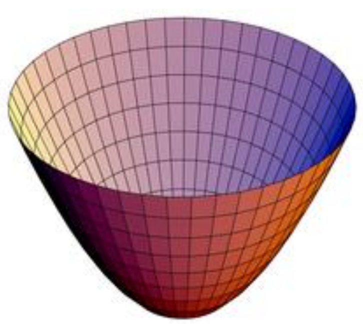

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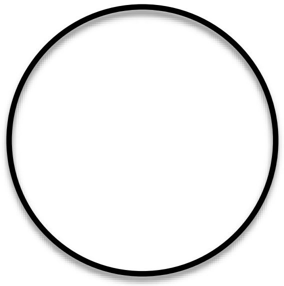

$$f(x, y) = x^2 + y^2$$

can be written in matrix form like this…

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

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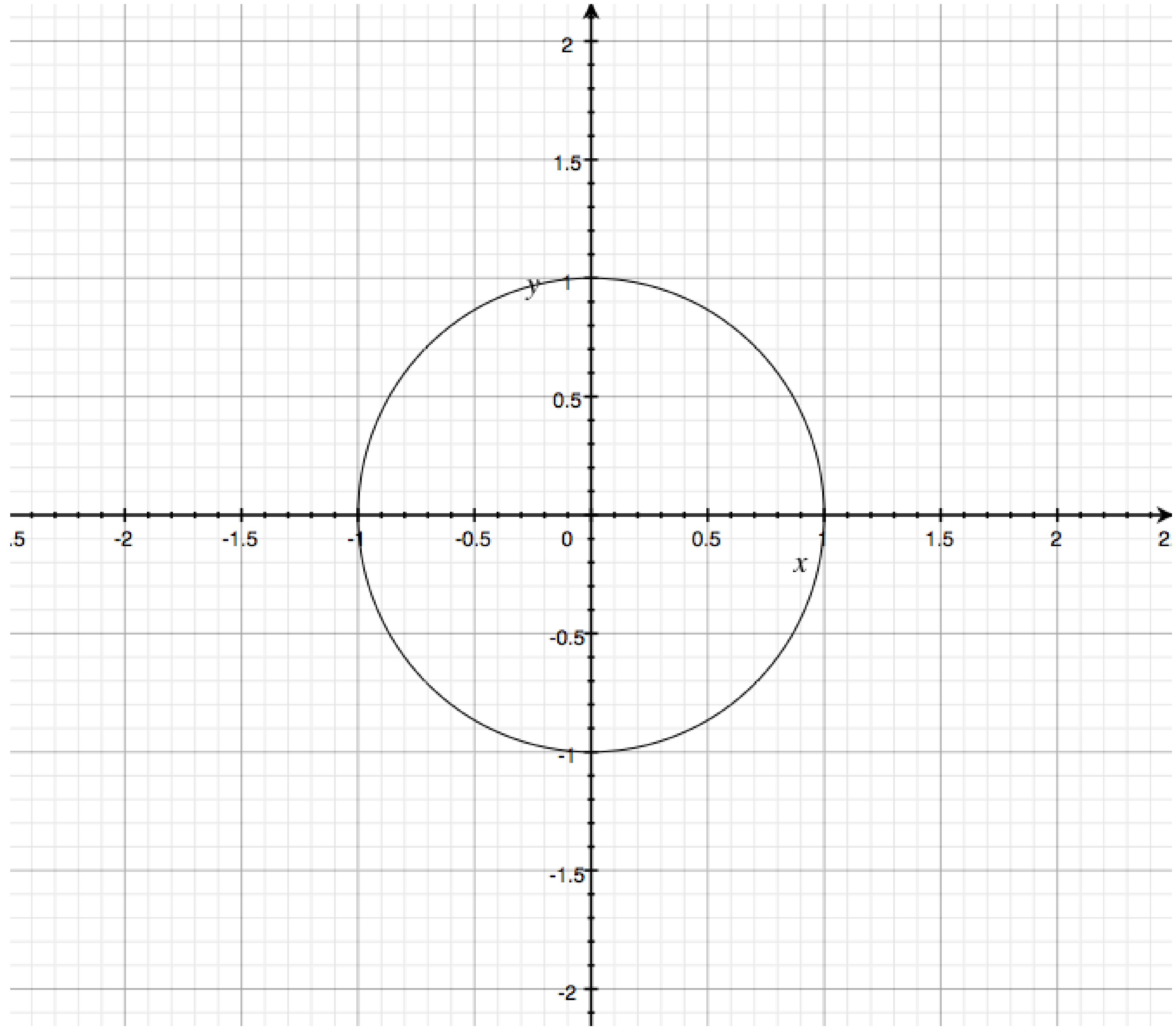
'sliced at 1'



*What happens if you **increase** coefficient on **x**?*

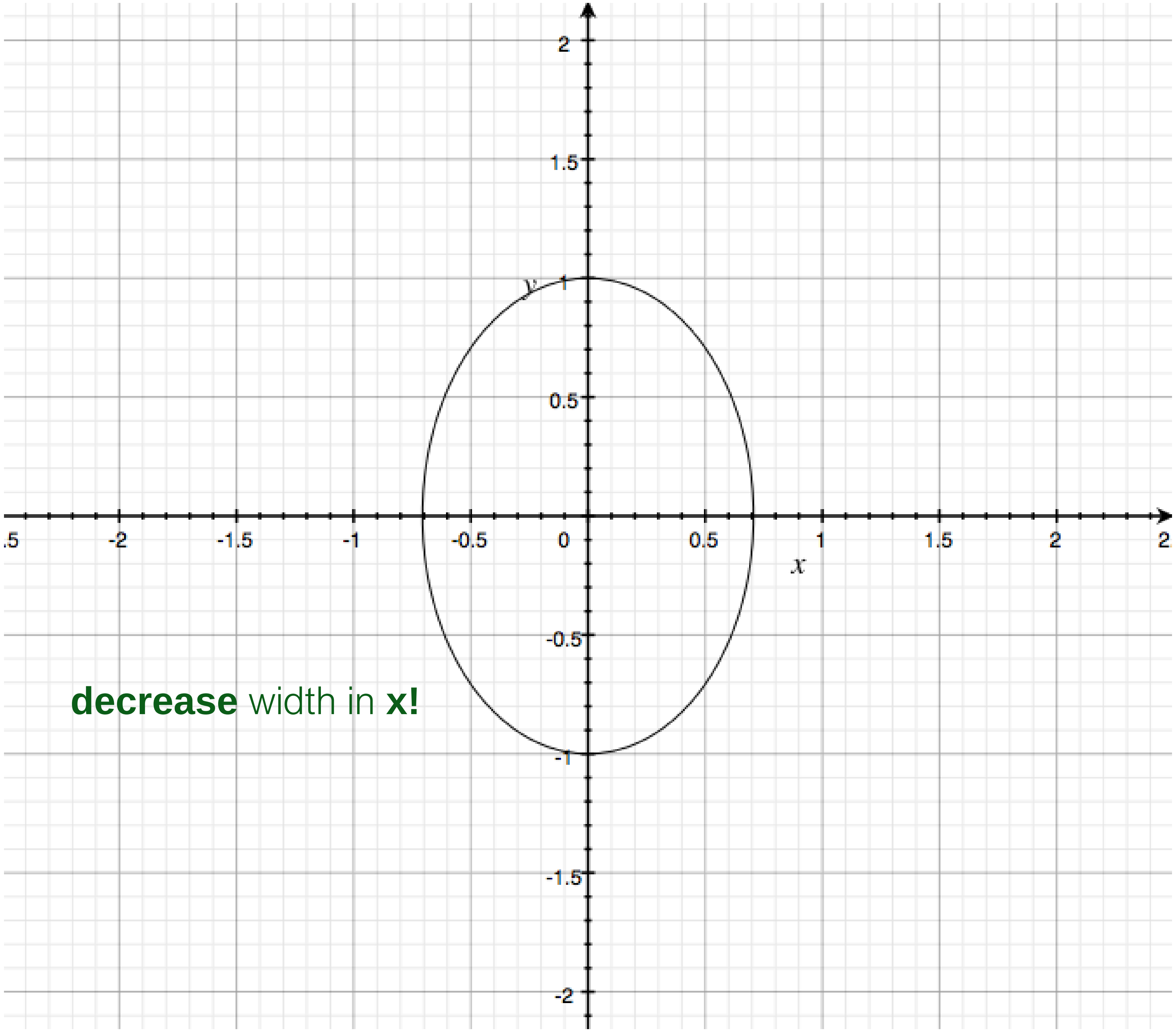
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1

What happens if you ***increase*** *coefficient on* ***x****?*

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1



*What happens if you **increase** coefficient on **y**?*

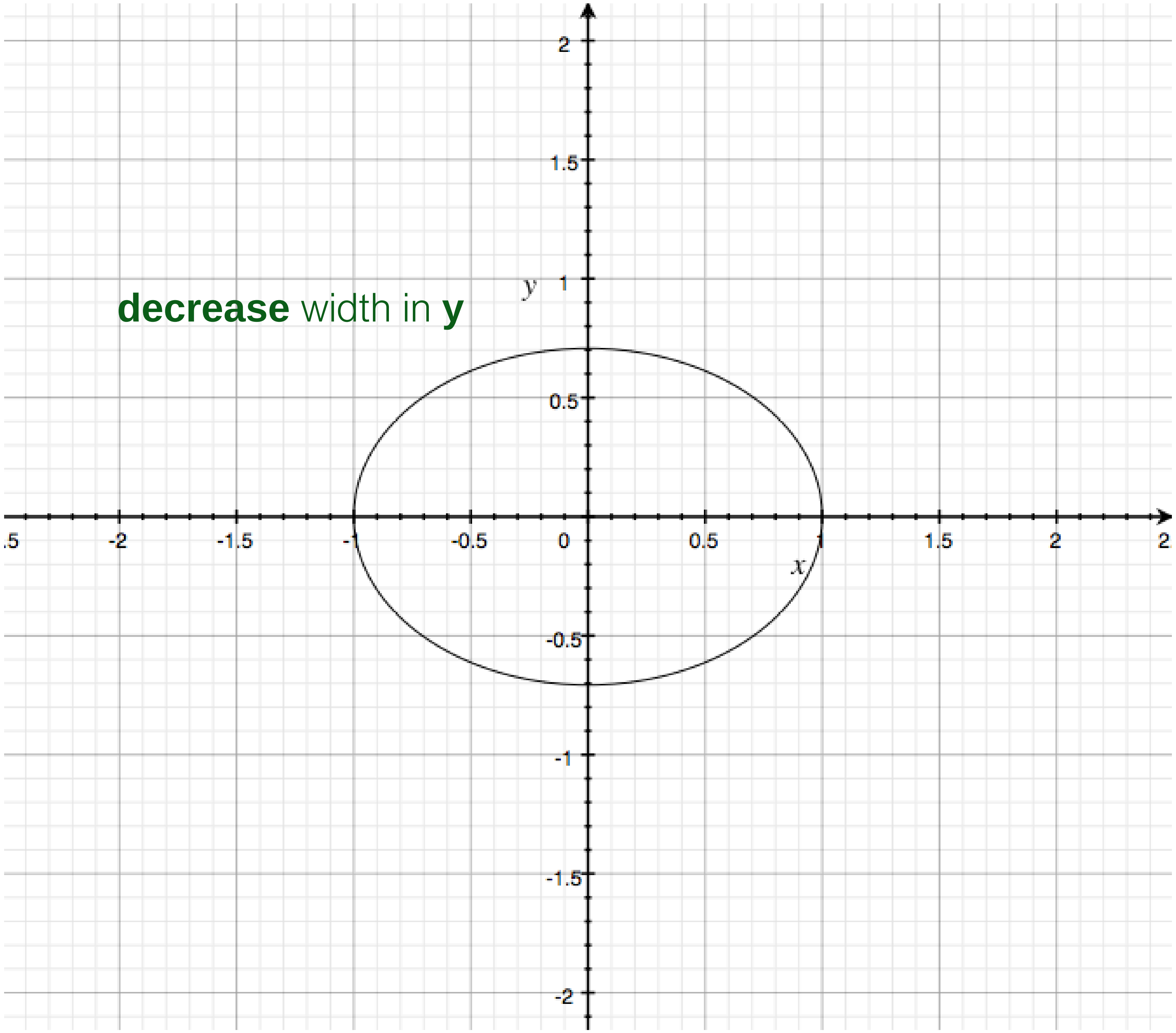
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1

What happens if you ***increase*** *coefficient on* ***y****?*

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1



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can be written in matrix form like this…

$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

What's the shape?
What are the eigenvectors?
What are the eigenvalues?

$$f(x, y) = x^2 + y^2$$

can be written in matrix form like this…

$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Result of Singular Value Decomposition (SVD)

eigenvectors

eigenvalues along diagonal

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^\top$$

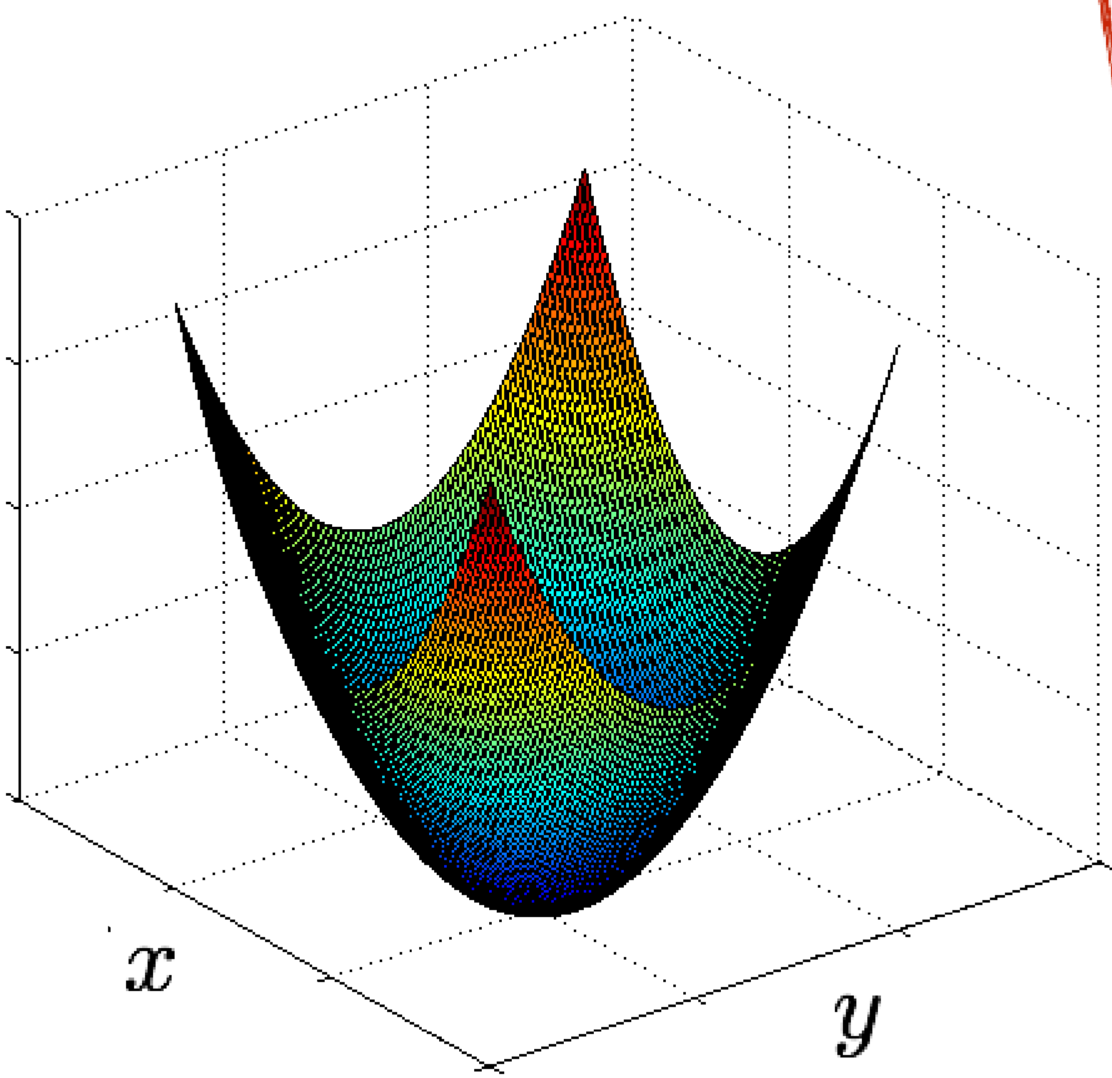
axis of the 'ellipse slice'

Inverse sqr of length of the quadratic along the axis

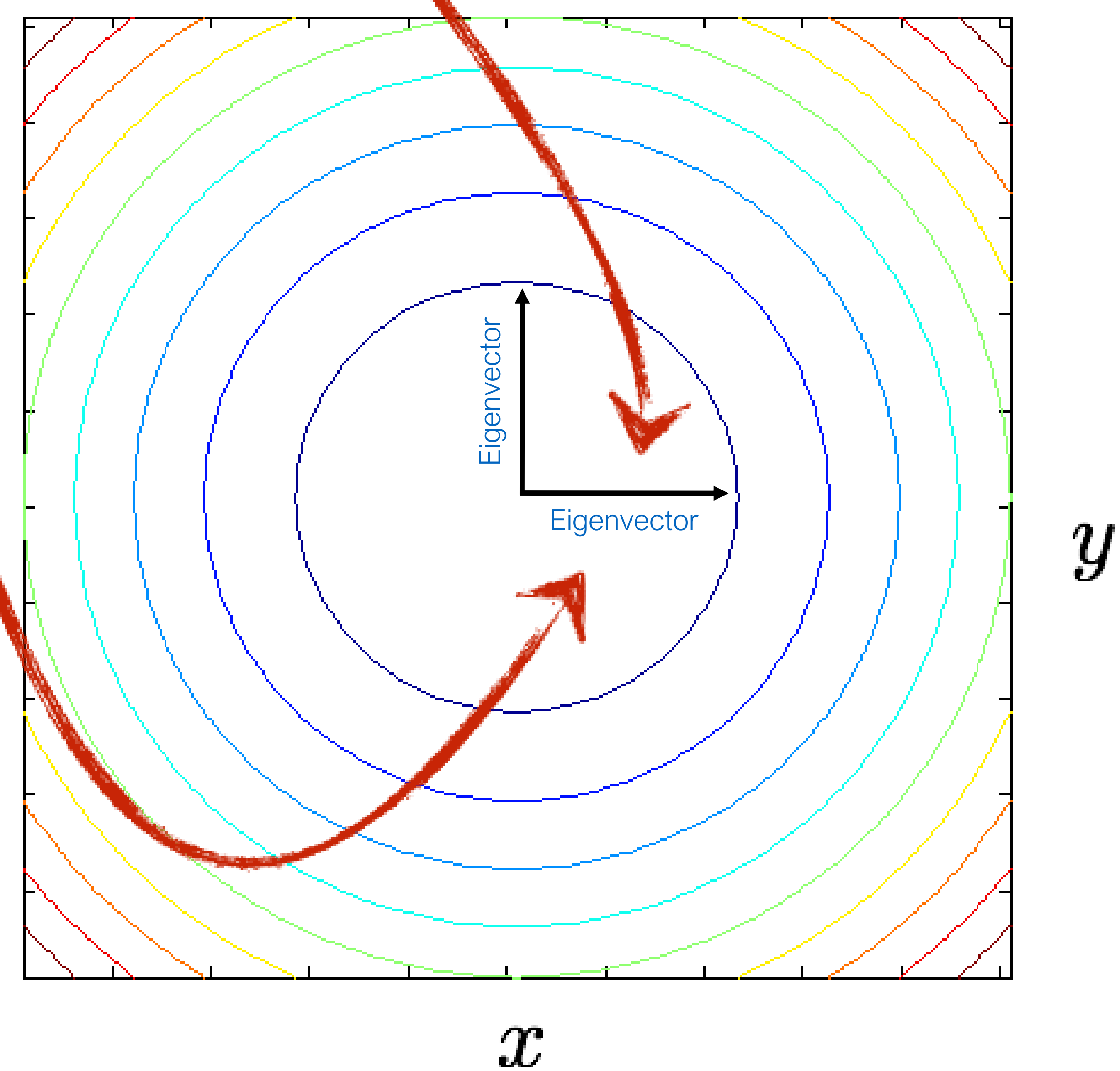
Eigenvectors Eigenvalues

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

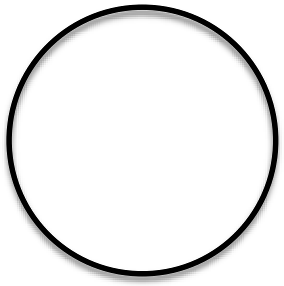
Eigenvectors



Inverse sqr of the size of the axis

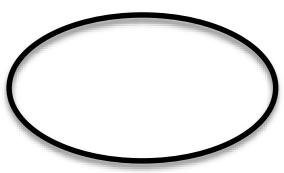


Recall:



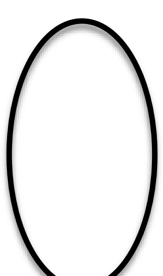
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

you can smash this bowl in the **y** direction



$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

you can smash this bowl in the **x** direction



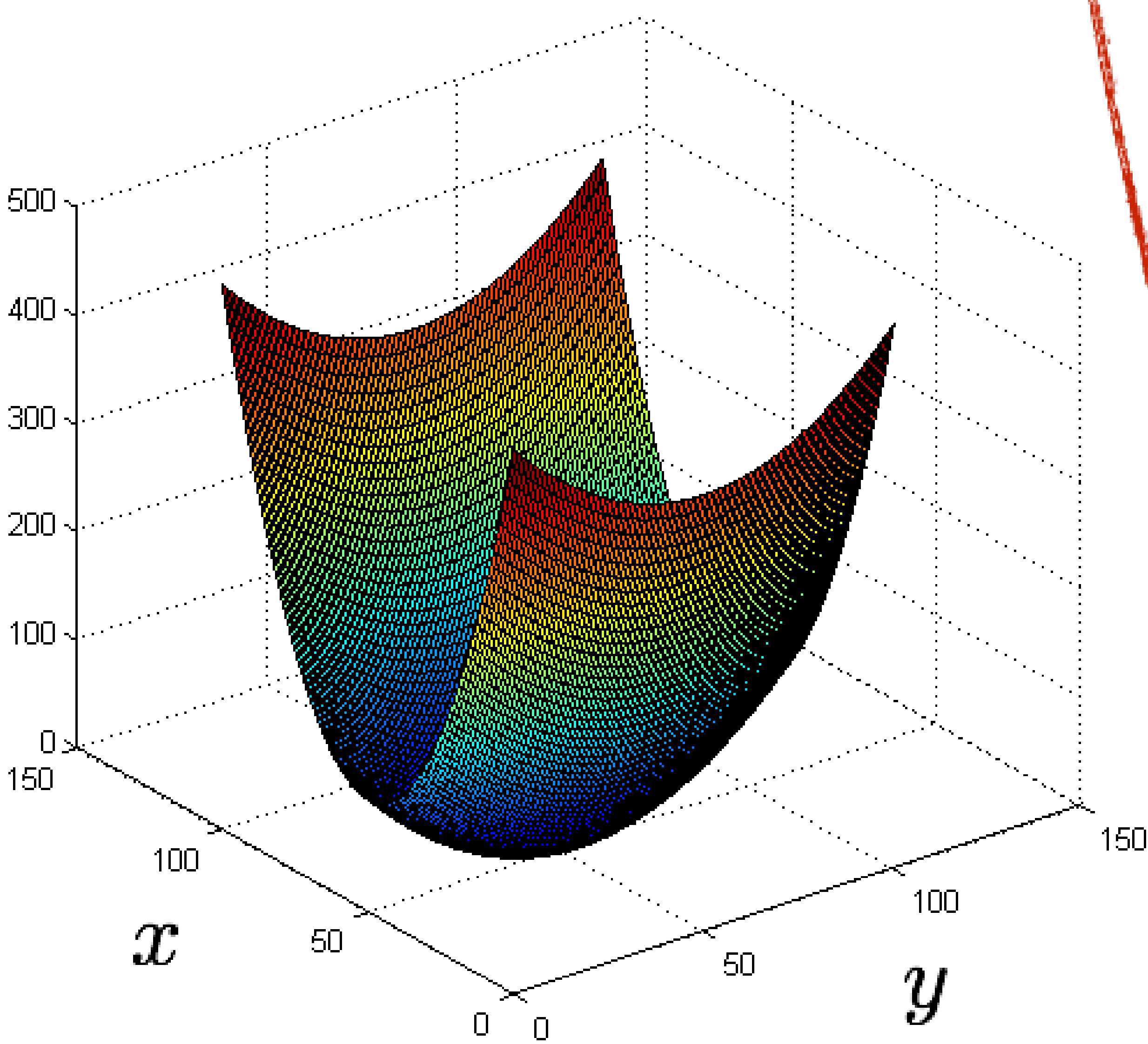
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

