# Motion Planning

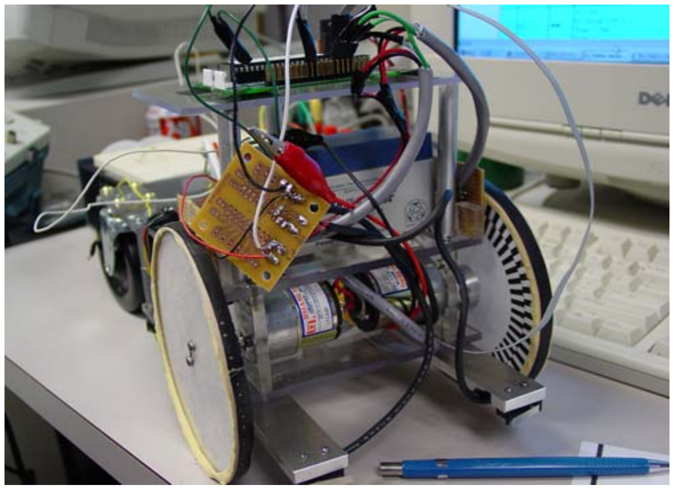
Howie CHoset

THE ROBOTICS INSTITUTE

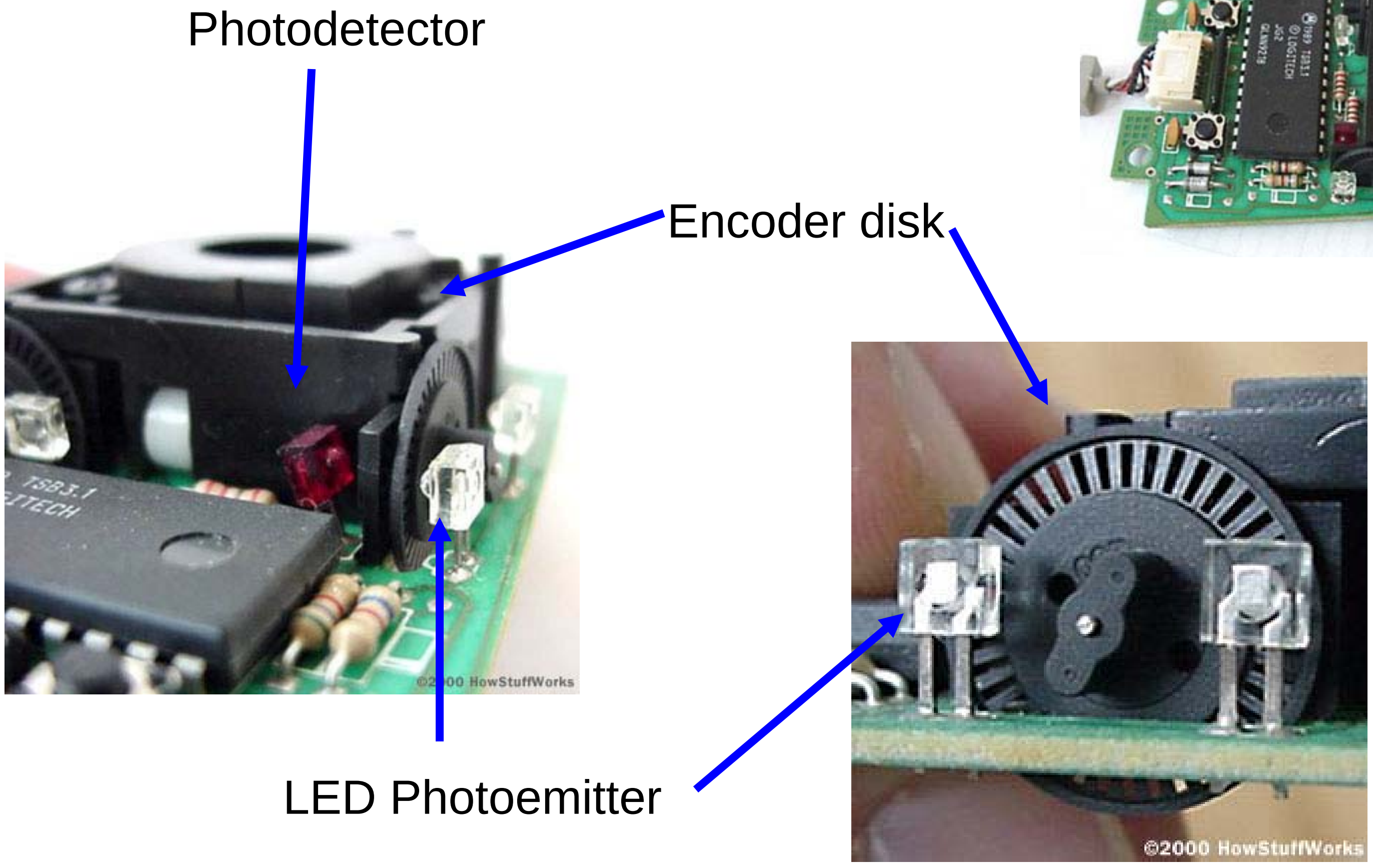
## Questions

- Where are we?
- Where do we go?
- Which is more important?

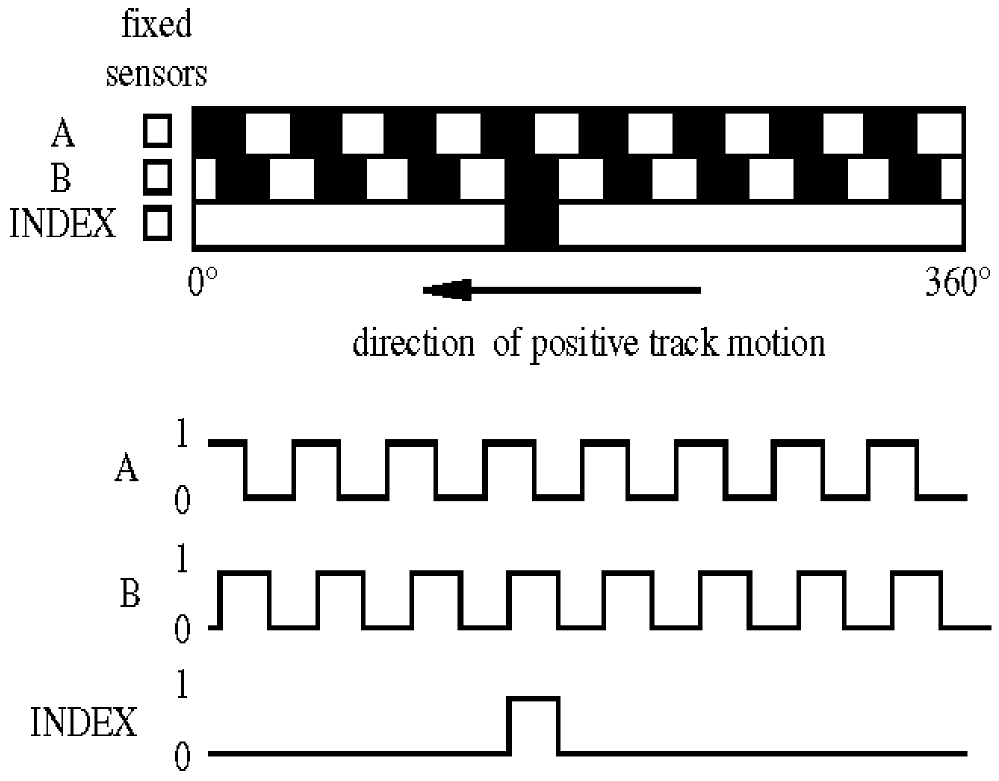
## Encoders

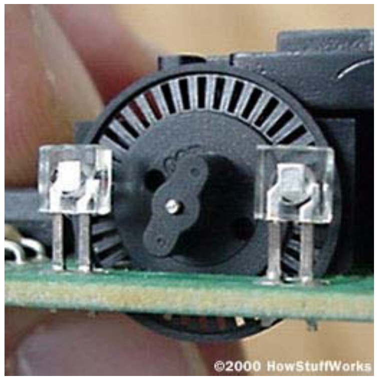


# Encoders – Incremental



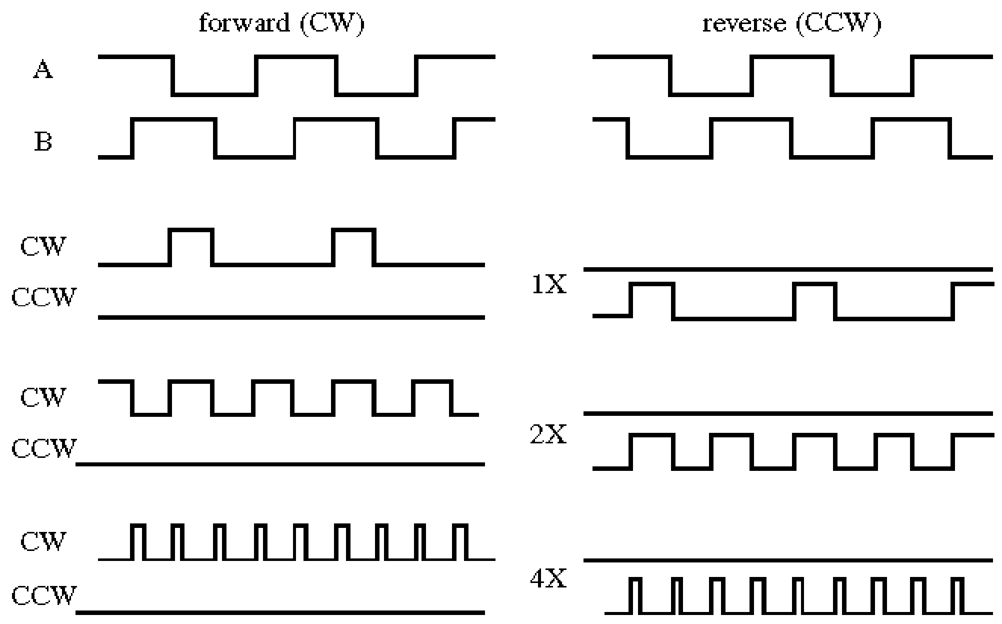
# Encoders - Incremental





# Encoders - Incremental

- Quadrature (resolution enhancing)



## Where are we?

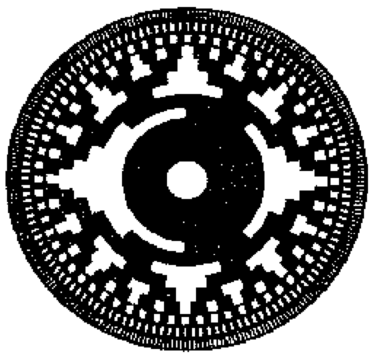
- If we know our encoder values after the motion, do we know where we are?

# Encoders - Absolute

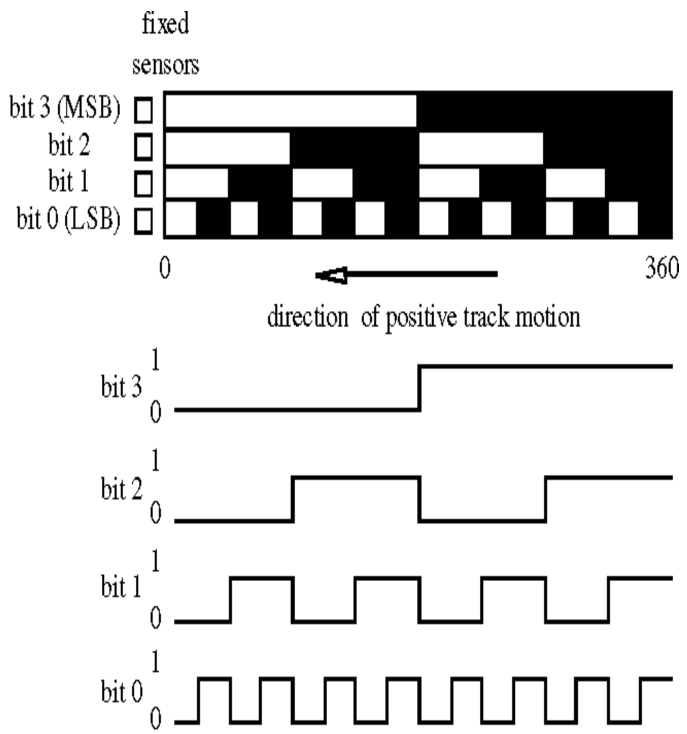
- More expensive
- Resolution = 360° / $2^N$

  where N is number of tracks

4 Bit Example



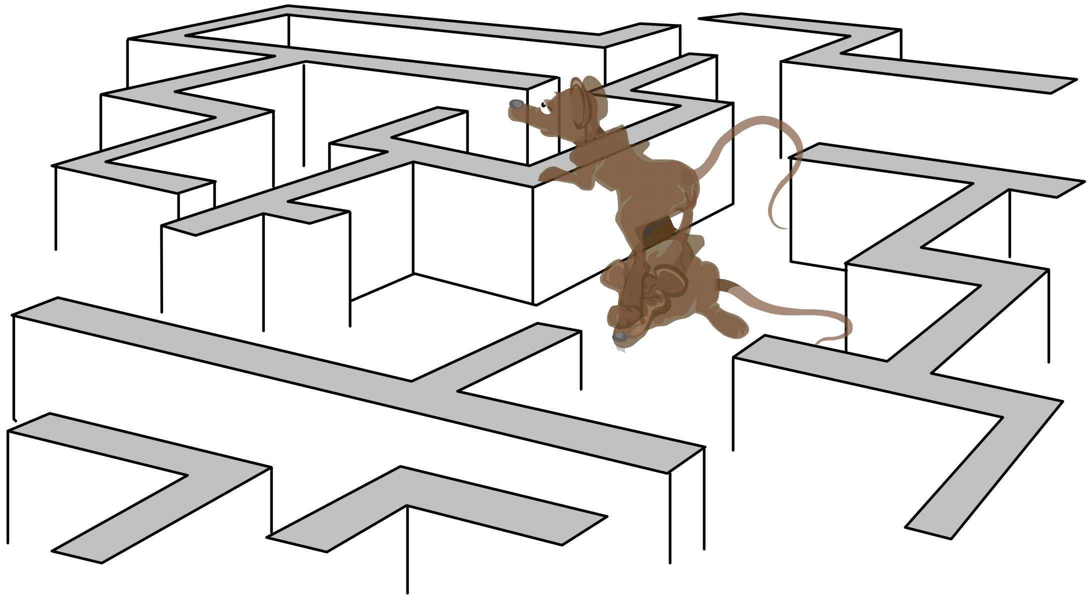
(b) actual disk (Courtesy of Parker Compumotor Division, Rohnert Park, CA)



(a) schematic and signals

# What is Motion Planning?

- Determining where to go



## Overview

- The Basics
  - Motion Planning Statement
  - The World and Robot
  - Configuration Space
  - Metrics
- Path Planning Algorithms
  - Start-Goal Methods
  - Map-Based Approaches
  - Cellular Decompositions
- Applications
  - Navigating Large Spaces
  - Coverage

## The World consists of...

- Obstacles
  - Already occupied spaces of the world
  - In other words, robots can’t go there
- Free Space
  - Unoccupied space within the world
  - Robots “might” be able to go here
  - To determine where a robot can go, we need to discuss what a *Configuration Space* is