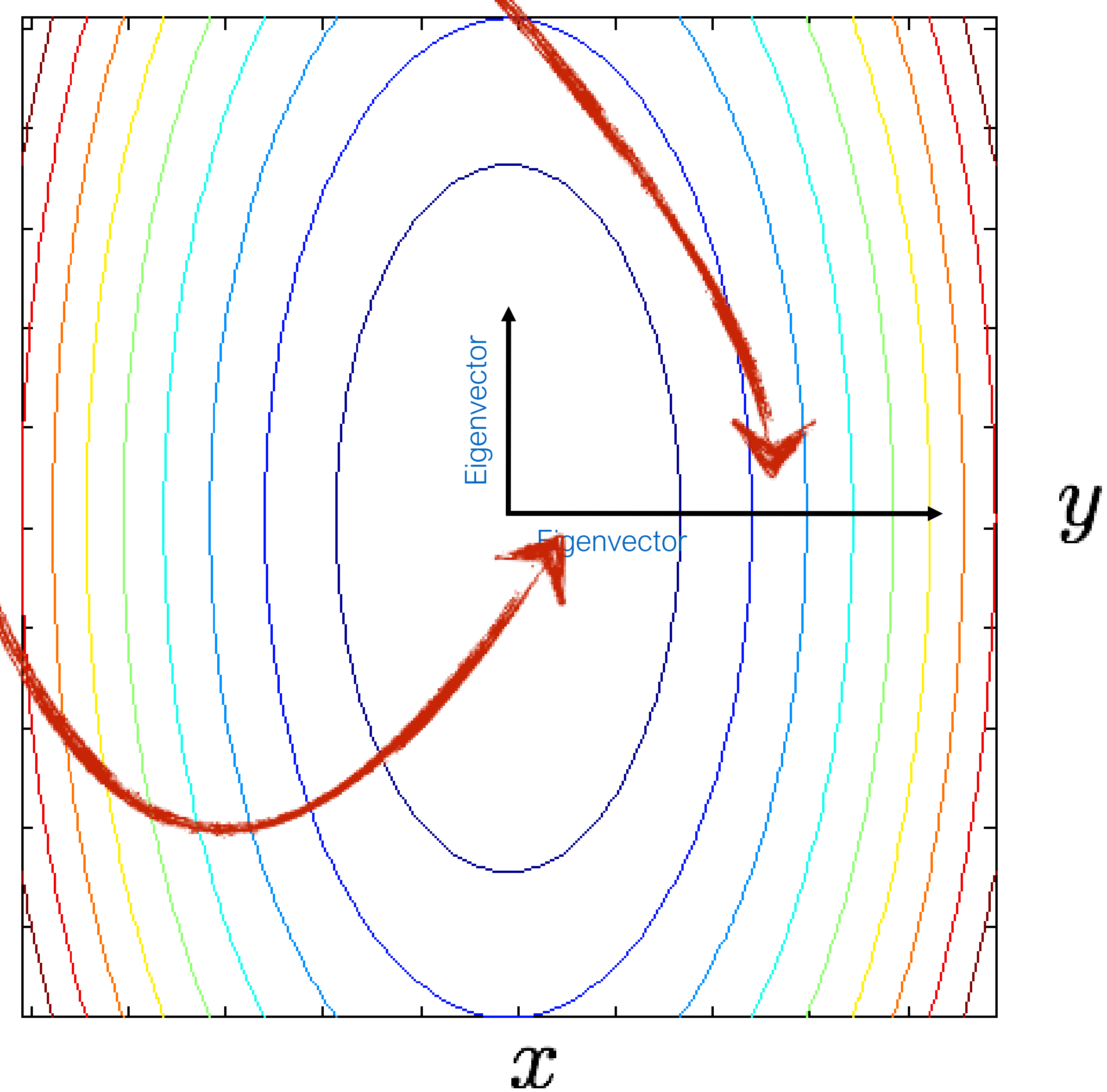
Eigenvalues

$$\mathbf{A} = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

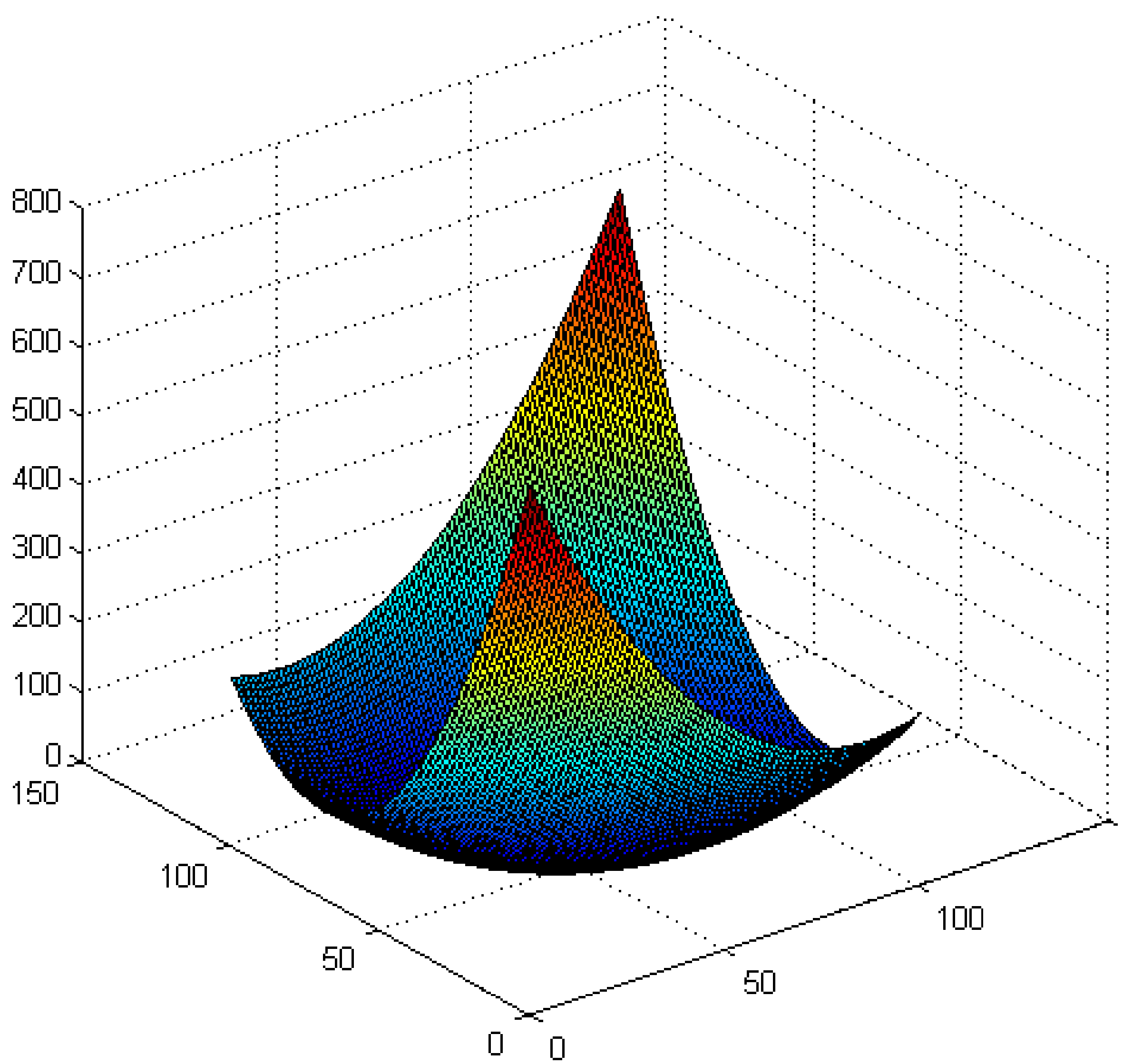
Eigenvectors Eigenvectors

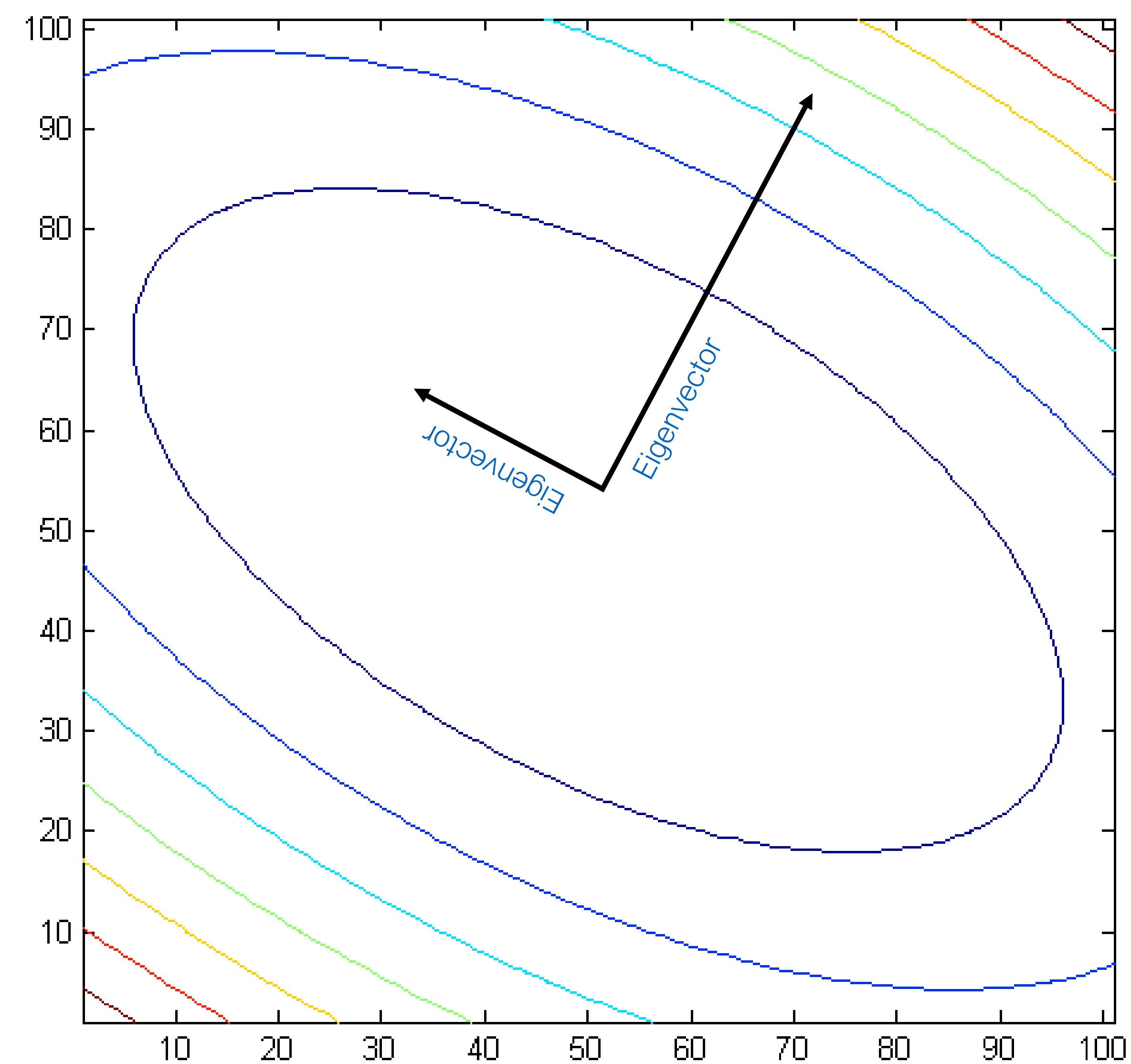
Inverse sqr of length of axis



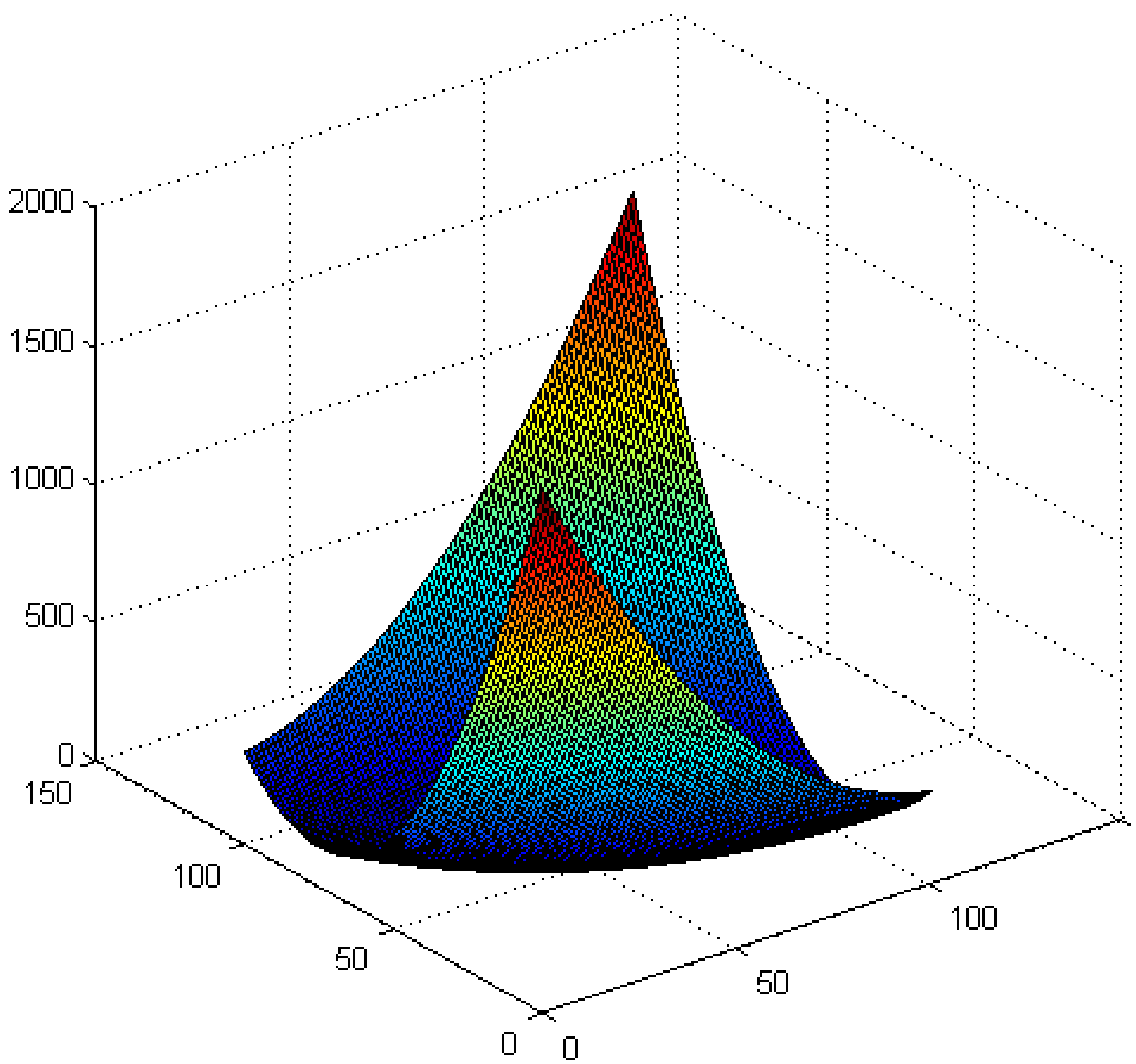


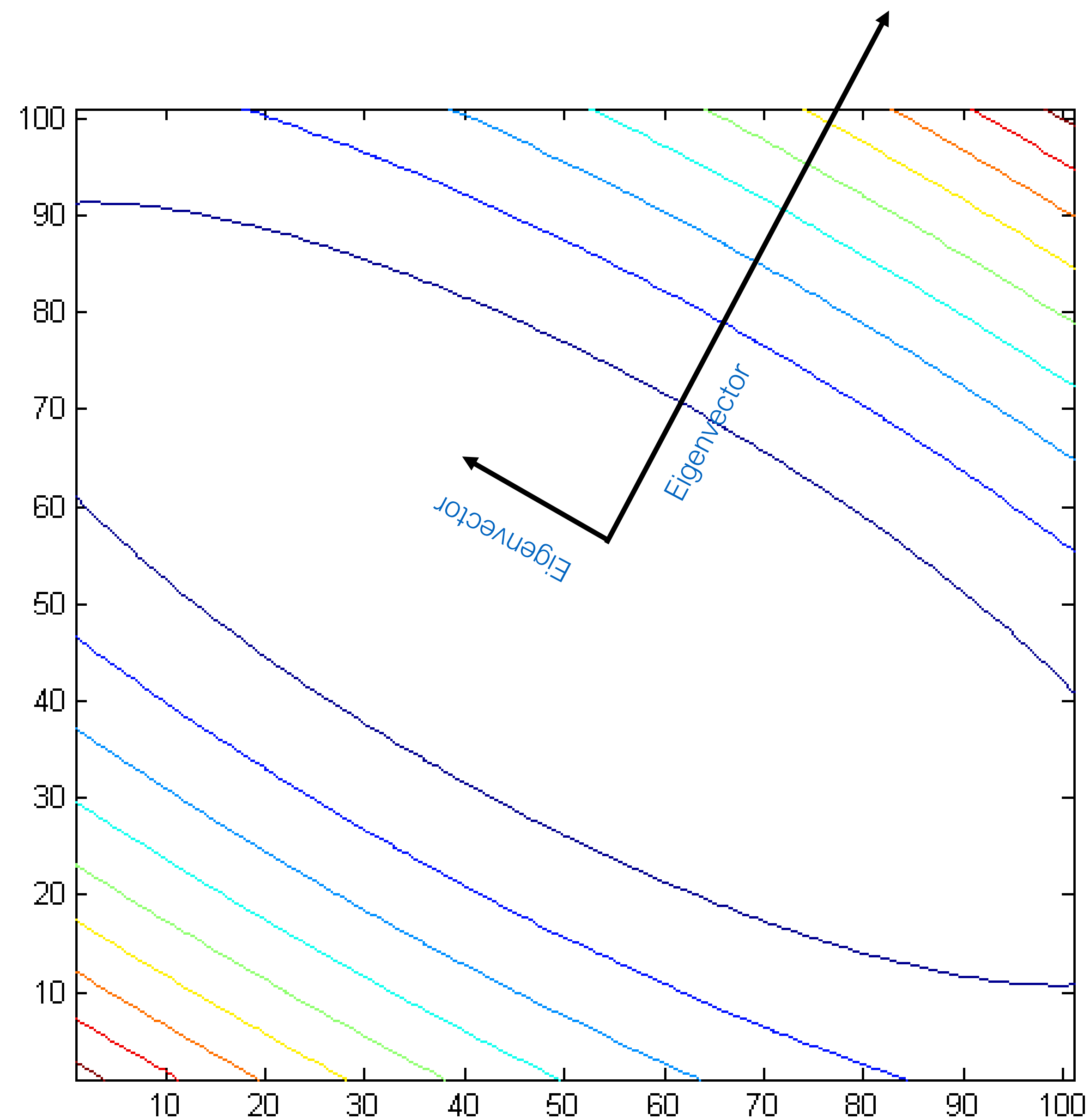
$$\mathbf{A} = \begin{bmatrix} 3.25 & 1.30 \\ 1.30 & 1.75 \end{bmatrix} = \underbrace{\begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}}_{\text{Eigenvectors}} \overbrace{\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}}^{\text{Eigenvalues}} \underbrace{\begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}^T}_{\text{Eigenvectors}}$$





$$\mathbf{A} = \begin{bmatrix} 7.75 & 3.90 \\ 3.90 & 3.25 \end{bmatrix} = \underbrace{\begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}}_{\text{Eigenvectors}} \overbrace{\begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}}^{\text{Eigenvalues}} \underbrace{\begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}^T}_{\text{Eigenvectors}}$$





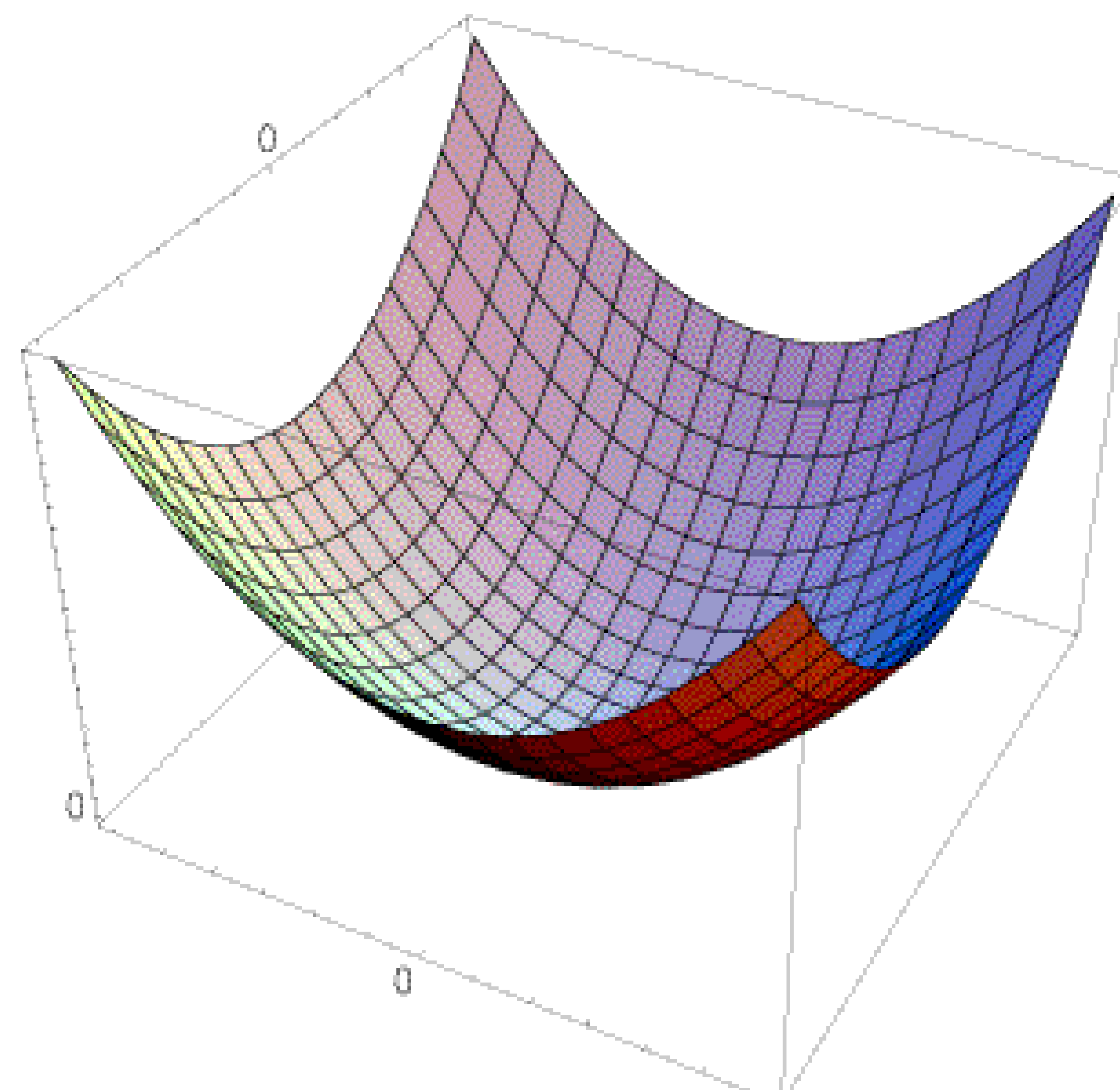
We will need this to understand the…

Error function for Harris Corners

The surface *E*(*u*,*v*) is locally approximated by a quadratic form

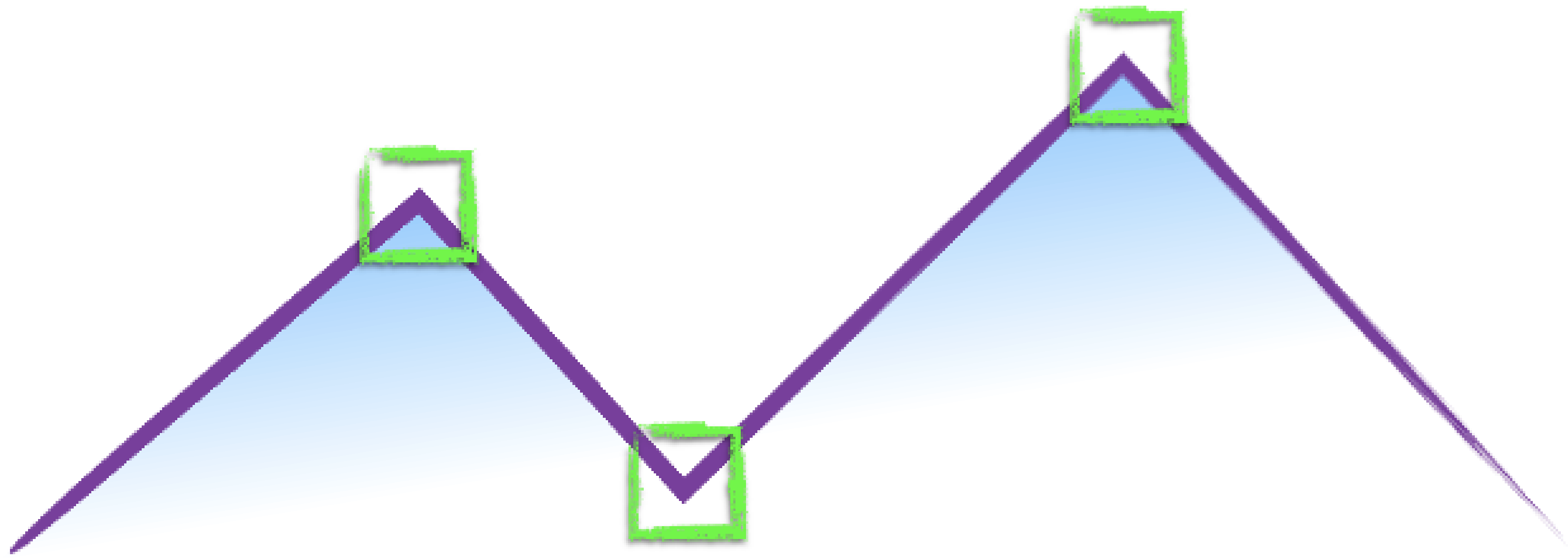
$$E(u,v) \approx [u \ \ v] \ M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



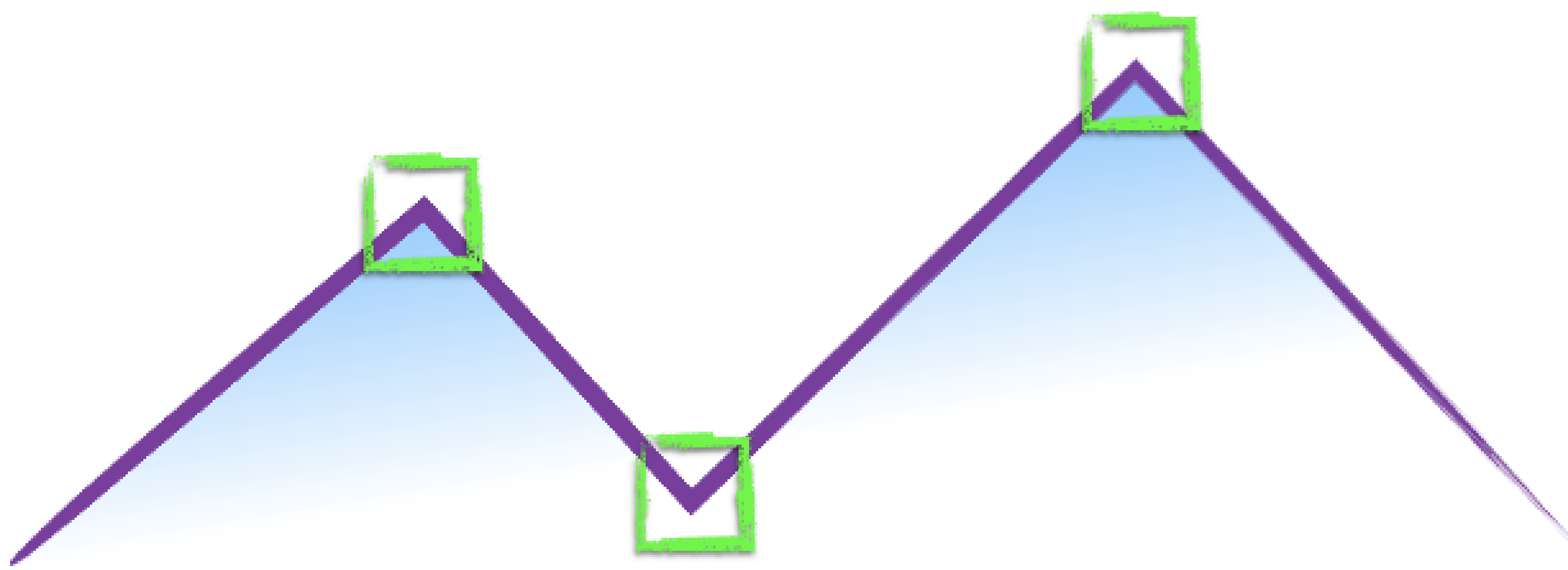
Harris corner detector

How do you find a corner?



How do you find a corner?

[Moravec 1980]



Easily recognized by looking through a small window

Shifting the window should give large change in intensity

Easily recognized by looking through a small window

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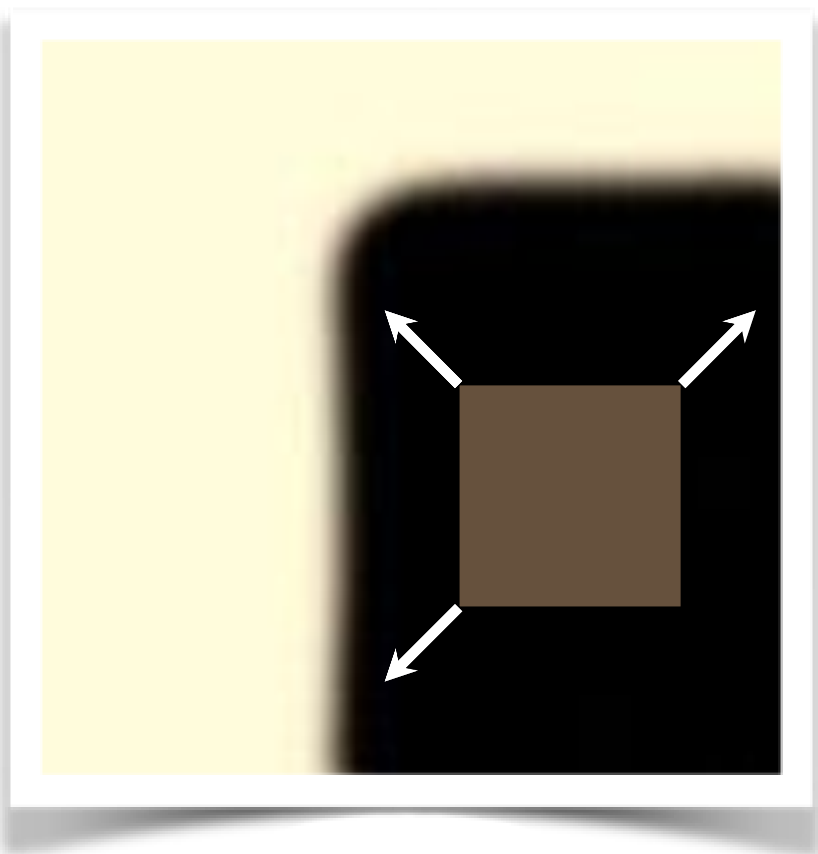
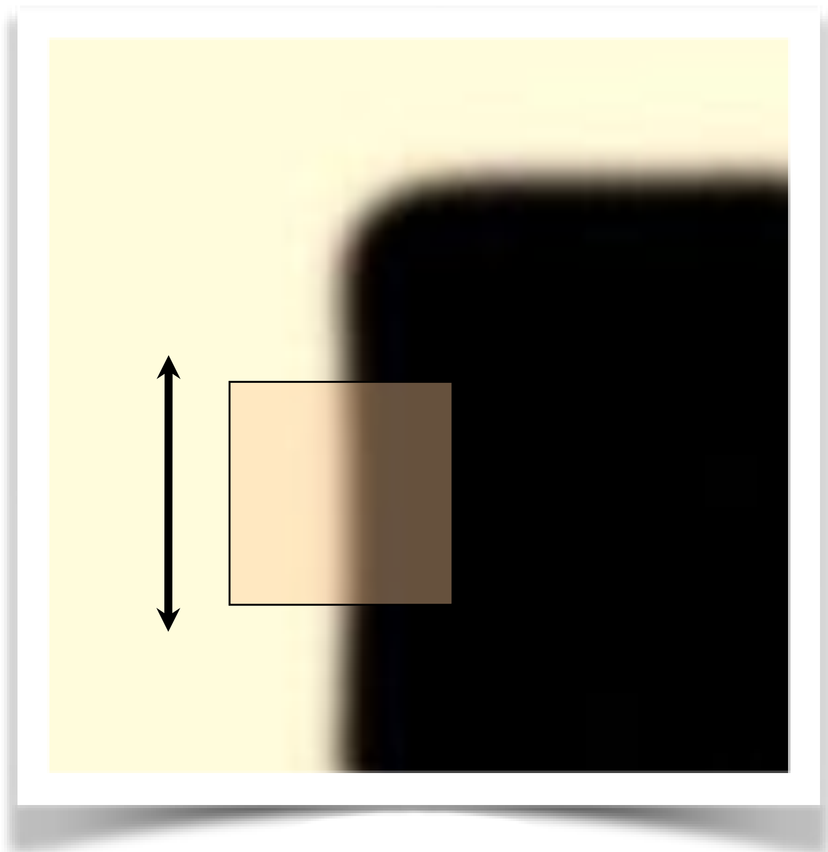
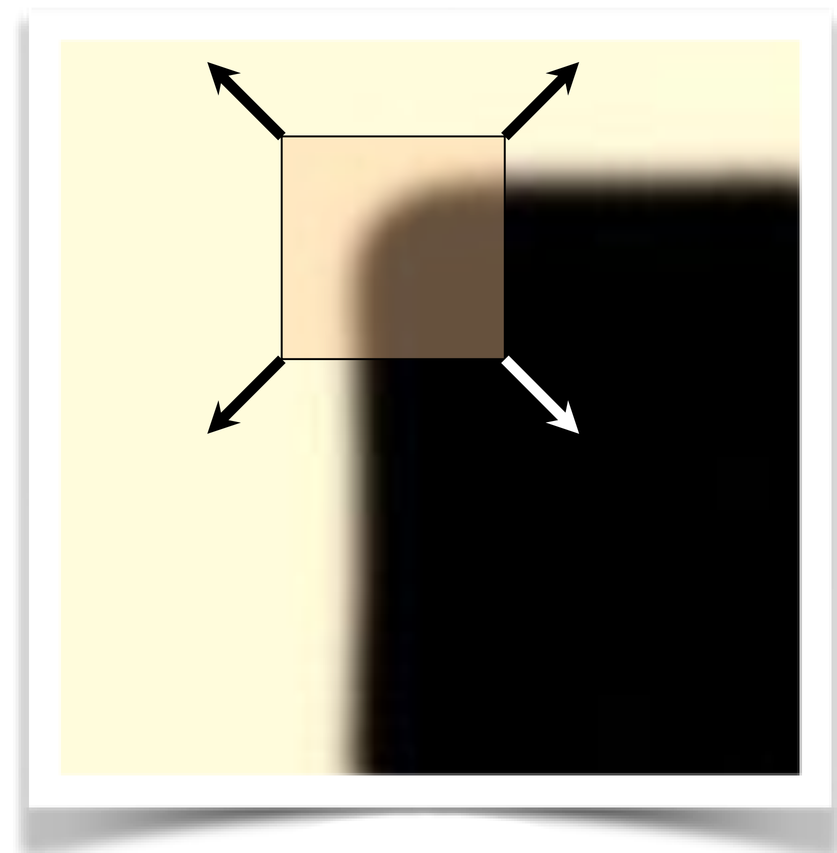


"flat" region:
no change in all directions



"edge":
no change along the edge direction



"corner":
significant change in all directions

[Moravec 1980]

Design a program to detect corners

(hint: use image gradients)

Finding corners
(a.k.a. PCA)

1. Compute image gradients over small region

$I_x = \frac{\partial I}{\partial x}$ $\qquad$ $I_y = \frac{\partial I}{\partial y}$

2. Subtract mean from each image gradient

3. Compute the covariance matrix

$$\begin{bmatrix} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{bmatrix}$$

4. Compute eigenvectors and eigenvalues

5. Use threshold on eigenvalues to detect corners

1. Compute image gradients over a small region
(not just a single pixel)

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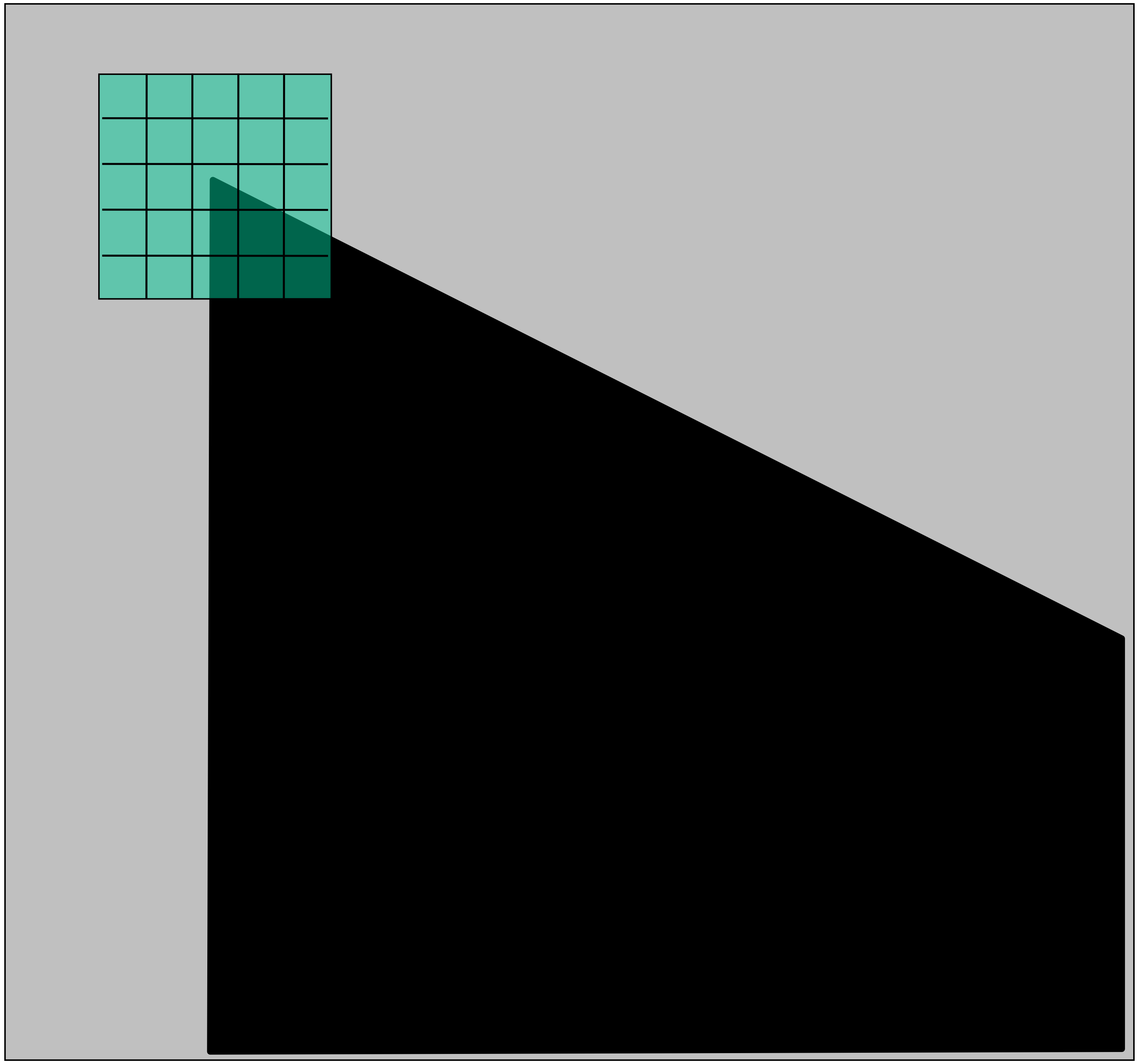
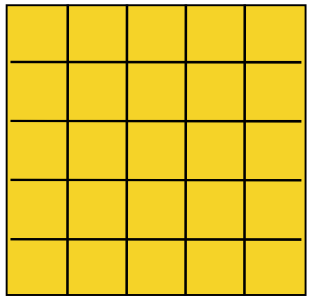
(not just a single pixel)



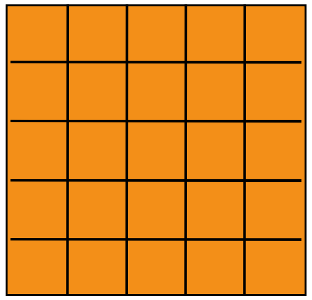
array of x gradients

$$I_x = \frac{\partial I}{\partial x}$$



array of y gradients

$$I_y = \frac{\partial I}{\partial y}$$

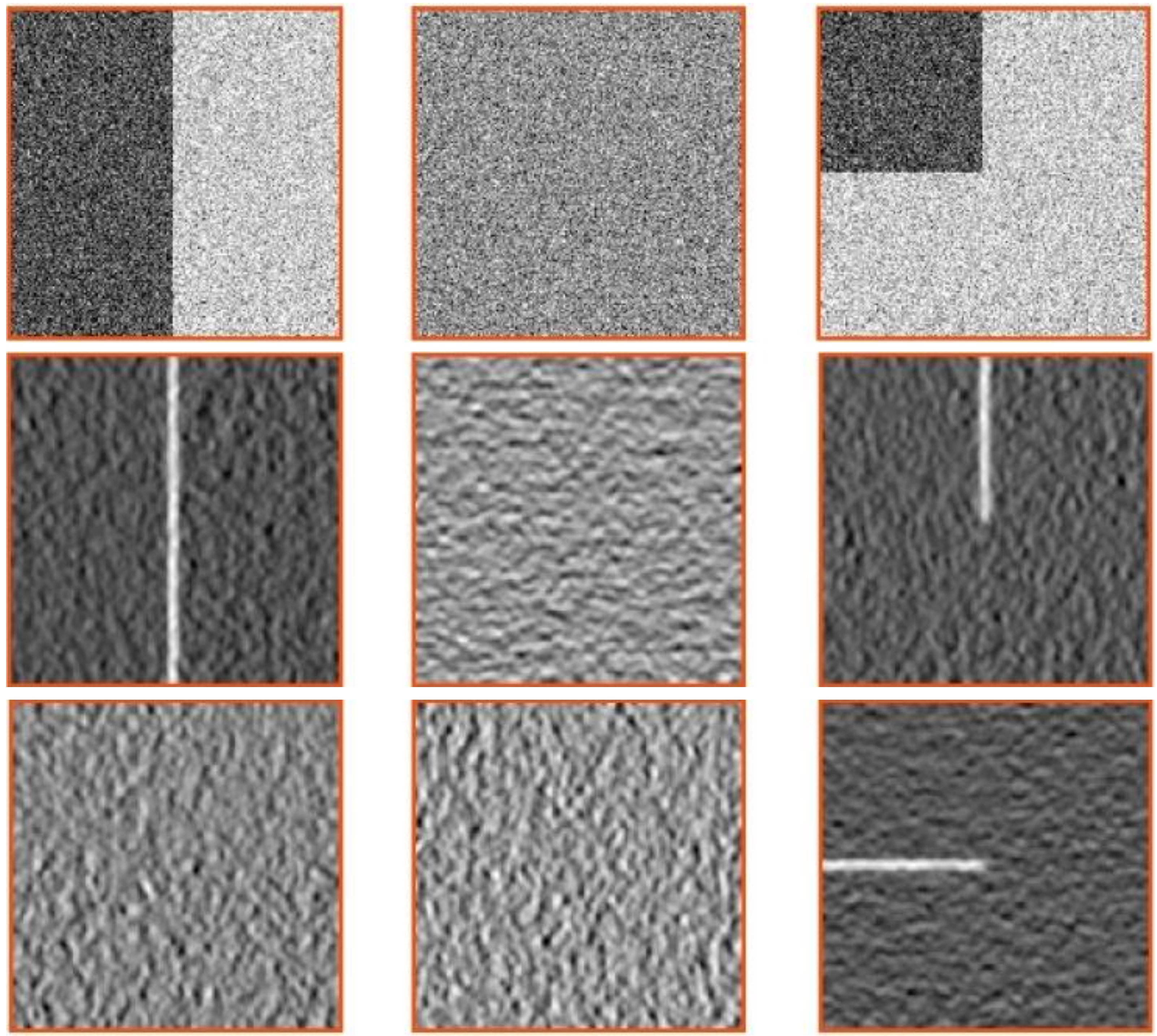


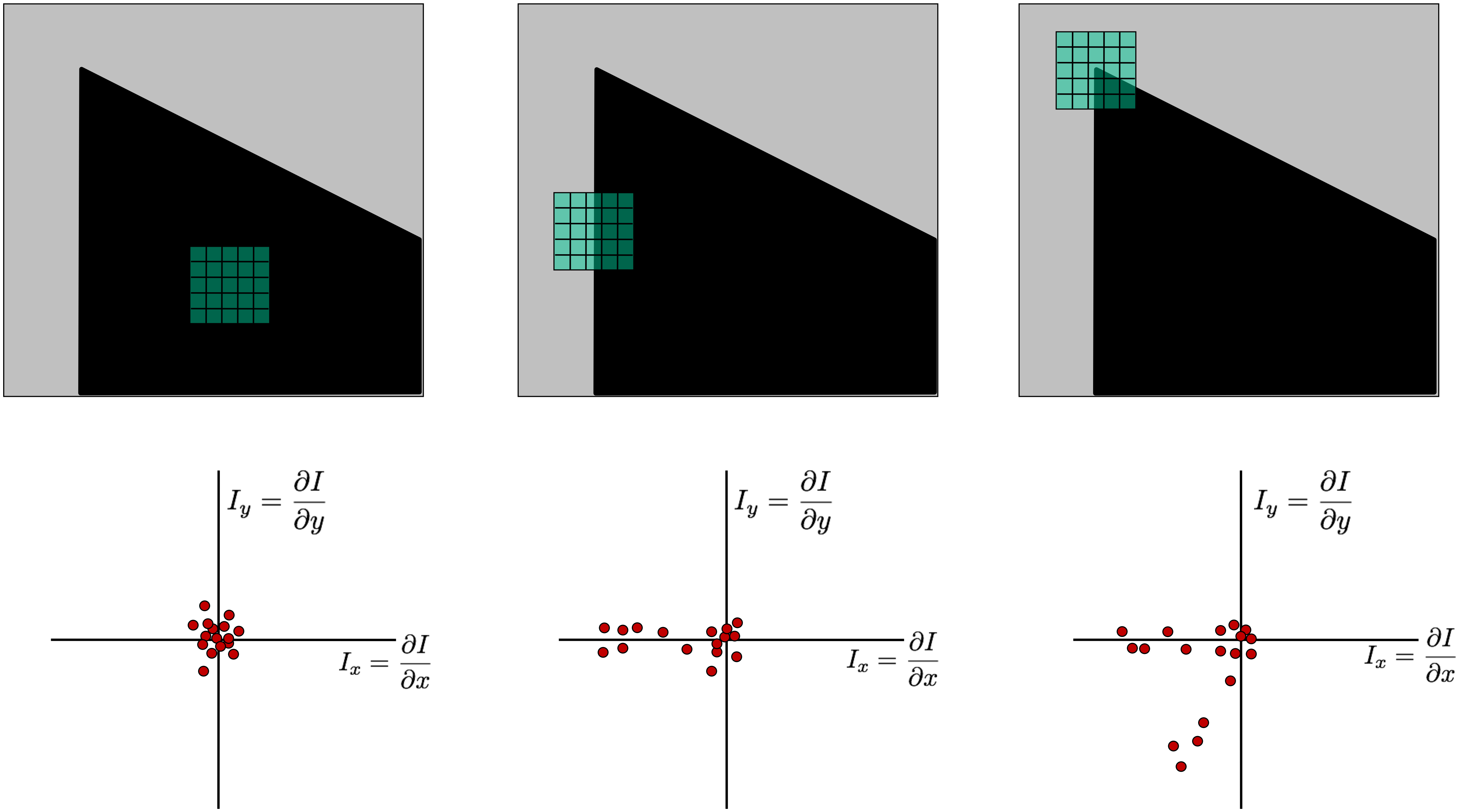
visualization of gradients

image

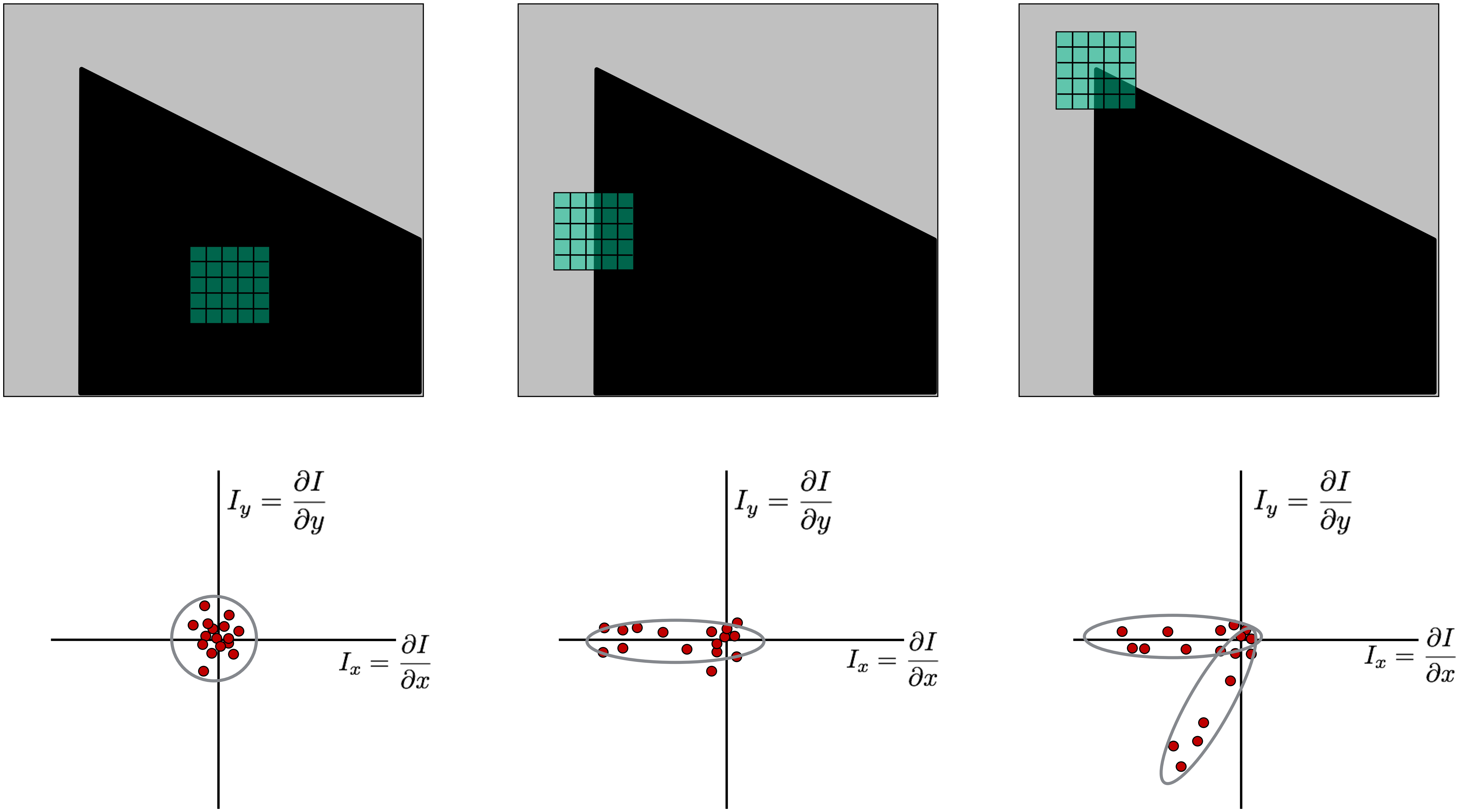
X derivative

Y derivative

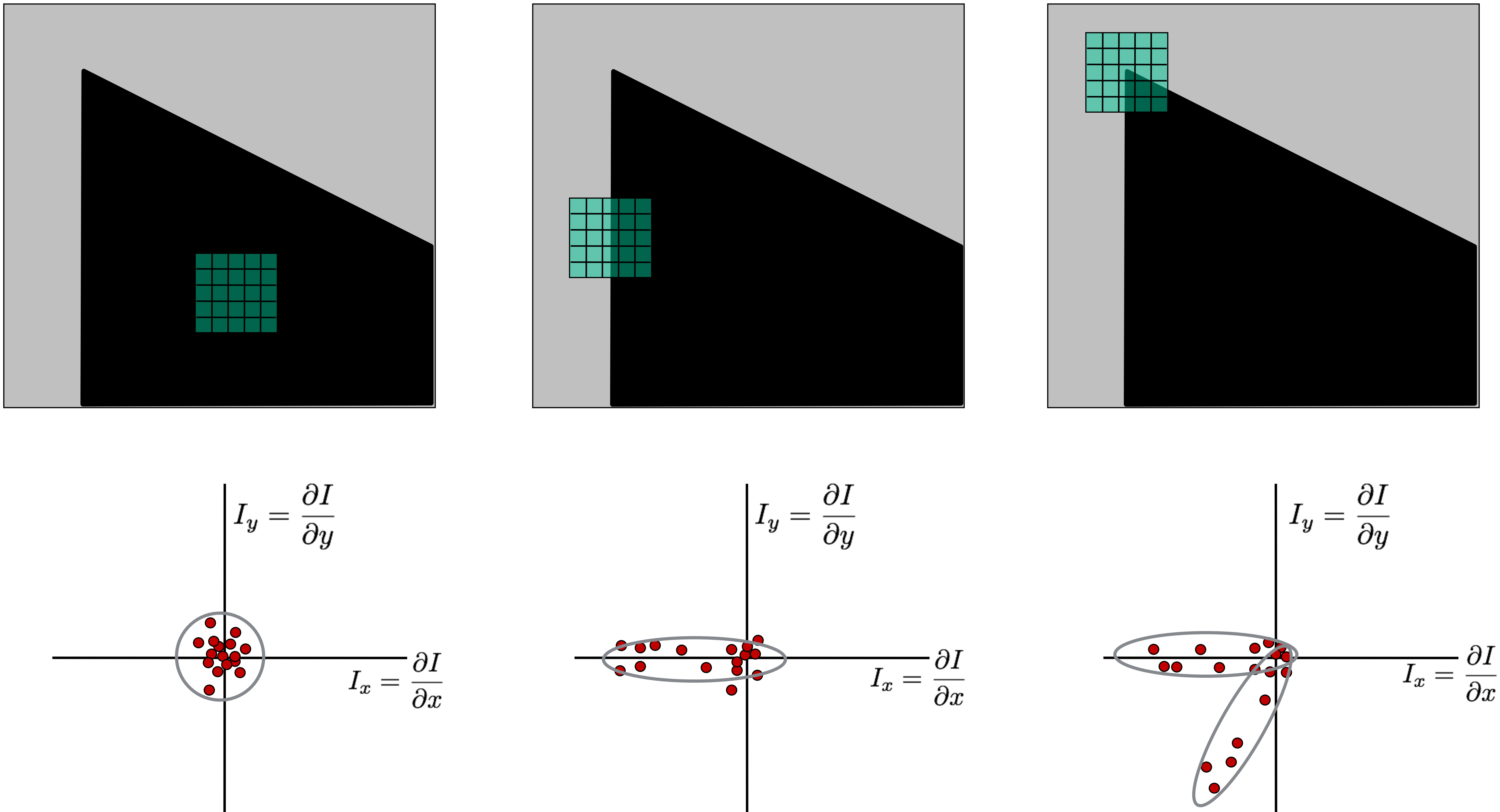




What does the distribution tell you about the region?



distribution reveals edge orientation and magnitude



How do you quantify orientation and magnitude?

2. Subtract the mean from each image gradient

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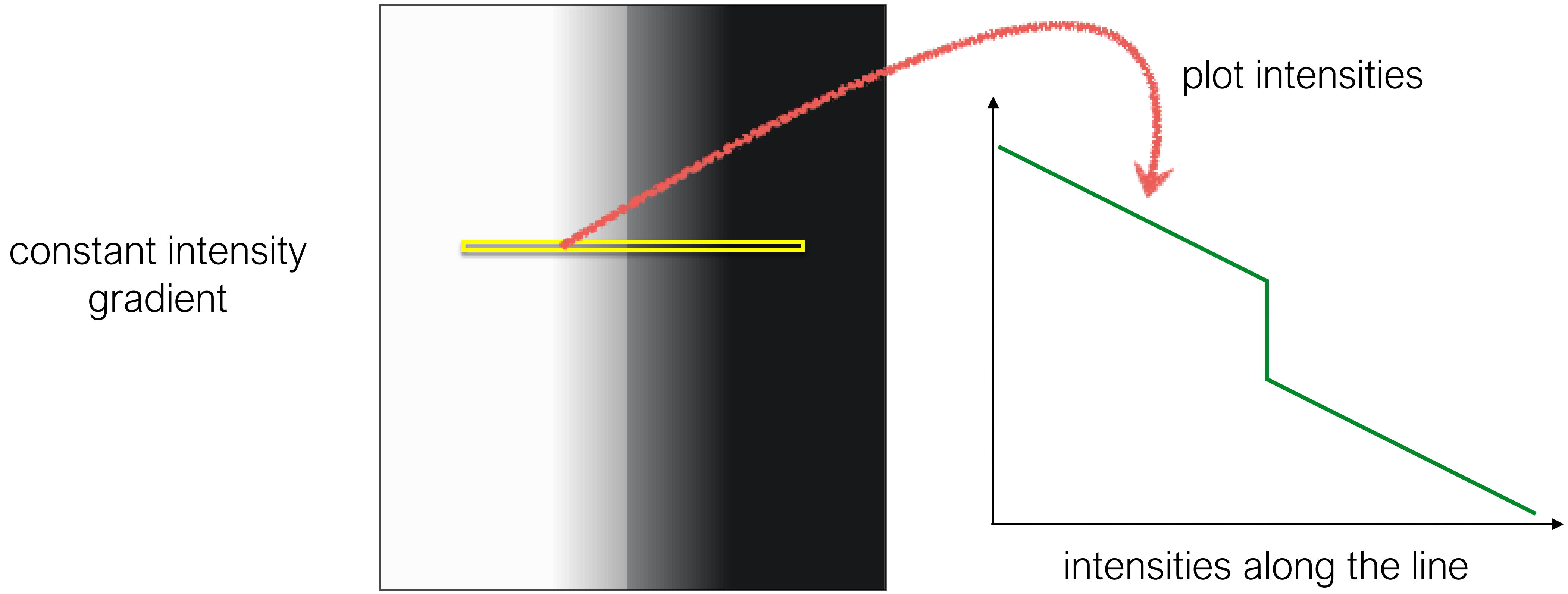

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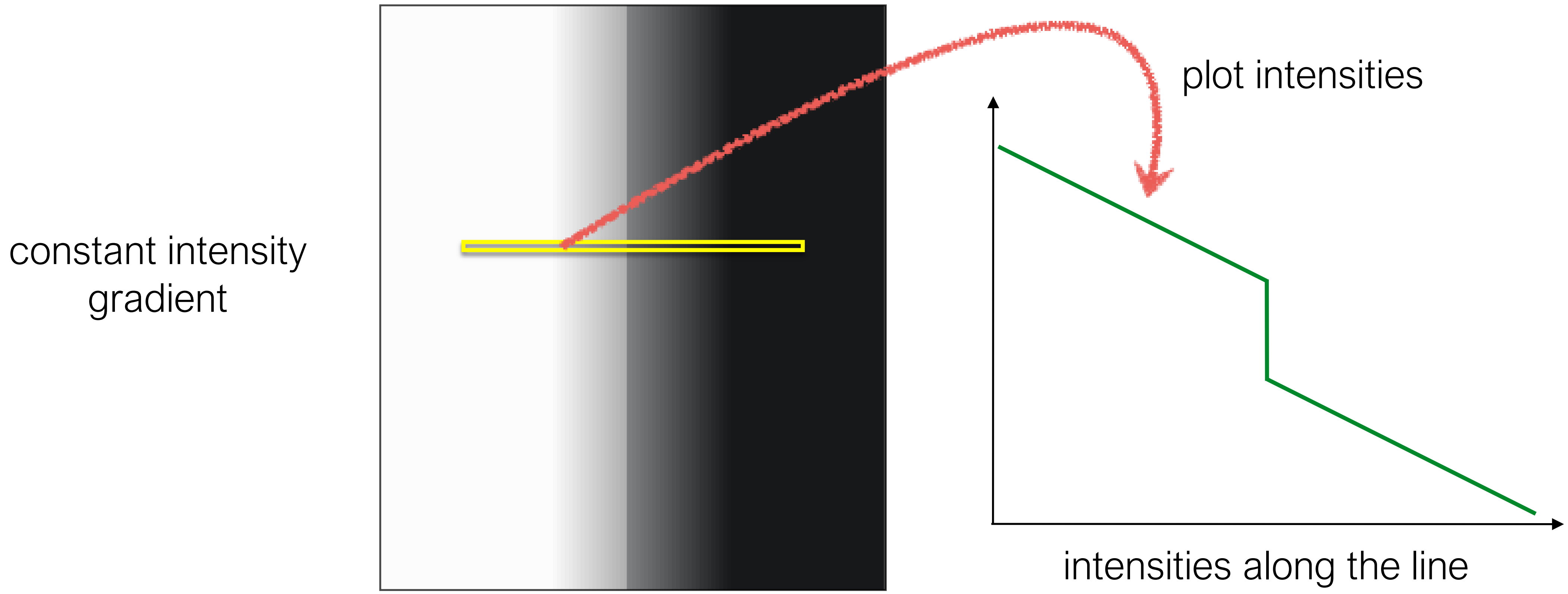
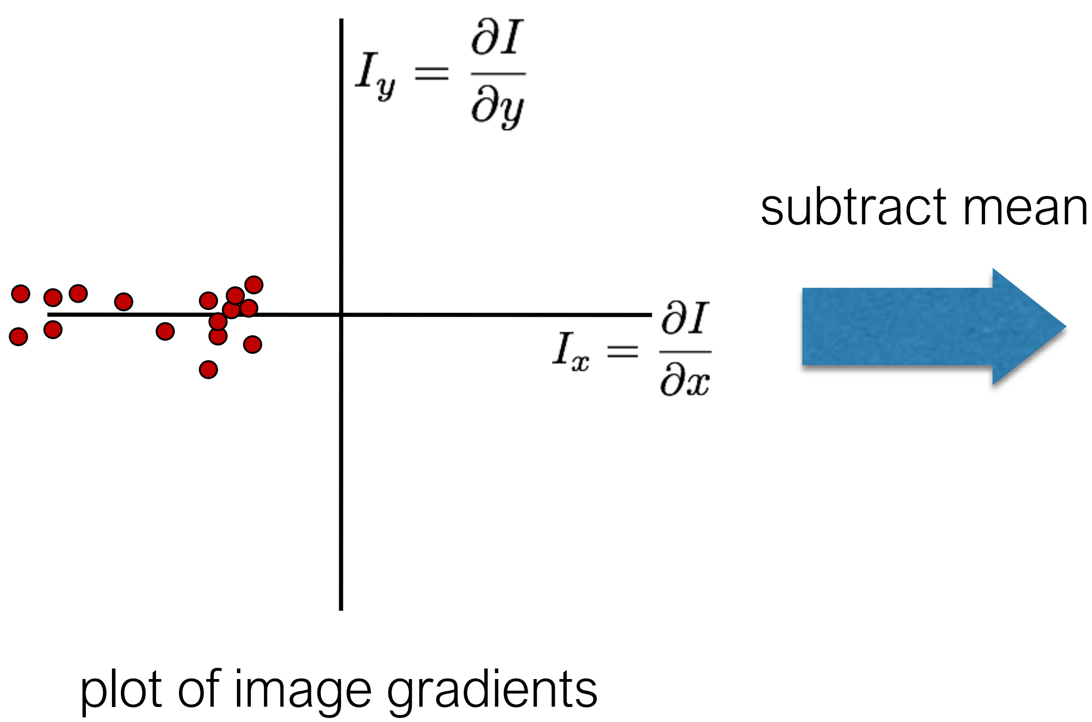
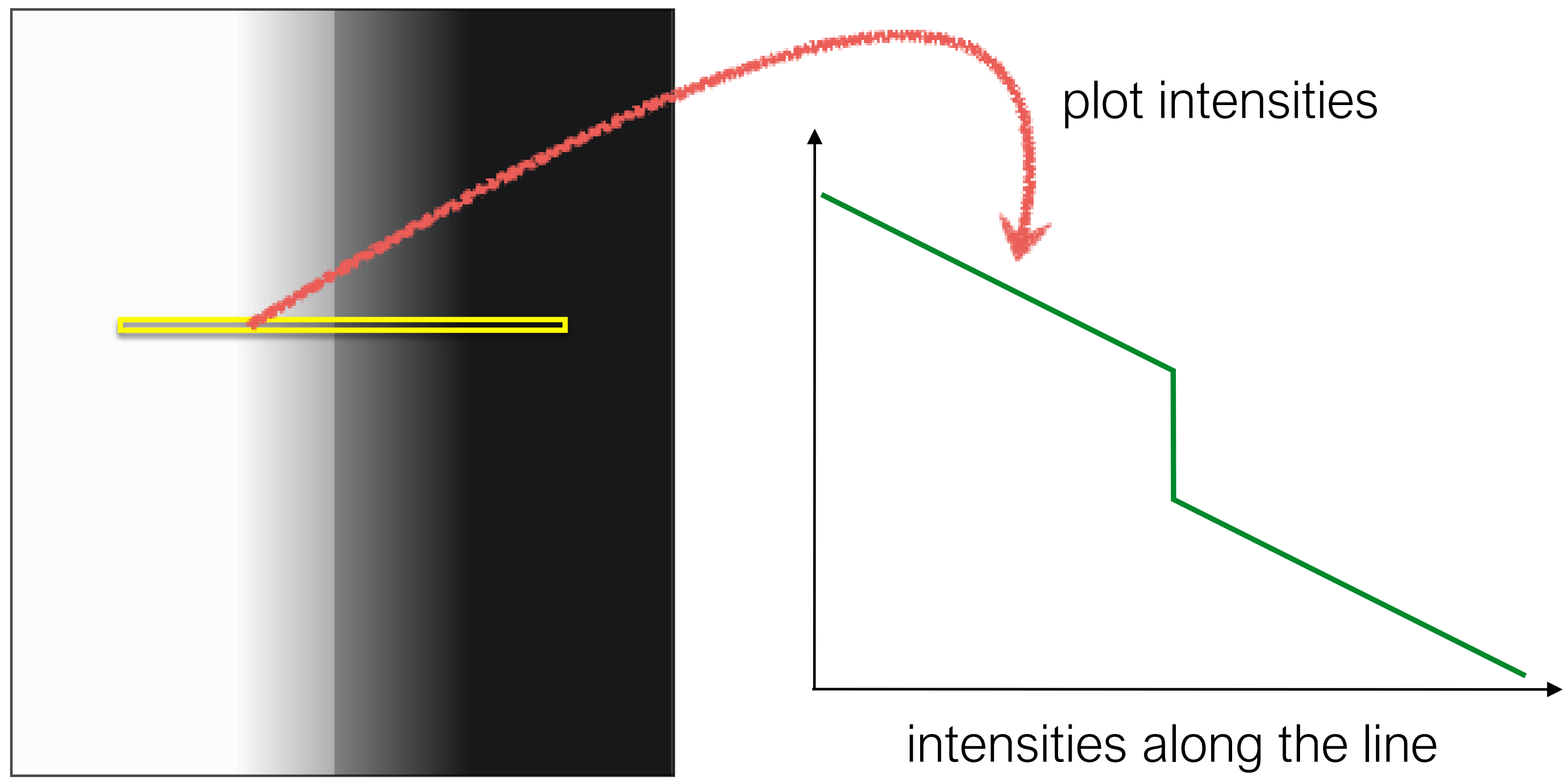
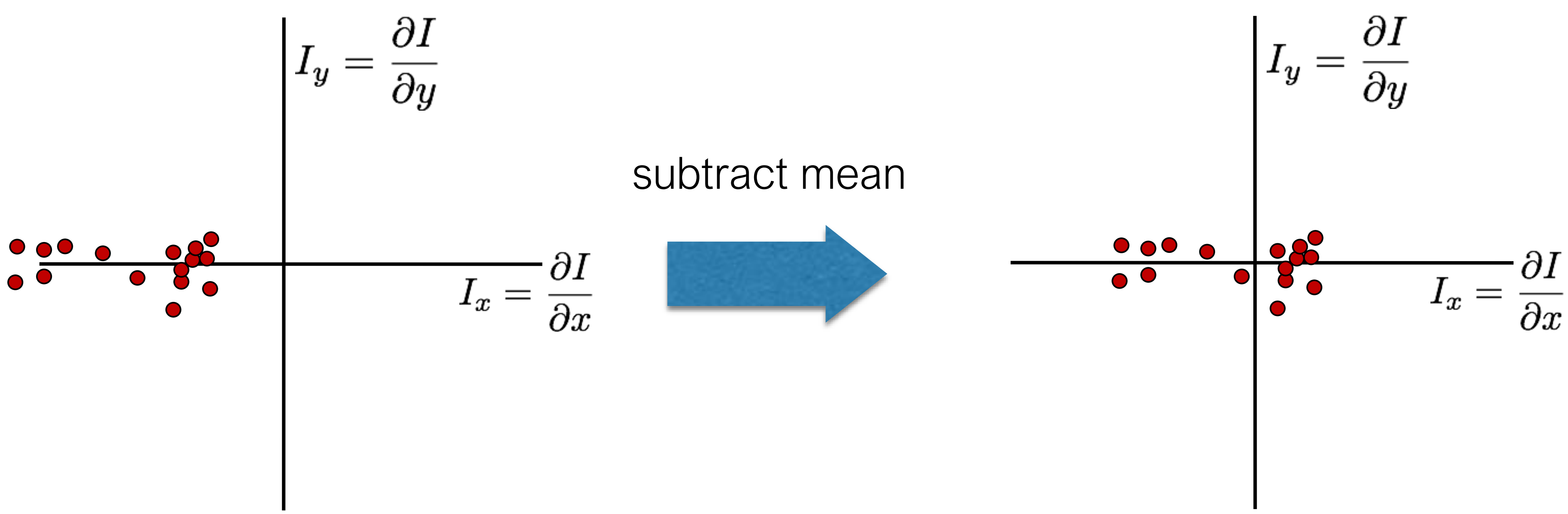




2. Subtract the mean from each image gradient

constant intensity
gradient





plot of image gradients

data is centered
('DC' offset is removed)

3. Compute the covariance matrix

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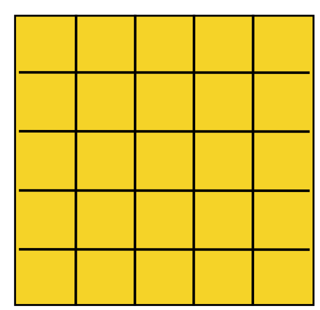
$$\begin{bmatrix} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{bmatrix}$$

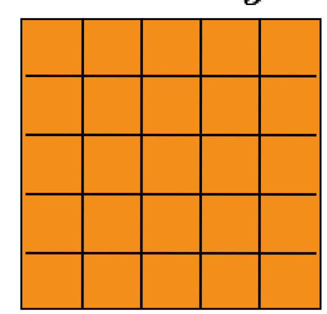
$$I_x = \frac{\partial I}{\partial x} \qquad I_y = \frac{\partial I}{\partial y}$$

$\sum\limits_{p \in P} I_x I_y$ =sum(



array of x gradients

.*



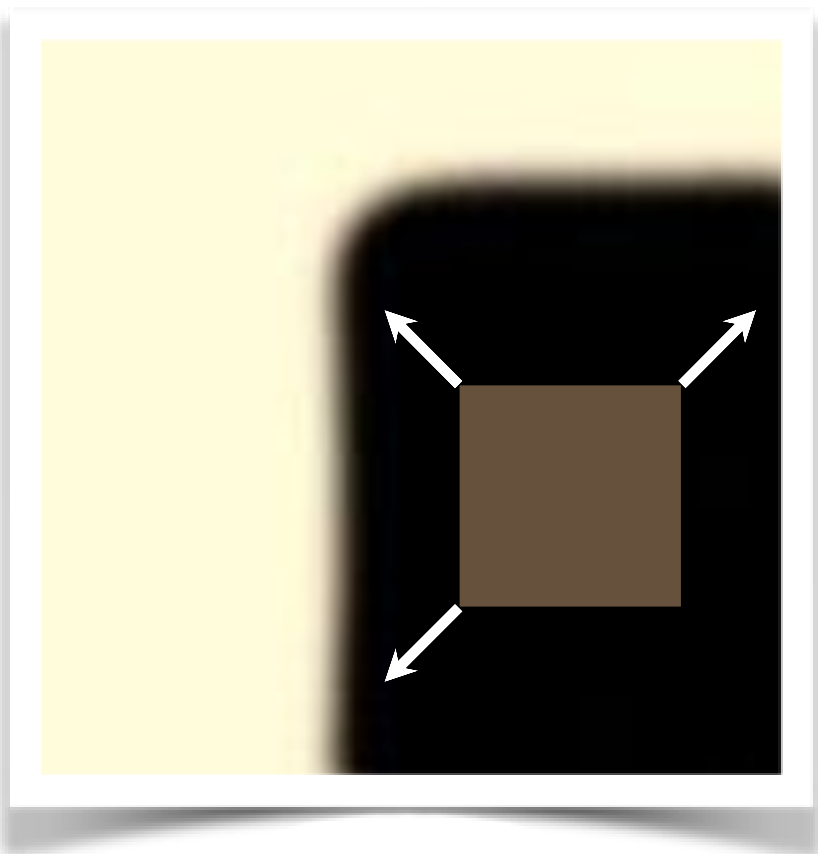
array of y gradients

)

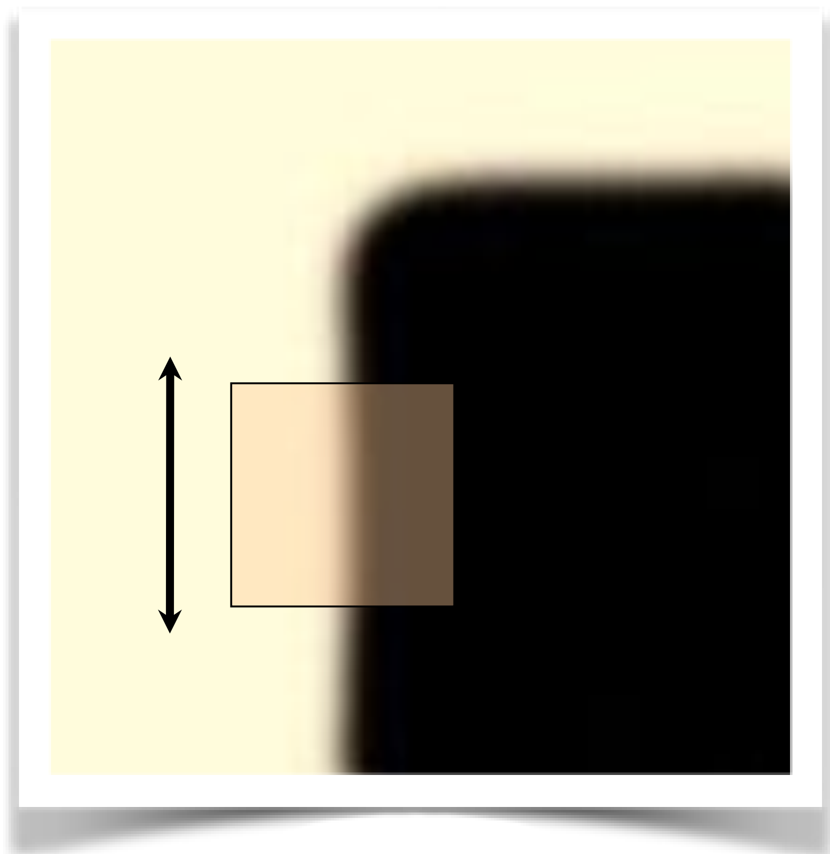
Where does this covariance matrix come from?

Easily recognized by looking through a small window

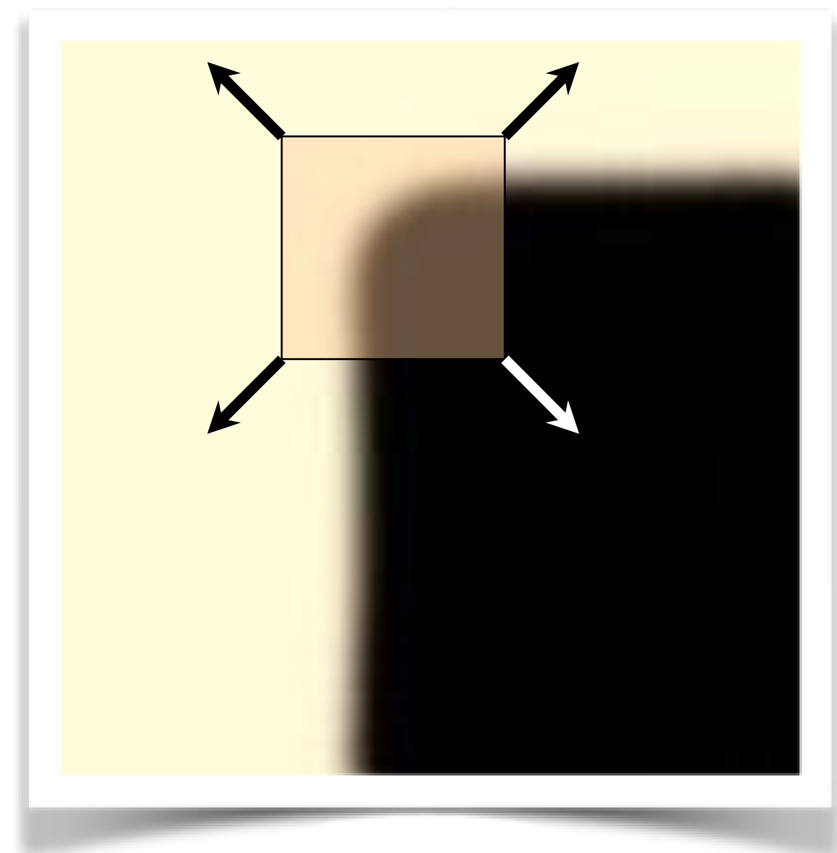
Shifting the window should give large change in intensity



"flat" region:
no change in all directions



"edge":
no change along the edge direction



"corner":
significant change in all directions

[Moravec 1980]

Error function

Change of intensity for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

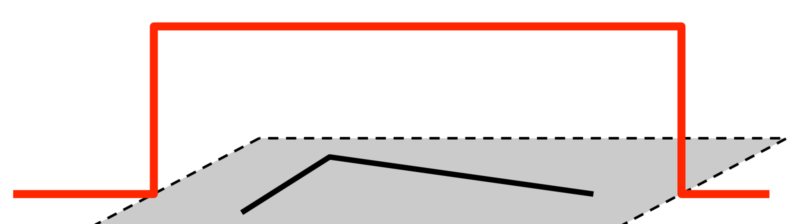
Error function

Window function

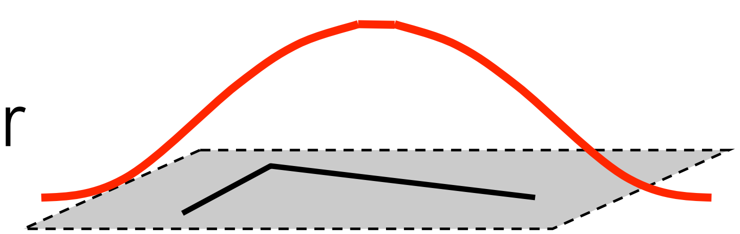
Shifted intensity

Intensity

Window function $w(x,y)$ =



or



1 in window, 0 outside

Gaussian

Error function approximation

Change of intensity for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

First-order Taylor expansion of I(x,y) about (0,0)
(bilinear approximation for small shifts)

Bilinear approximation

For small shifts $[u,v]$ we have a 'bilinear approximation':

Change in appearance for a shift [u,v]

$$E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

'second moment' matrix
'structure tensor'

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

By computing the gradient covariance matrix…

$$\begin{bmatrix} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{bmatrix}$$

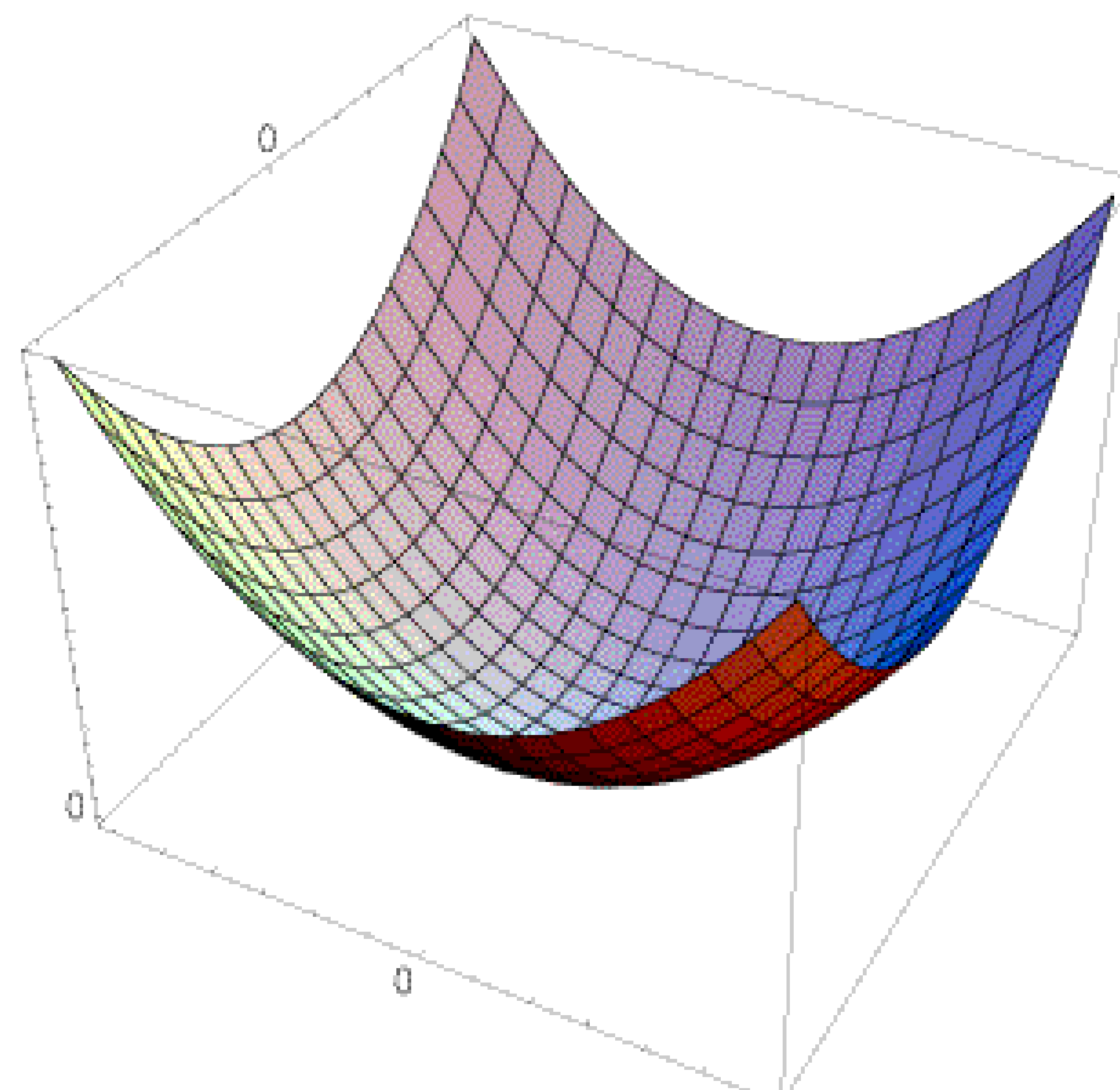
we are fitting a quadratic to the gradients over a small image region

Visualization of a quadratic

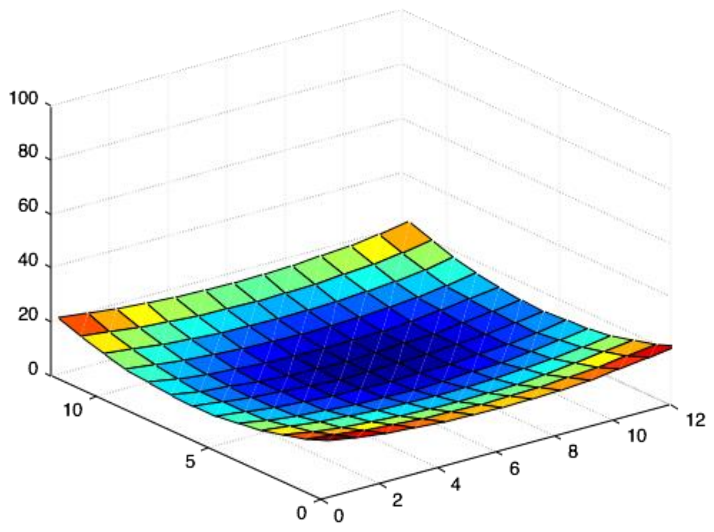
The surface *E*(*u*,*v*) is locally approximated by a quadratic form

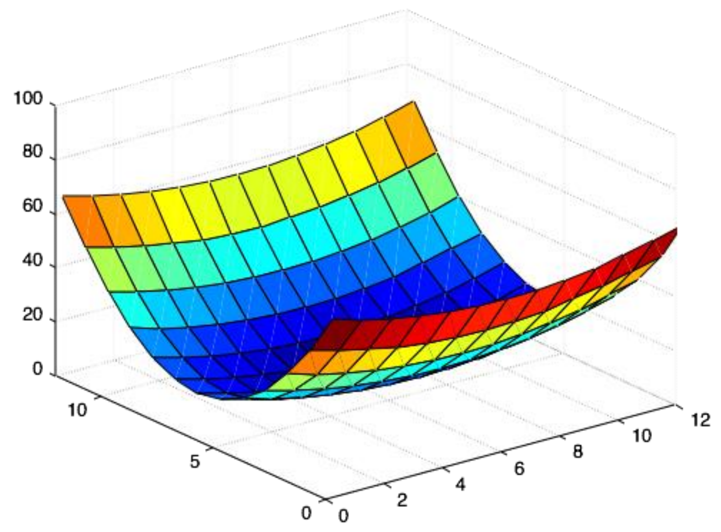
$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

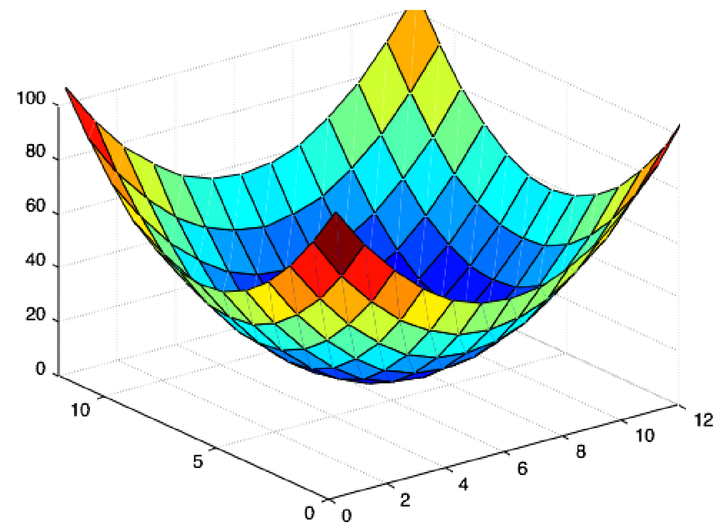
$$M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Which error surface indicates a good image feature?