## Motion Planning Statement

If $\mathbf{W}$ denotes the robot's workspace,

And $\mathbf{C_i}$ denotes the i'th obstacle,

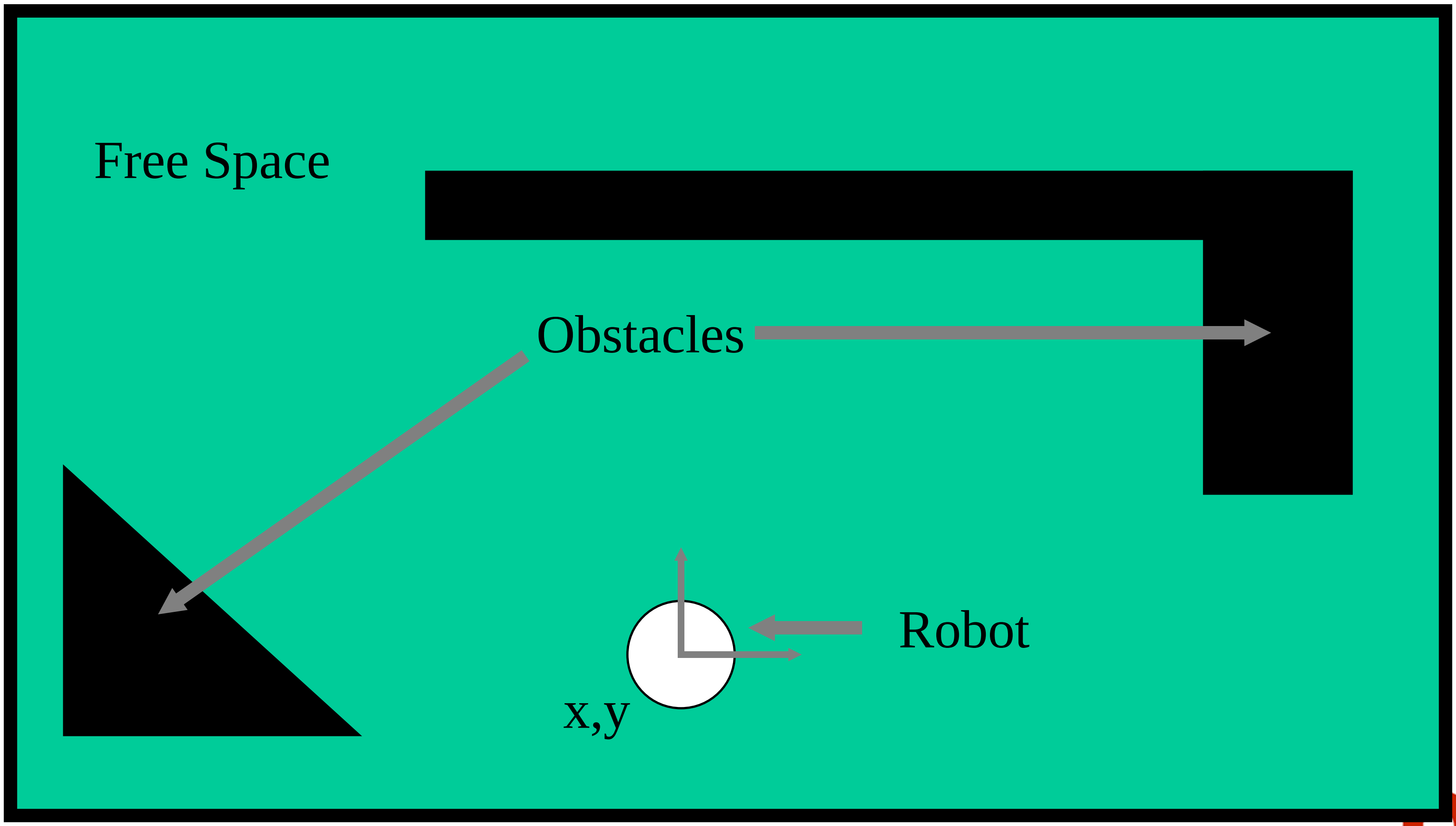
Then the robot's free space, **FS,** is defined as:

$$\mathbf{FS = W - ( \bigcup C_i )}$$

And a path $\mathbf{c} \in \mathbf{C^0}$ is $\mathbf{c : [0,1] \rightarrow FS}$

where $\mathbf{c(0)}$ is $\mathbf{q_{start}}$ and $\mathbf{c(1)}$ is $\mathbf{q_{goal}}$

## Example of a World (and Robot)



## What is a good path?

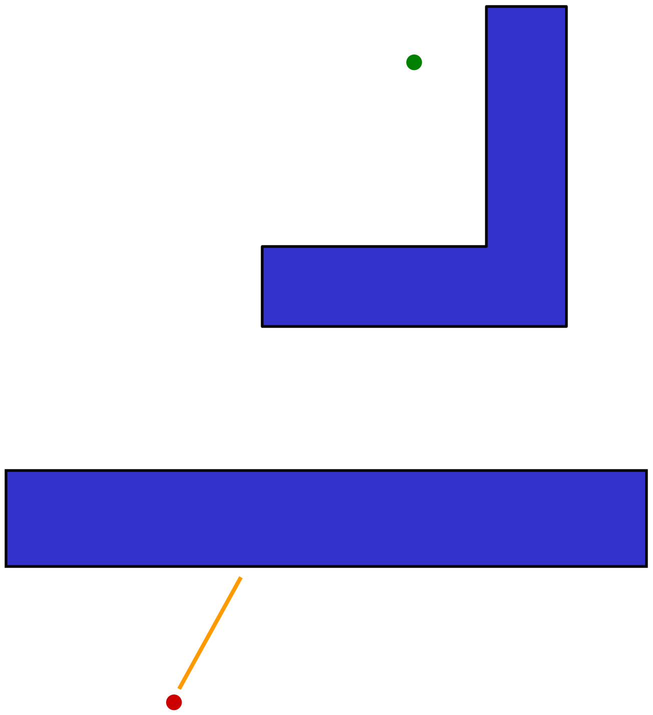
## Basics: Metrics

- There are many different ways to measure a path:
  - Time
  - Distance traveled
  - Expense
  - Distance from obstacles
  - Etc…

# Bug 1

But some computing power!
- known direction to goal
- otherwise local sensing

walls/obstacles & **encoders**



"Bug 1" algorithm

1) head toward goal

2) if an obstacle is encountered, circumnavigate it *and* remember how close you get to the goal

3) return to that closest point (by wall-following) and continue

Vladimir Lumelsky & Alexander Stepanov: Algorithmica 1987

# Bug 1

But some computing power!

- known direction to goal
- otherwise local sensing

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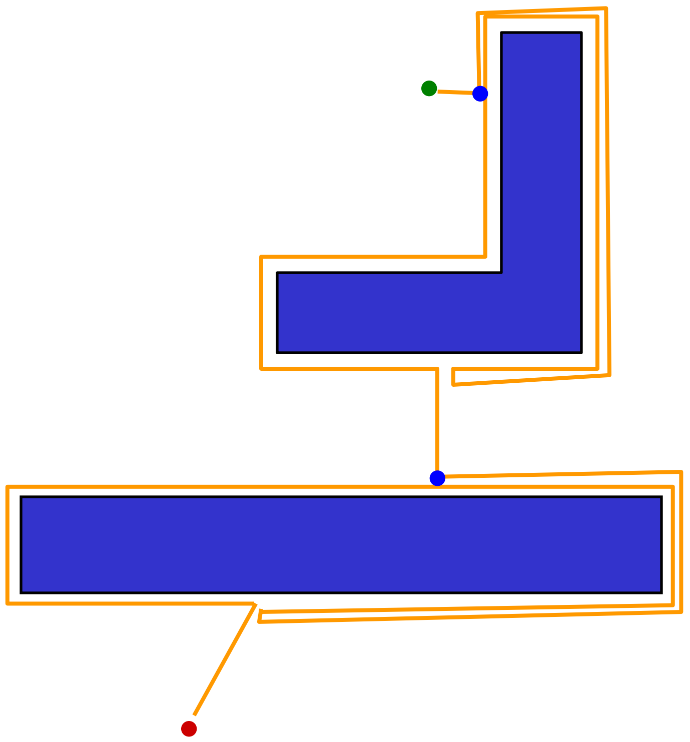

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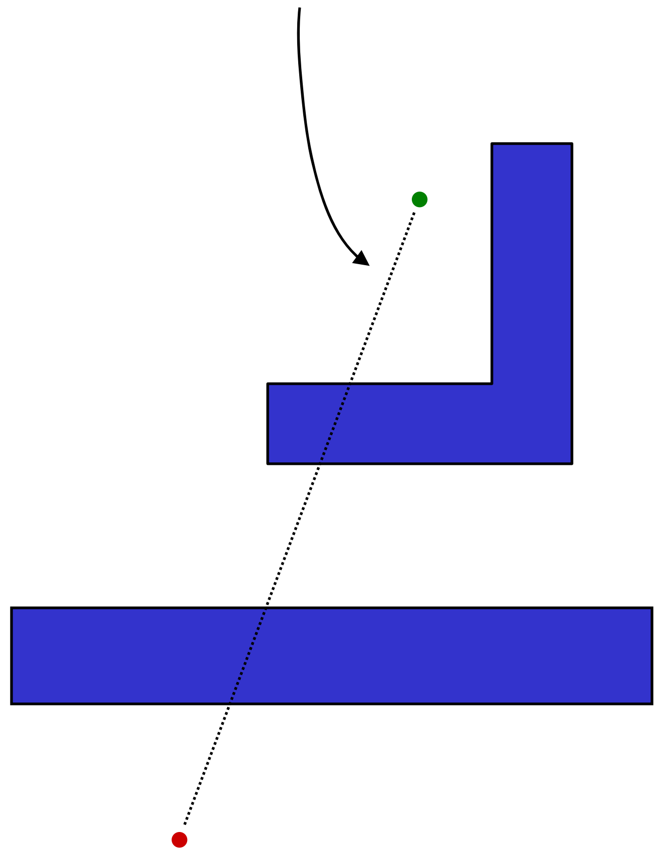
Vladimir Lumelsky & Alexander Stepanov: Algorithmica 1987

# Bug2

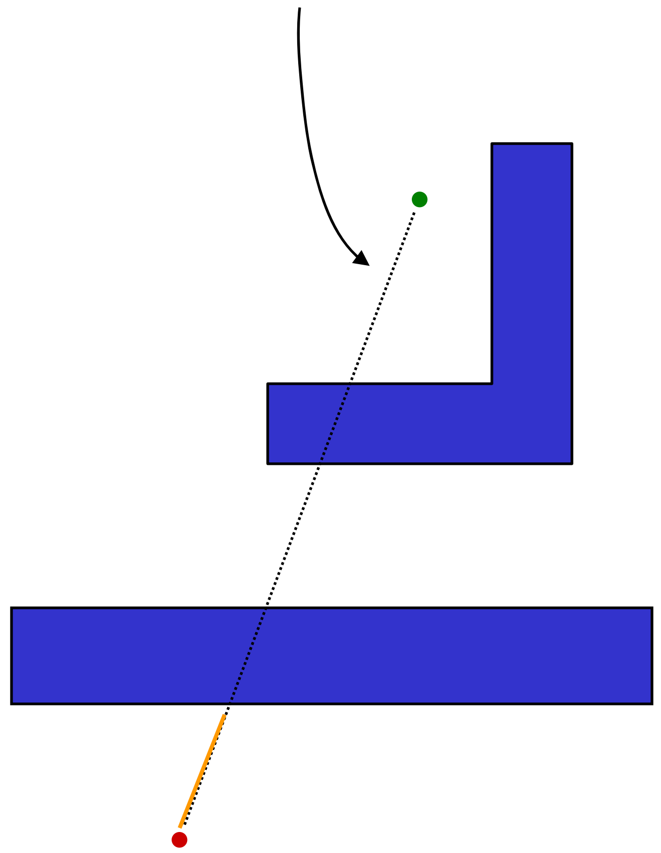
"Bug 2" Algorithm

Call the line from the starting point to the goal the ***m-line***



# A better bug?

Call the line from the starting point to the goal the ***m-line***

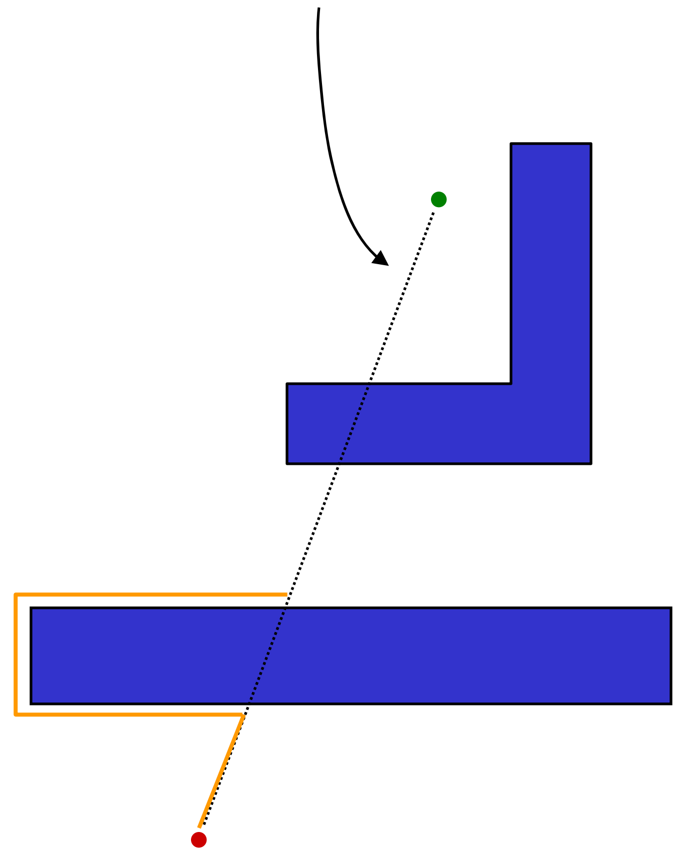


"Bug 2" Algorithm

1) head toward goal on the *m-line*

# A better bug?

Call the line from the starting point to the goal the ***m-line***

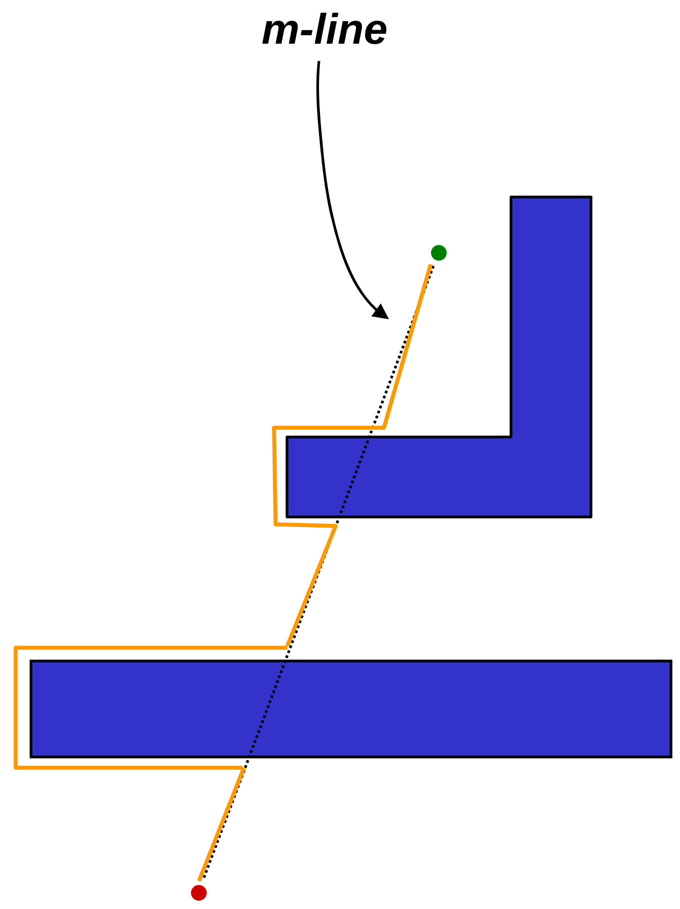


"Bug 2" Algorithm

1) head toward goal on the *m-line*

2) if an obstacle is in the way, follow it until you encounter the m-line again.

# A better bug?



"Bug 2" Algorithm

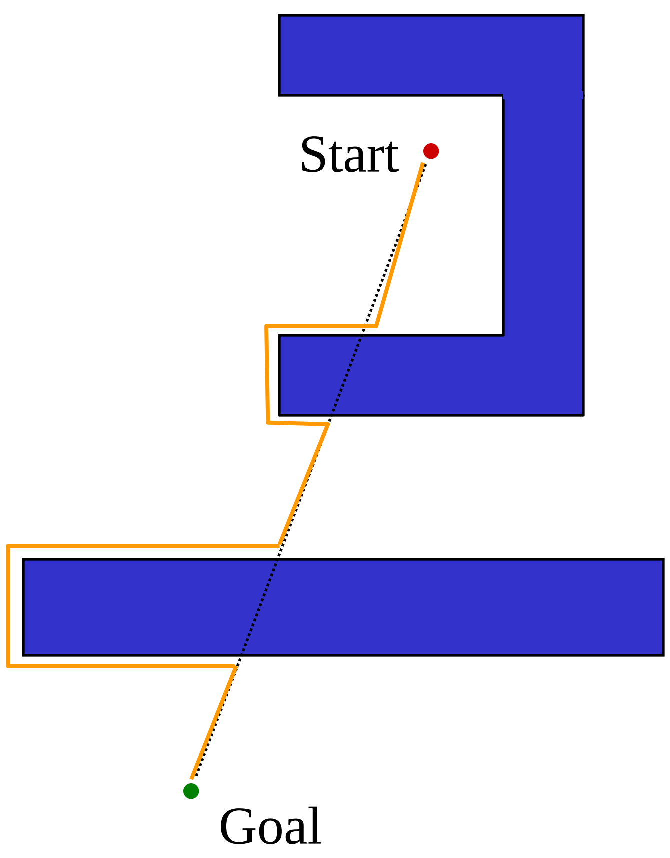
1) head toward goal on the *m-line*

2) if an obstacle is in the way, follow it until you encounter the m-line again.

3) Leave the obstacle and continue toward the goal

OK ?

# A better bug?



## "Bug 2" Algorithm

1) head toward goal on the *m-line*

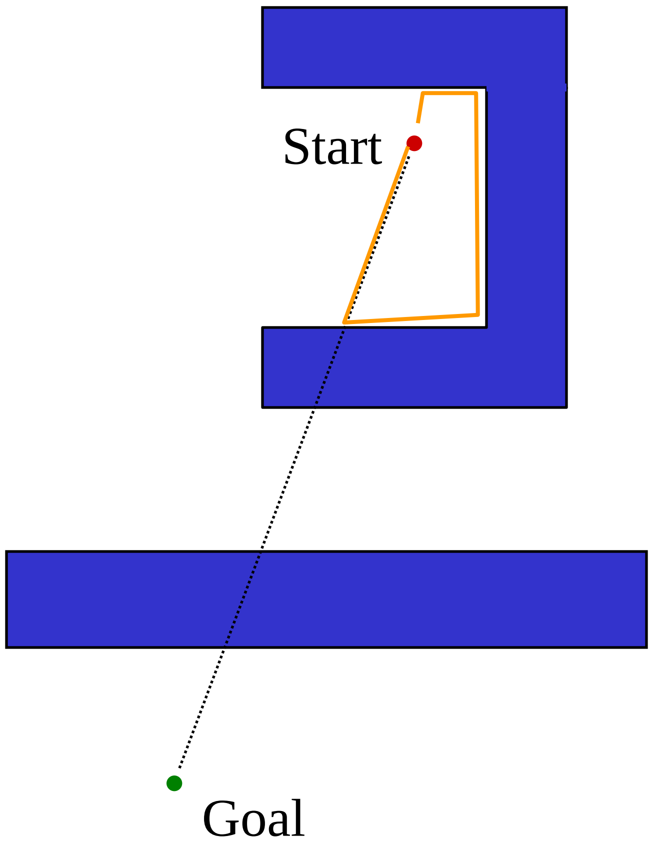
2) if an obstacle is in the way, follow it until you encounter the m-line again.

3) Leave the obstacle and continue toward the goal

Better or worse than Bug1?

# A better bug?

“Bug 2” Algorithm



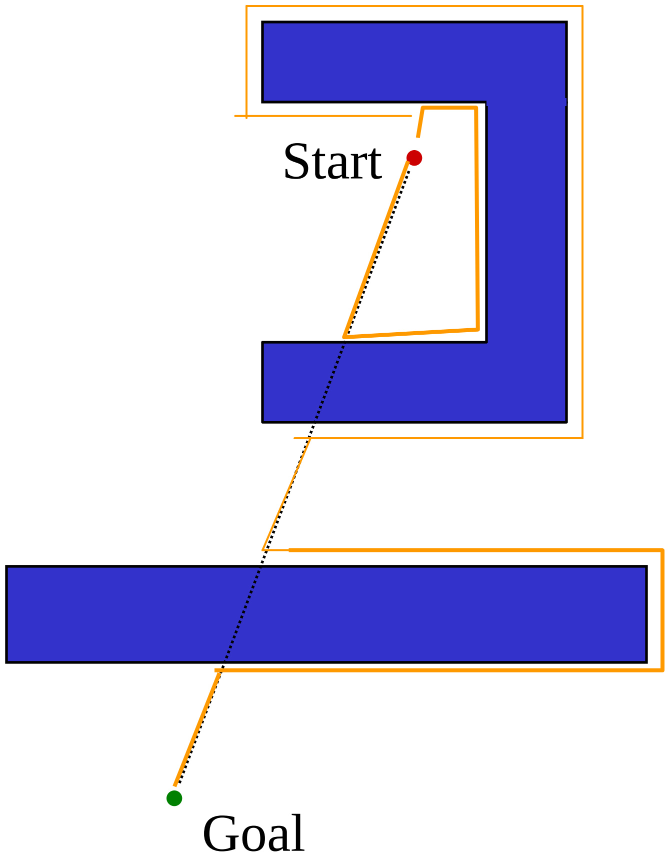
1) head toward goal on the *m-line*

2) if an obstacle is in the way, follow it until you encounter the m-line again.

3) Leave the obstacle and continue toward the goal

NO! How do we fix this?

# A better bug?



"Bug 2" Algorithm

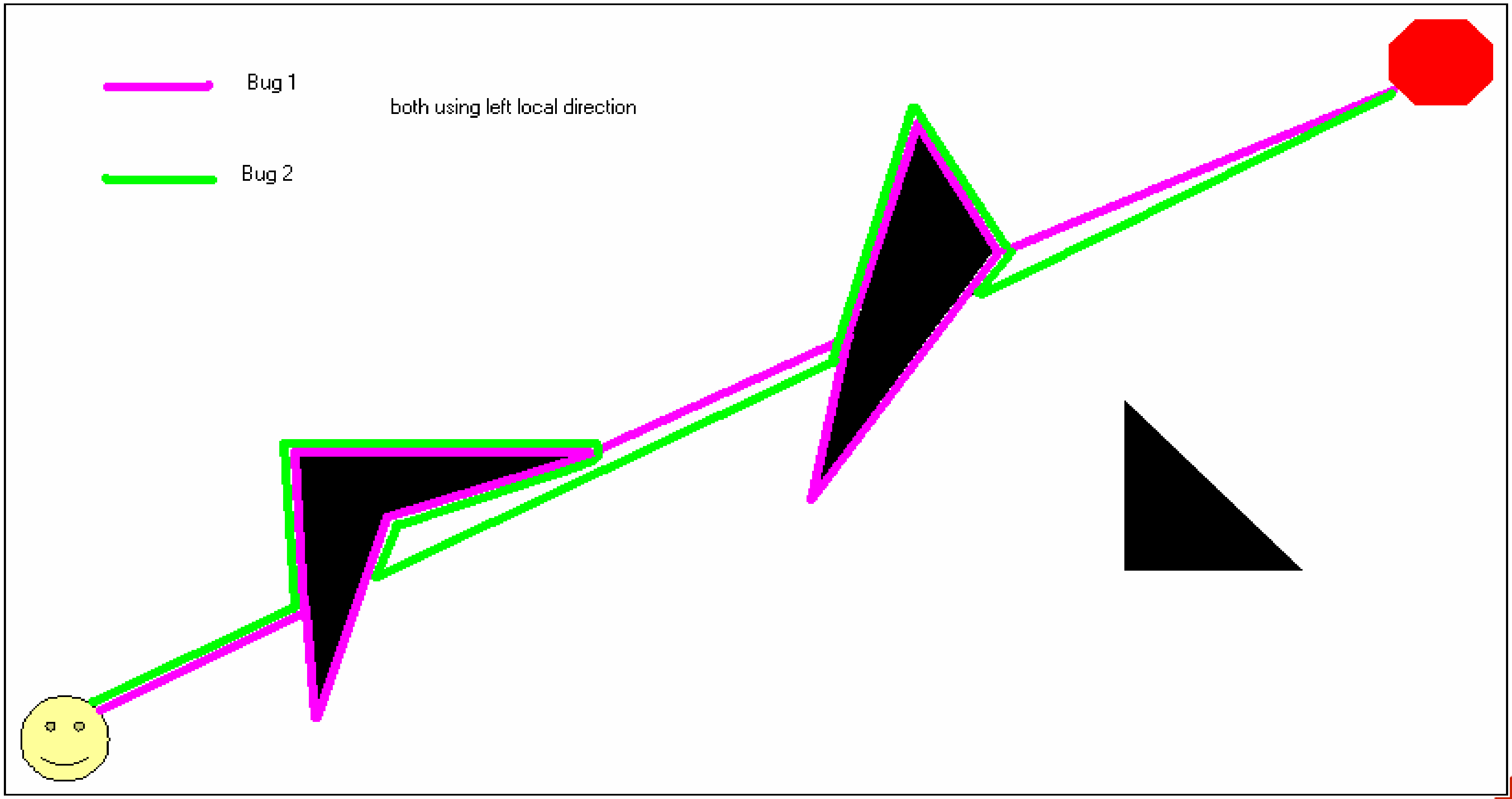
1) head toward goal on the *m-line*

2) if an obstacle is in the way, follow it until you encounter the m-line again ***closer to the goal***.

3) Leave the obstacle and continue toward the goal

Better or worse than Bug1?

# Start-Goal Algorithm: Lumelsky Bug Algorithms



## Lumelsky Bug Algorithms

- Unknown obstacles, known start and goal.
- Simple "bump" sensors, encoders.
- Choose arbitrary direction to turn (left/right) to make all turns, called "local direction"
- Motion is like an ant walking around:
  - In Bug 1 the robot goes all the way around each obstacle encountered, recording the point nearest the goal, then goes around again to leave the obstacle from that point
  - In Bug 2 the robot goes around each obstacle encountered until it can continue on its previous path toward the goal

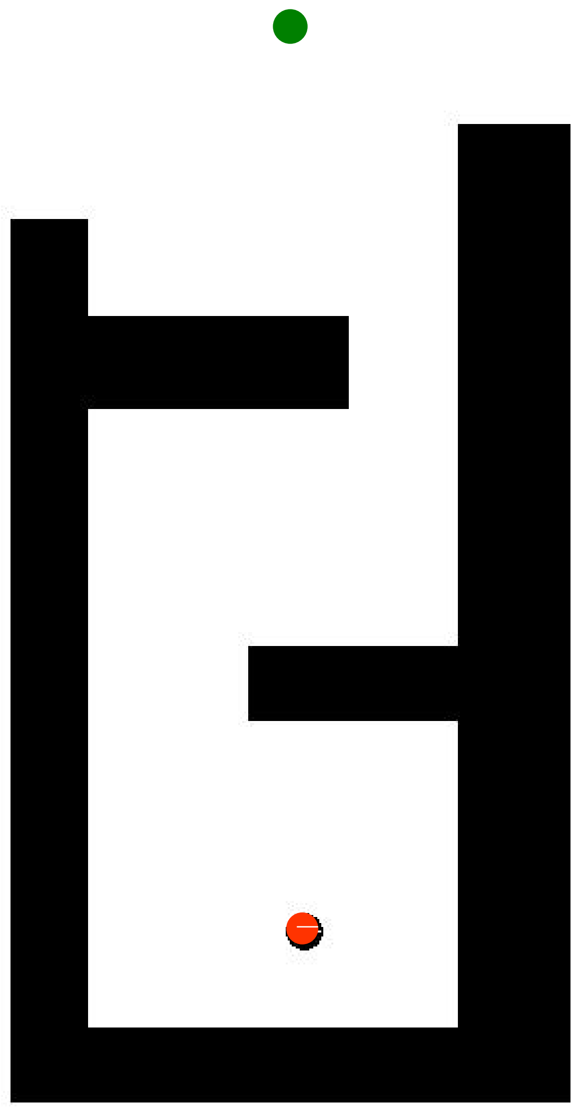
## Assumptions?

## Assumptions

- Size of robot
- Perfect sensing
- Perfect control
- Localization (heading)

What else?

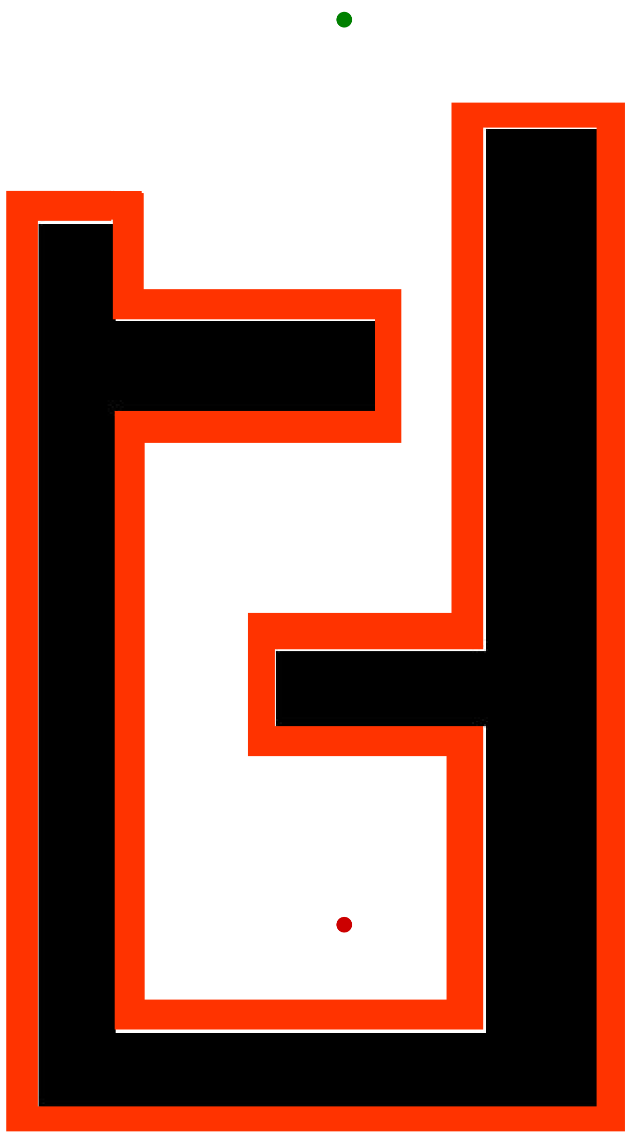
## What is the position of the robot?



Expand obstacle(s)