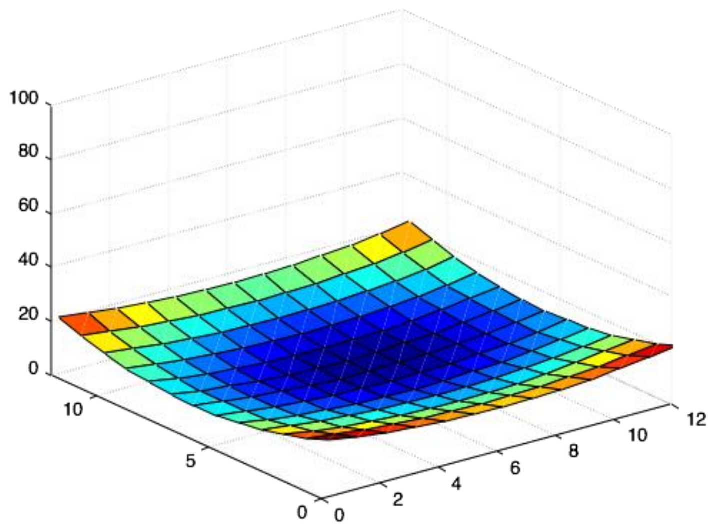




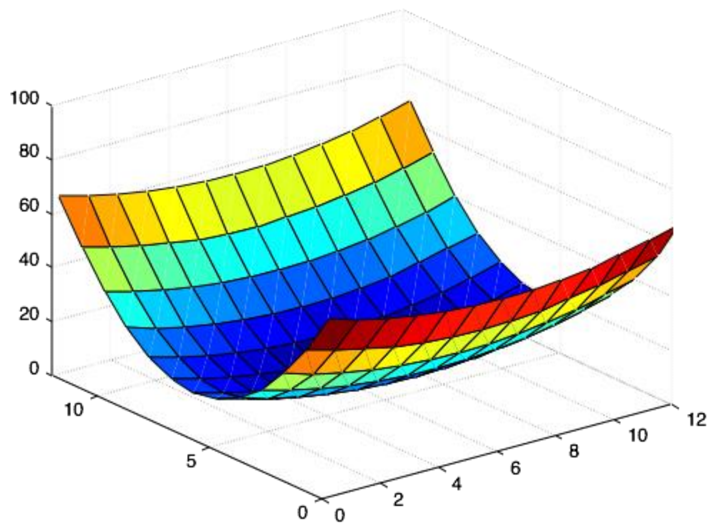


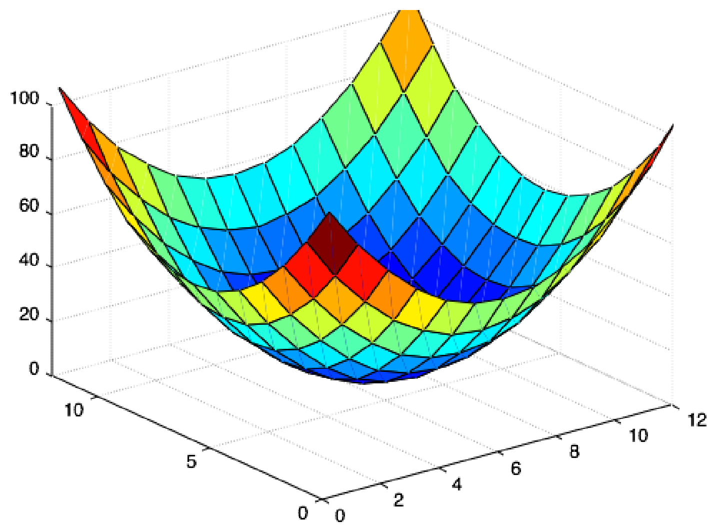
What kind of image patch do these surfaces represent?

Which error surface indicates a good image feature?

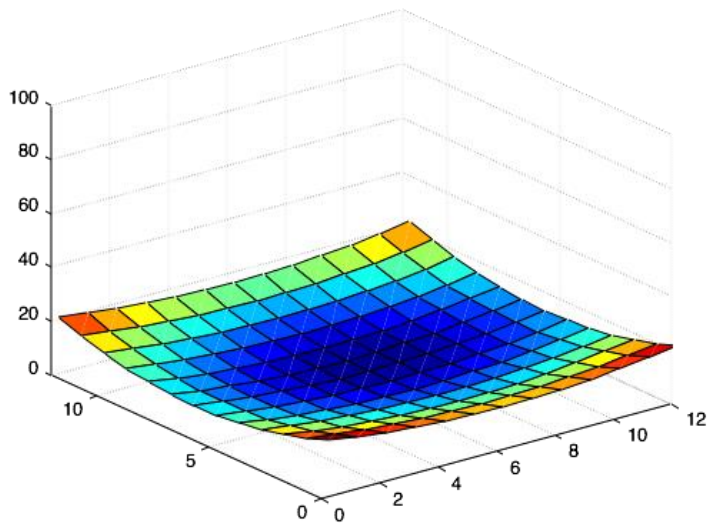


flat

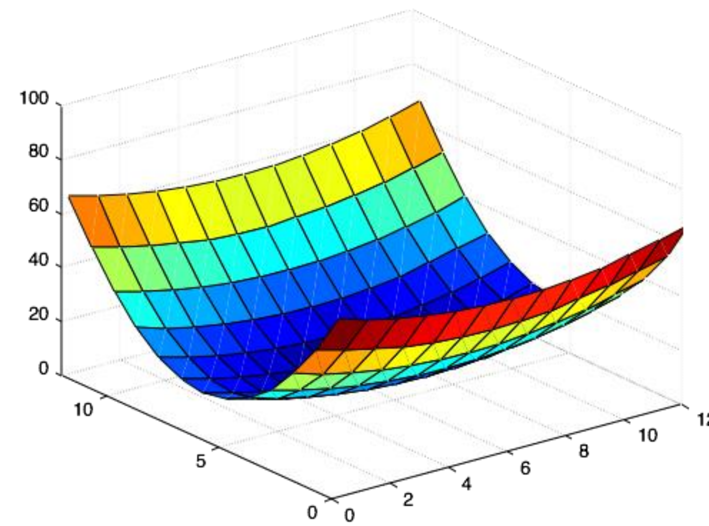




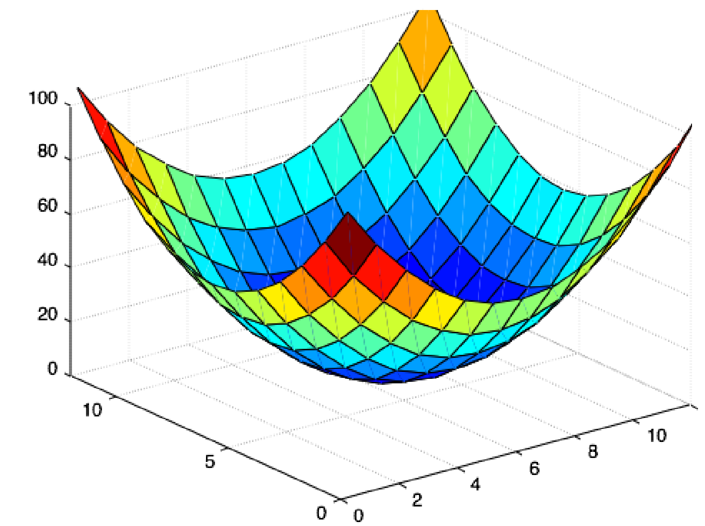
Which error surface indicates a good image feature?



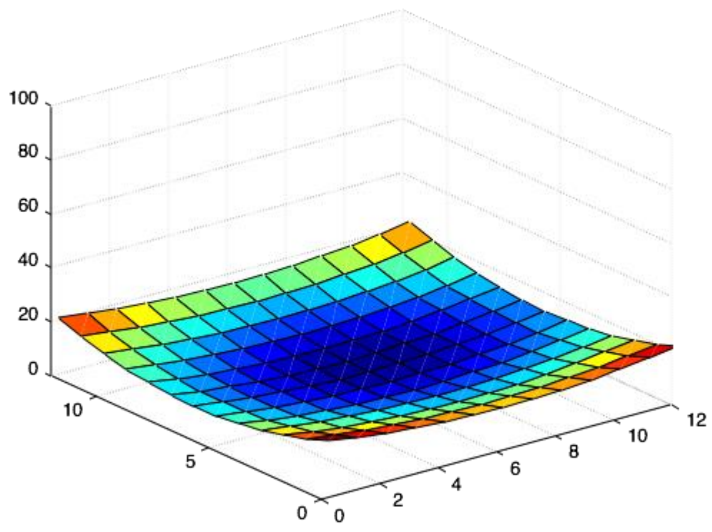
flat



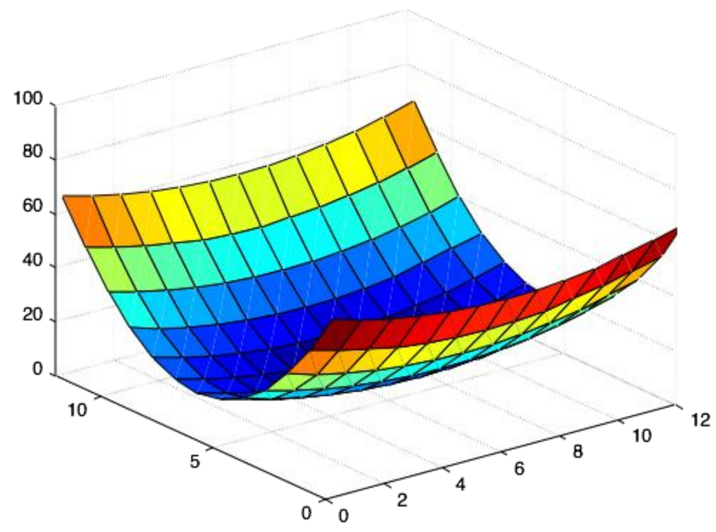
edge
‘line’



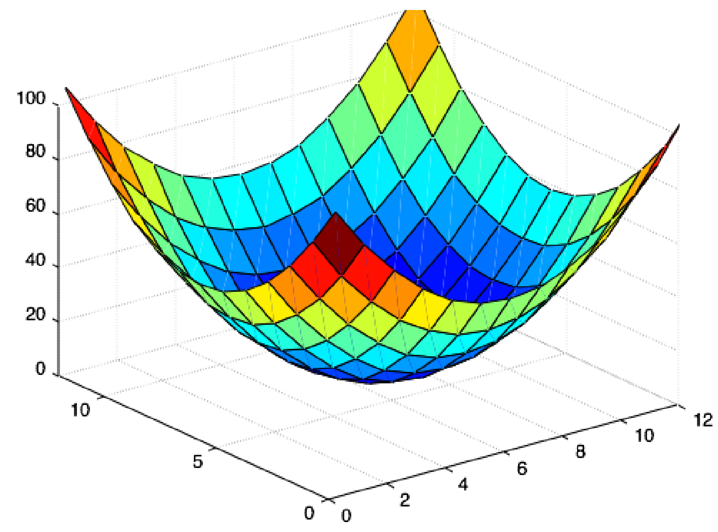
Which error surface indicates a good image feature?



flat



edge
‘line’



corner
‘dot’

4. Compute eigenvalues and eigenvectors

4. Compute eigenvalues and eigenvectors

eigenvalue

$$M\boldsymbol{e} = \lambda\boldsymbol{e}$$

eigenvector

$$(M - \lambda I)\boldsymbol{e} = 0$$

4. Compute eigenvalues and eigenvectors

eigenvalue

$$M\boldsymbol{e} = \lambda\boldsymbol{e}$$

eigenvector

$$(M - \lambda I)\boldsymbol{e} = 0$$

1. Compute the determinant of $M - \lambda I$

(returns a polynomial)

4. Compute eigenvalues and eigenvectors

eigenvalue

$$\boldsymbol{M}\boldsymbol{e} = \lambda\boldsymbol{e}$$

eigenvector

$$(M - \lambda I)\boldsymbol{e} = 0$$

1. Compute the determinant of $M - \lambda I$

(returns a polynomial)

2. Find the roots of polynomial $\det(M - \lambda I) = 0$

(returns eigenvalues)

4. Compute eigenvalues and eigenvectors

eigenvalue

$$\underset{\text{eigenvector}}{M\boldsymbol{e}} = \lambda \boldsymbol{e}$$

eigenvector

$$(M - \lambda I)\boldsymbol{e} = 0$$

1. Compute the determinant of $M - \lambda I$
(returns a polynomial)

2. Find the roots of polynomial $\det(M - \lambda I) = 0$
(returns eigenvalues)

3. For each eigenvalue, solve $(M - \lambda I)\boldsymbol{e} = 0$
(returns eigenvectors)

```
eig(M)
```

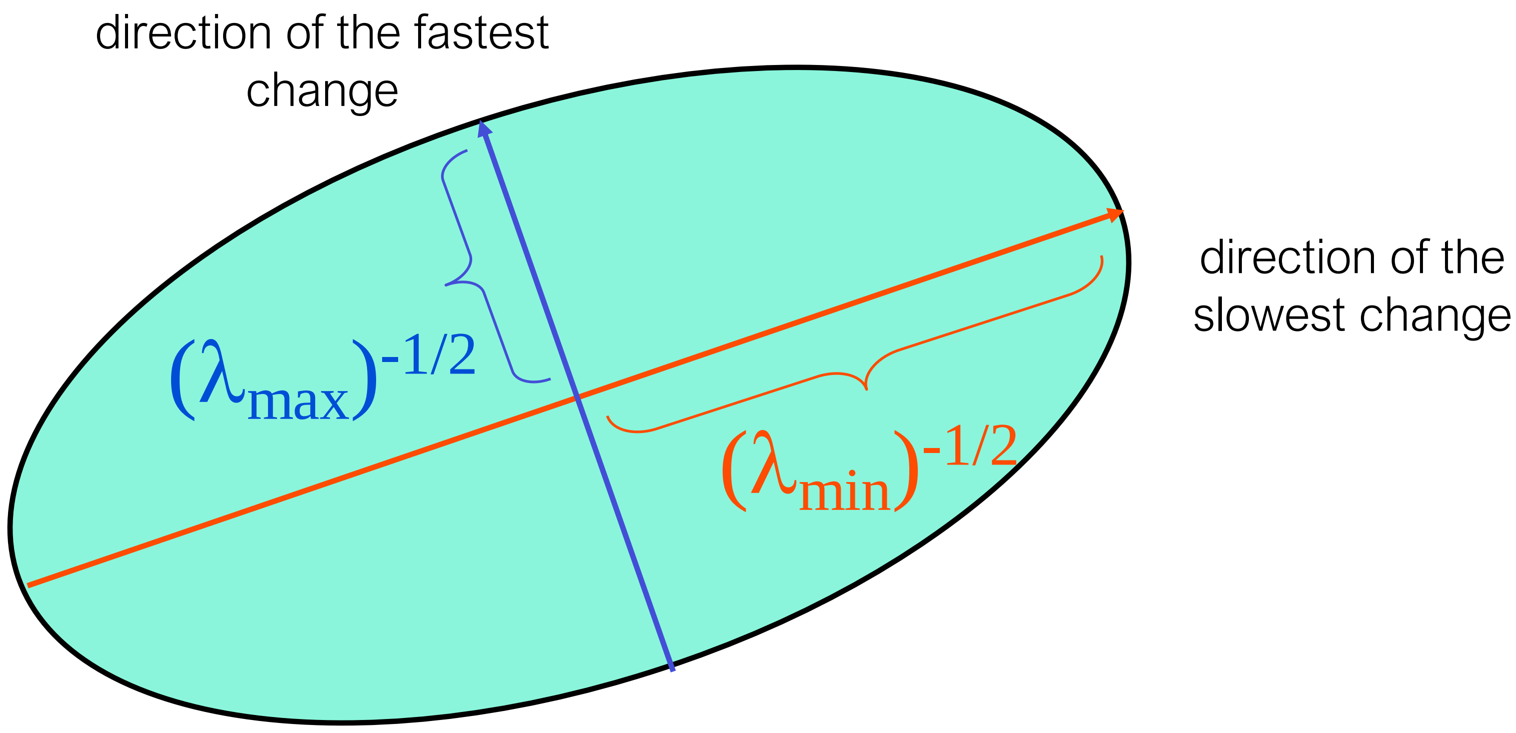
Visualization as an ellipse

Since M is symmetric, we have $$M = R^{-1}\begin{bmatrix}\lambda_1 & 0 \\ 0 & \lambda_2\end{bmatrix}R$$

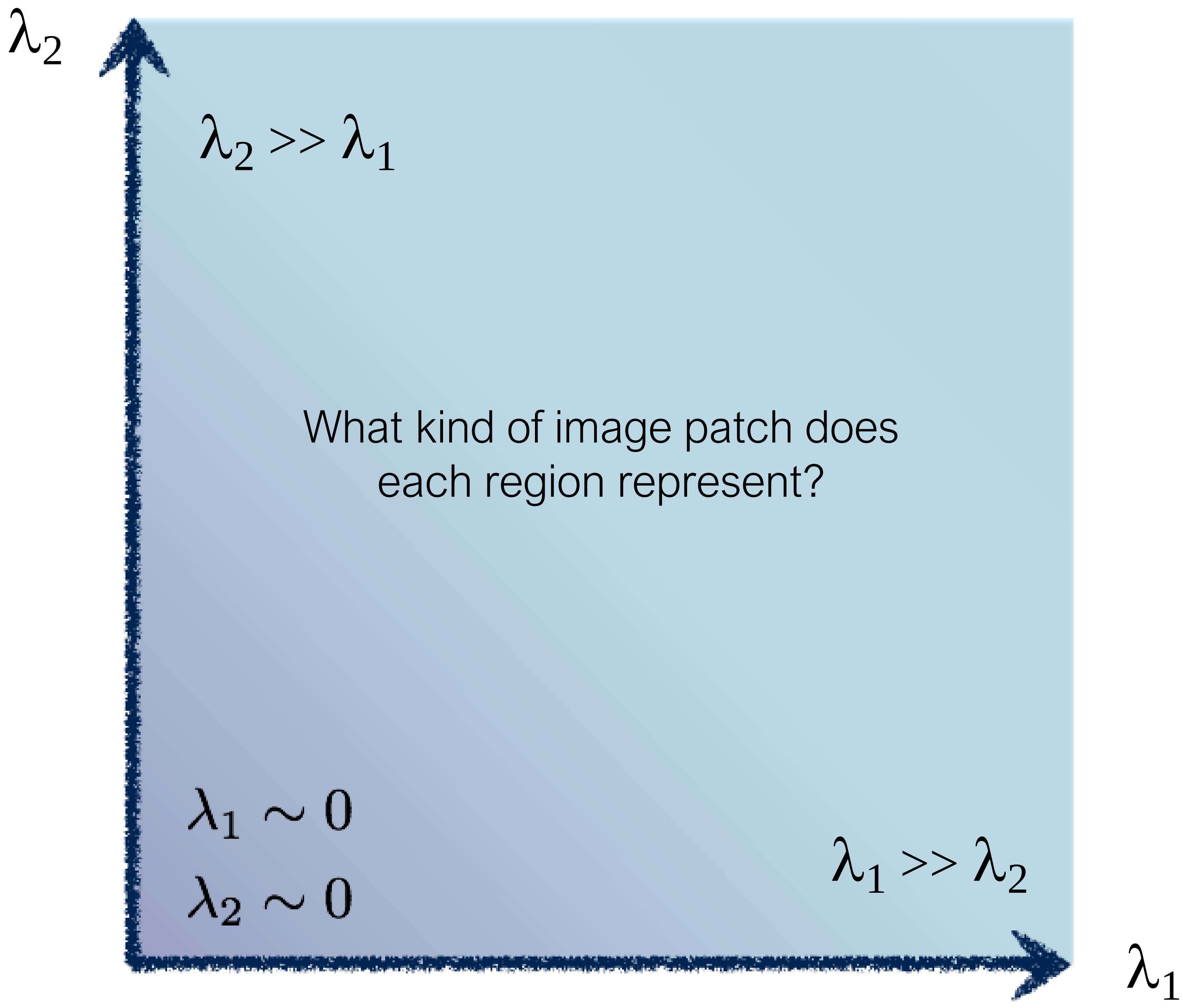
We can visualize *M* as an ellipse with axis lengths determined by the eigenvalues and orientation determined by *R*

Ellipse equation:

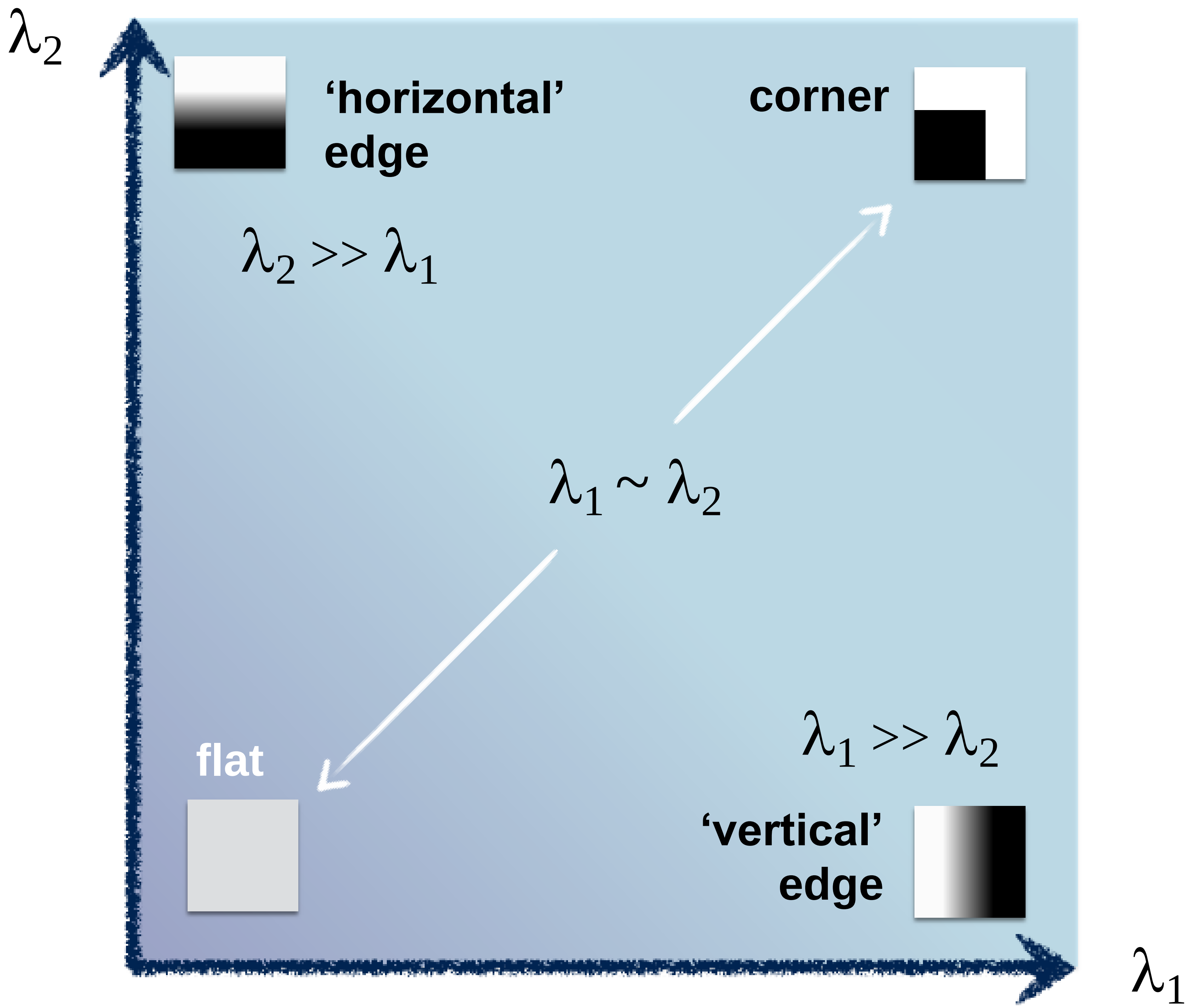
$$[u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



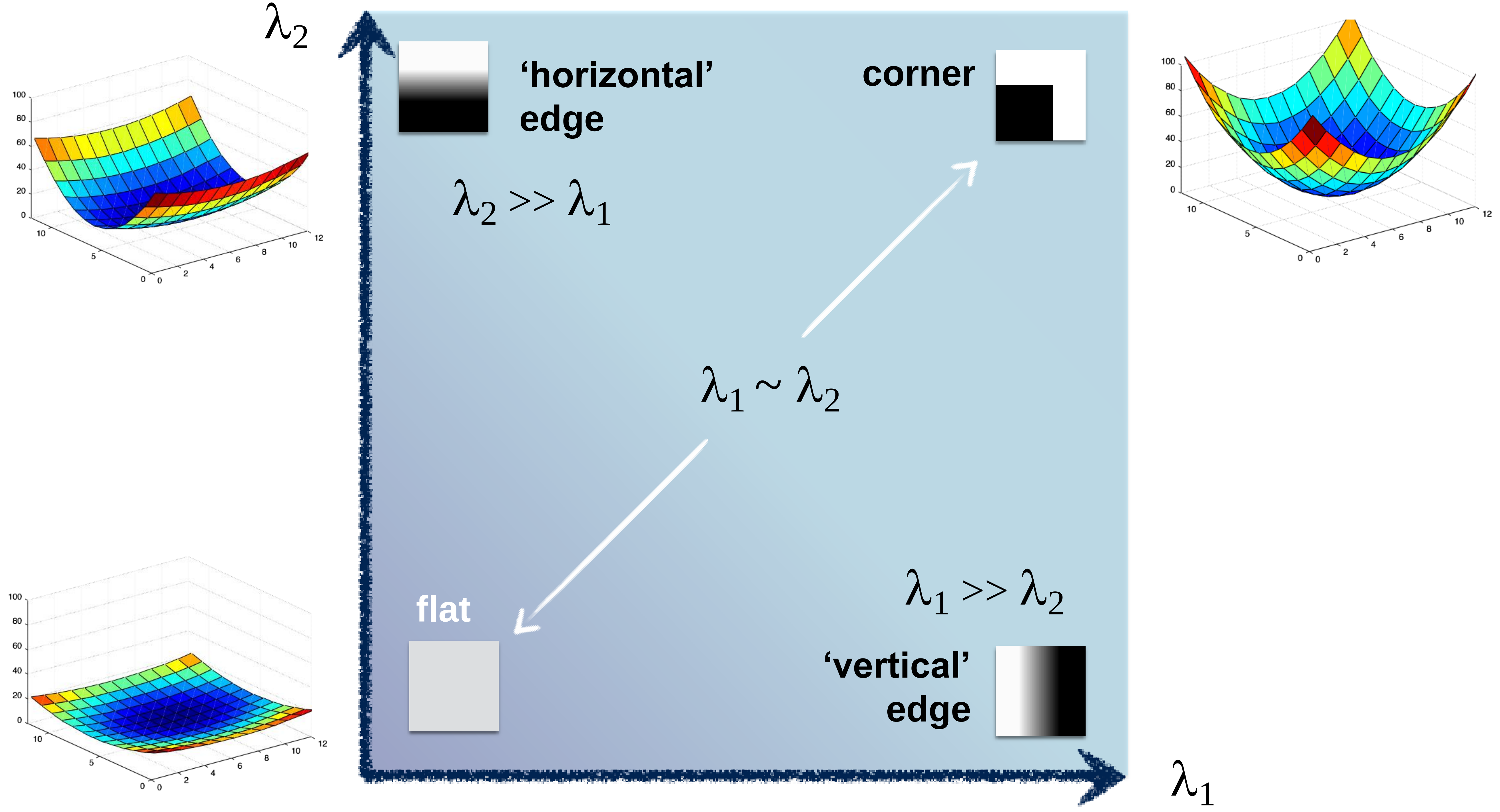
interpreting eigenvalues



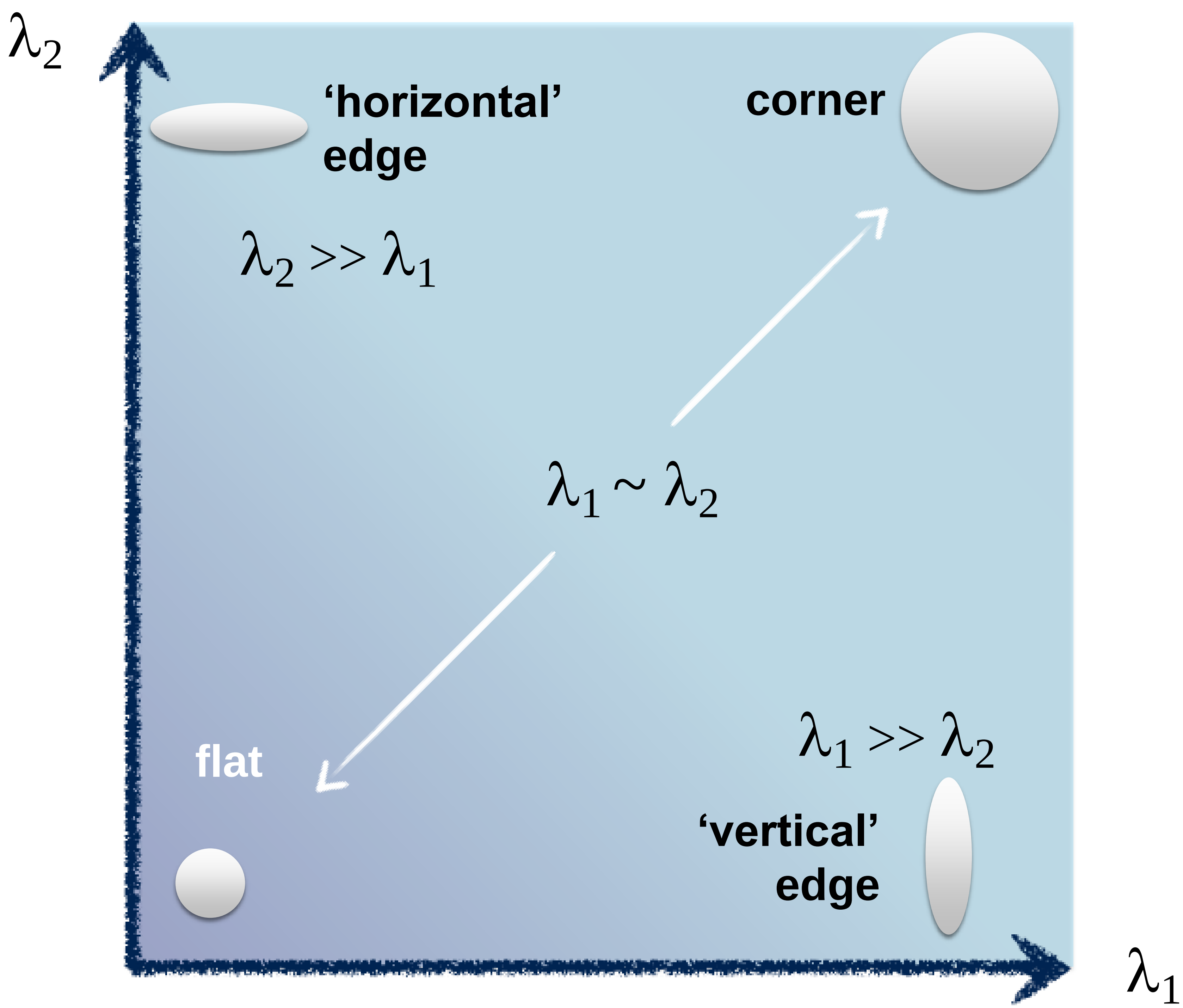
interpreting eigenvalues



interpreting eigenvalues



interpreting eigenvalues



5. Use threshold on eigenvalues to detect corners

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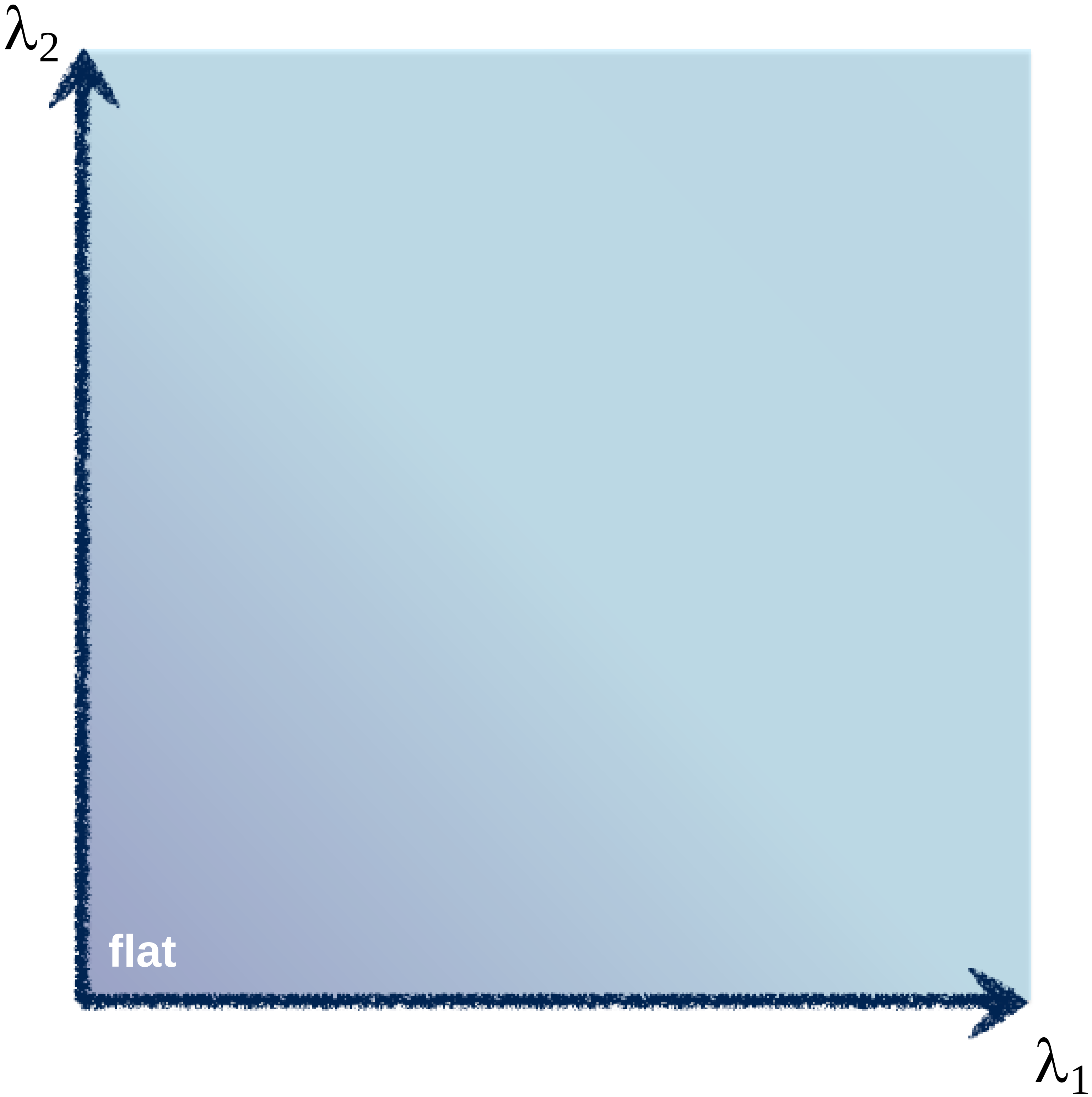
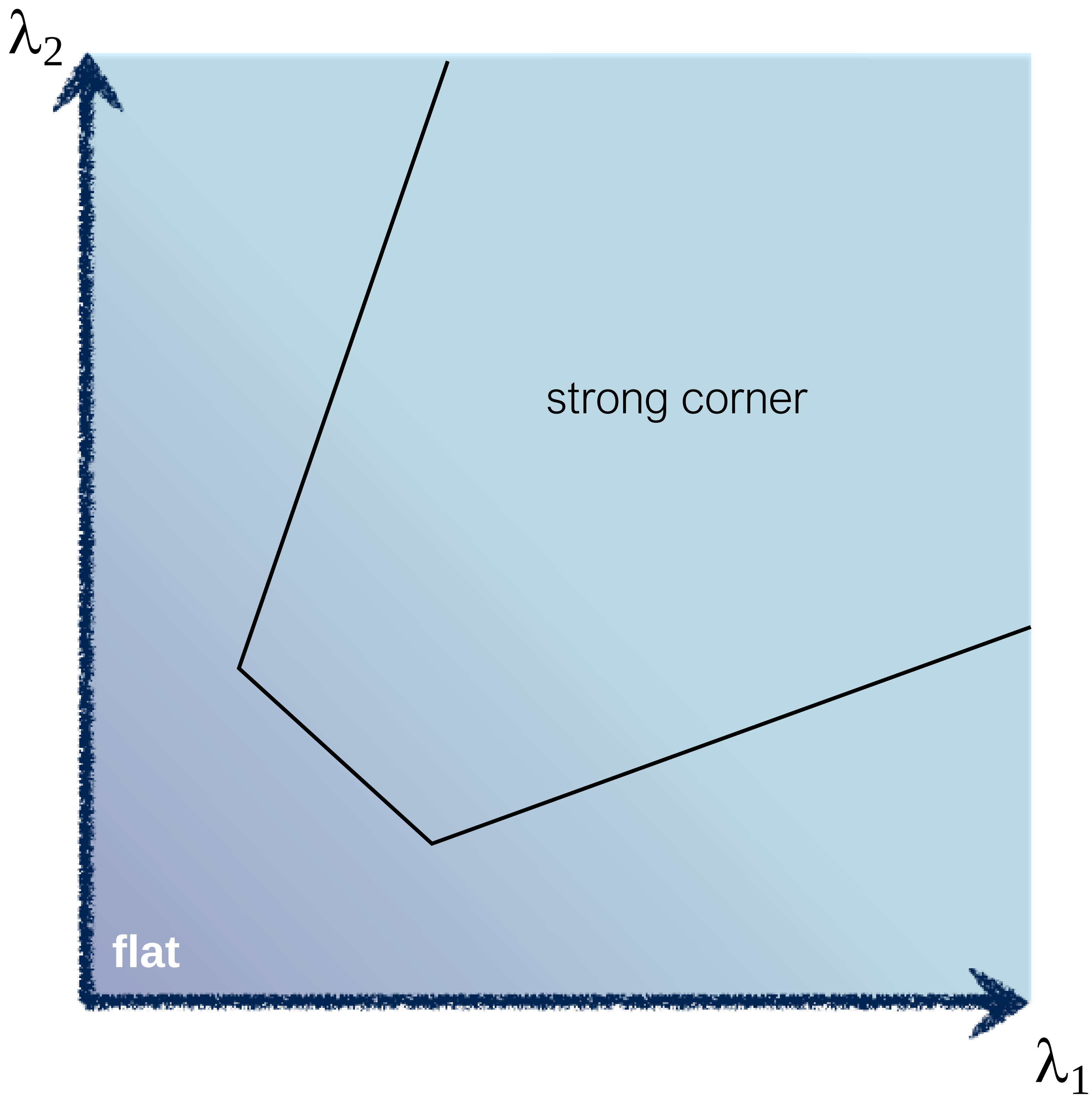