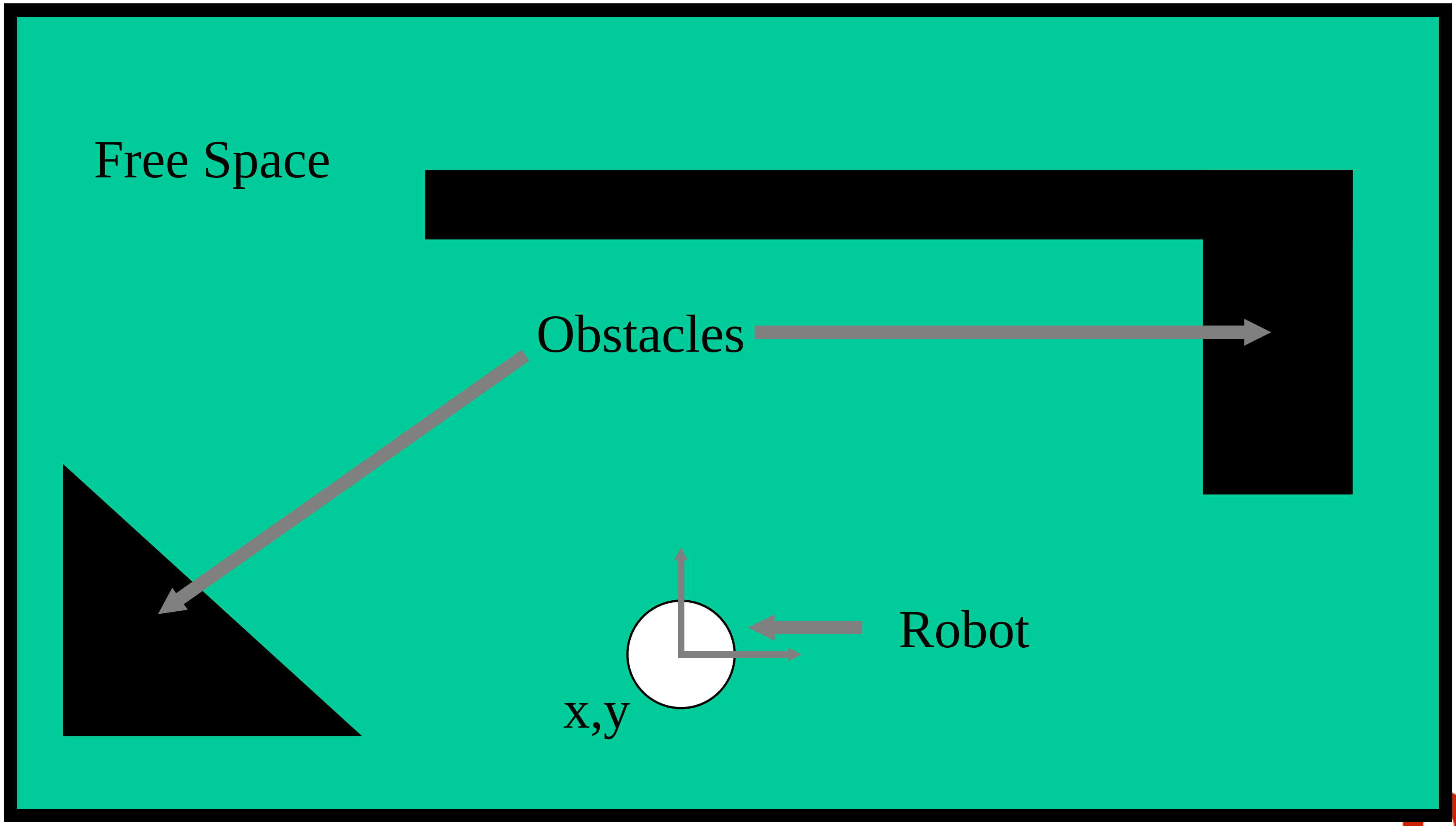
$\longrightarrow$

Reduce robot

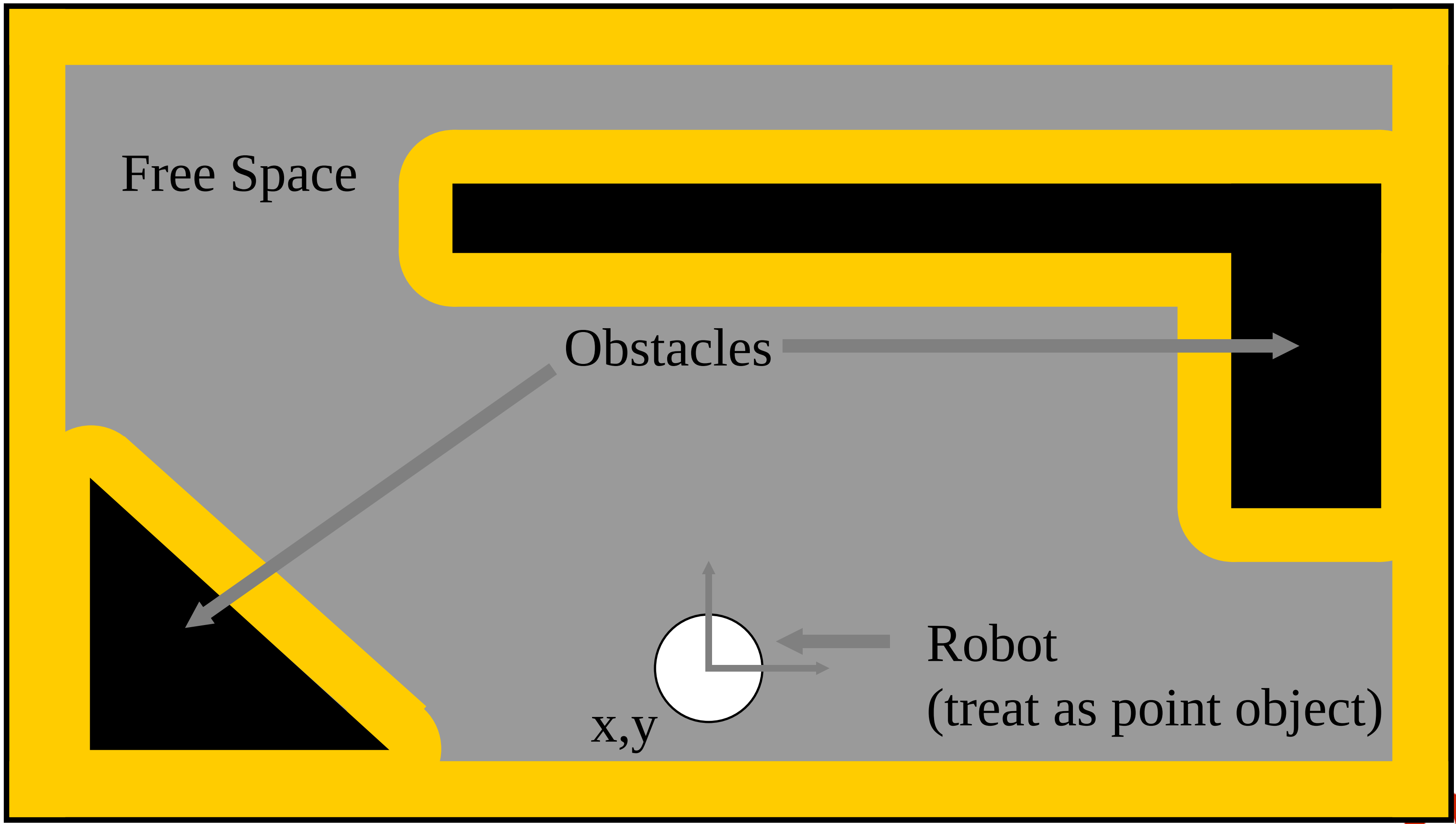


not quite right ...

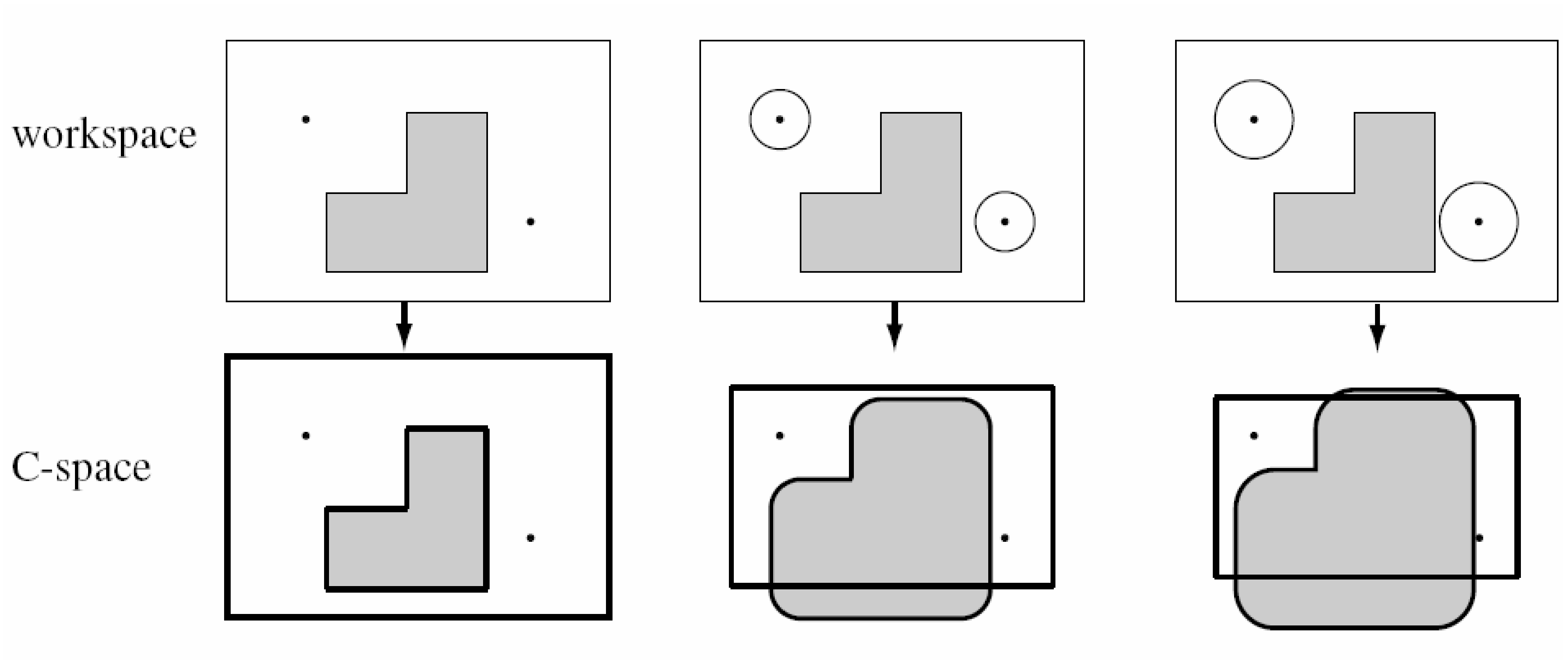
# Example of a World (and Robot)



## Configuration Space: Accommodate Robot Size

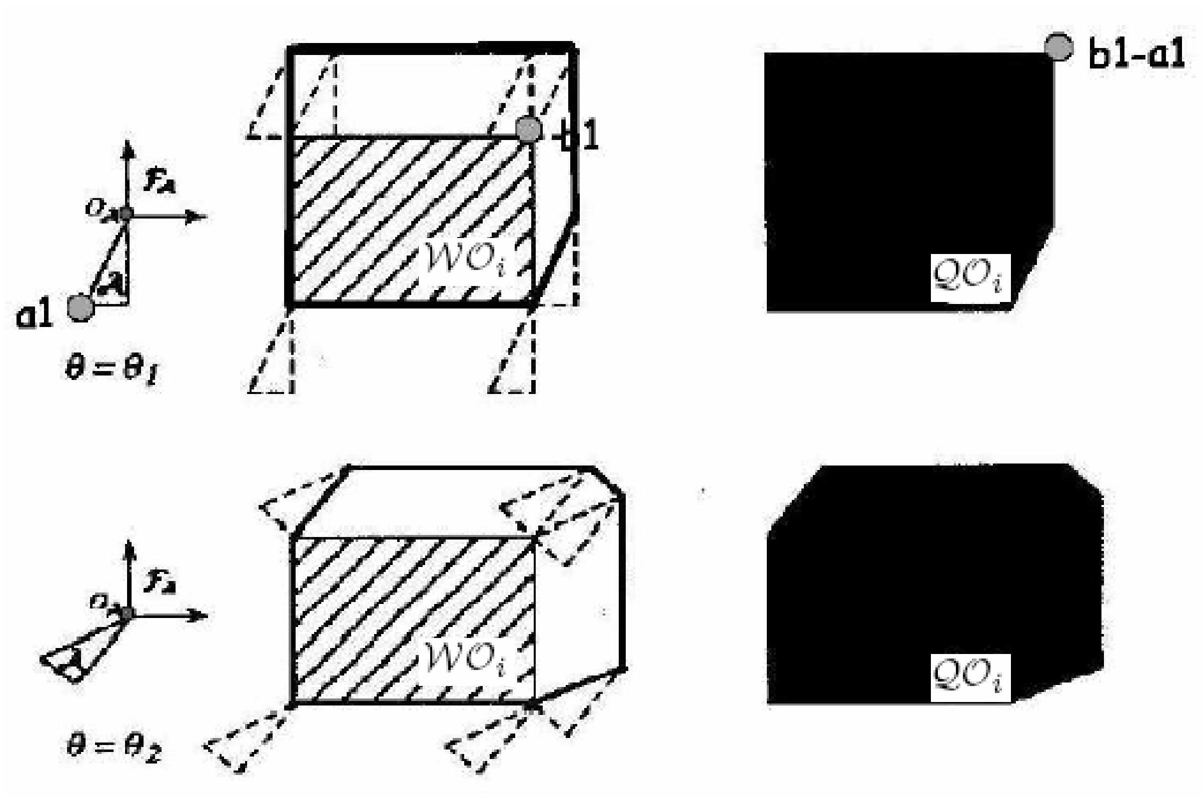


## Trace Boundary of Workspace



Pick a reference point…

# Translate-only, non-circularly



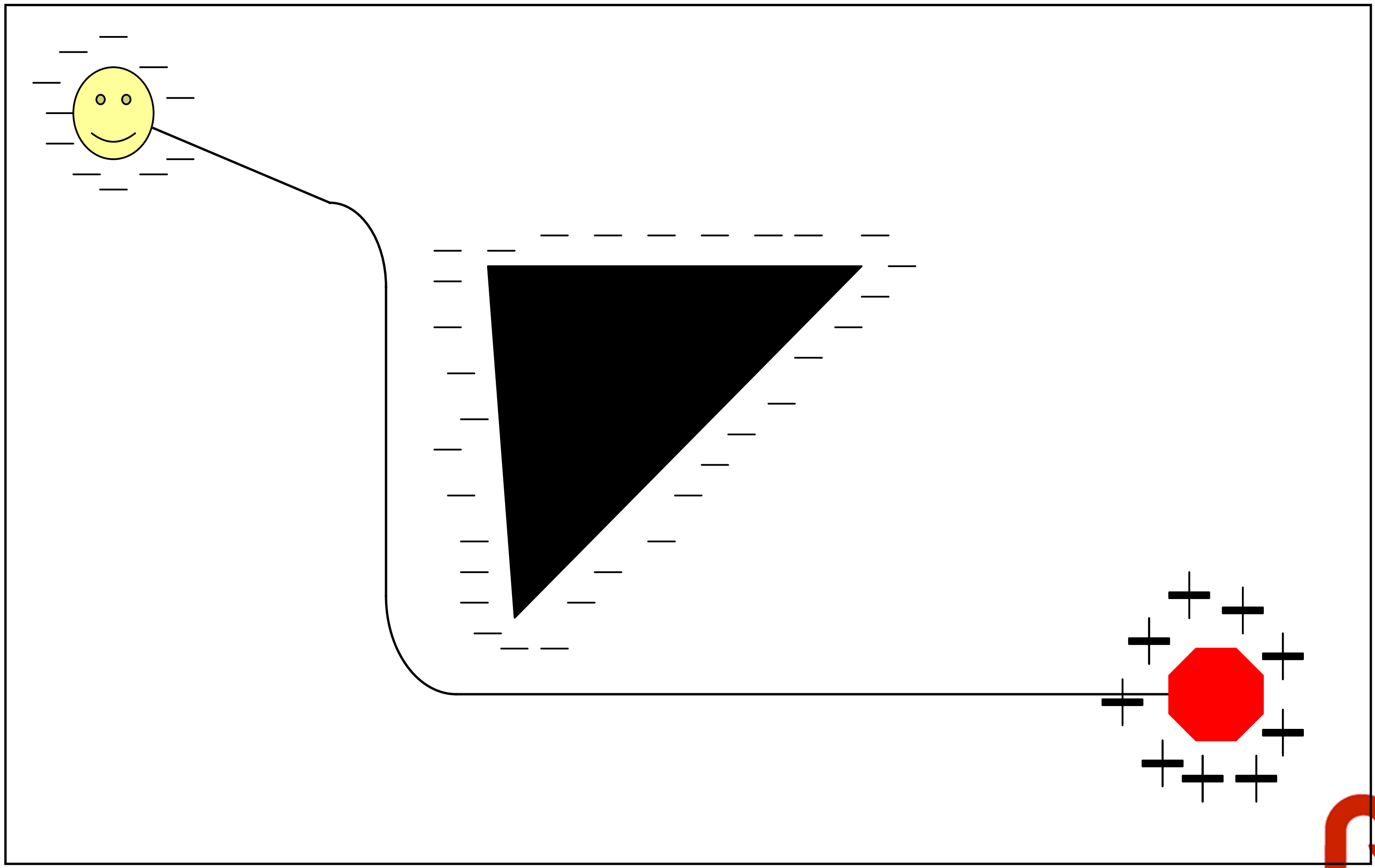
$$QO_i = \{q \in Q \mid R(q) \bigcap WO_i \neq \emptyset\}.$$

Pick a reference point…

## The Configuration Space

- What it is
  - A set of “reachable” areas constructed from knowledge of both the robot and the world
- How to create it
  - First abstract the robot as a point object. Then, enlarge the obstacles to account for the robot’s footprint and degrees of freedom
  - In our example, the robot was circular, so we simply enlarged our obstacles by the robot’s radius (*note the curved vertices*)

# Start-Goal Algorithm: Potential Functions



## Attractive/Repulsive Potential Field

$$U(q) = U_{\text{att}}(q) + U_{\text{rep}}(q)$$

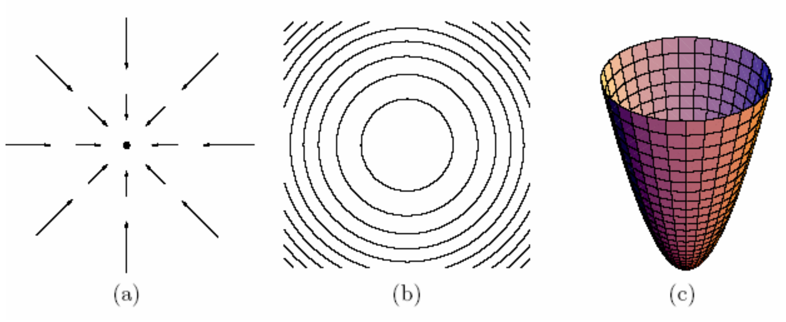
– $U_{att}$ is the “attractive” potential --- move to the goal

– $U_{rep}$ is the “repulsive” potential --- avoid obstacles

# Artificial Potential Field Methods: Attractive Potential

Quadratic Potential $\longrightarrow$

$$U_{\text{att}}(q) = \frac{1}{2}\zeta d^2(q, q_{\text{goal}}),$$


(a) (b) (c)

$$\begin{aligned} F_{\text{att}}(q) = \nabla U_{\text{att}}(q) &= \nabla\left(\frac{1}{2}\zeta d^2(q, q_{\text{goal}})\right), \\ &= \frac{1}{2}\zeta \nabla d^2(q, q_{\text{goal}}), \\ &= \zeta(q - q_{\text{goal}}), \end{aligned}$$

## Distance

$$d : R^2 \times R^2 \rightarrow R$$

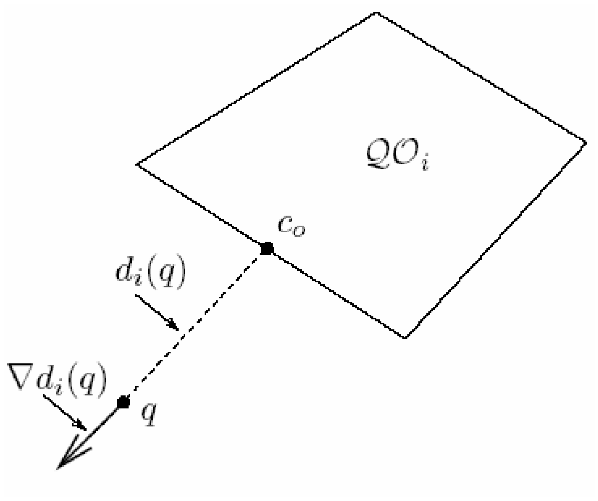
| | | |
|---|---|---|
| L1 Metric | (diamond) | $d(a,b) = \lvert a_x - b_x \rvert + \lvert a_y - b_y \rvert$ |
| L2 Metrix | (circle) | $d(a,b) = \sqrt{(a_x - b_x)^2 + (a_y - b_y)^2}$ |