Think of a function to score 'cornerness'

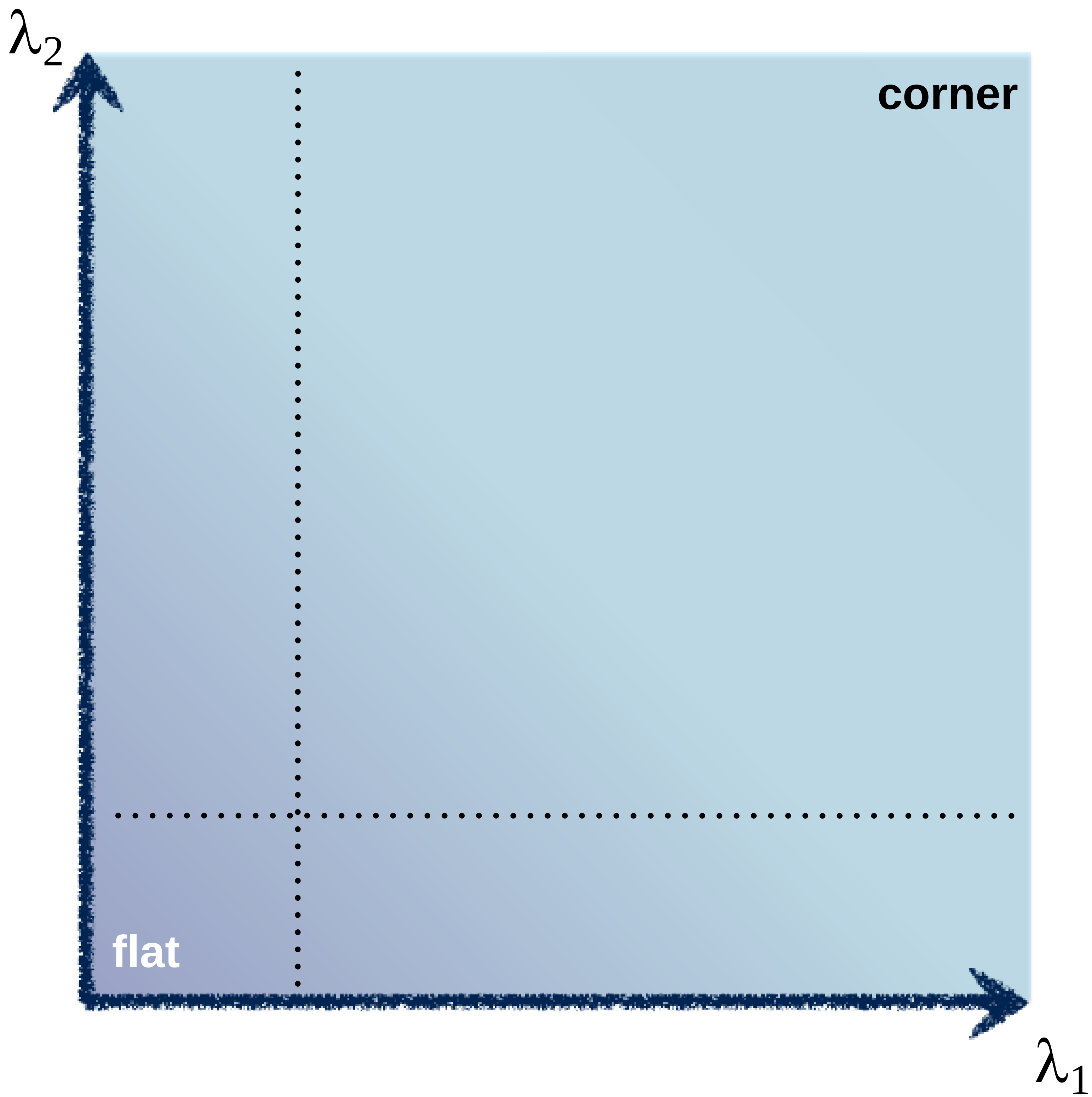
5. Use threshold on eigenvalues to detect corners



Think of a function to score 'cornerness'

5. Use threshold on eigenvalues to detect corners

^
(a function of)



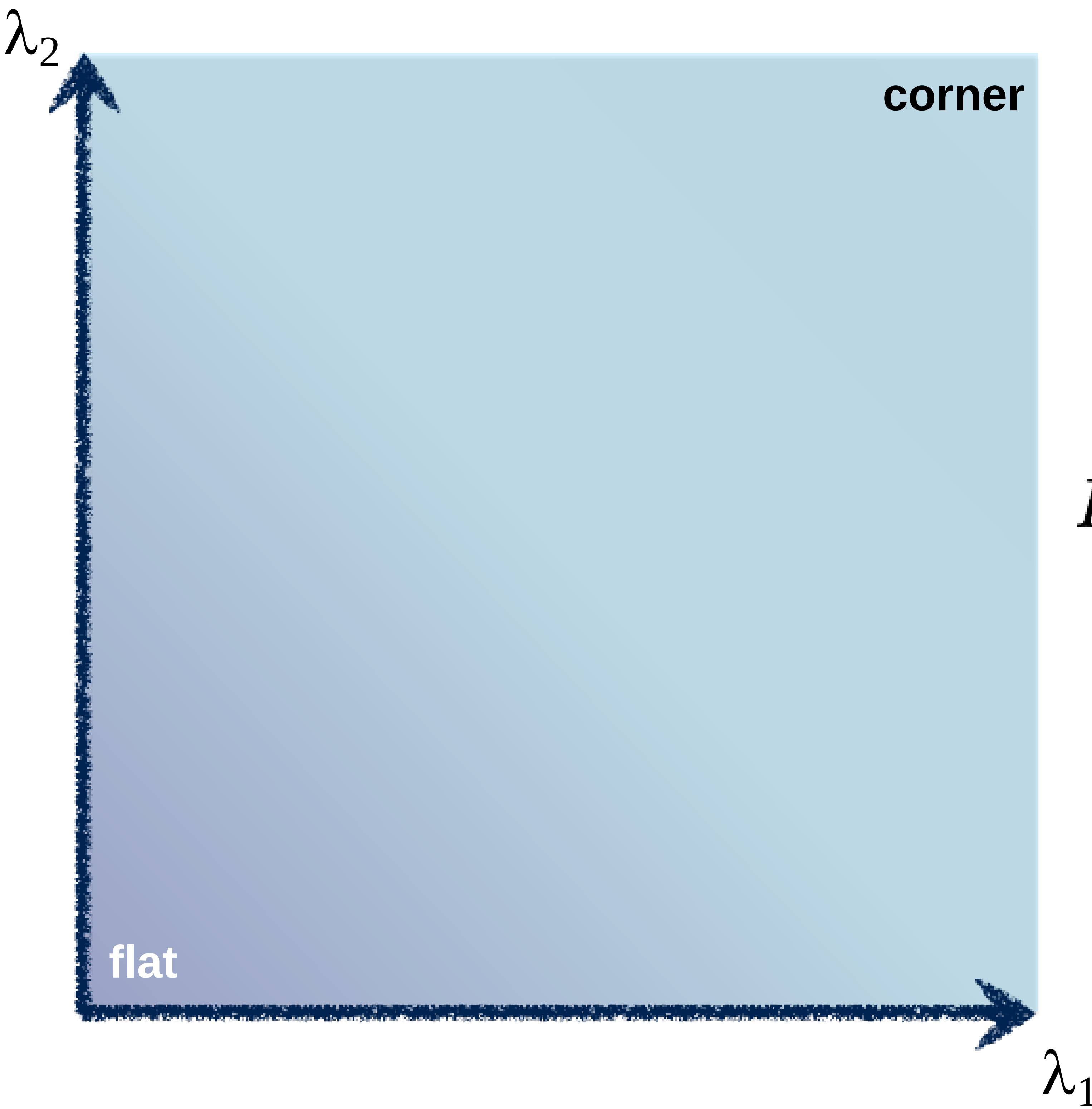
Use the smallest eigenvalue as the response function

$$R = \min(\lambda_1, \lambda_2)$$

5. Use threshold on eigenvalues to detect corners

^

(a function of)



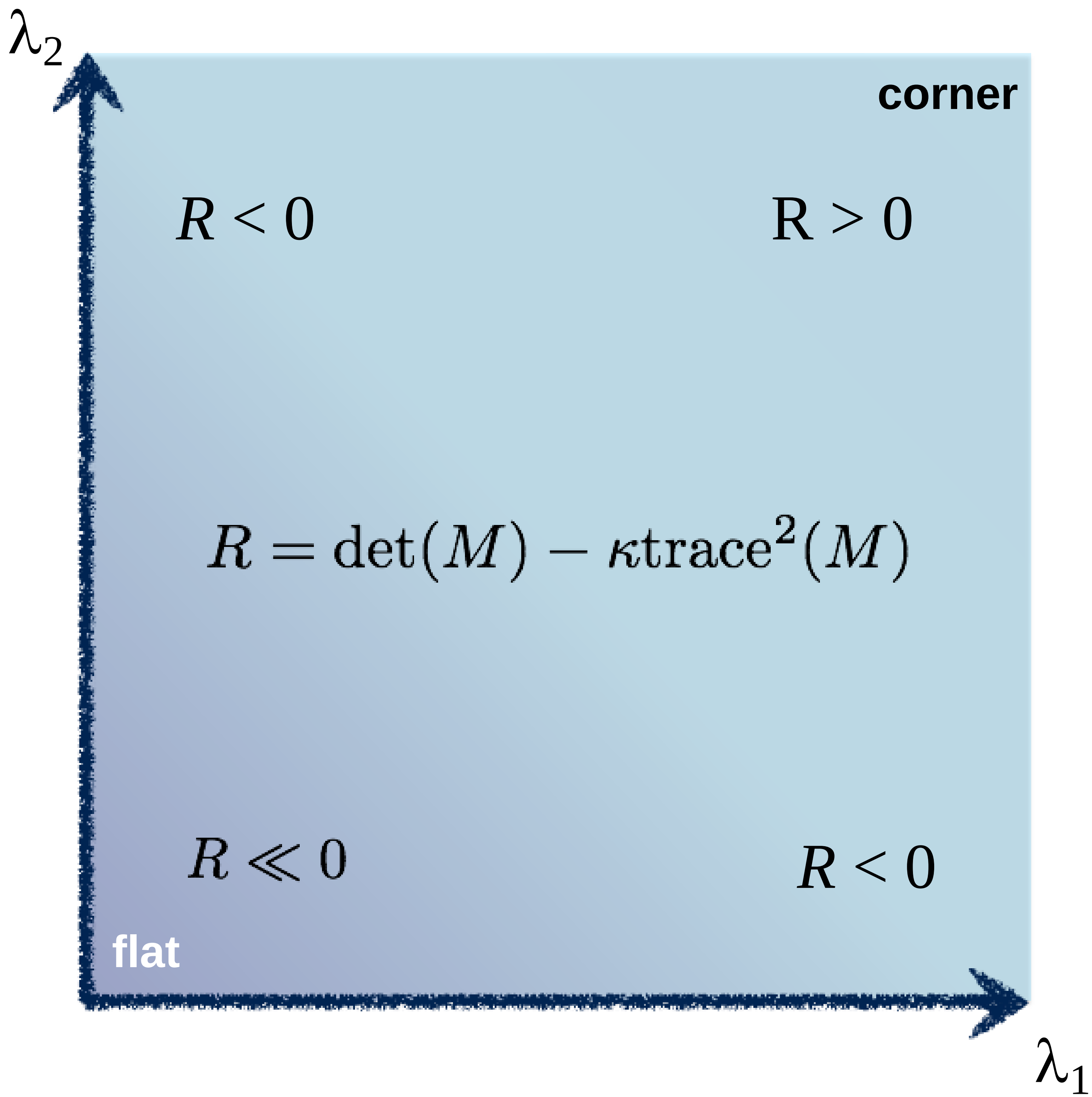
Eigenvalues need to be bigger than one.

$$R = \lambda_1 \lambda_2 - \kappa(\lambda_1 + \lambda_2)^2$$

Can compute this more efficiently…

5. Use threshold on eigenvalues to detect corners

(a function of)



$$\det M = \lambda_1 \lambda_2$$

$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

$$det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

$$trace\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + d$$

Harris & Stephens (1988)

$$R = \det(M) - \kappa \operatorname{trace}^2(M)$$

Kanade & Tomasi (1994)

$$R = \min(\lambda_1, \lambda_2)$$

Nobel (1998)

$$R = \frac{\det(M)}{\operatorname{trace}(M) + \epsilon}$$

Harris Detector

C.Harris and M.Stephens. "A Combined Corner and Edge Detector."1988.

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \qquad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x^2} = I_x \cdot I_x \qquad I_{y^2} = I_y \cdot I_y \qquad I_{xy} = I_x \cdot I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x^2} = G_{\sigma'} * I_{x^2} \qquad S_{y^2} = G_{\sigma'} * I_{y^2} \qquad S_{xy} = G_{\sigma'} * I_{xy}$$

Harris Detector

C.Harris and M.Stephens. “A Combined Corner and Edge Detector.”1988.

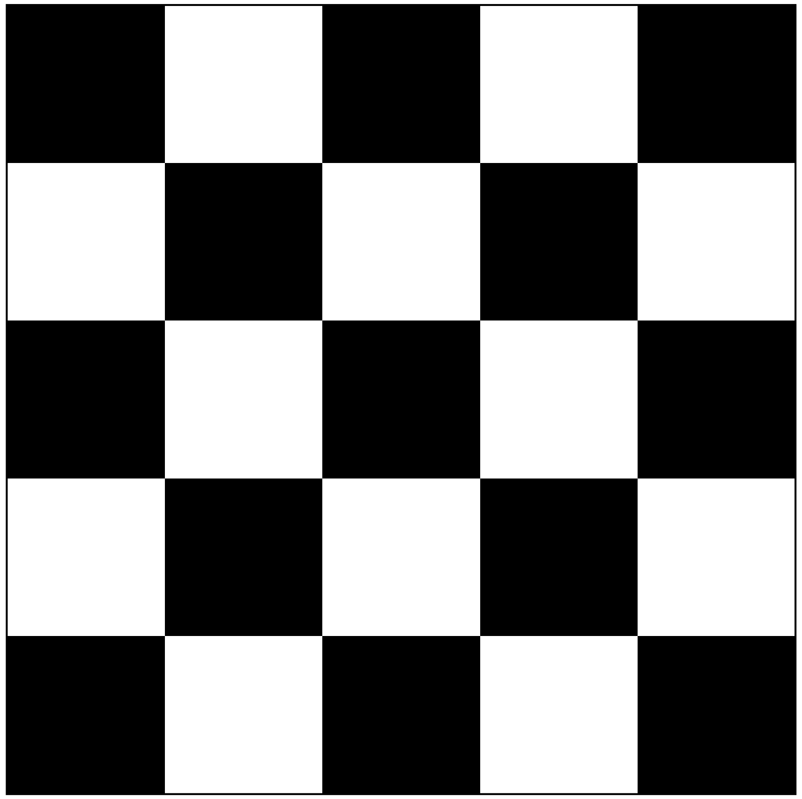
4. Define the matrix at each pixel

$$M(x,y) = \begin{bmatrix} S_{x^2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y^2}(x,y) \end{bmatrix}$$

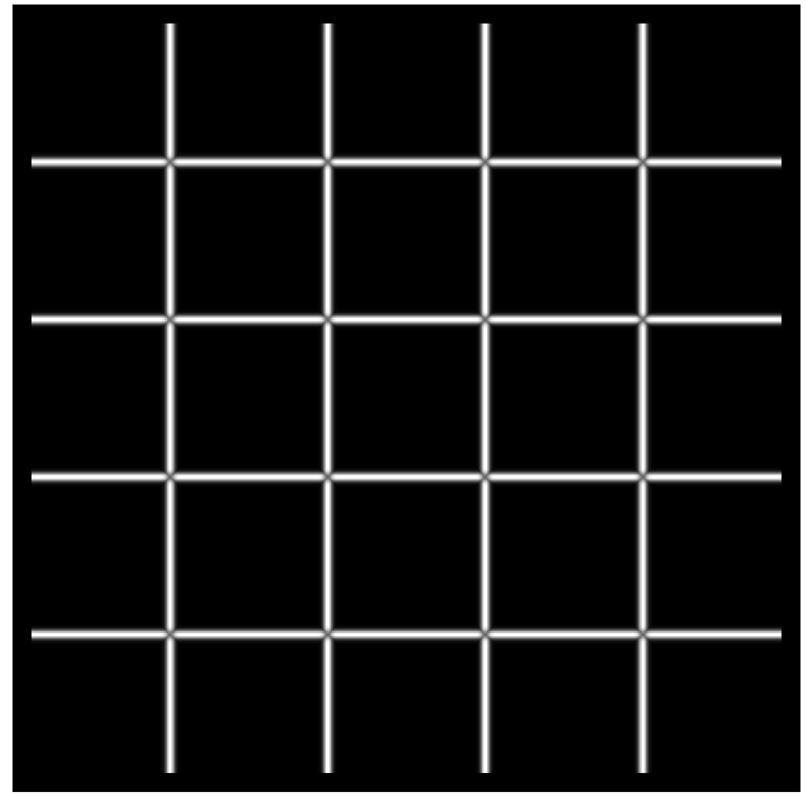
5. Compute the response of the detector at each pixel

$$R = \det M - k(\mathrm{trace} M)^2$$

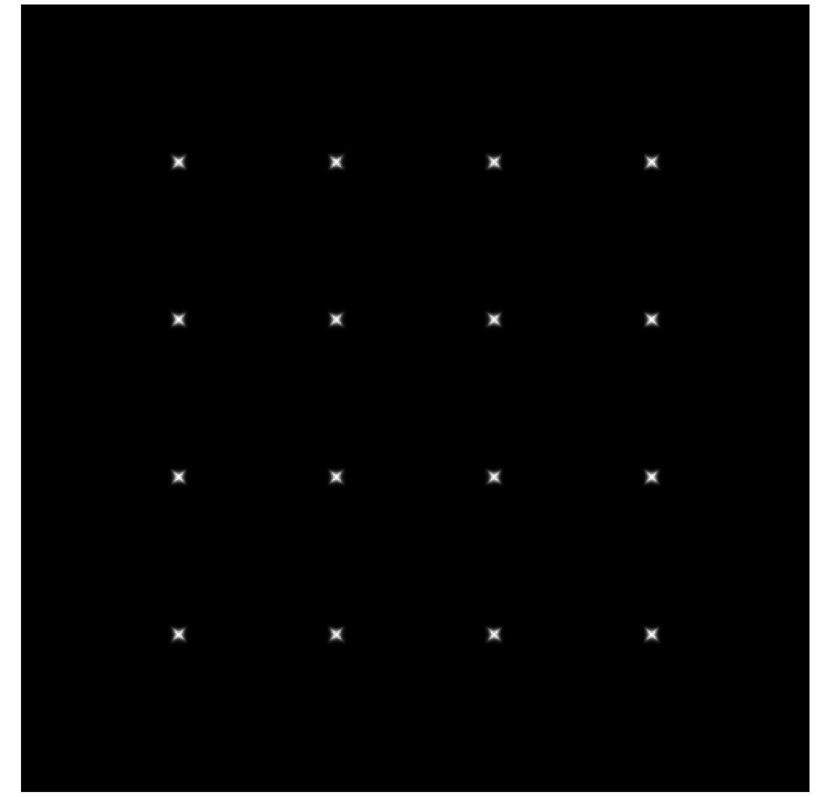
6. Threshold on value of R; compute non-max suppression.



I



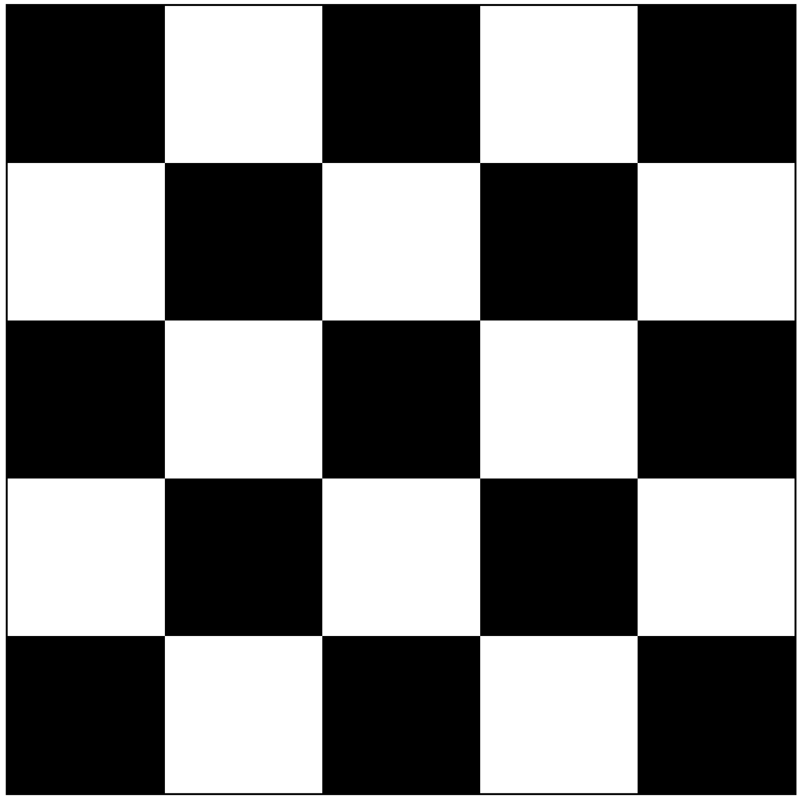
$\lambda_{\max}$



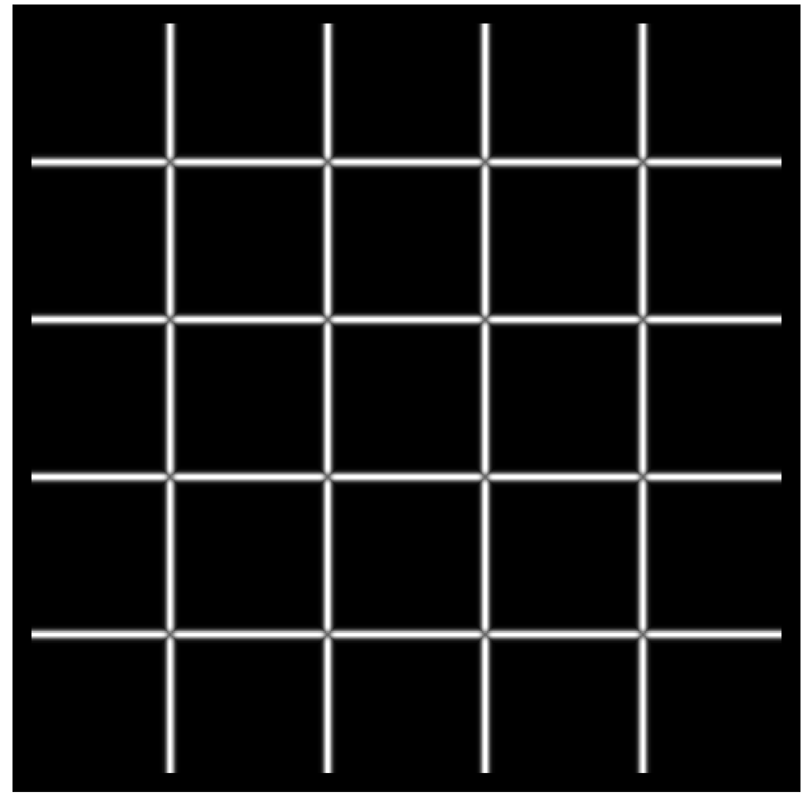
$\lambda_{\min}$

Yet another option: $$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

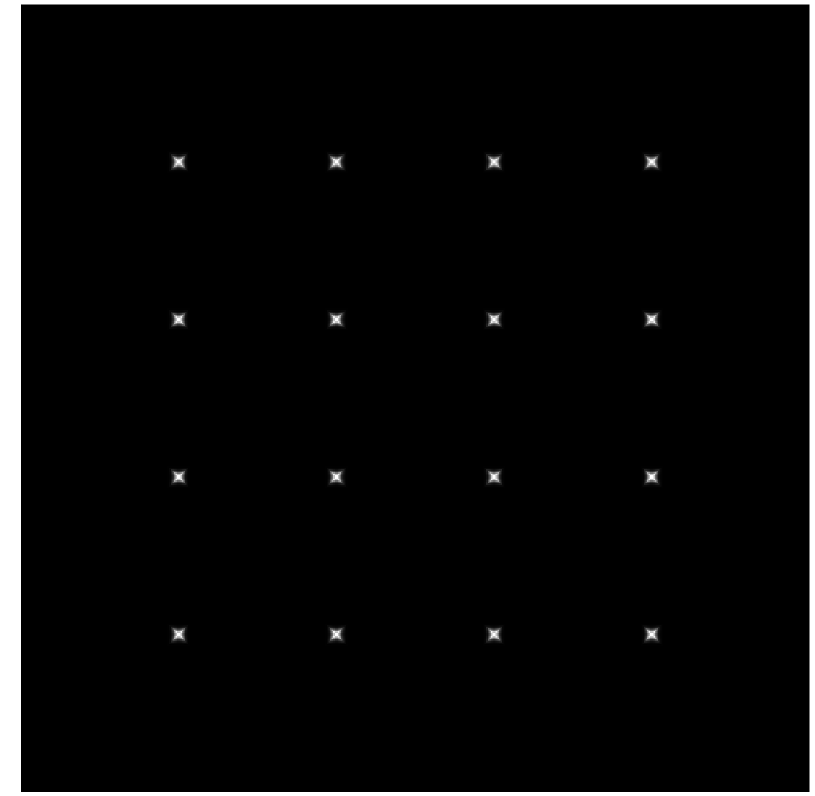
How do you write this equivalently using determinant and trace?



I



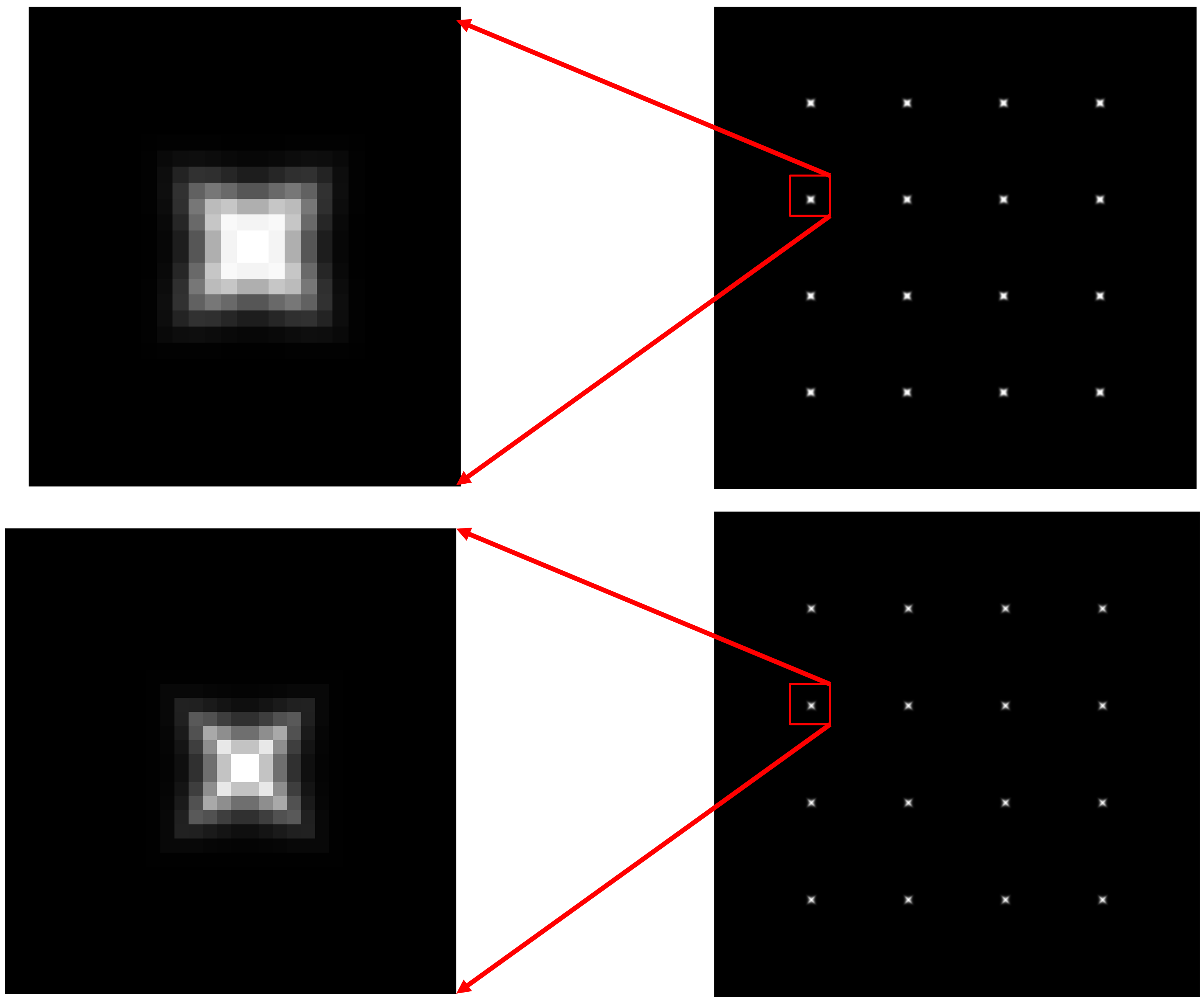
$\lambda_{\max}$



$\lambda_{\min}$

Yet another option: $$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} = \frac{determinant(H)}{trace(H)}$$

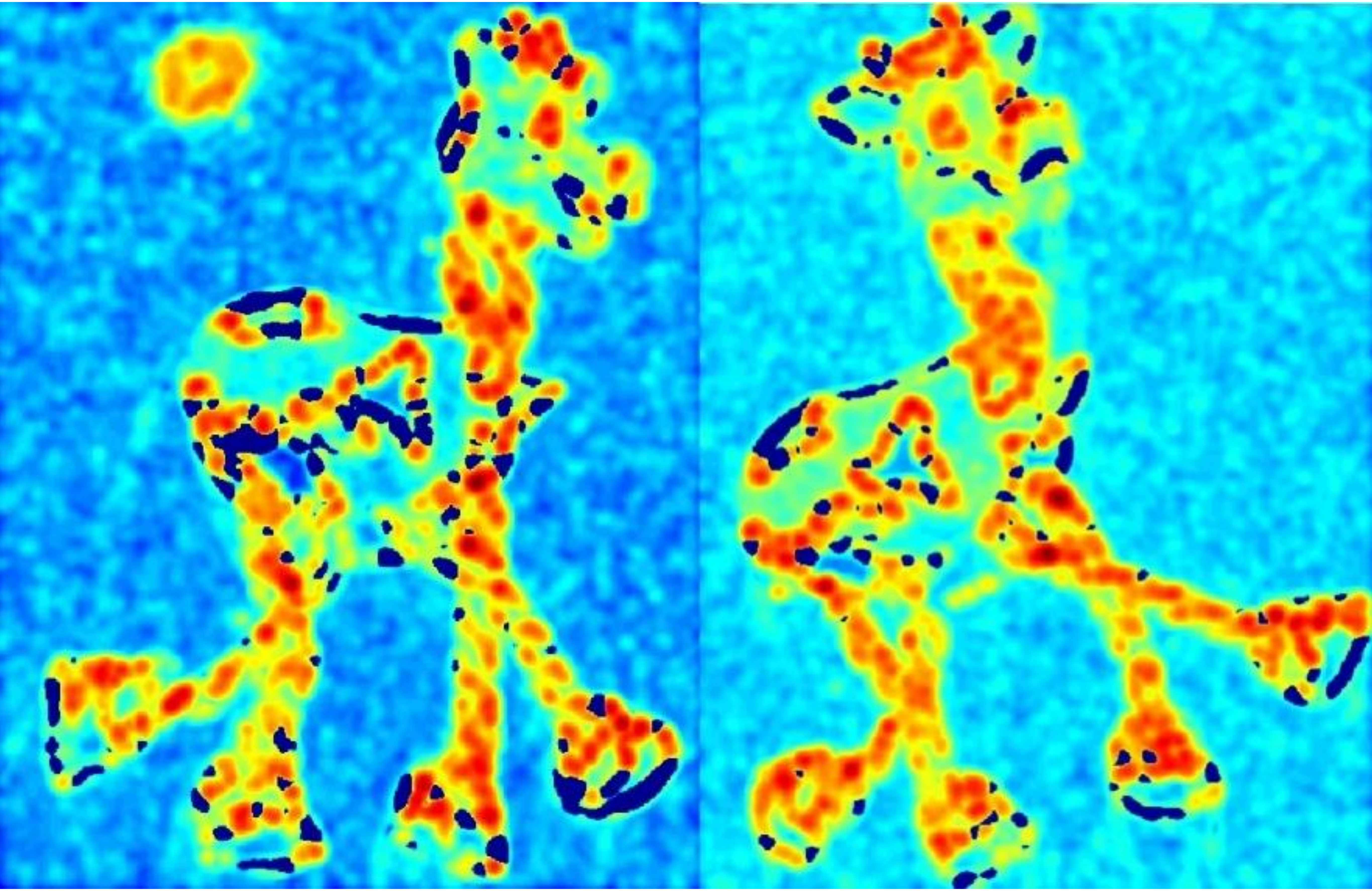
Different criteria



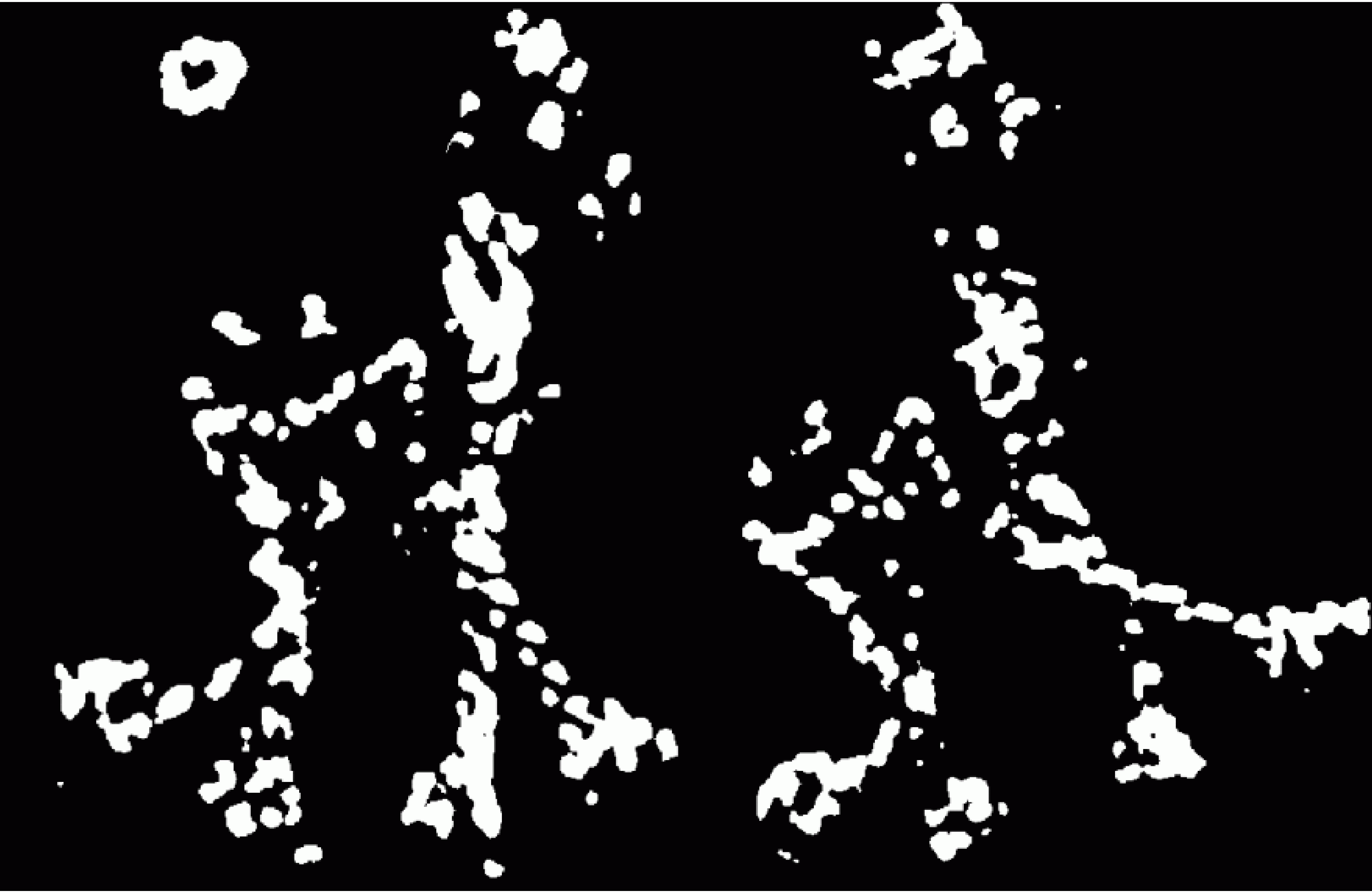
Harris criterion

$\lambda_{\min}$

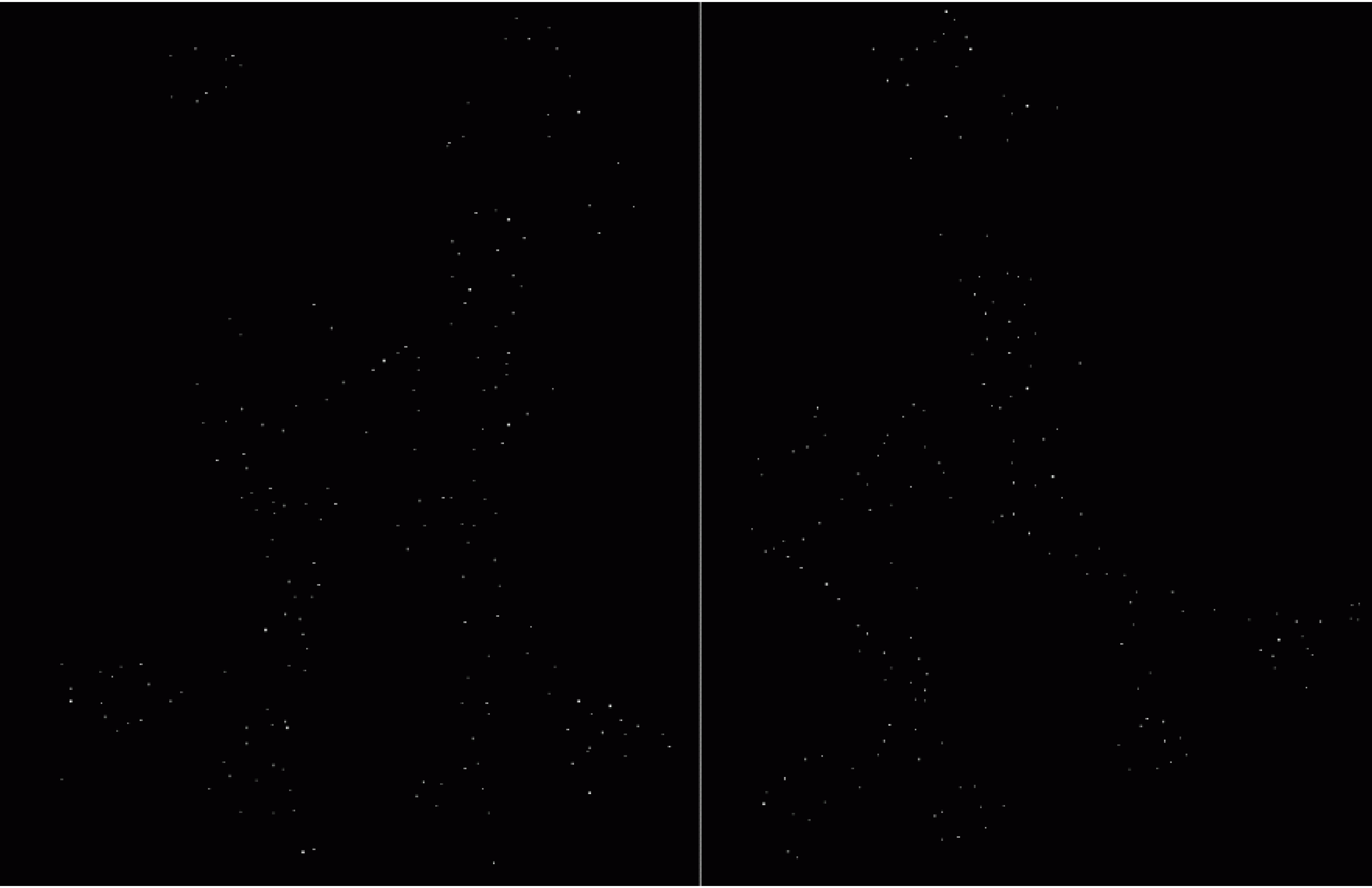
Corner response



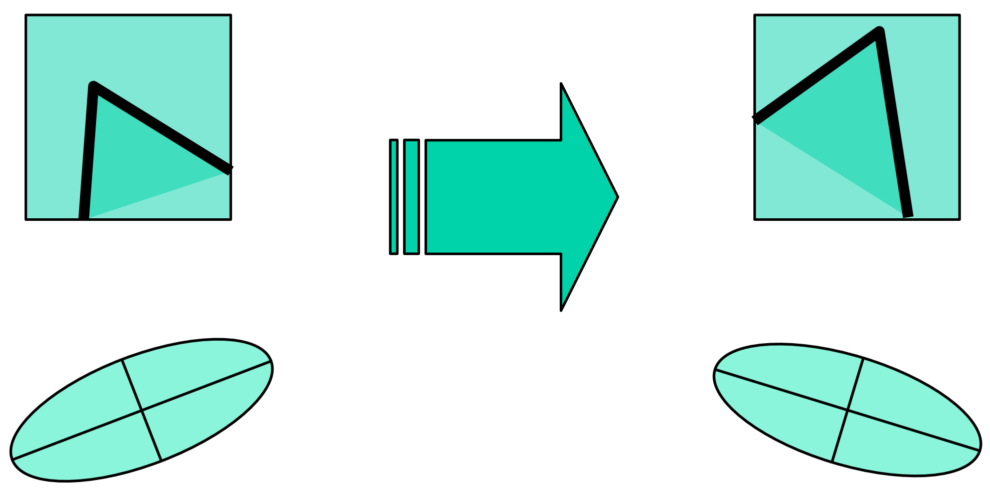

Thresholded corner response



Non-maximal suppression



Harris corner response is invariant to rotation



Ellipse rotates but its shape (**eigenvalues**) remains the same

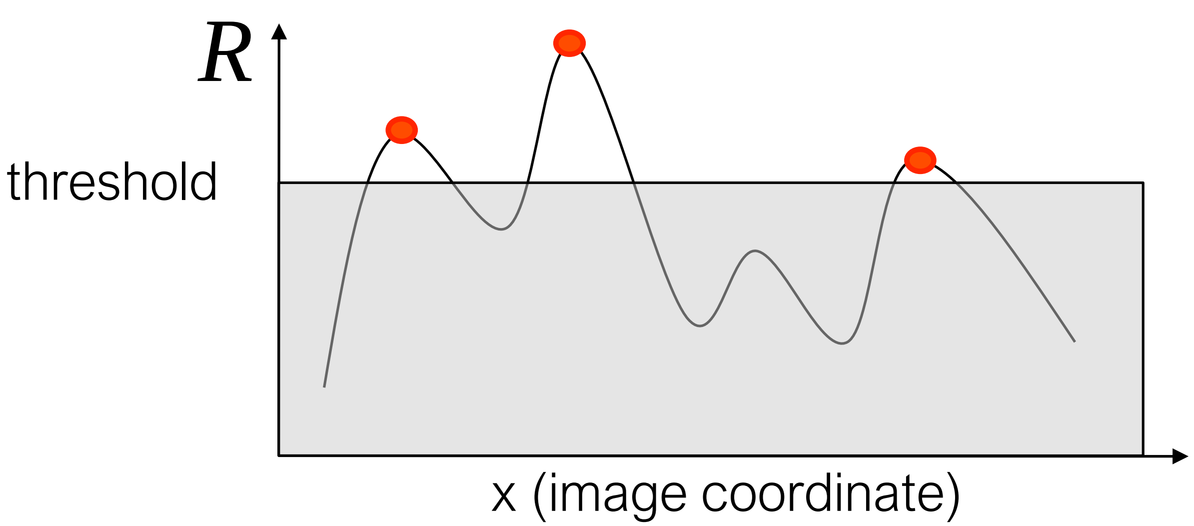
Corner response R is invariant to image rotation

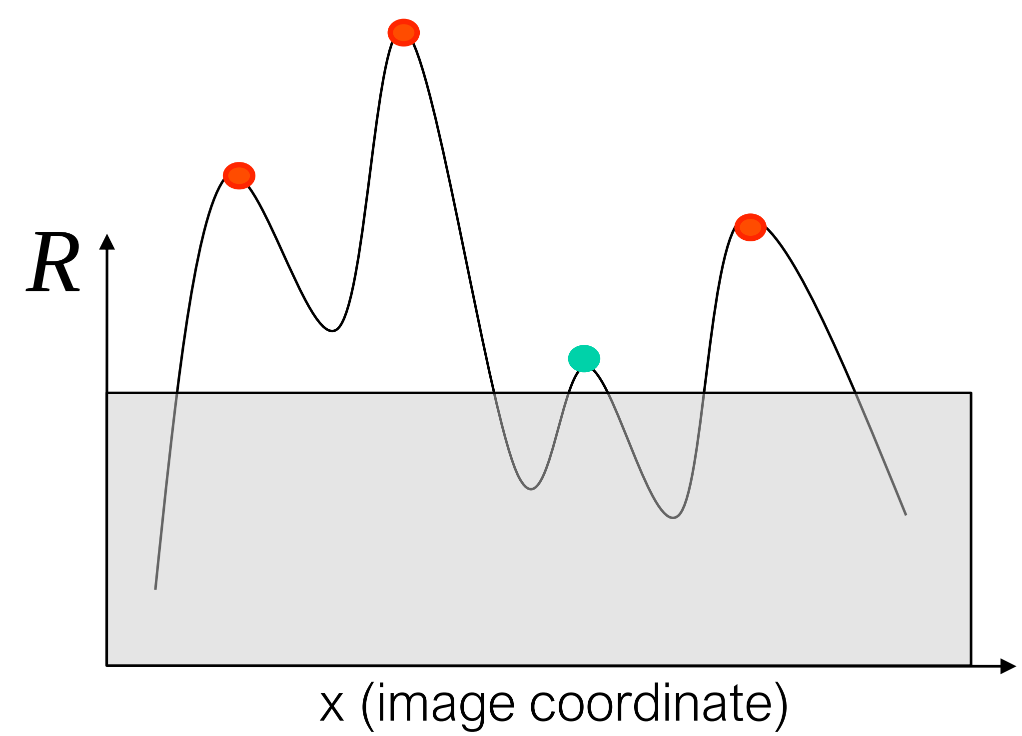
Harris corner response is invariant to intensity changes

Partial invariance to *affine intensity* change

- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

- Intensity scale: $I \rightarrow a\, I$





The Harris detector is not invariant to changes in …

The Harris corner detector is <u>not</u> invariant to scale

Multi-scale detection

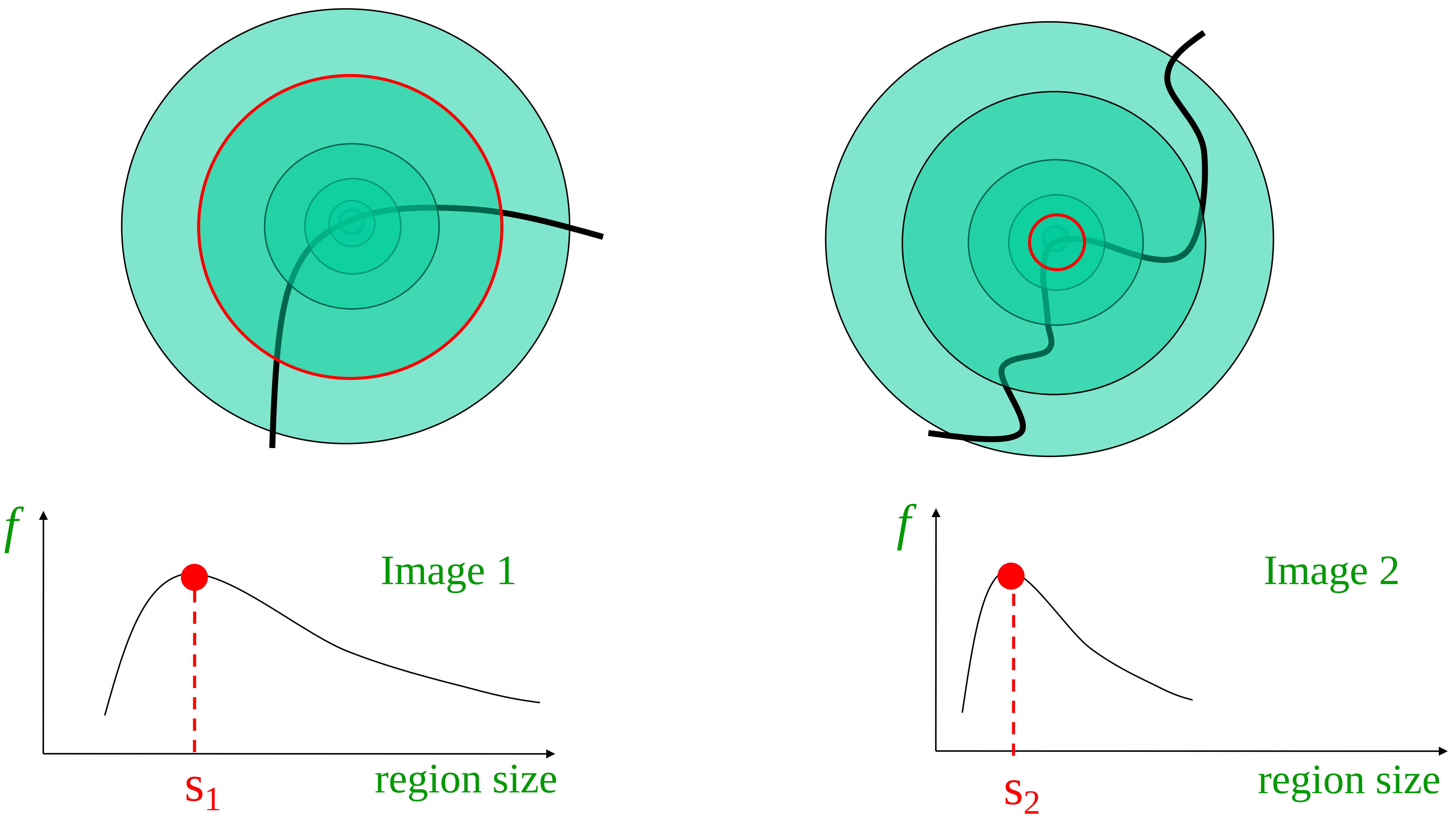
How can we make a feature detector scale-invariant?

How can we automatically select the scale?

Multi-scale blob detection

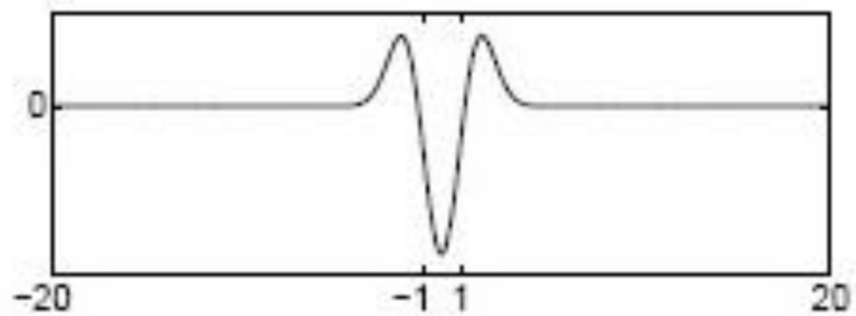
Intuitively…

Find local maxima in both **position** and **scale**

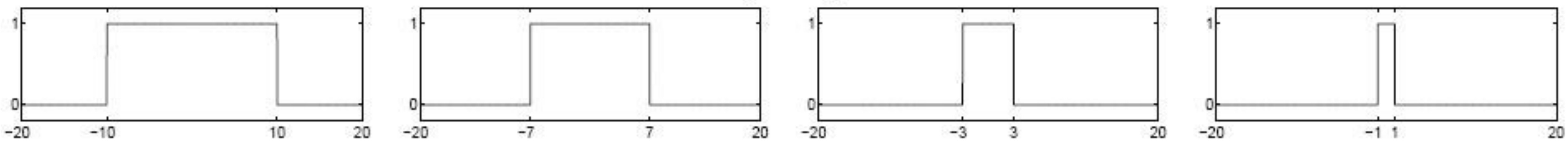


Formally…

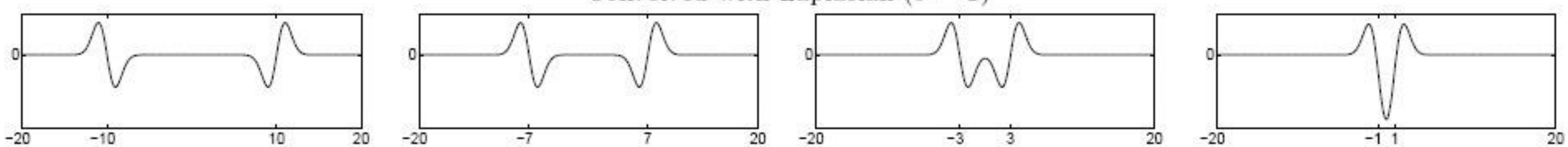
Laplacian filter



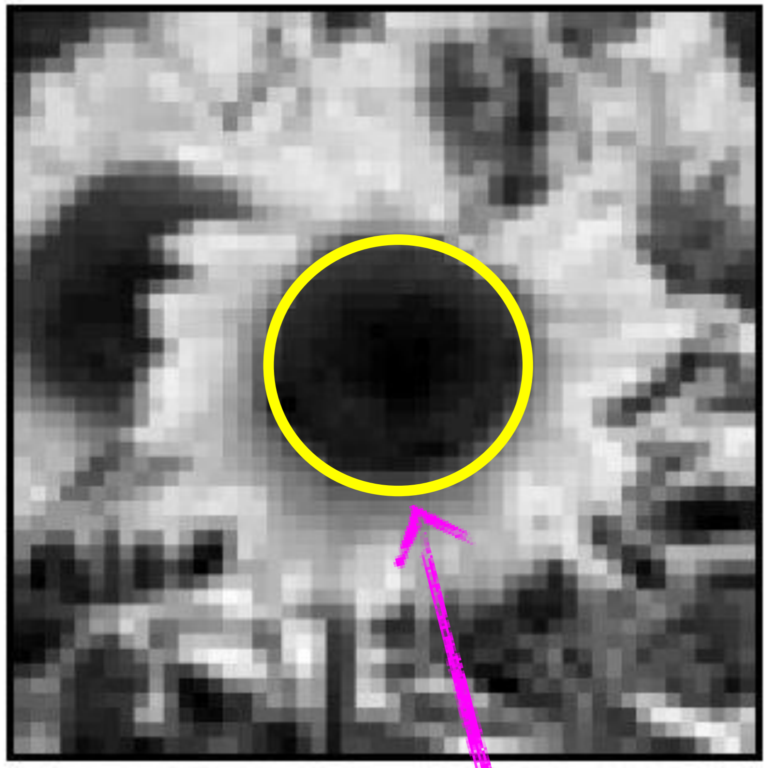
Original signal

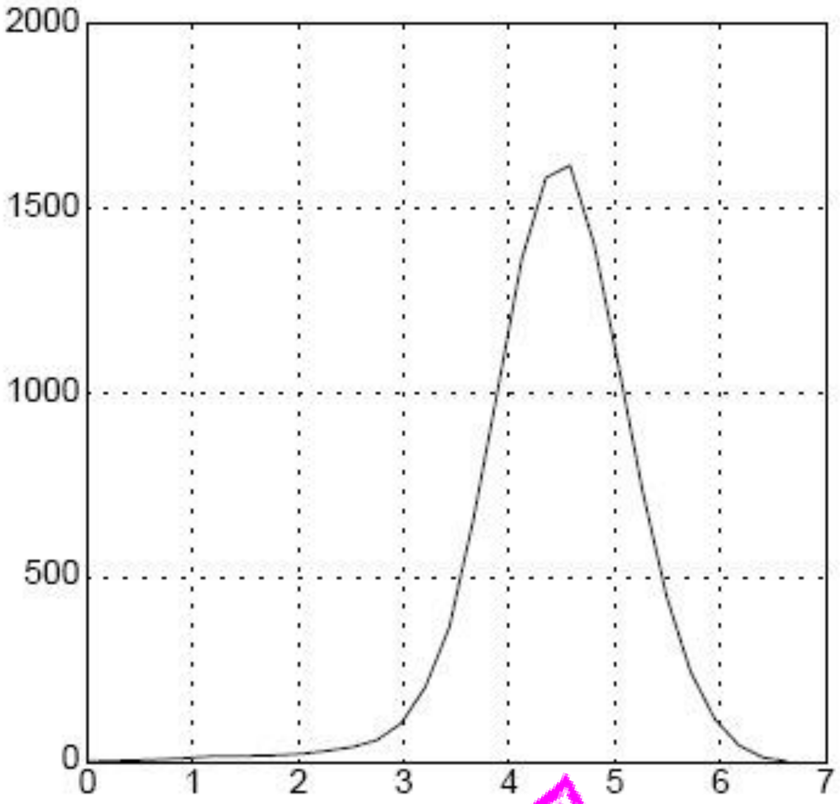


Convolved with Laplacian ($\sigma = 1$)



Highest response when the signal has the same **characteristic scale** as the filter characteristic scale - the scale that produces peak filter response

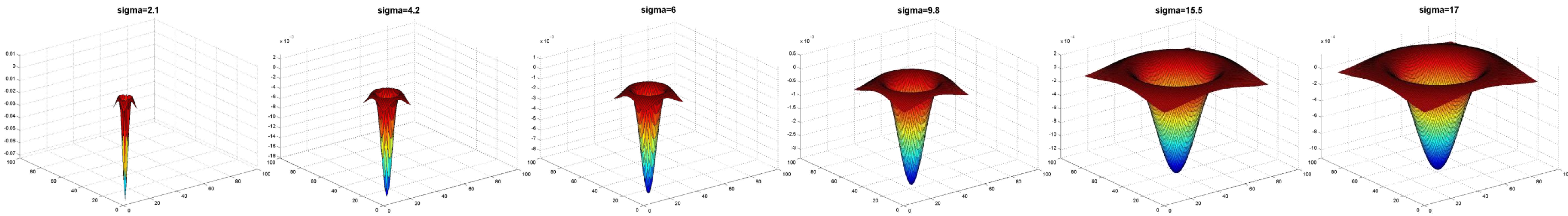




characteristic scale

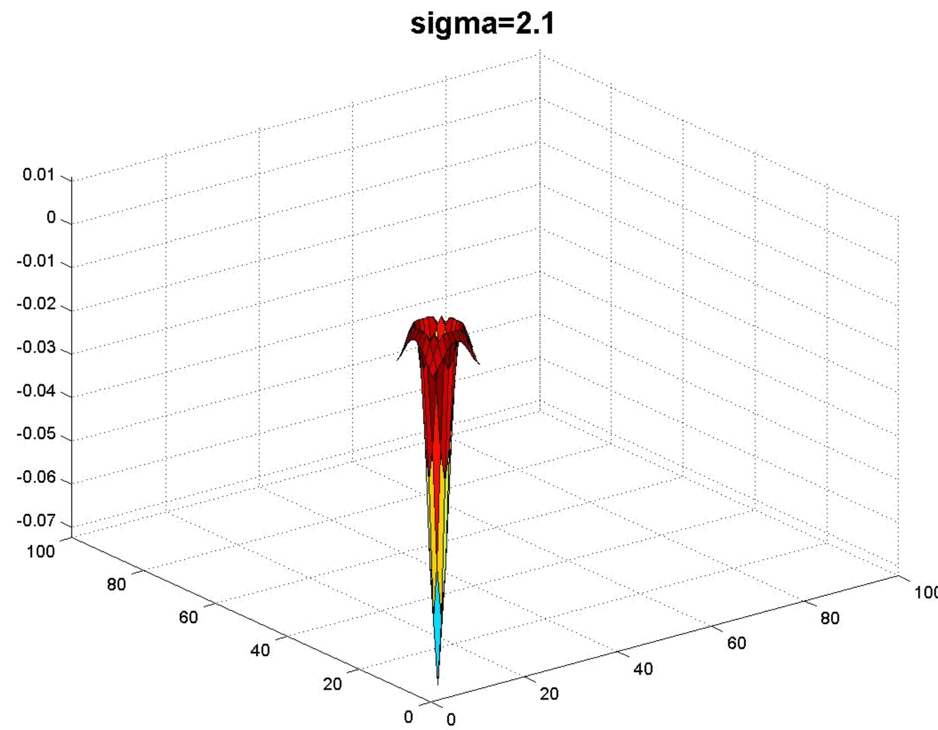
we need to search over characteristic scales

What happens if you apply different Laplacian filters?