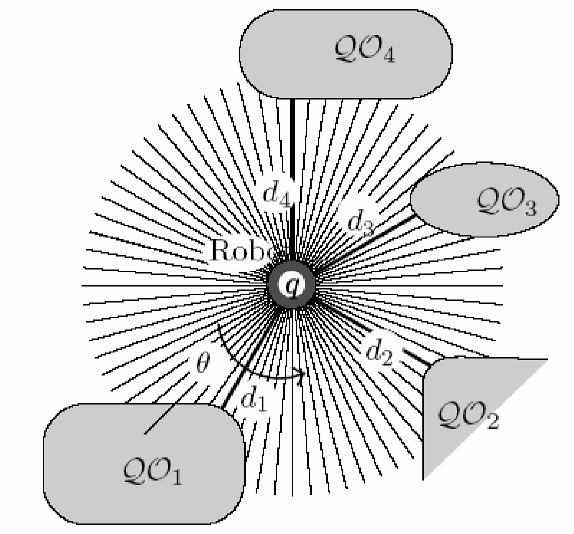
# Distance to Obstacle(s)

$$d_i(q) = \min_{c \in \mathcal{QO}_i} d(q, c)$$

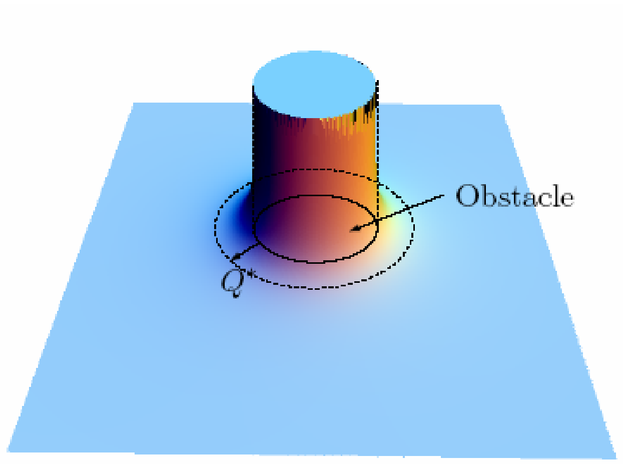
$$\nabla d_i(q) = \frac{q - c}{d(q, c)}$$

$$D(q) = \min d_i(q)$$





# The Repulsive Potential

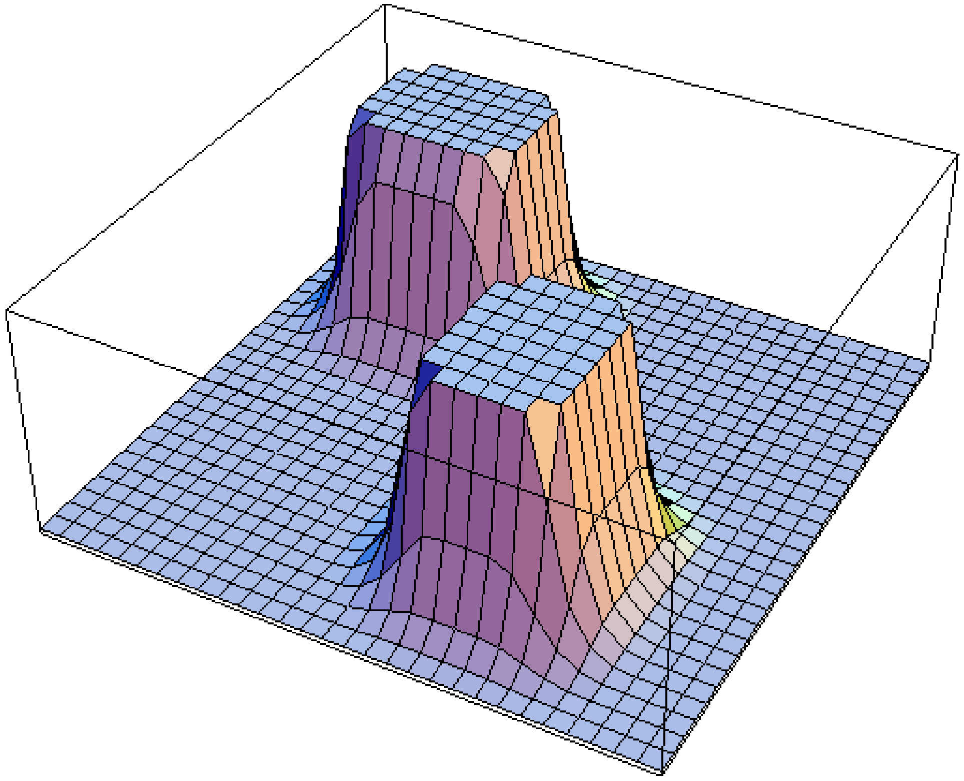


$$U_{\text{rep}}(q) = \begin{cases} \frac{1}{2}\eta(\frac{1}{D(q)} - \frac{1}{Q^*})^2, & D(q) \le Q^*, \\ 0, & D(q) > Q^*, \end{cases}$$

whose gradient is

$$\nabla U_{\text{rep}}(q) = \begin{cases} \eta\left(\frac{1}{Q^*} - \frac{1}{D(q)}\right)\frac{1}{D^2(q)}\nabla D(q), & D(q) \le Q^*, \\ 0, & D(q) > Q^*, \end{cases}$$

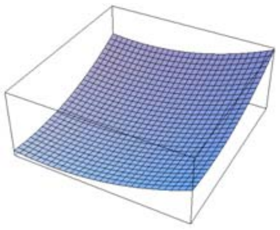
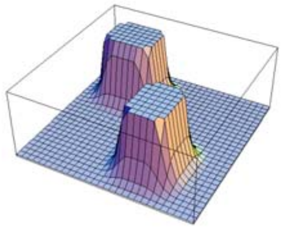
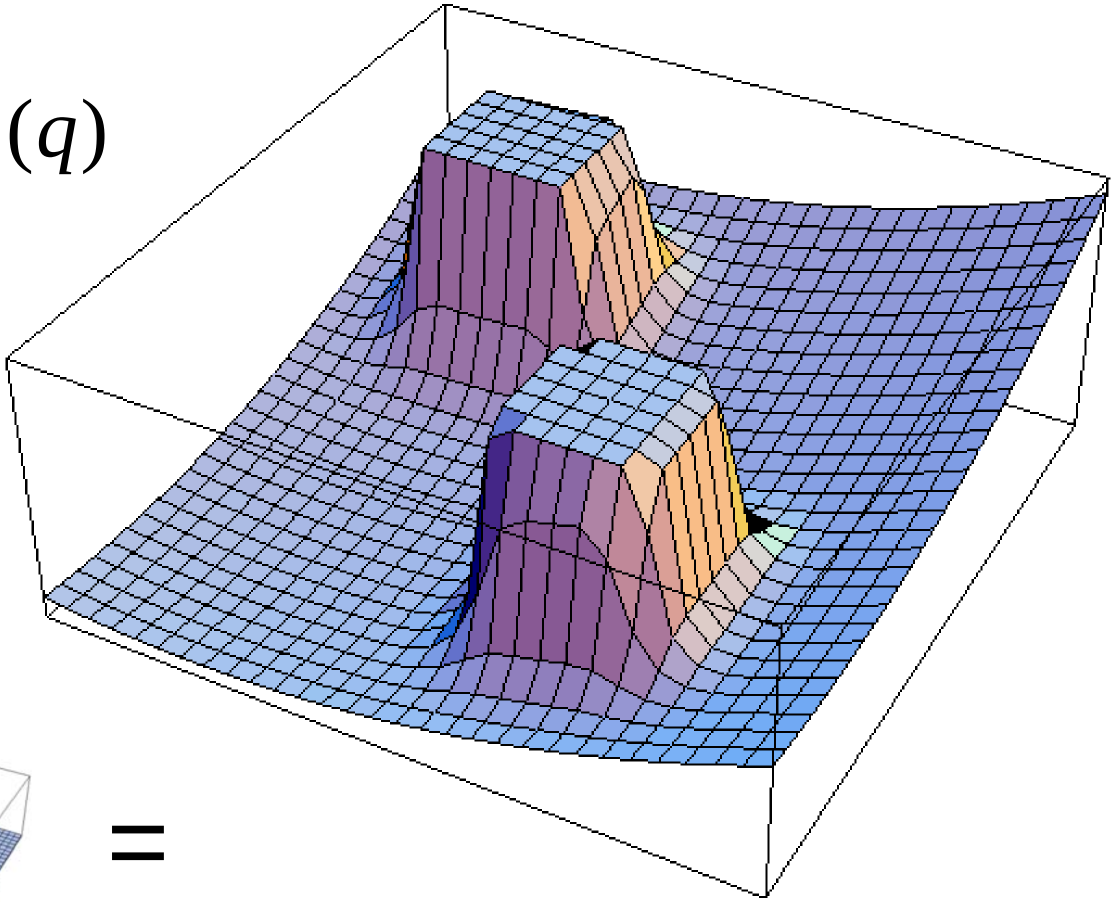
# Repulsive Potential



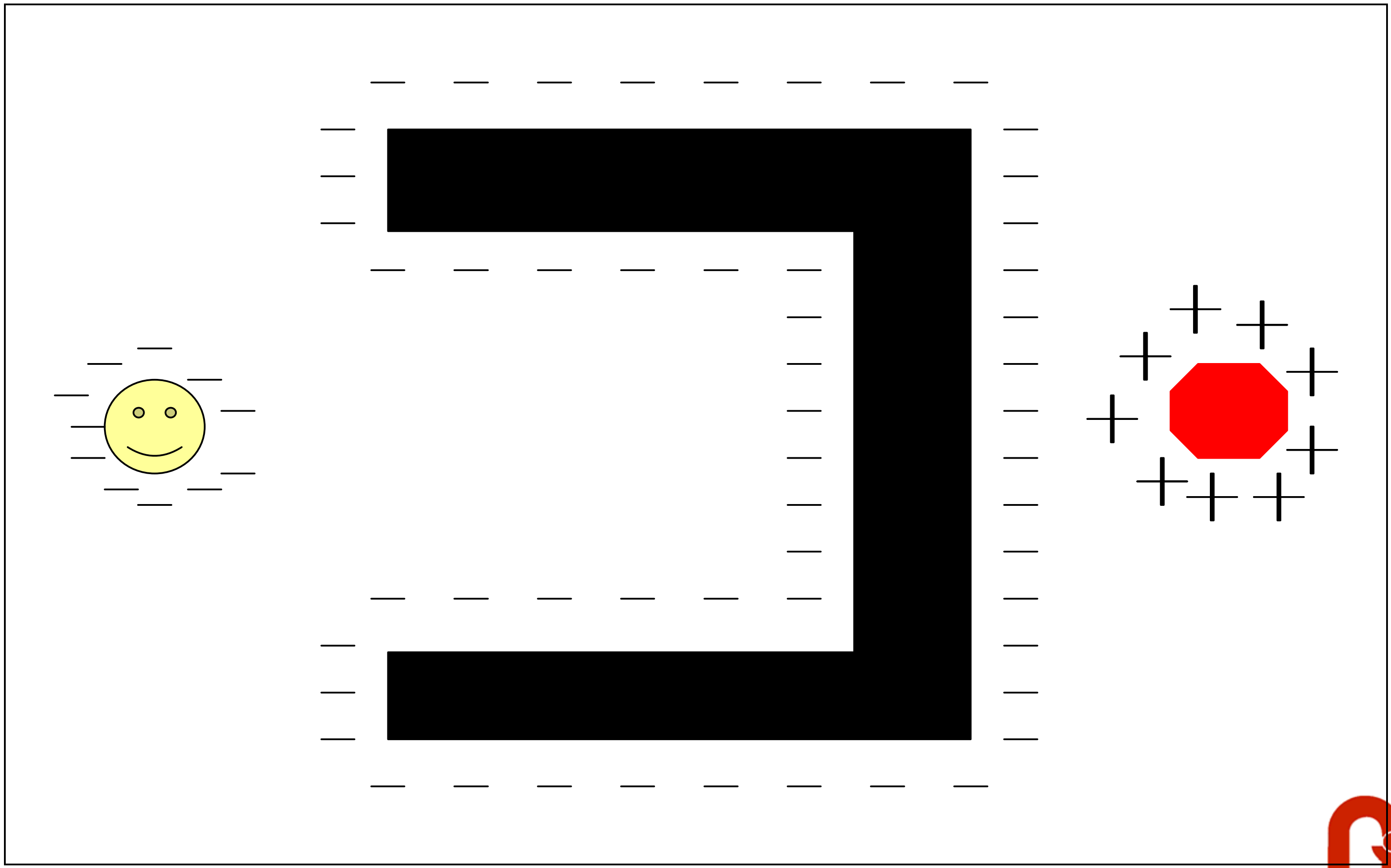
# Total Potential Function

$$U(q) = U_{\text{att}}(q) + U_{\text{rep}}(q)$$

$$F(q) = -\nabla U(q)$$

 +  = 

# Local Minimum Problem with the Charge Analogy

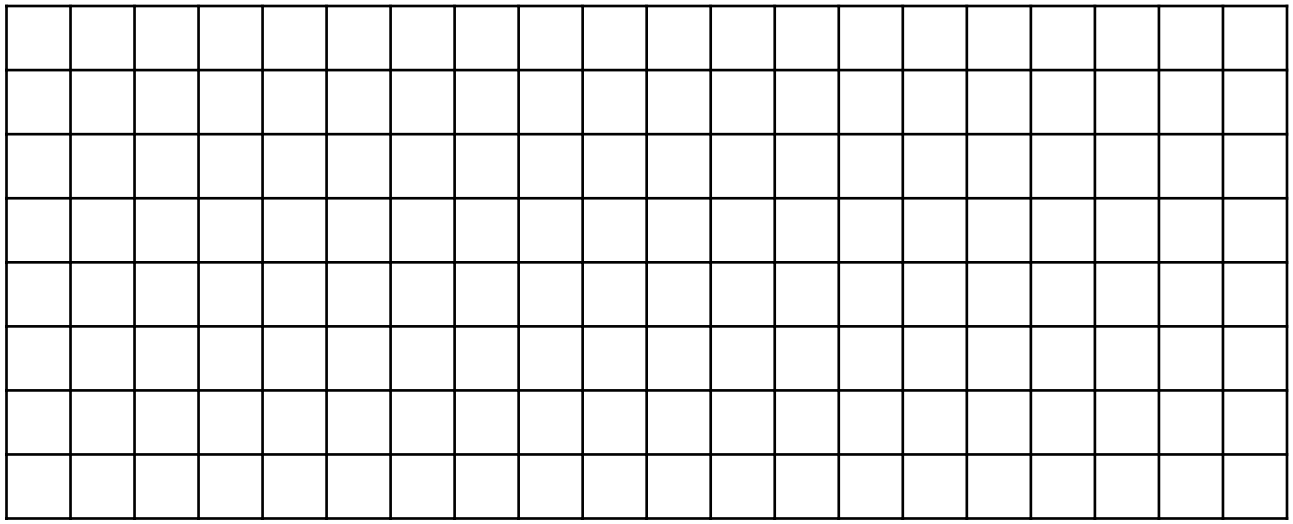


## The Wavefront Planner

- A common algorithm used to determine the shortest paths between two points
  - In essence, a breadth first search of a graph
- For simplification, we'll present the world as a two-dimensional grid
- Setup:
  - Label free space with 0
  - Label start as START
  - Label the destination as 2

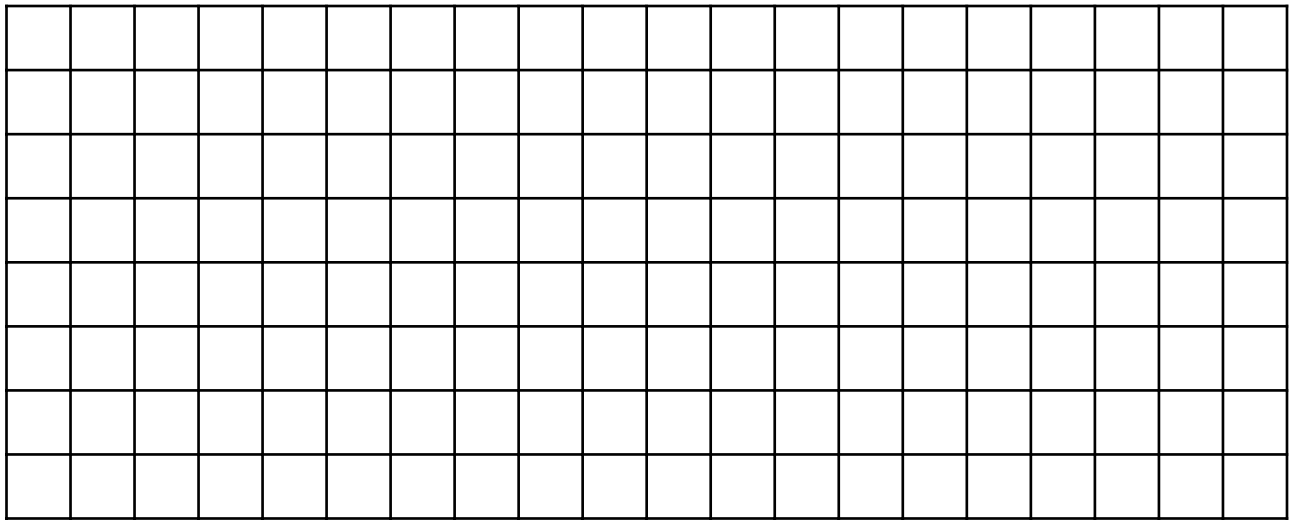
## Representations

- World Representation
  - You could always use a large region and distances
  - However, a grid can be used for simplicity



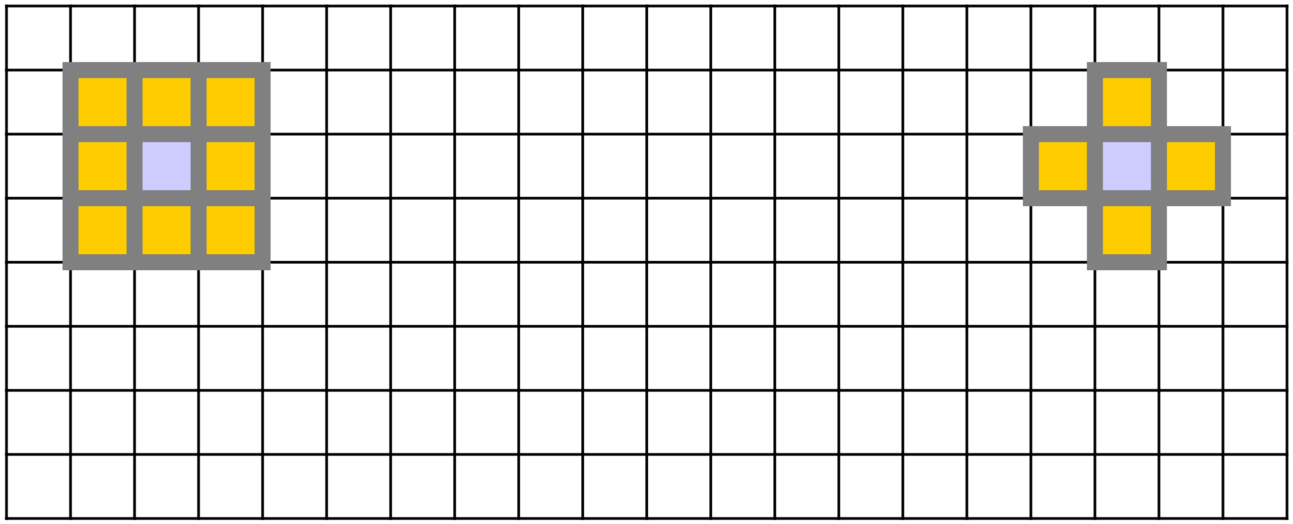
## Representations: A Grid

- Distance is reduced to discrete steps
  - For simplicity, we’ll assume distance is uniform
- Direction is now limited from one adjacent cell to another
  - Time to revisit Connectivity (Remember Vision?)

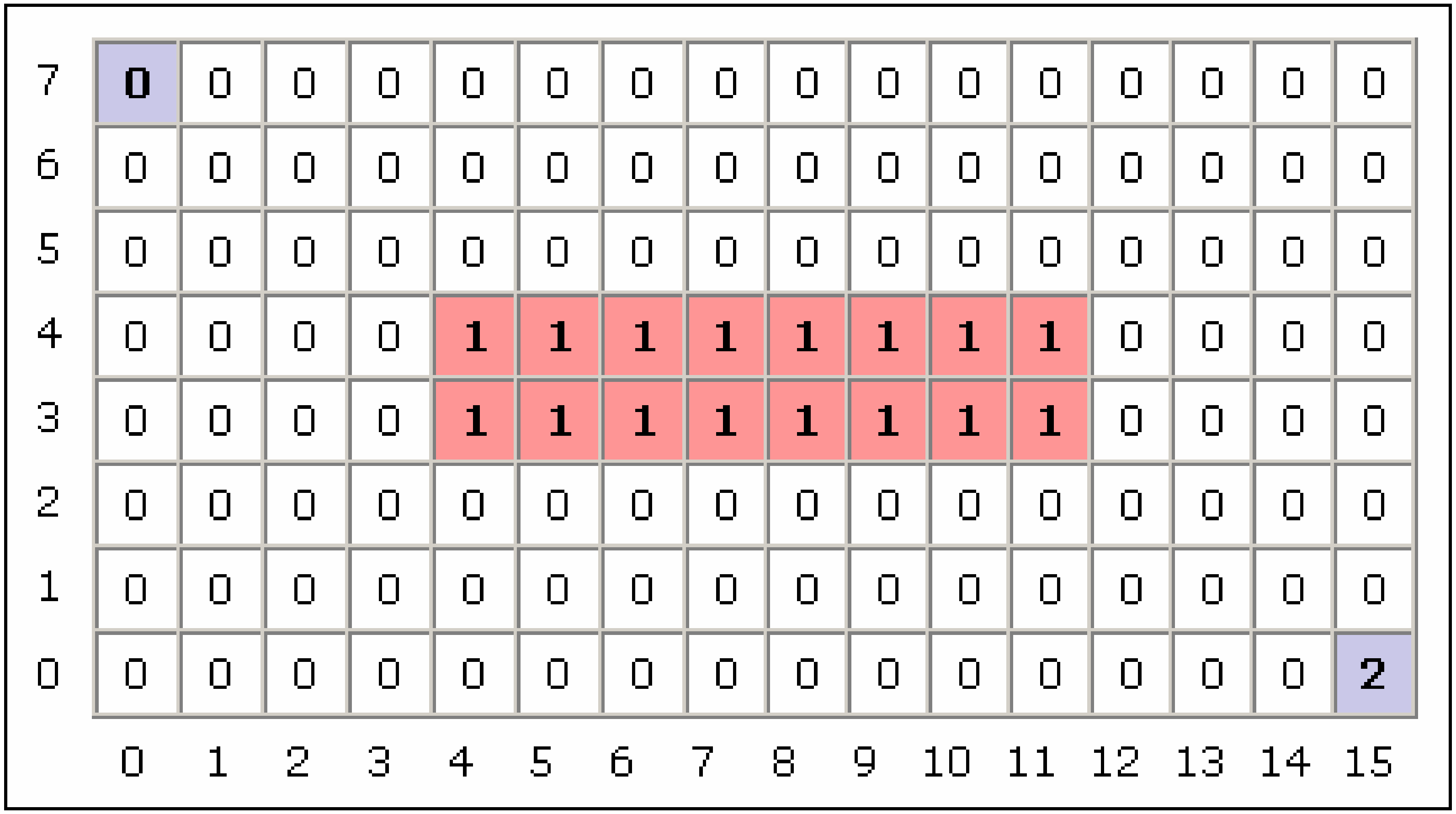


# Representations: Connectivity

- 8-Point Connectivity
- 4-Point Connectivity
  - *(approximation of the L1 metric)*



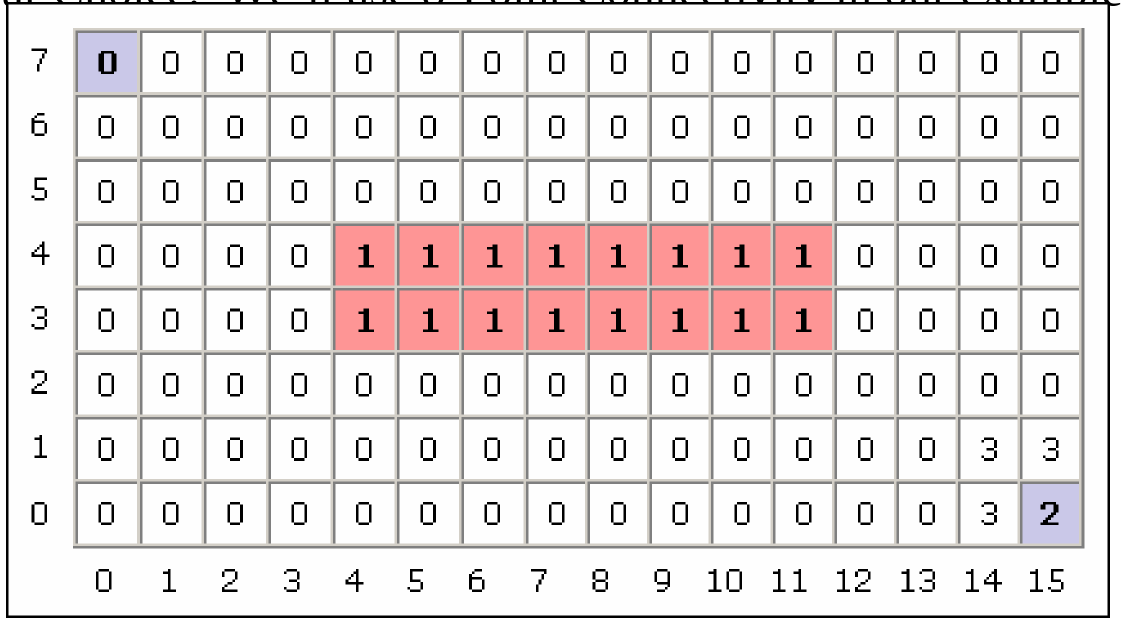
# The Wavefront Planner: Setup



| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |

# The Wavefront in Action (Part 1)

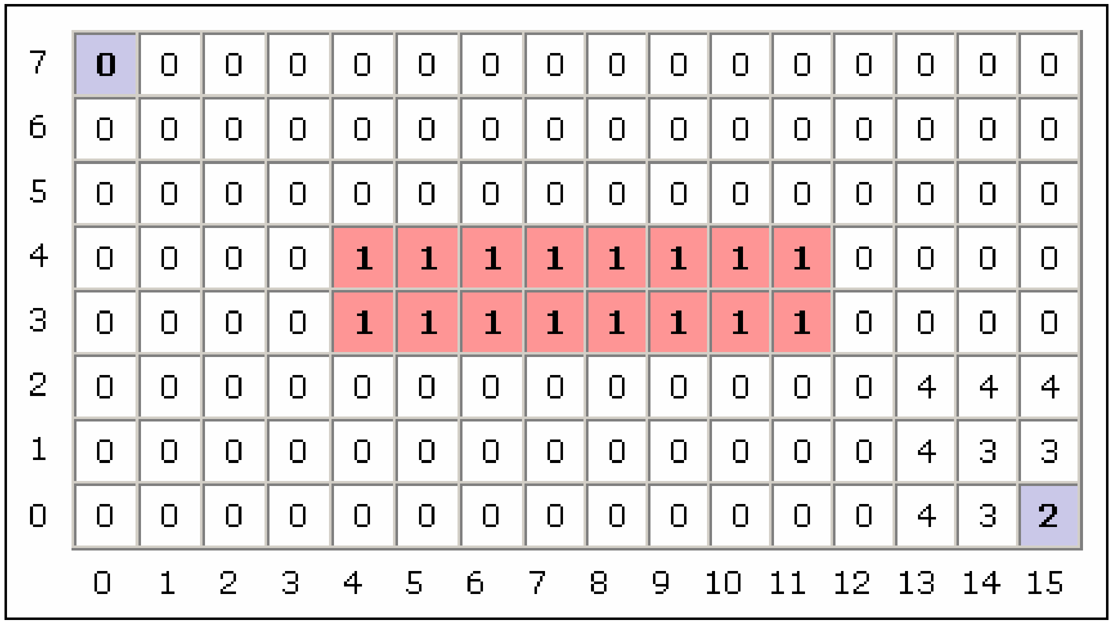
- Starting with the goal, set all adjacent cells with "0" to the current cell + 1
  - 4-Point Connectivity or 8-Point Connectivity?
  - Your Choice. We'll use 8-Point Connectivity in our example



| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 3 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | **2** |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |

## The Wavefront in Action (Part 2)

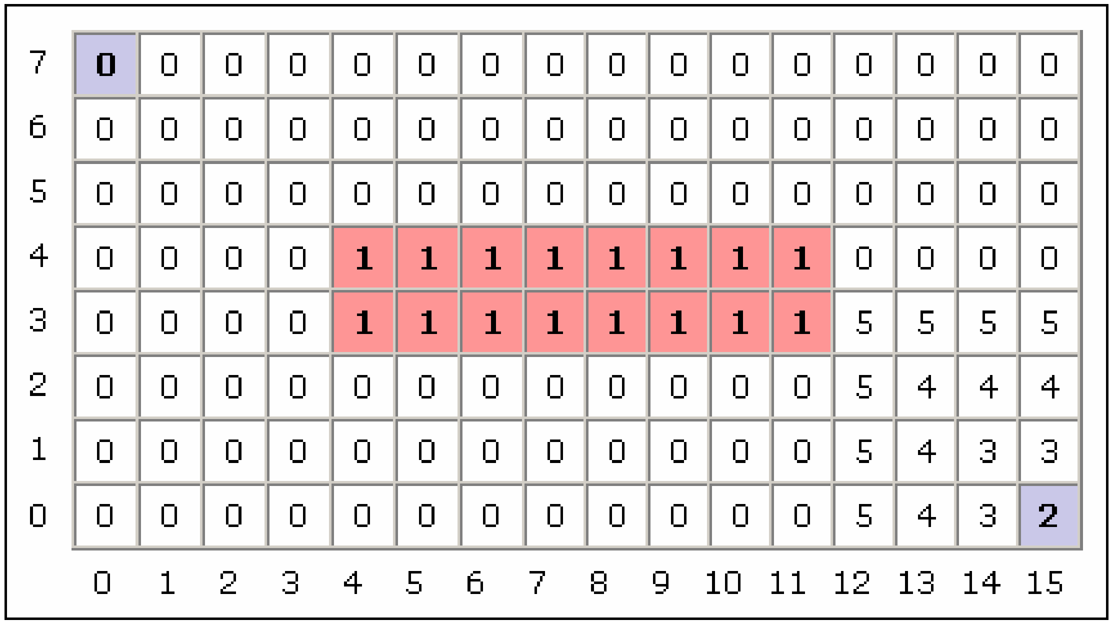
- Now repeat with the modified cells
  - This will be repeated until no 0’s are adjacent to cells with values >= 2
    - 0’s will only remain when regions are unreachable



| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 4 | 4 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 3 | 3 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 3 | **2** |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |

# The Wavefront in Action (Part 3)

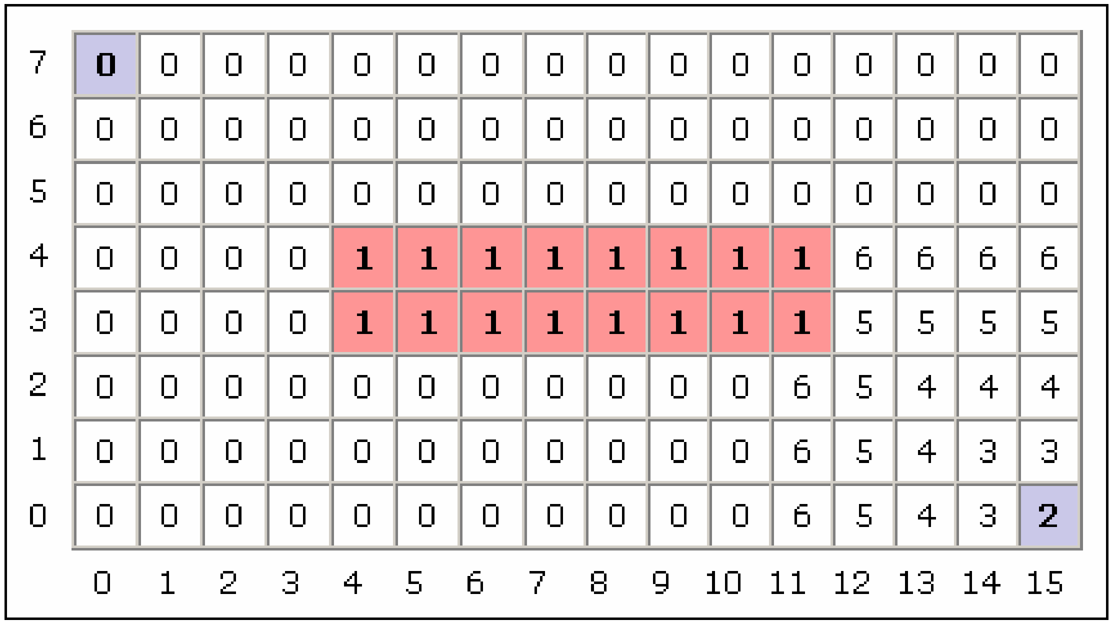
- Repeat again...



| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 5 | 5 | 5 | 5 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 4 | 4 | 4 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 4 | 3 | 3 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 4 | 3 | **2** |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |

# The Wavefront in Action (Part 4)

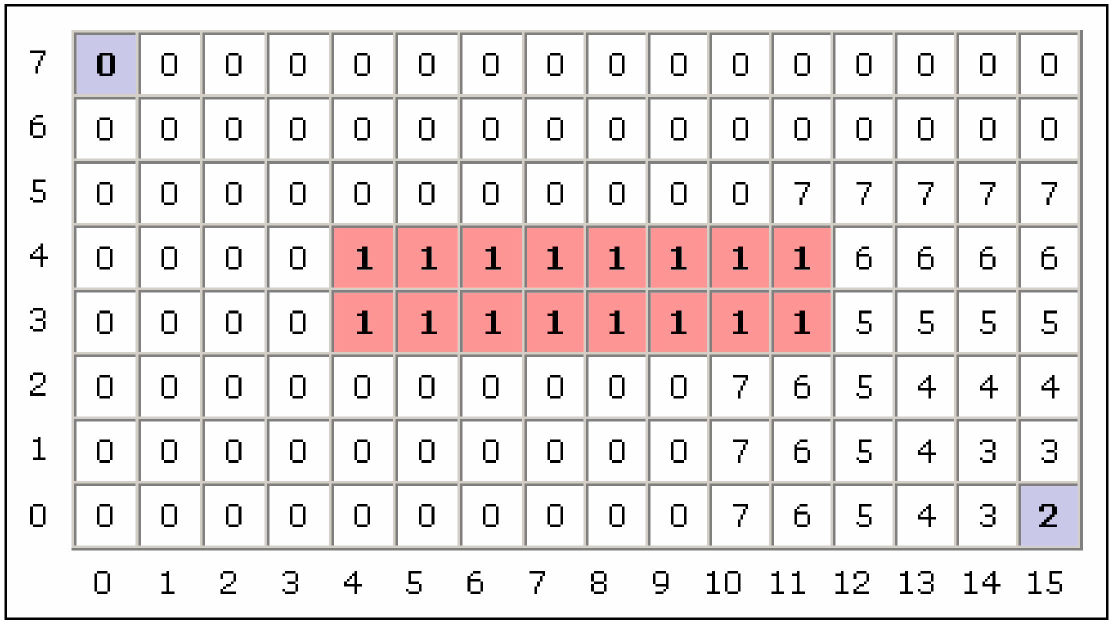
- And again…



| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 6 | 6 | 6 | 6 |
| 3 | 0 | 0 | 0 | 0 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 5 | 5 | 5 | 5 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 5 | 4 | 4 | 4 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 5 | 4 | 3 | 3 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 5 | 4 | 3 | **2** |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |

# The Wavefront in Action (Part 5)

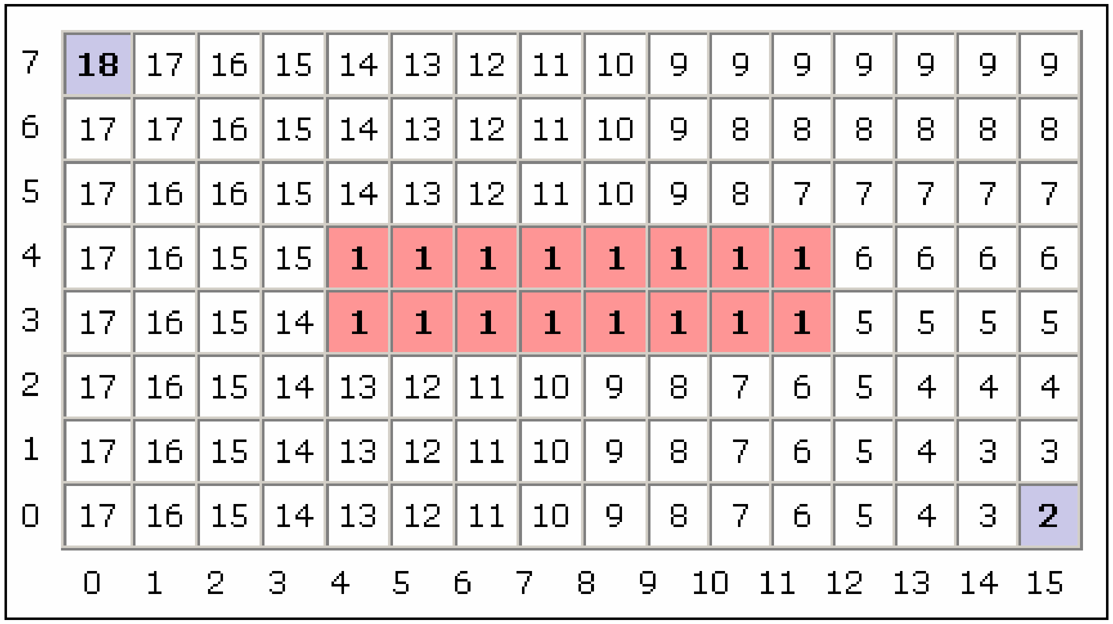
- And again until…



|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 7 | 7 | 7 | 7 |
| 4 | 0 | 0 | 0 | 0 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 6 | 6 | 6 | 6 |
| 3 | 0 | 0 | 0 | 0 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 5 | 5 | 5 | 5 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 6 | 5 | 4 | 4 | 4 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 6 | 5 | 4 | 3 | 3 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 6 | 5 | 4 | 3 | **2** |

# The Wavefront in Action (Done)

- You’re done
  - Remember, 0’s should only remain if unreachable regions exist

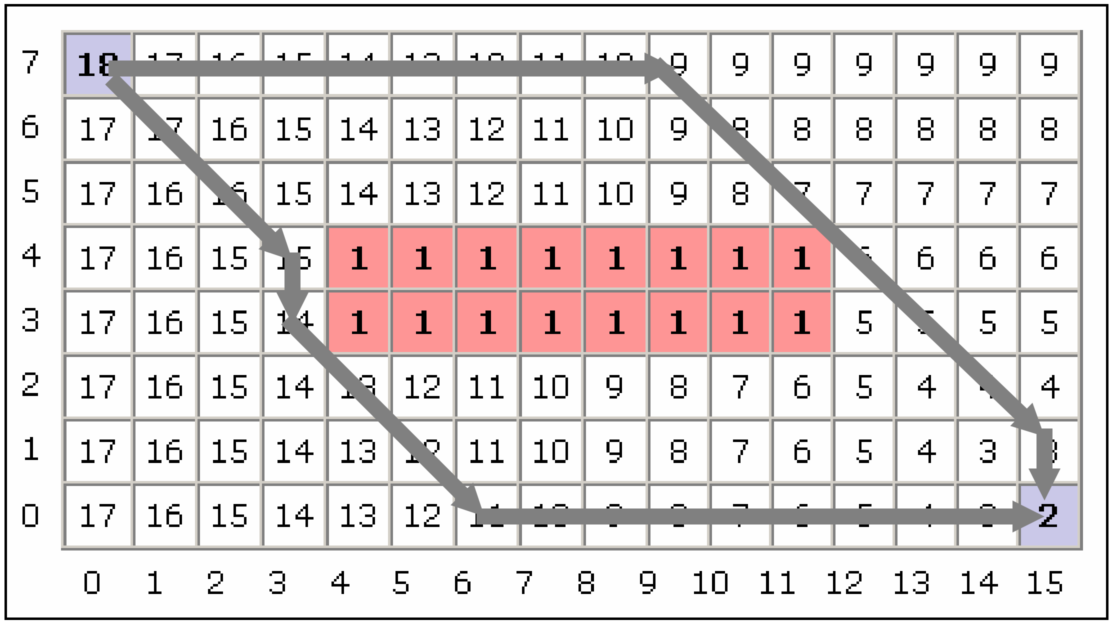


| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | **18** | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 6 | 17 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 8 | 8 | 8 | 8 | 8 |
| 5 | 17 | 16 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 7 | 7 | 7 | 7 |
| 4 | 17 | 16 | 15 | 15 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 6 | 6 | 6 | 6 |
| 3 | 17 | 16 | 15 | 14 | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | 5 | 5 | 5 | 5 |
| 2 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 4 |
| 1 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 |
| 0 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | **2** |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |

# The Wavefront, Now What?

- To find the shortest path, according to your metric, simply always move toward a cell with a lower number
  - The numbers generated by the Wavefront planner are roughly proportional to their distance from the goal

Two possible shortest paths shown



| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 6 | 17 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 8 | 8 | 8 | 8 | 8 |
| 5 | 17 | 16 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 7 | 7 | 7 | 7 |
| 4 | 17 | 16 | 15 | 15 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 6 | 6 | 6 | 6 |
| 3 | 17 | 16 | 15 | 14 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 5 | 5 | 5 | 5 |
| 2 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 4 | 4 |
| 1 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 3 |
| 0 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |

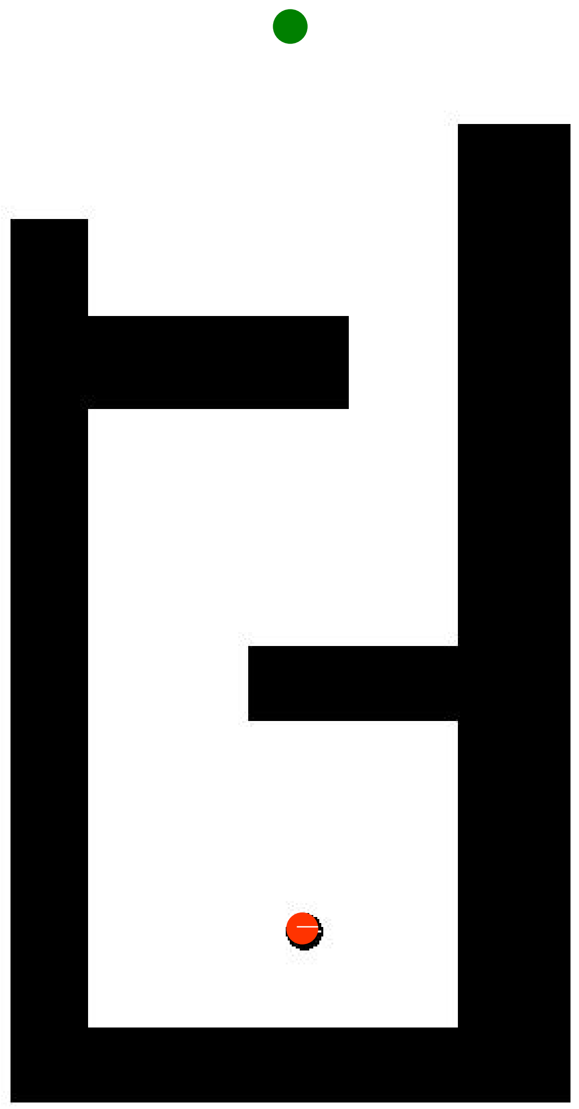
## Wavefront (Overview)

- Divide the space into a grid.
- Number the squares starting at the start in either 4 or 8 point connectivity starting at the goal, increasing till you reach the start.
- Your path is defined by any uninterrupted sequence of decreasing numbers that lead to the goal.

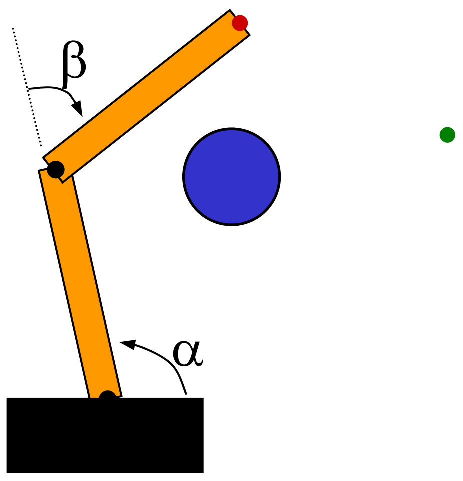
## Return to Configuration Spaces

- Non-Euclidean
- Non-Planar

# What if the robot is not a point?



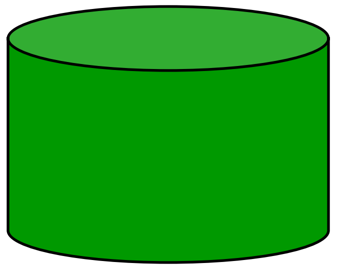
The Scout should probably not be modeled as a point...

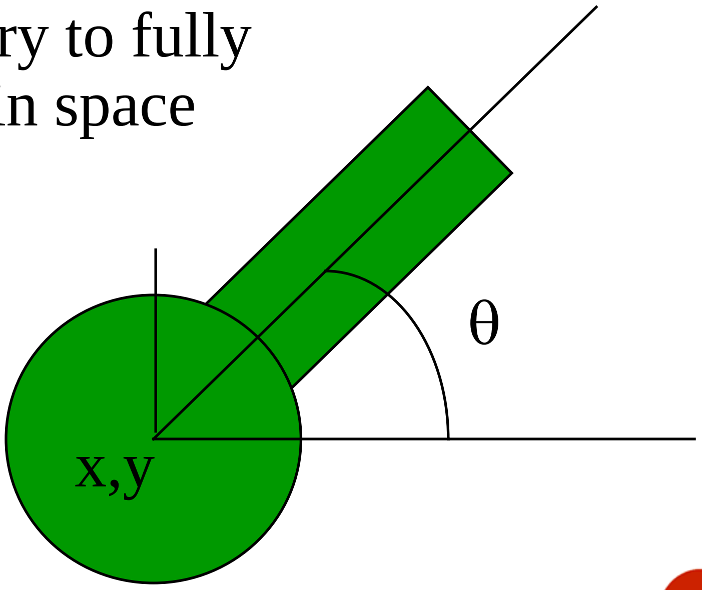


Nor should robots with extended linkages that may contact obstacles...

# Configuration Space: the robot has...

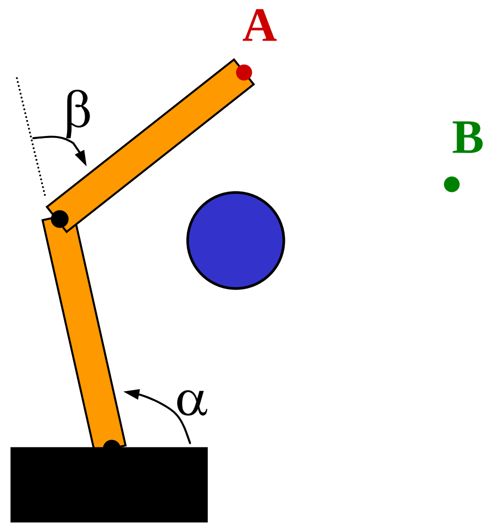
- A Footprint
  - The amount of space a robot occupies
- Degrees of Freedom
  - The number of variables necessary to fully describe a robot’s configuration in space
    - You’ll cover this more in depth later
    - *fun with non-holonomic constraints, etc*



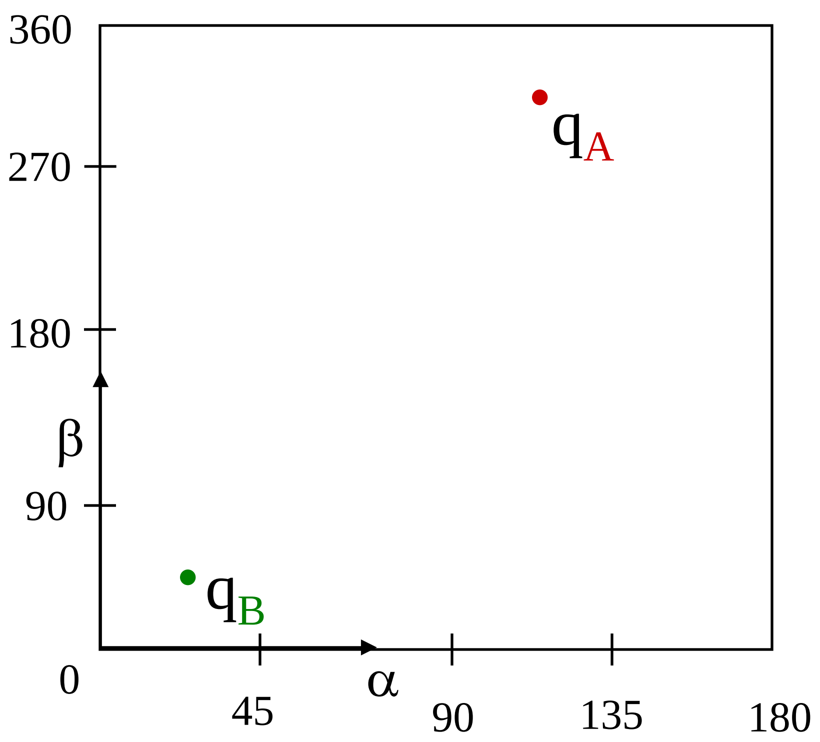


# Configuration Space "Quiz"

Where do we put ● ?