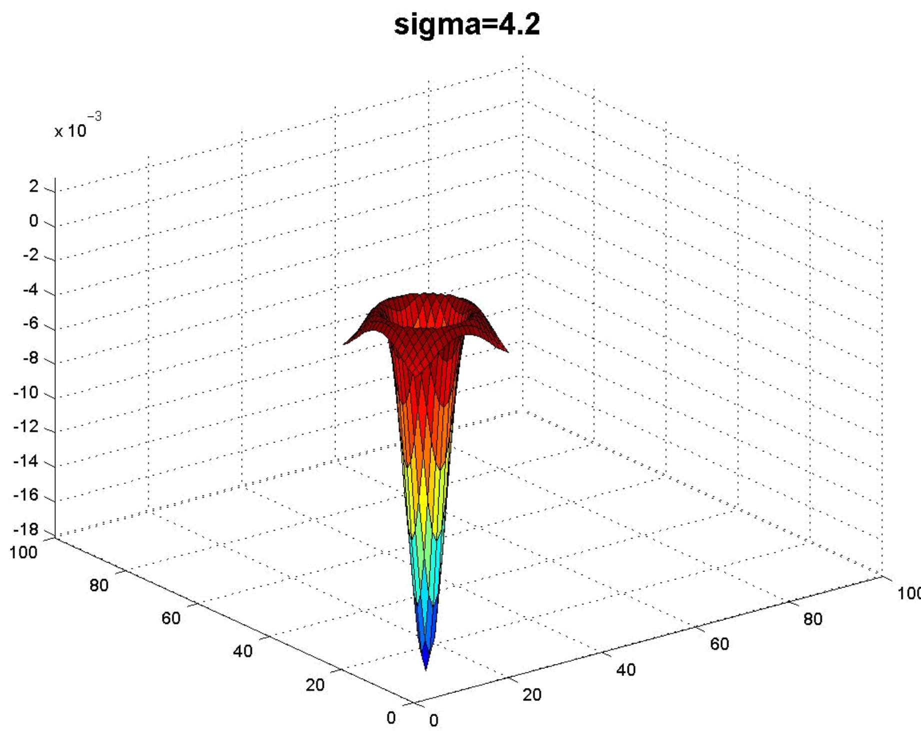


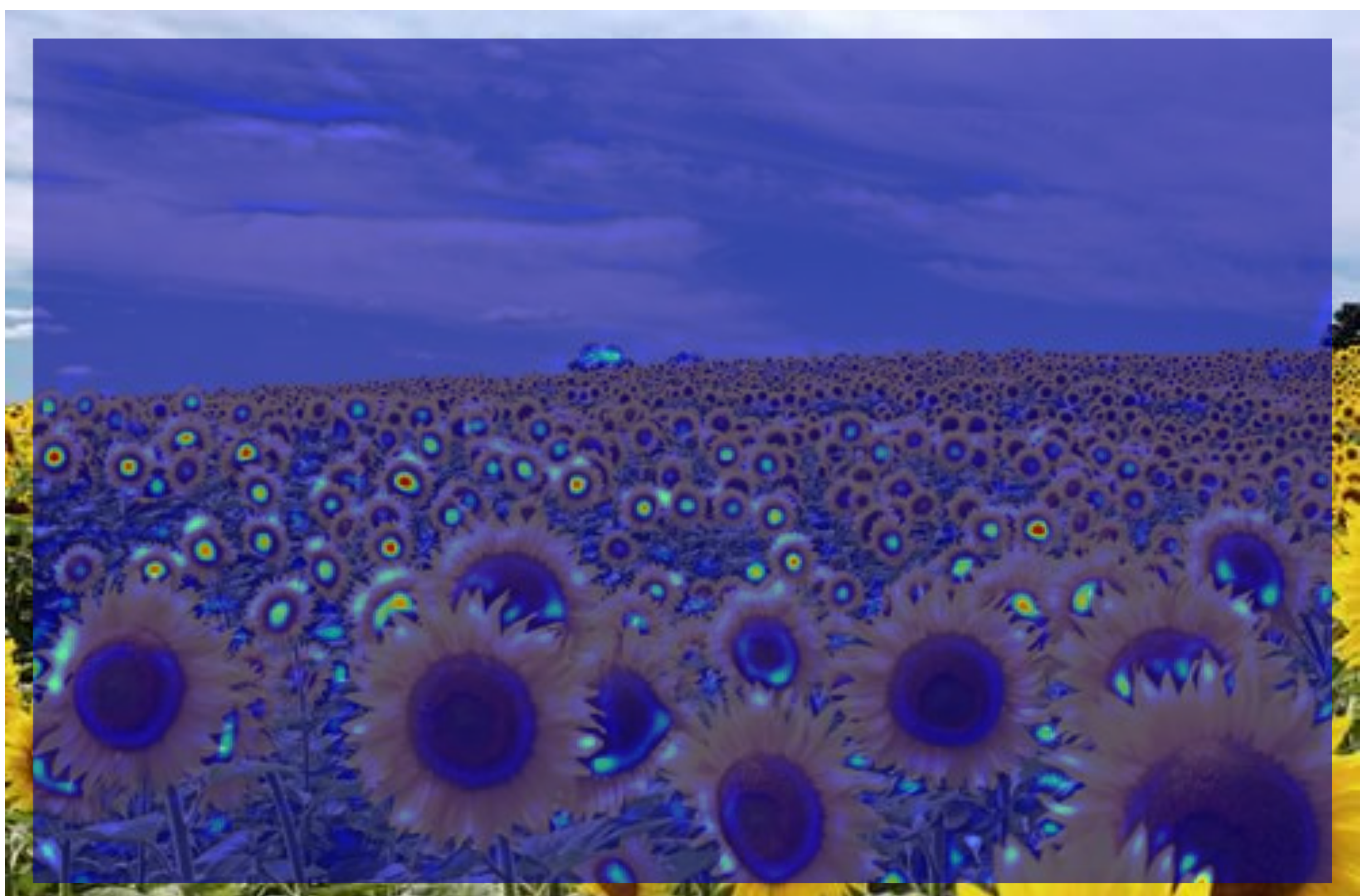
Full size

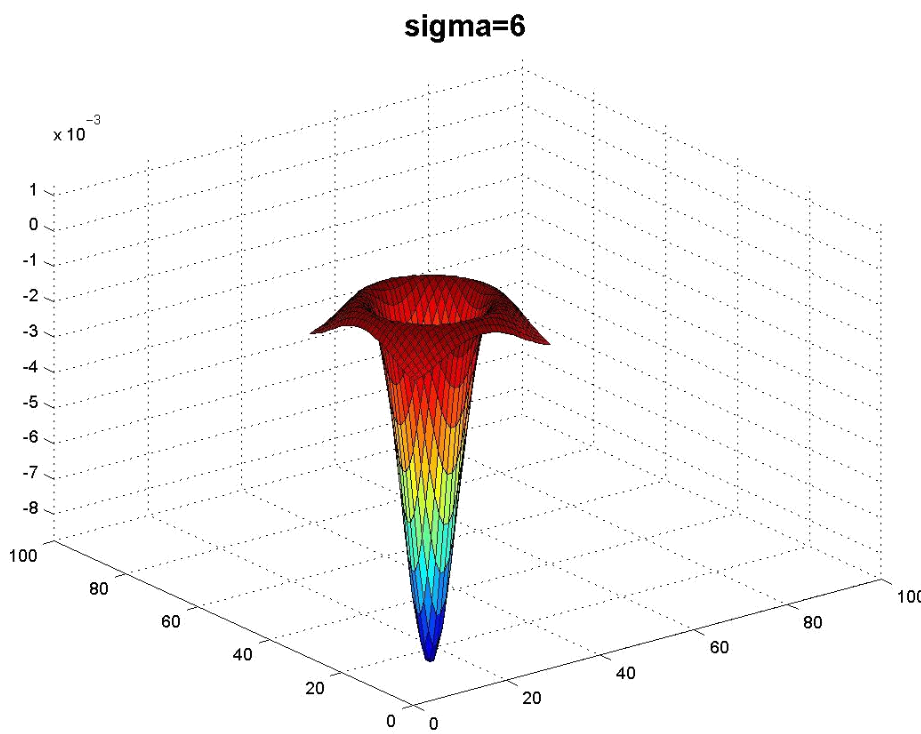
3/4 size

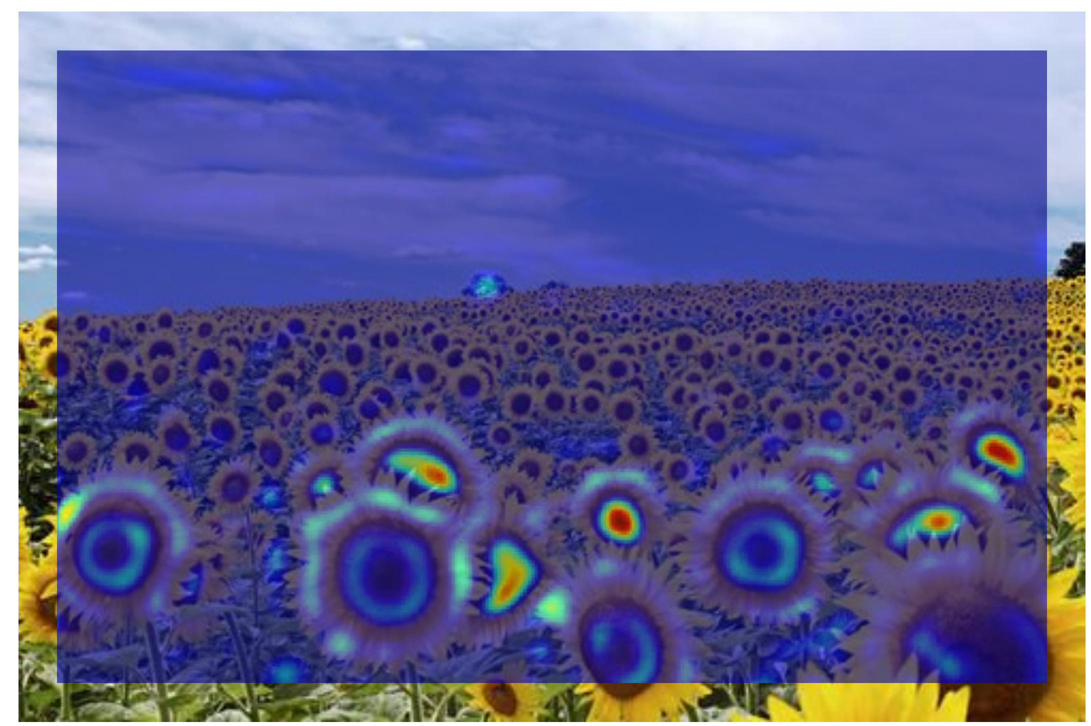


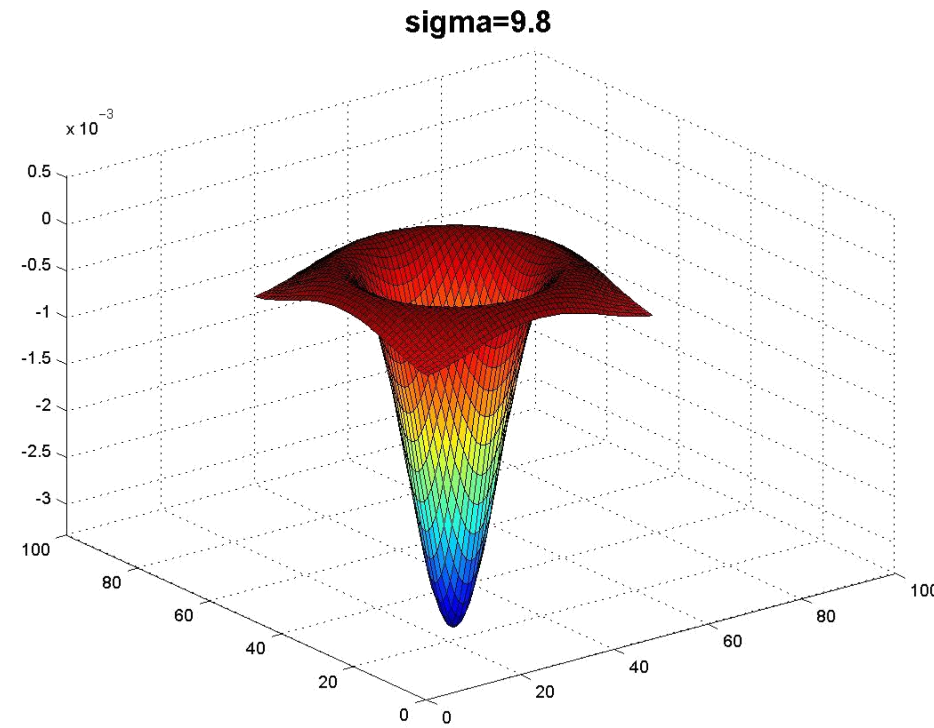
jet color scale
blue: low, red: high

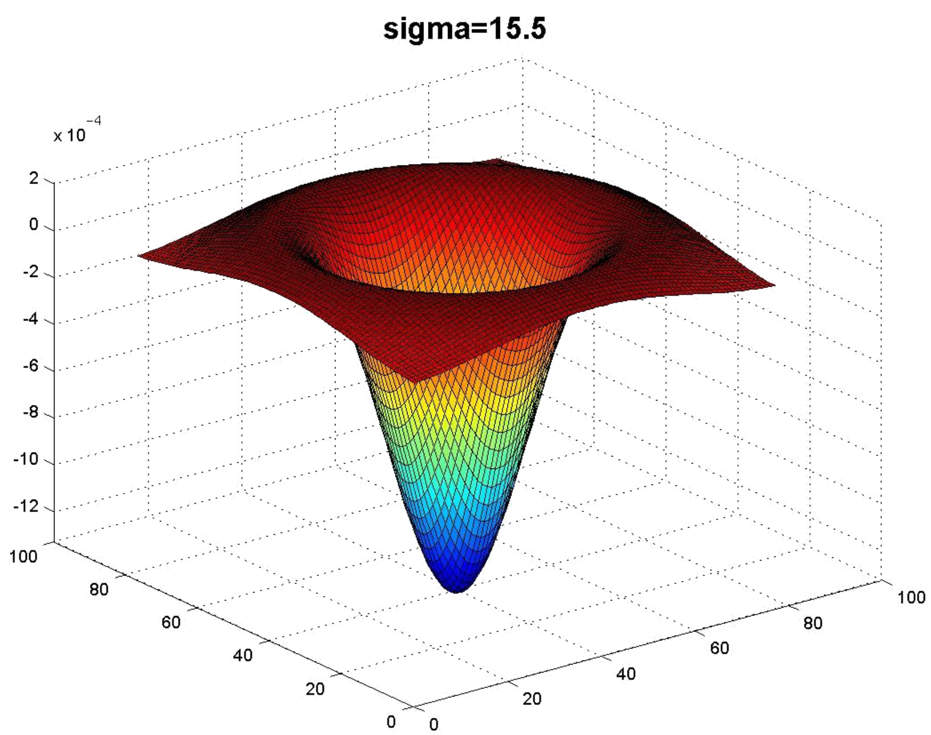

sigma=4.2
x 10^-3
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
100
80
60
40
20
0
0
20
40
60
80
100

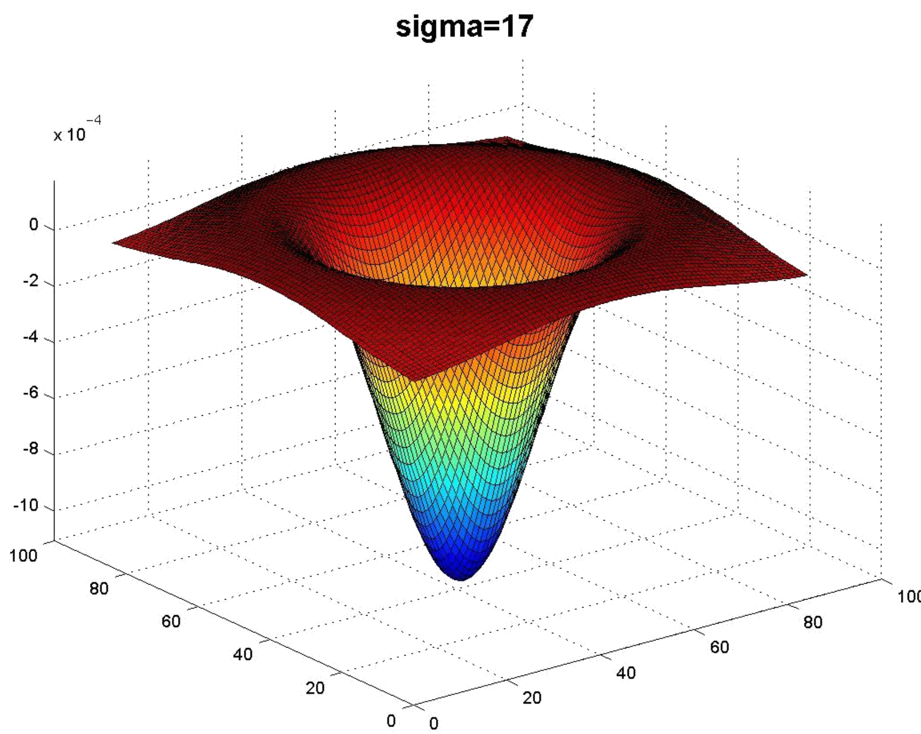



sigma=6
x 10^-3
1
0
-1
-2
-3
-4
-5
-6
-7
-8
100
80
60
40
20
0
0
20
40
60
80
100




sigma=9.8
x 10^-3
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
100
80
60
40
20
0
0
20
40
60
80
100


sigma=15.5
x 10^-4
2
0
-2
-4
-6
-8
-10
-12
100
80
60
40
20
0
0
20
40
60
80
100

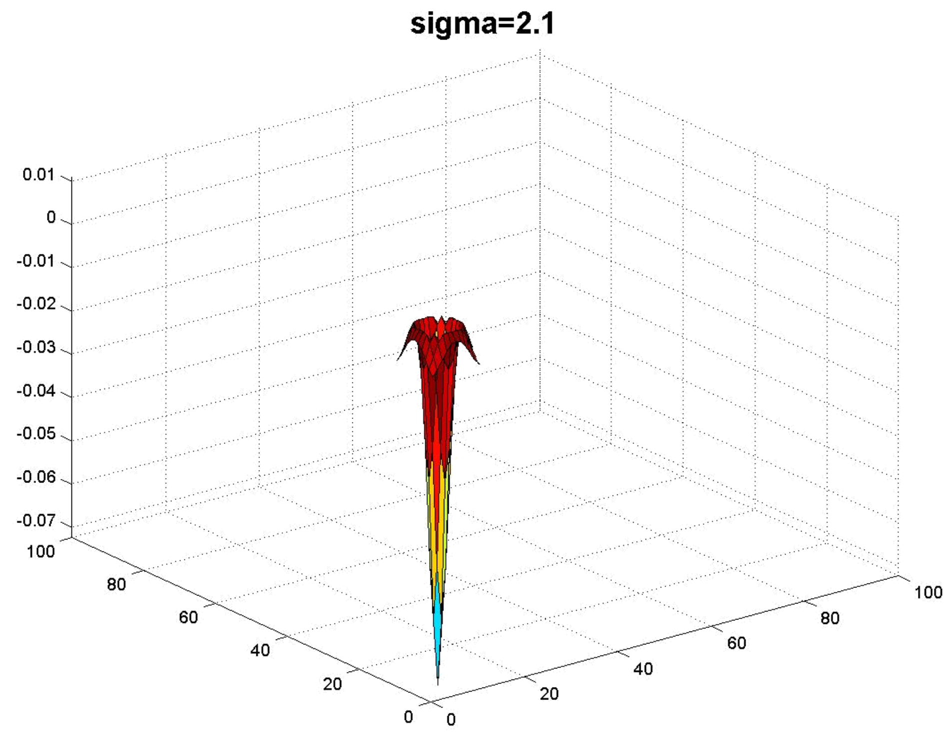


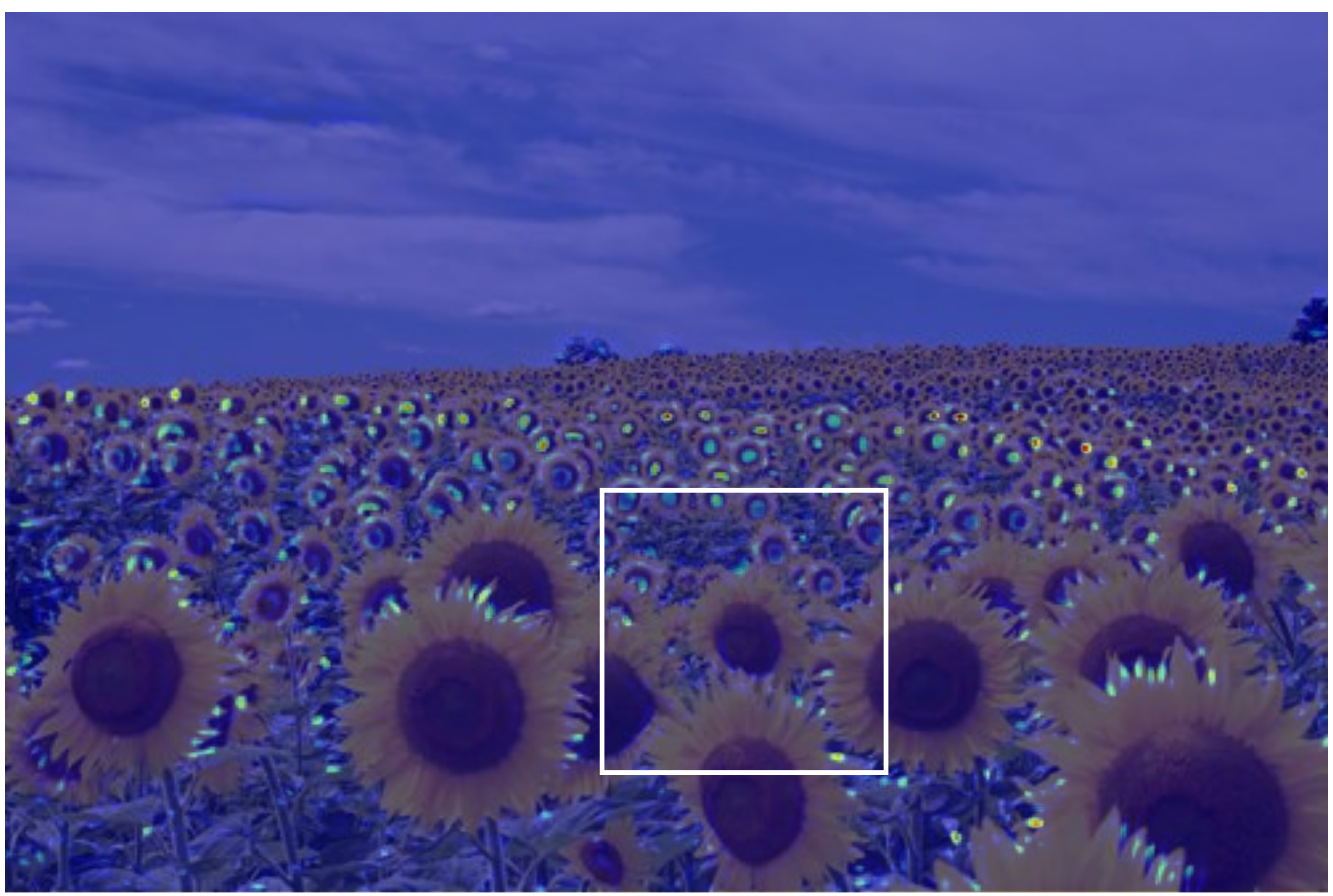
sigma=17
x 10^-4
0
-2
-4
-6
-8
-10
100
80
60
40
20
0
0
20
40
60
80
100

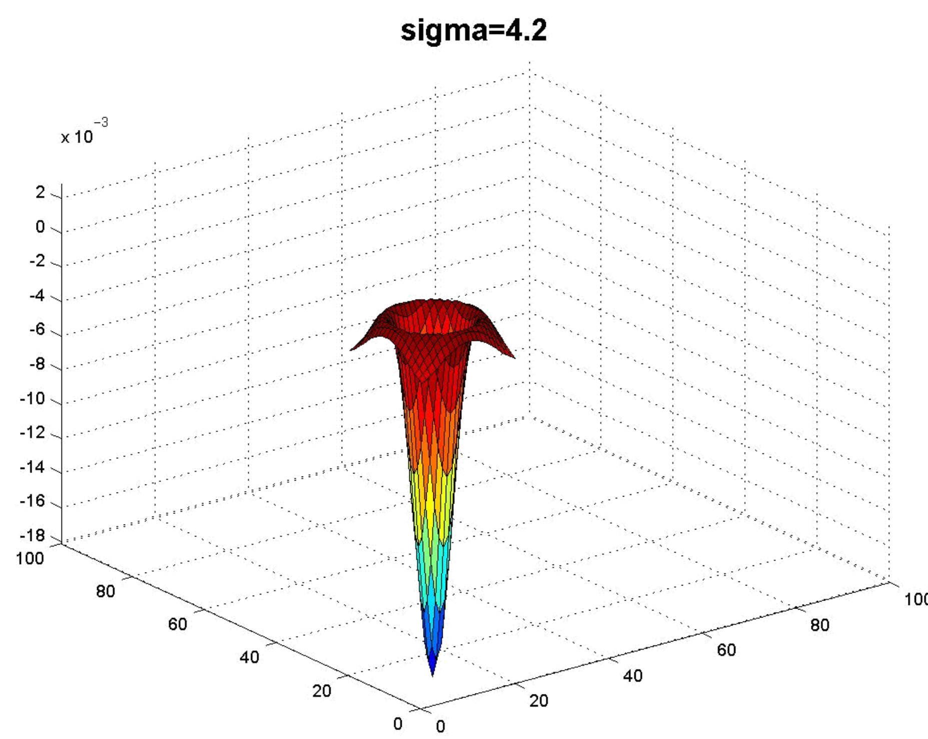
What happened when you applied different Laplacian filters?

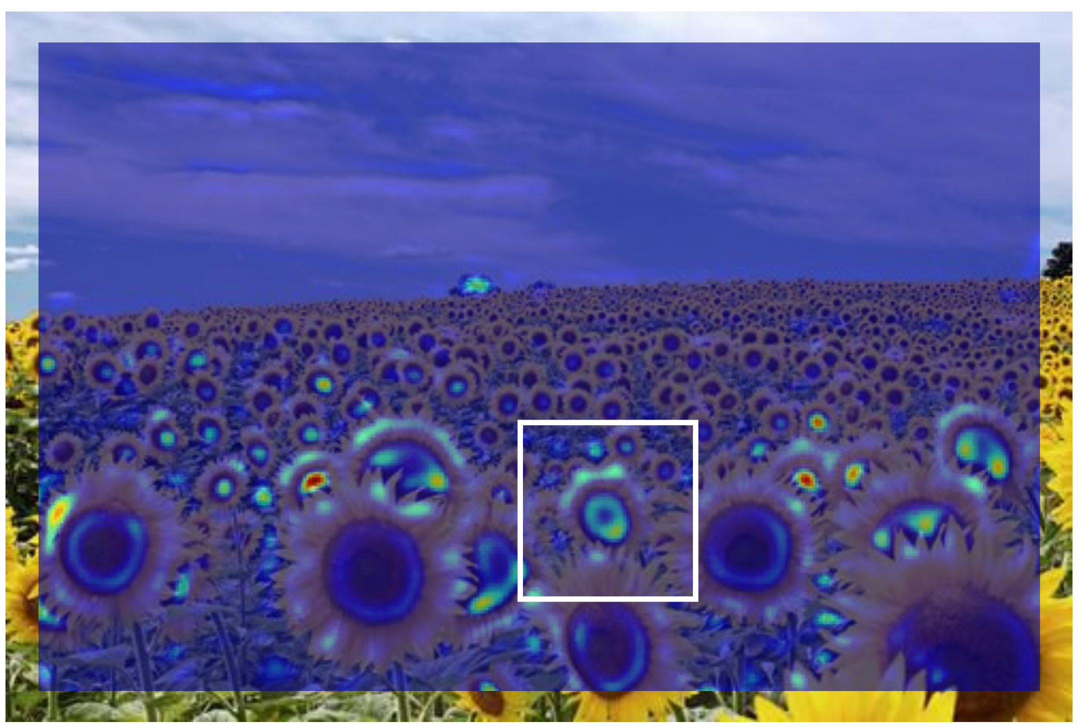
Full size

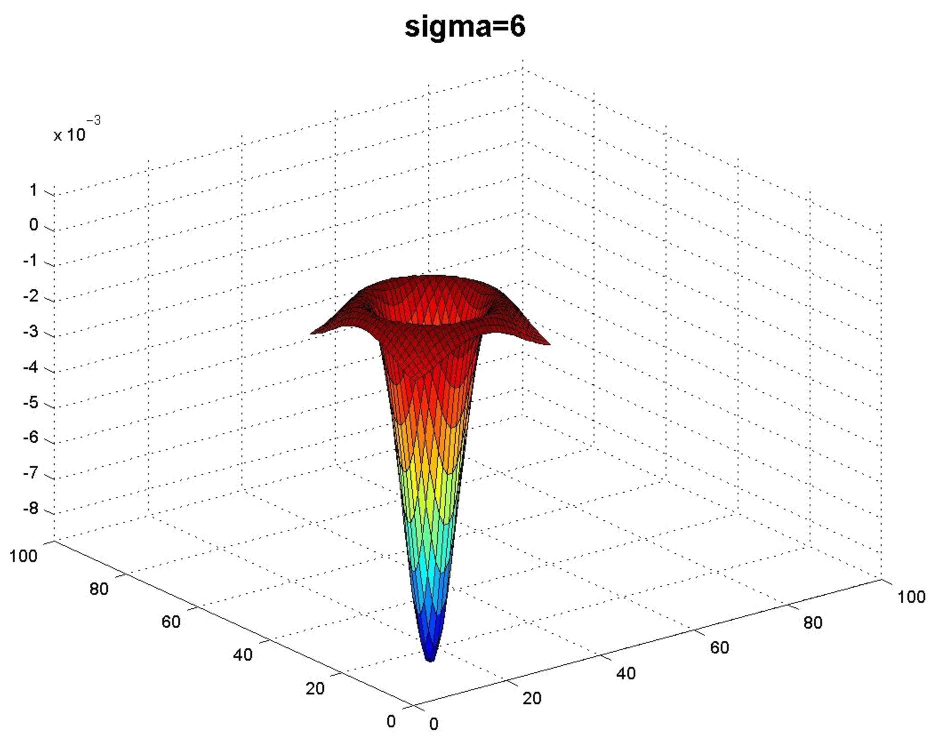
3/4 size

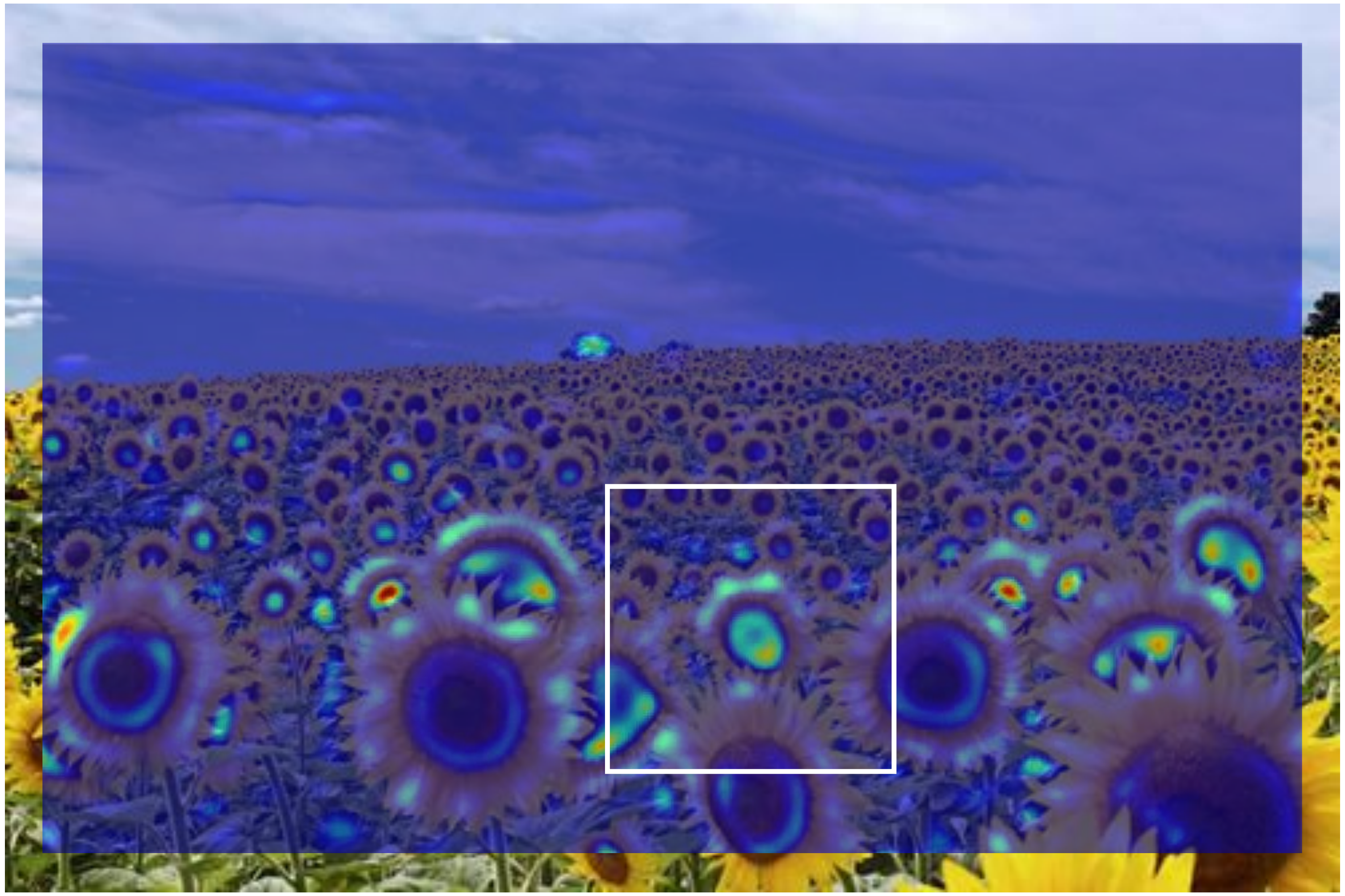

sigma=2.1
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
100
80
60
40
20
0
0
20
40
60
80
100

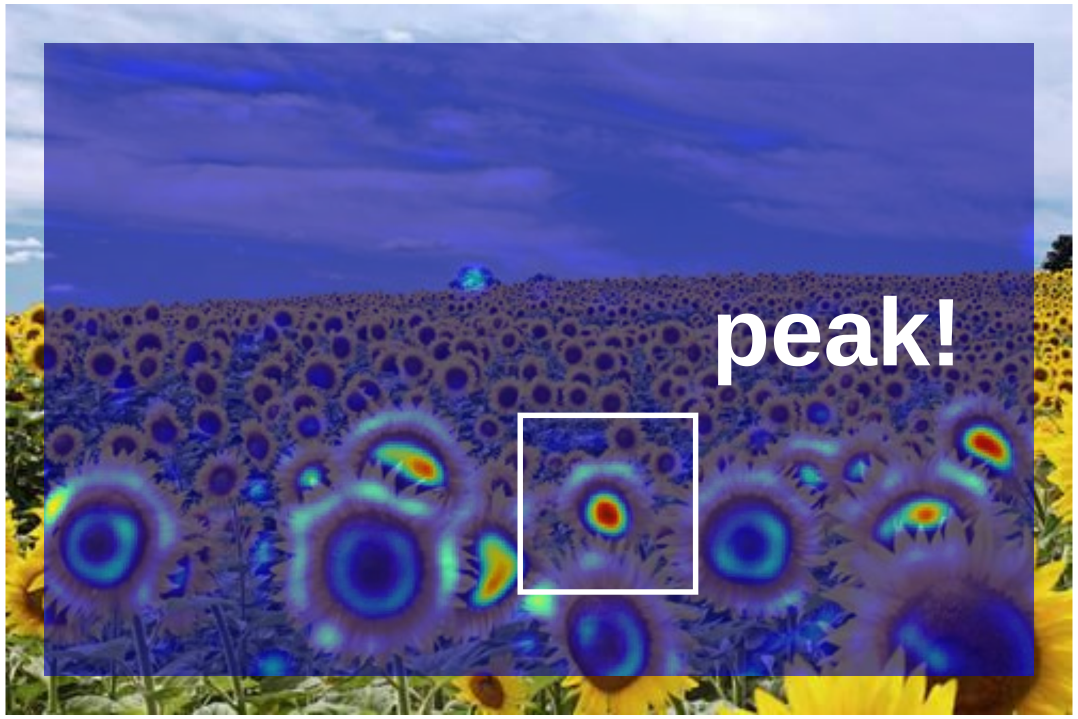


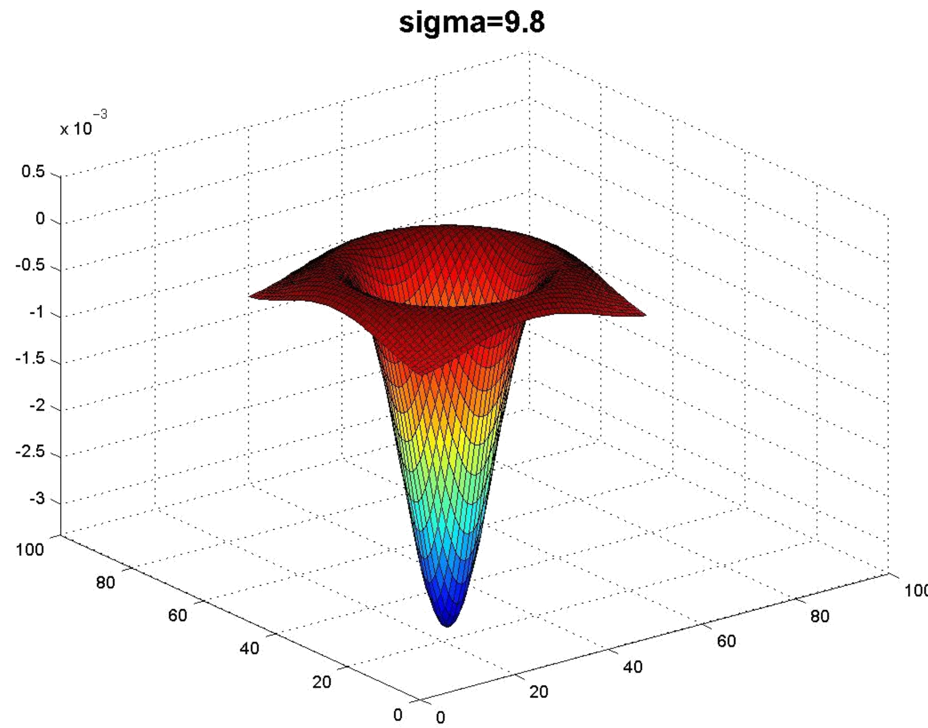

sigma=4.2
x 10^-3
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
100
80
60
40
20
0
0
20
40
60
80
100

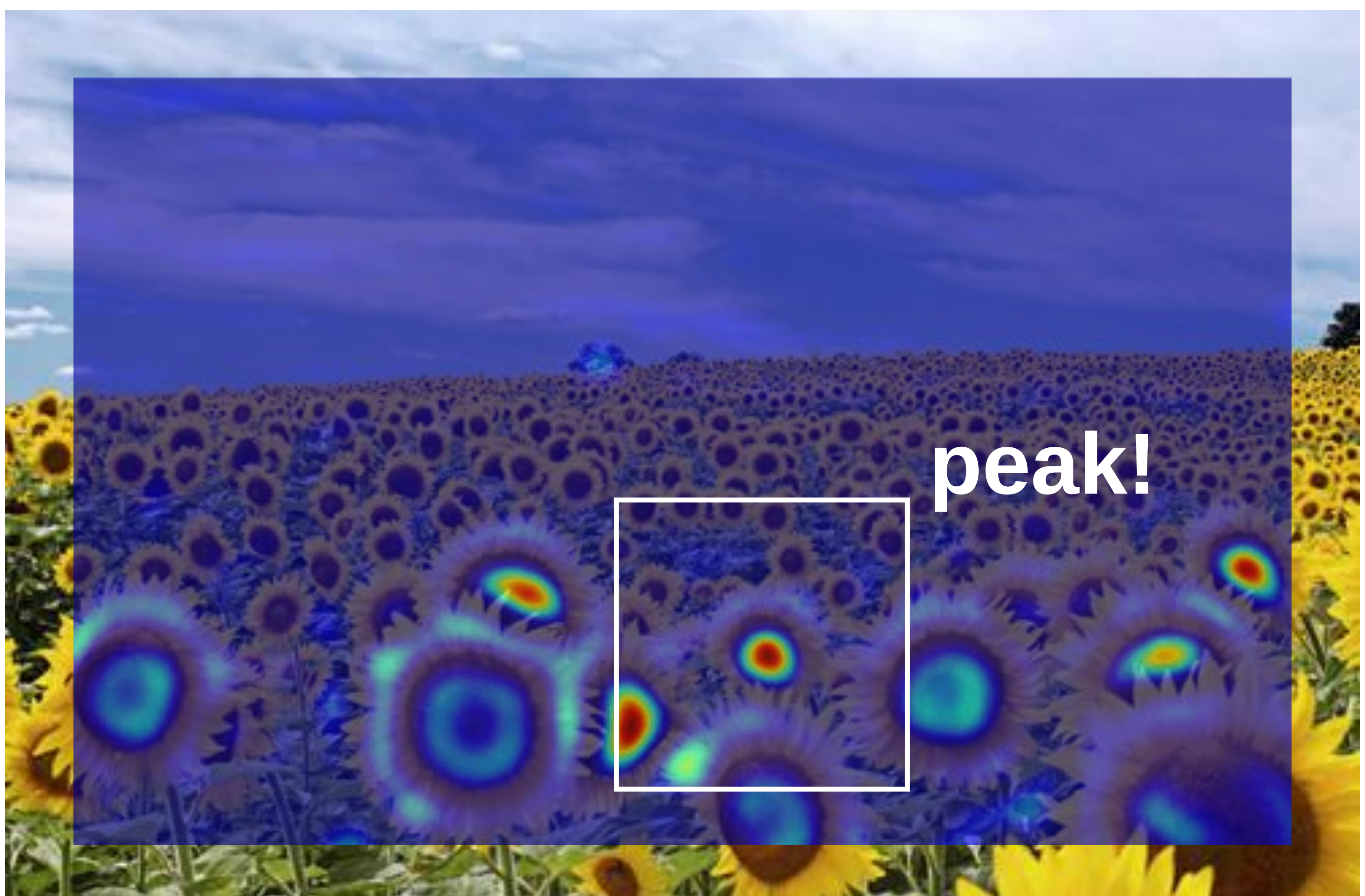


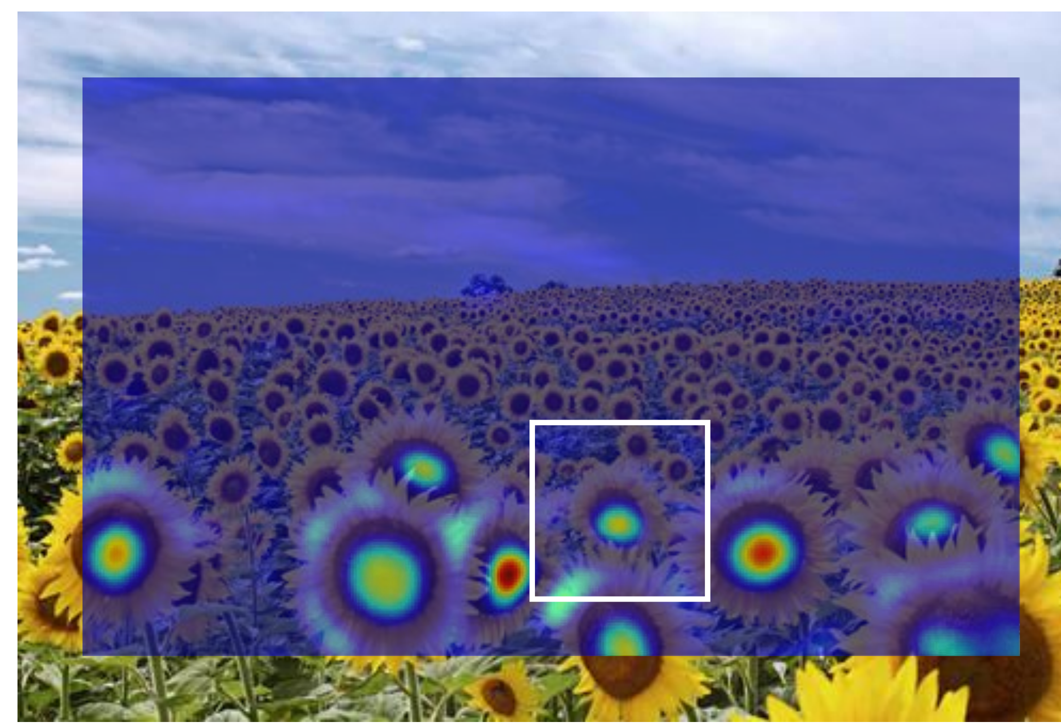

sigma=6
x 10^-3

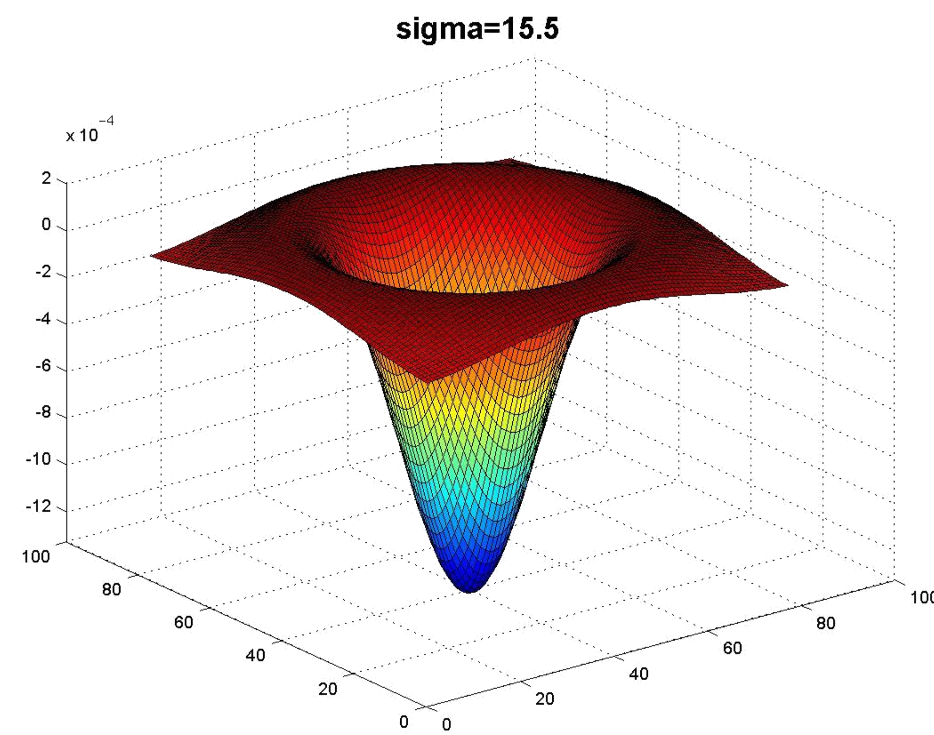


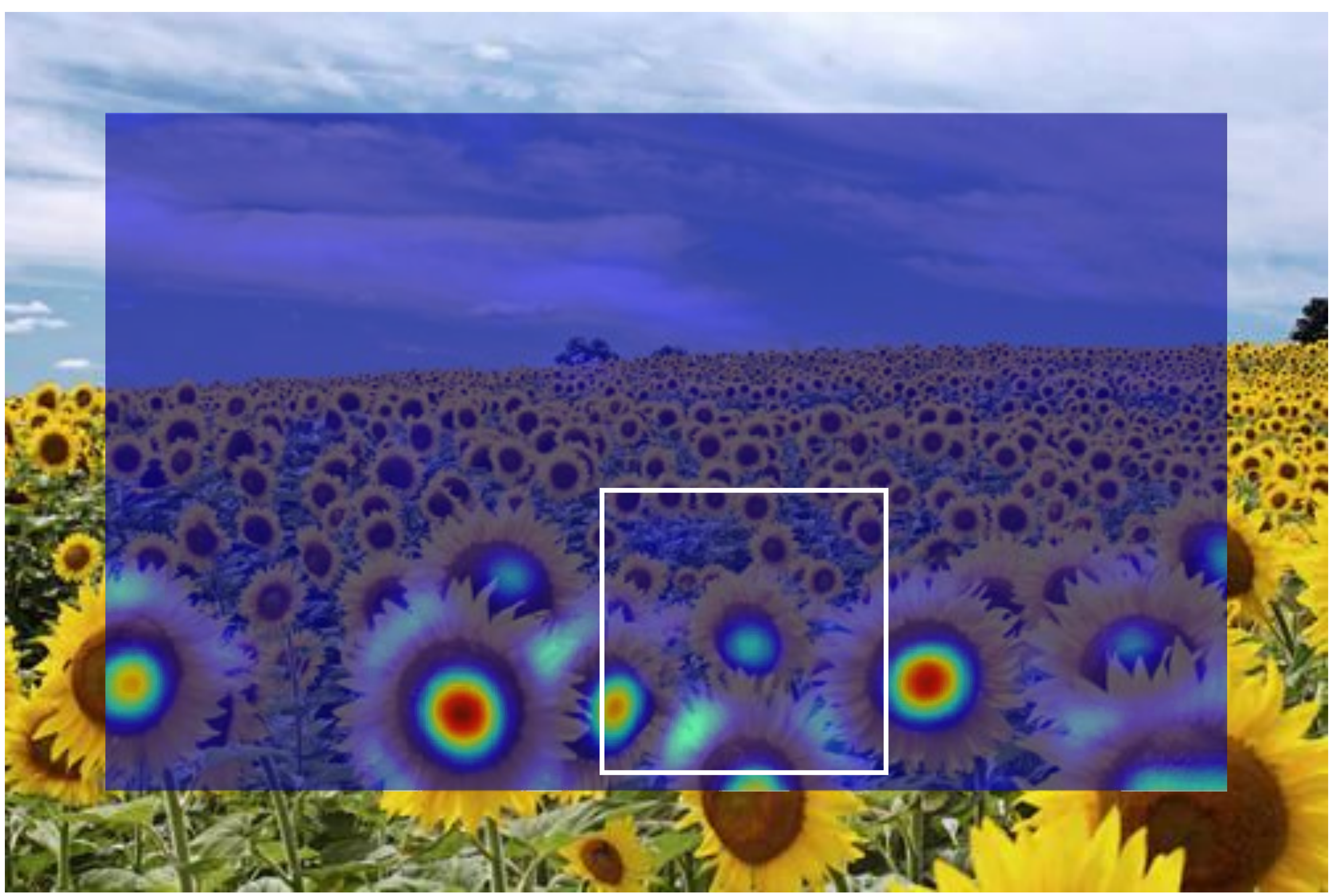

peak!

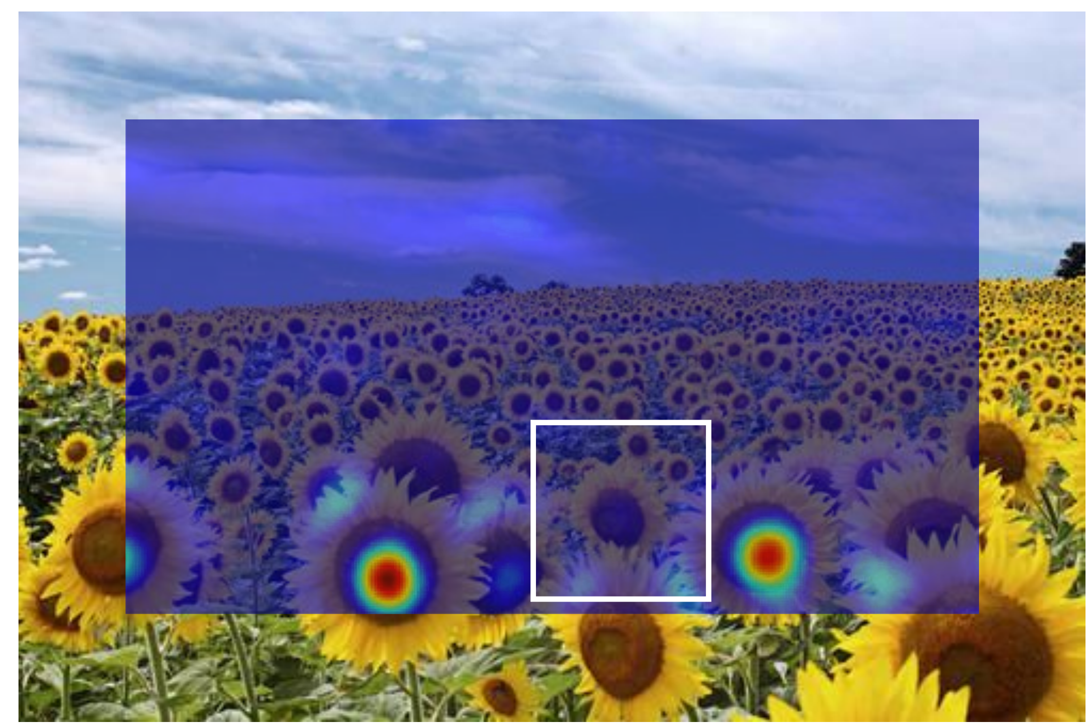

sigma=9.8
x 10^-3
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
100
80
60
40
20
0
0
20
40
60
80
100

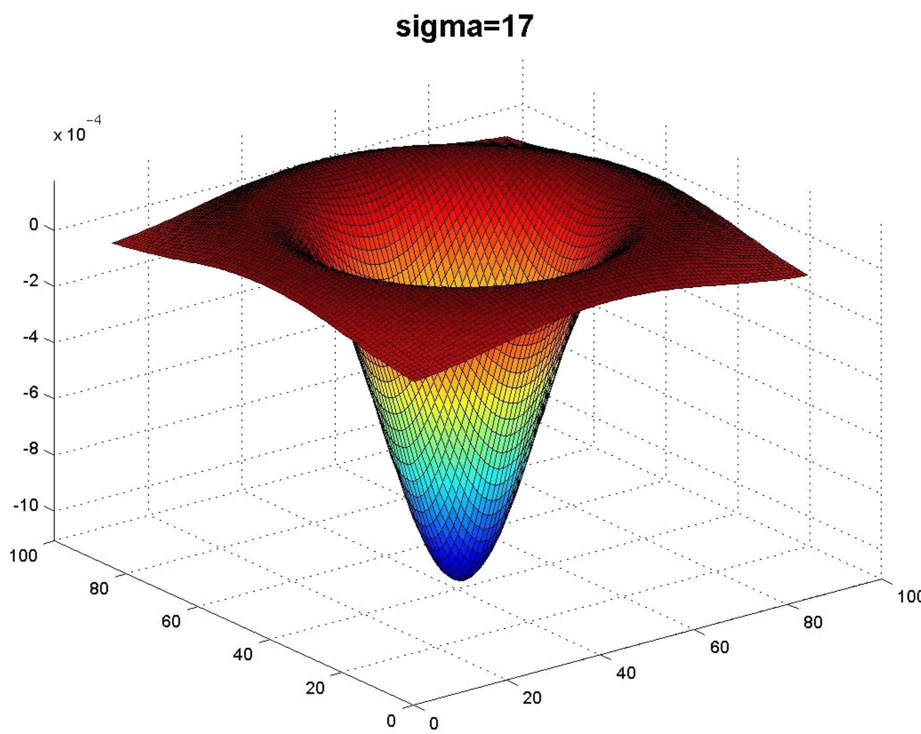

peak!



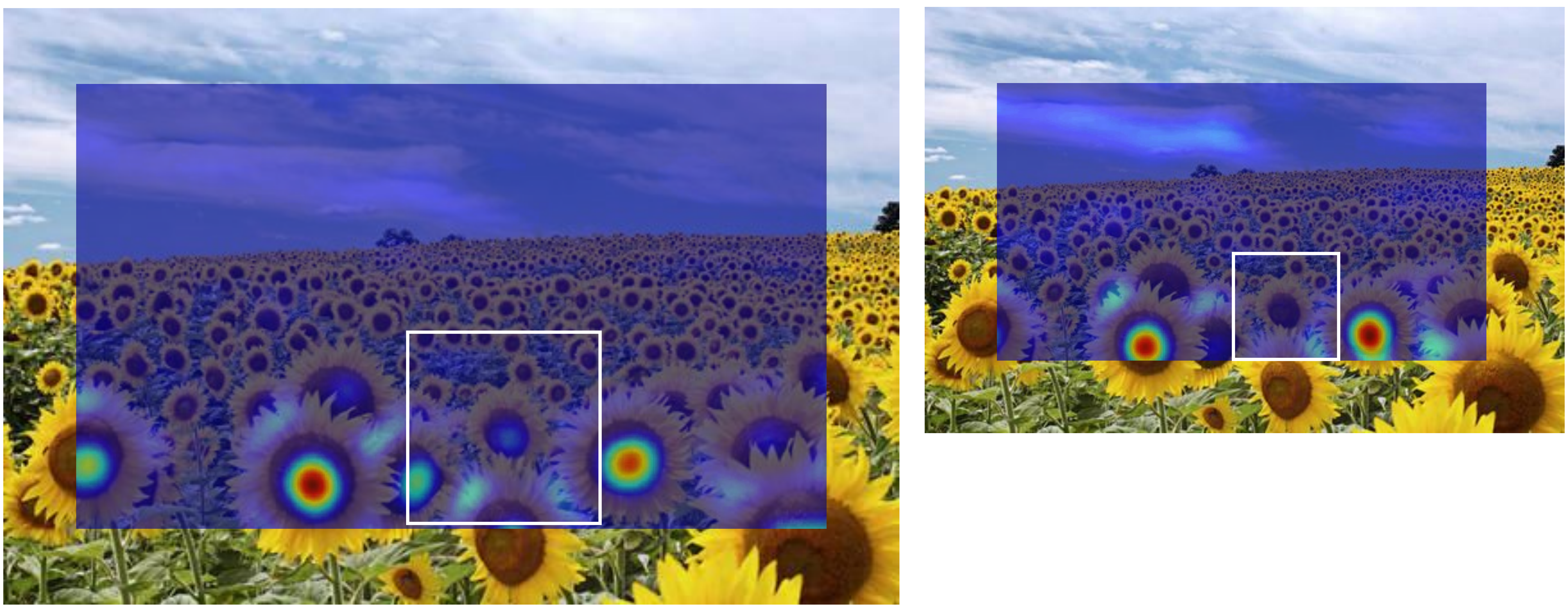

sigma=15.5
x 10^-4
2
0
-2
-4
-6
-8
-10
-12
100
80
60
40
20
0
0
20
40
60
80
100







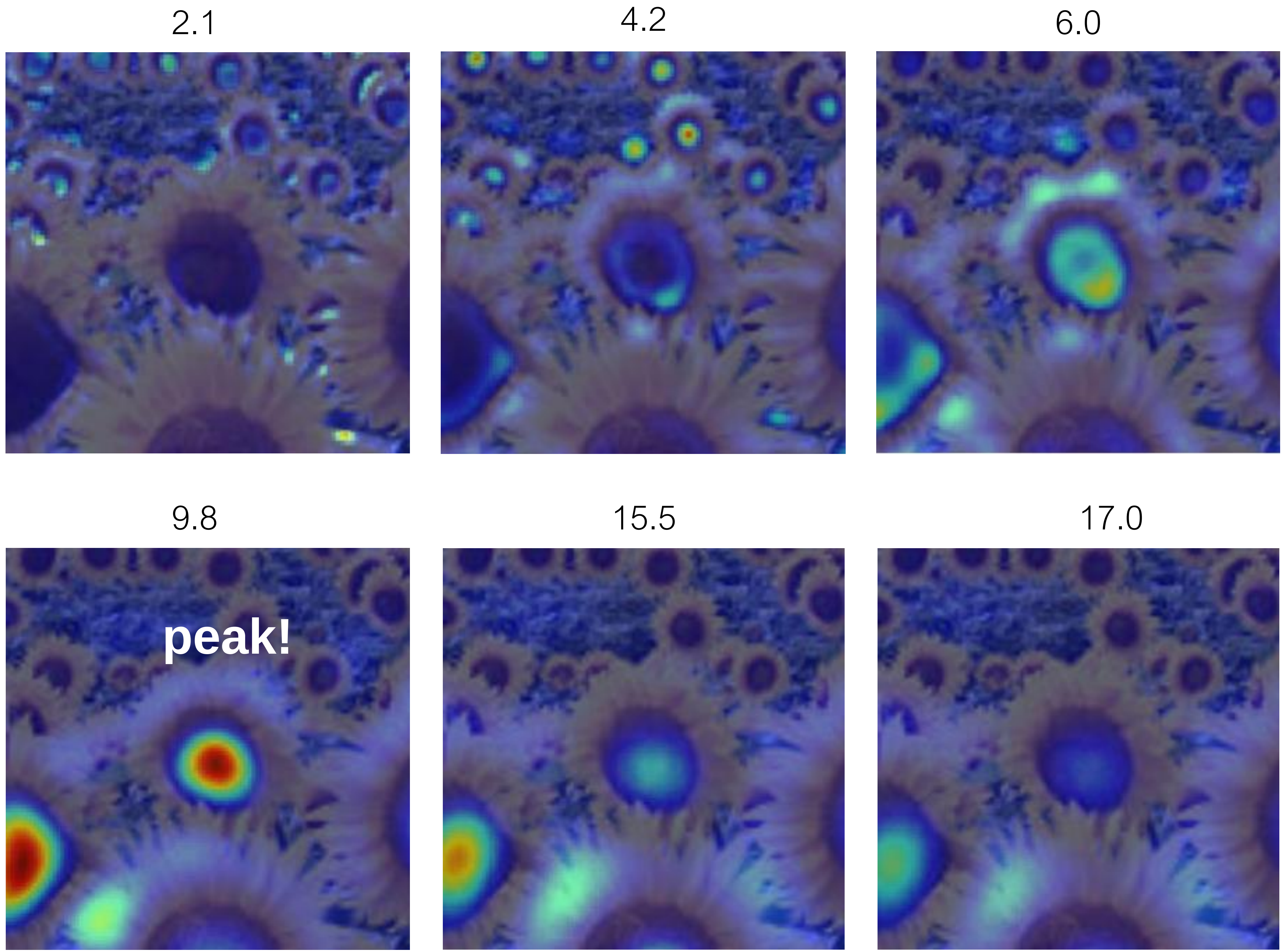
sigma=17
x 10^-4
0
-2
-4
-6
-8
-10
100
80
60
40
20
0
0
20
40
60
80
100

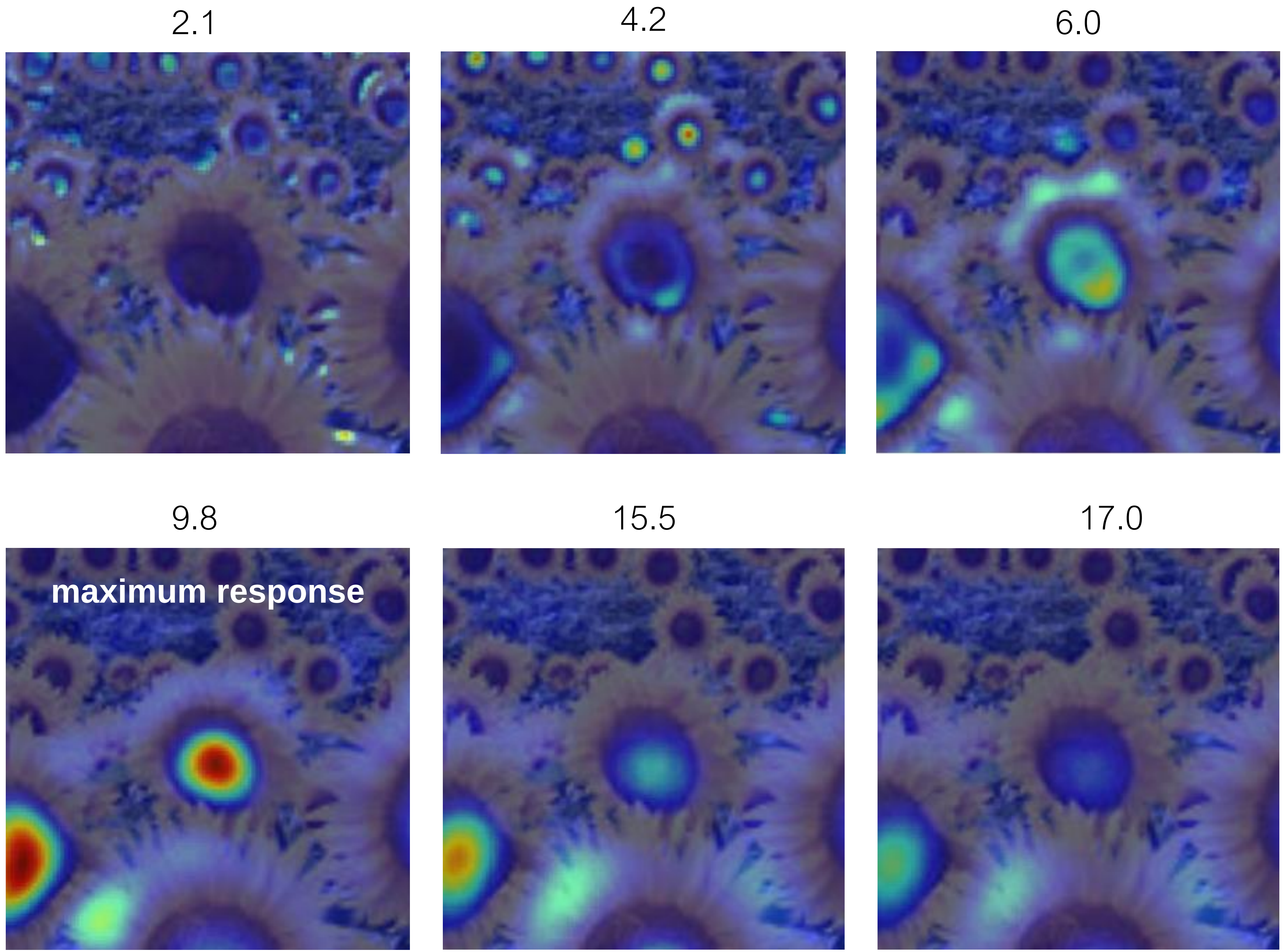


What happened when you applied different Laplacian filters?

Full size

3/4 size

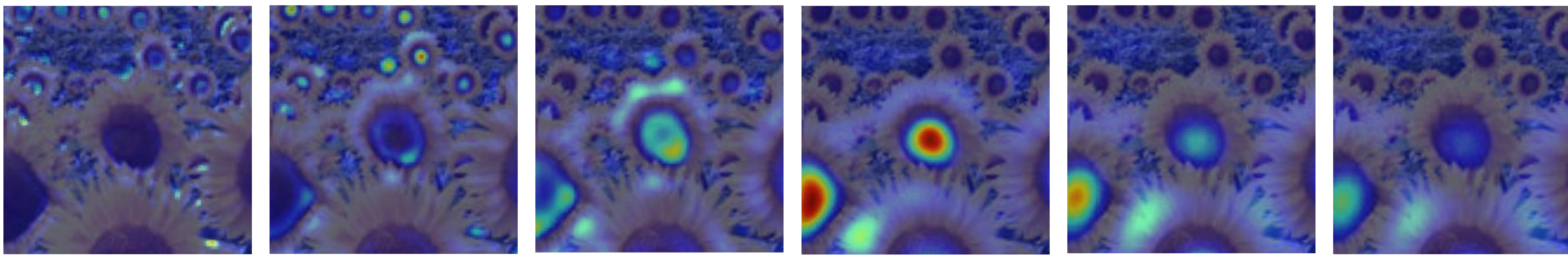

2.1
4.2
6.0
9.8
peak!
15.5
17.0



2.1
4.2
6.0
9.8
maximum response
15.5
17.0

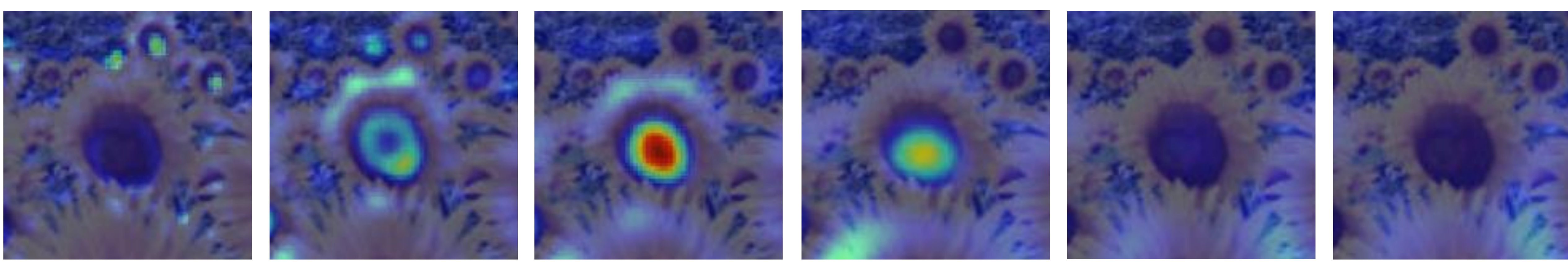
optimal scale

2.1 4.2 6.0 9.8 15.5 17.0



Full size image

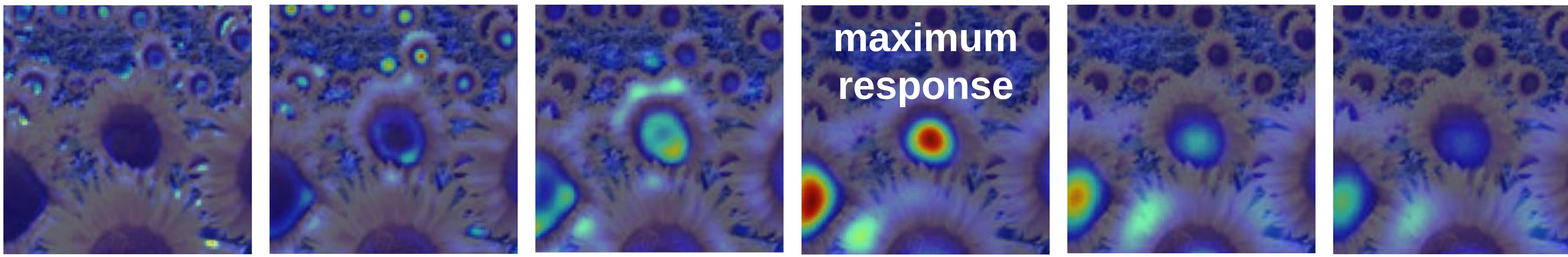
2.1 4.2 6.0 9.8 15.5 17.0



3/4 size image

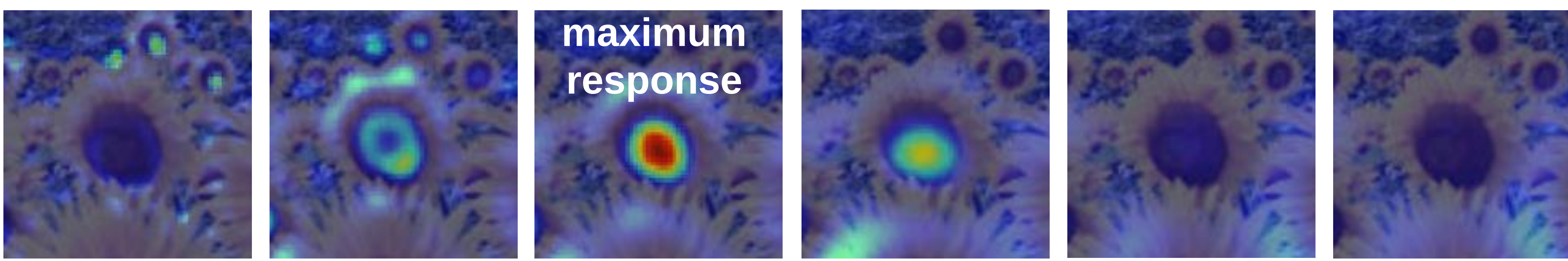
optimal scale

2.1 4.2 6.0 9.8 15.5 17.0

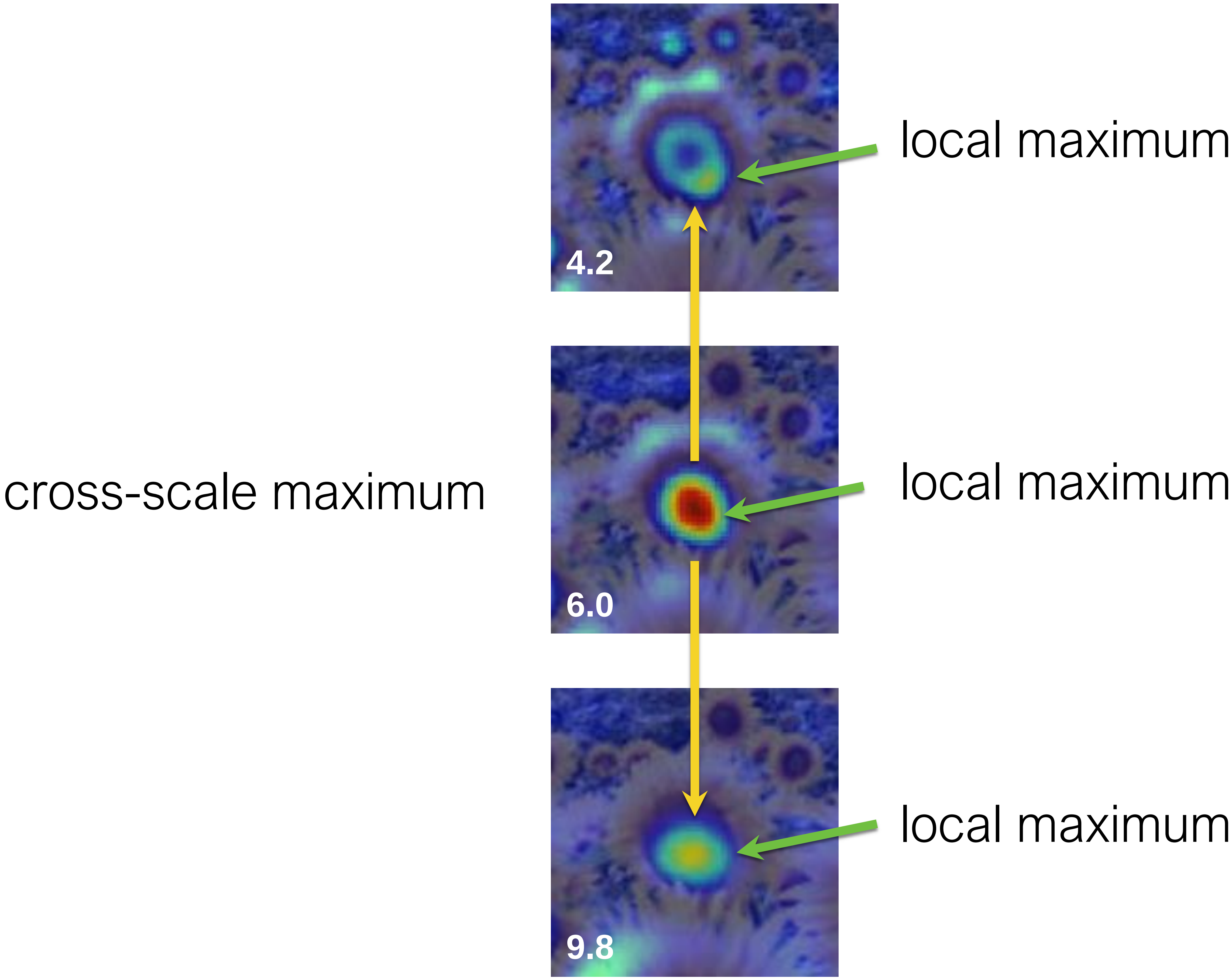


Full size image

2.1 4.2 6.0 9.8 15.5 17.0



3/4 size image



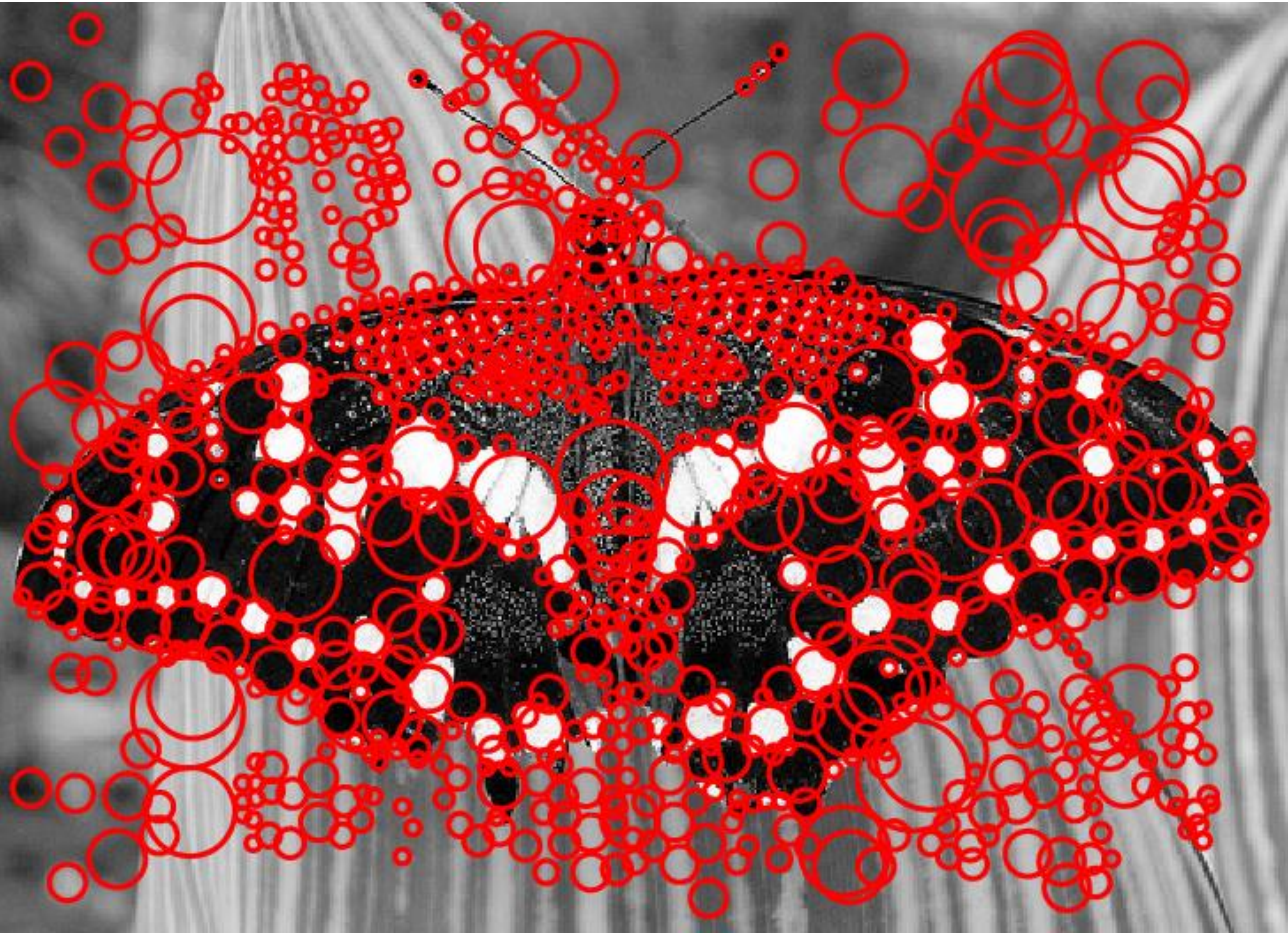
cross-scale maximum
local maximum
local maximum
local maximum
4.2
6.0
9.8

How would you implement scale selection?

implementation

```
For each level of the Gaussian pyramid

      compute feature response (e.g. Harris, Laplacian)


For each level of the Gaussian pyramid

     if local maximum and cross-scale

     save scale and location of feature
```

(x, y, s)



References

Basic reading:

- Szeliski textbook, Sections 4.1.