An obstacle in the robot's workspace



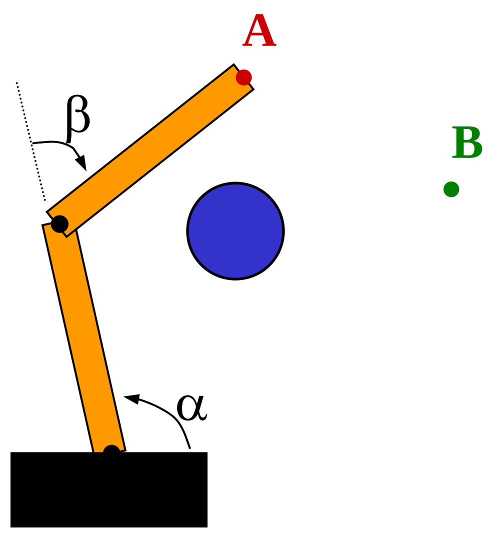
Torus

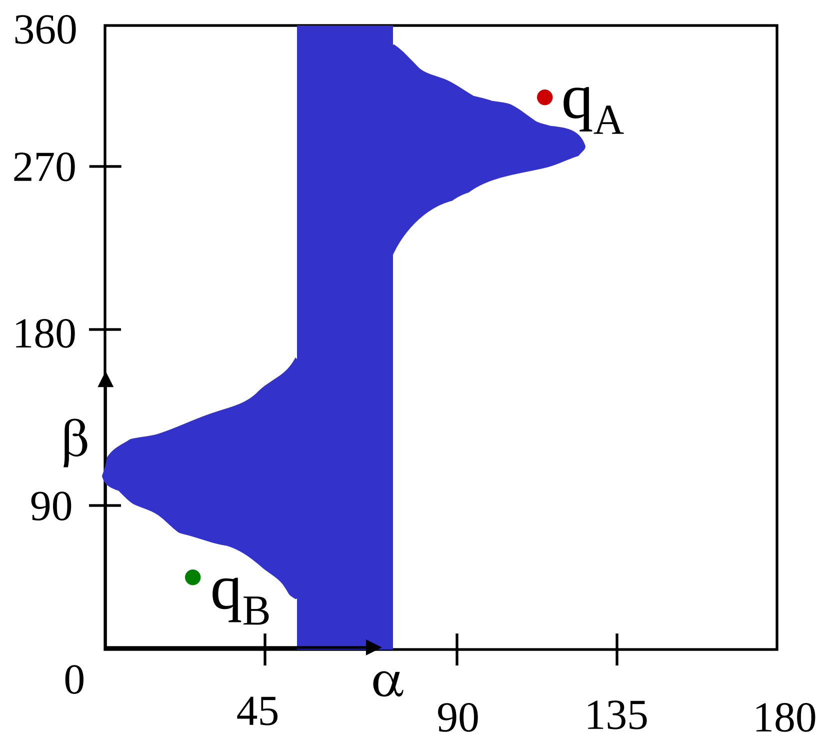
(wraps horizontally and vertically)

# Configuration Space Obstacle

Reference *configuration*

How do we get from A to B ?

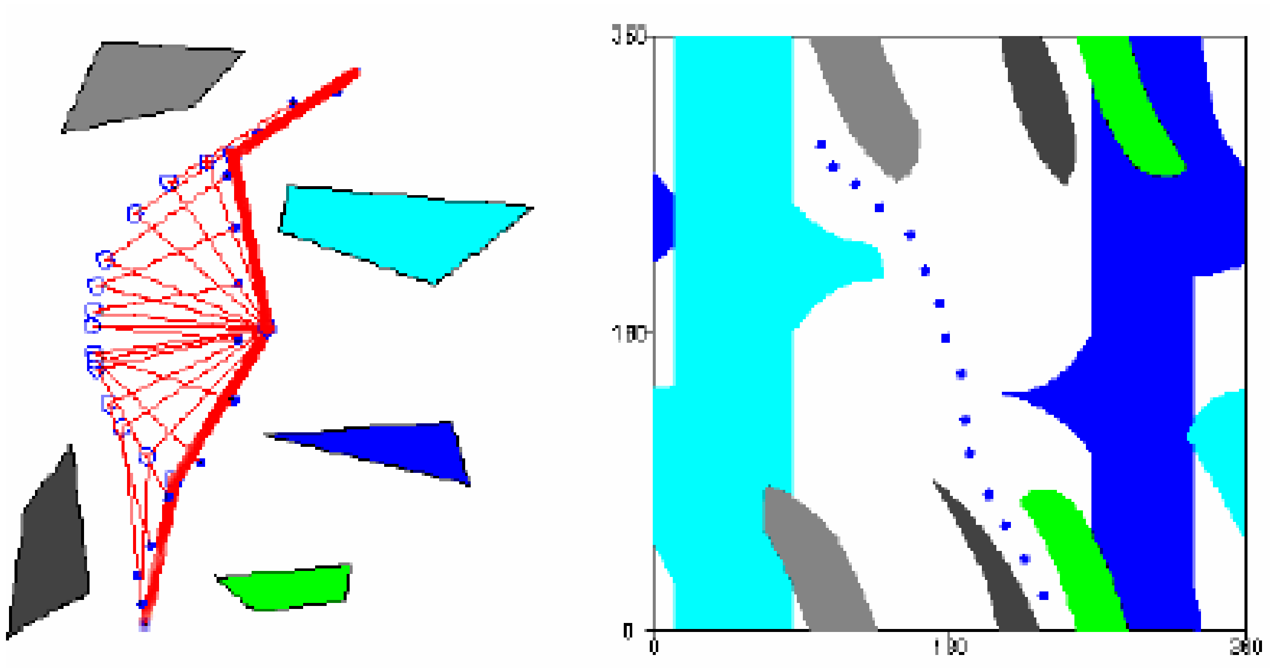




An obstacle in the robot's workspace

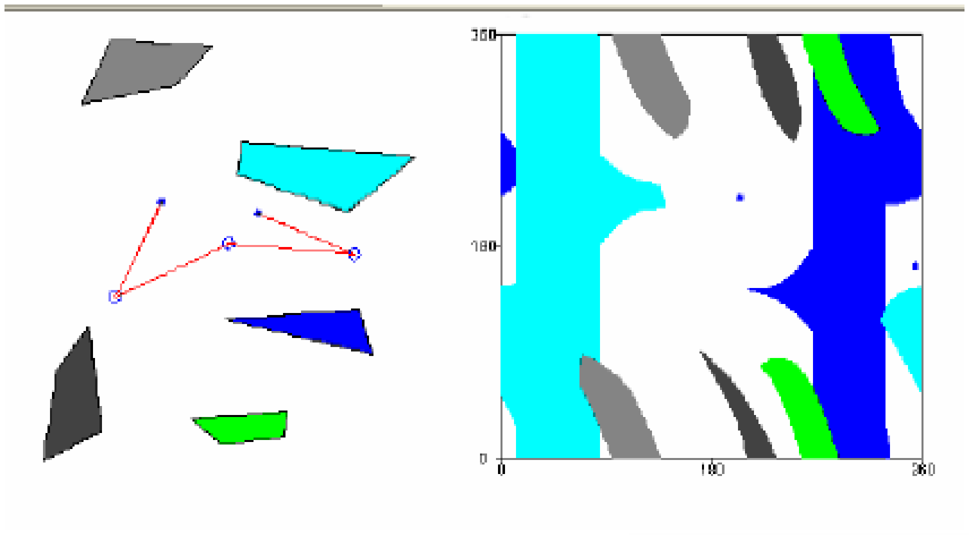
The C-space representation of this obstacle…

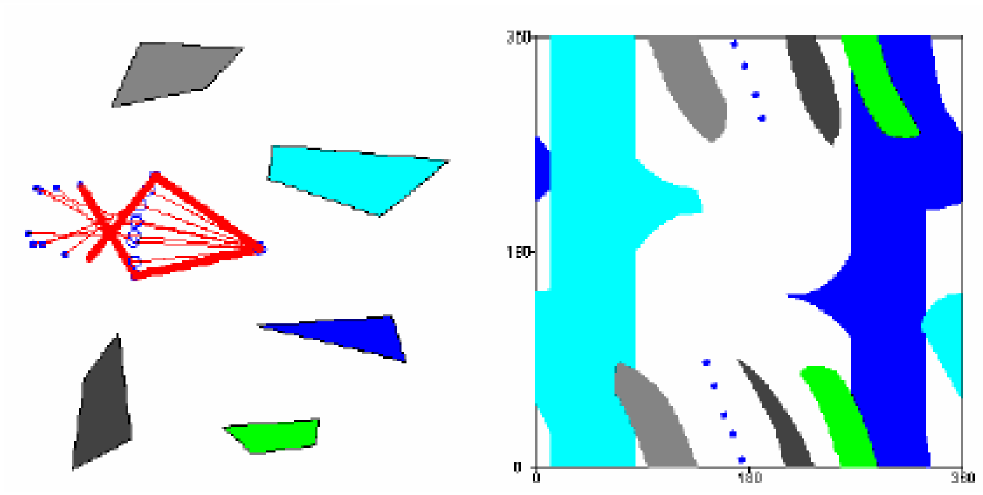
# Two Link Path



Thanks to Ken Goldberg

# Two Link Path





## More Example Configuration Spaces (contrasted with workspace)

- Free moving (no wheels) robot in plane:
  - workspace $\Re^2$
  - configuration space $\Re^2$

- 3-joint revolute arm in the plane
  - Workspace, a torus of outer radius L1 + L2 + L3
  - configuration space $T^3 = S^1 \times S^1 \times S^1$

- 2-joint revolute arm with a prismatic joint in the plane
  - workspace disc of radius L1 + L2 + L3
  - configuration space $T^2 \times \Re$

- 3-joint revolute arm mounted on a mobile robot (holonomic)
  - workspace is a "sandwich" of radius L1 + L2 + L3
  - $\Re^2 \times T^3$

- 3-joint revolute arm floating in space
  - workspace is $\Re^3$
  - configuration space is SE(3) x $T^3$

# Map-Based Approaches: Roadmap Theory

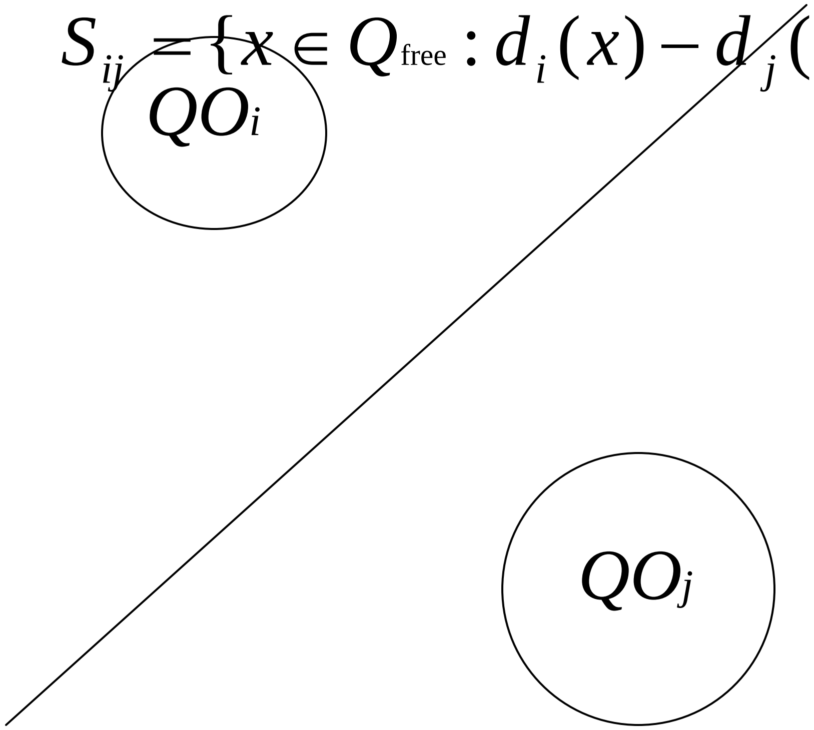
- Properties of a roadmap:
  - Accessibility: there exists a collision-free path from the start to the road map
  - Departability: there exists a collision-free path from the roadmap to the goal.
  - Connectivity: there exists a collision-free path from the start to the goal (on the roadmap).

- **a roadmap exists ⇔ a path exists**
- **Examples of Roadmaps**
  - **Generalized Voronoi Graph (GVG)**
  - **Visibility Graph**

## Two-Equidistant

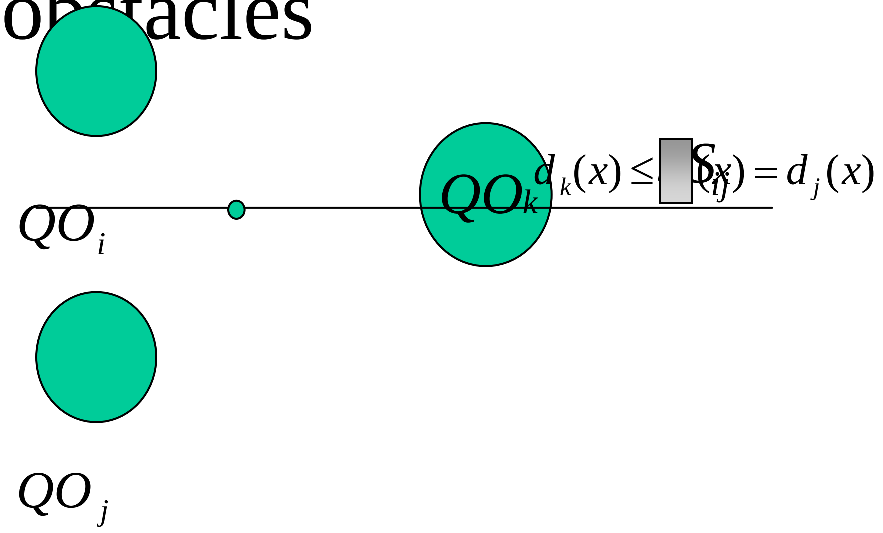
- *Two-equidistant surface*

$$S_{ij} = \{x \in Q_{\text{free}} : d_i(x) - d_j(x) = 0\}$$



# More Rigorous Definition

Going through obstacles

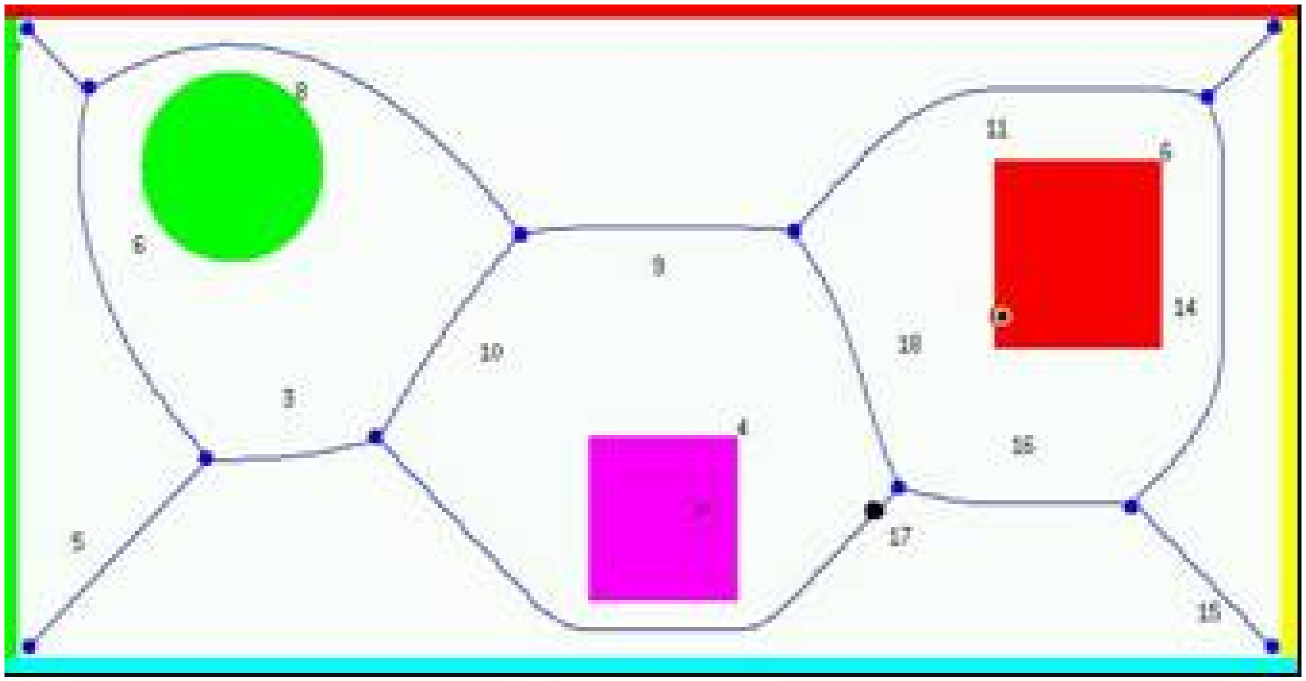


*Two-equidistant face*

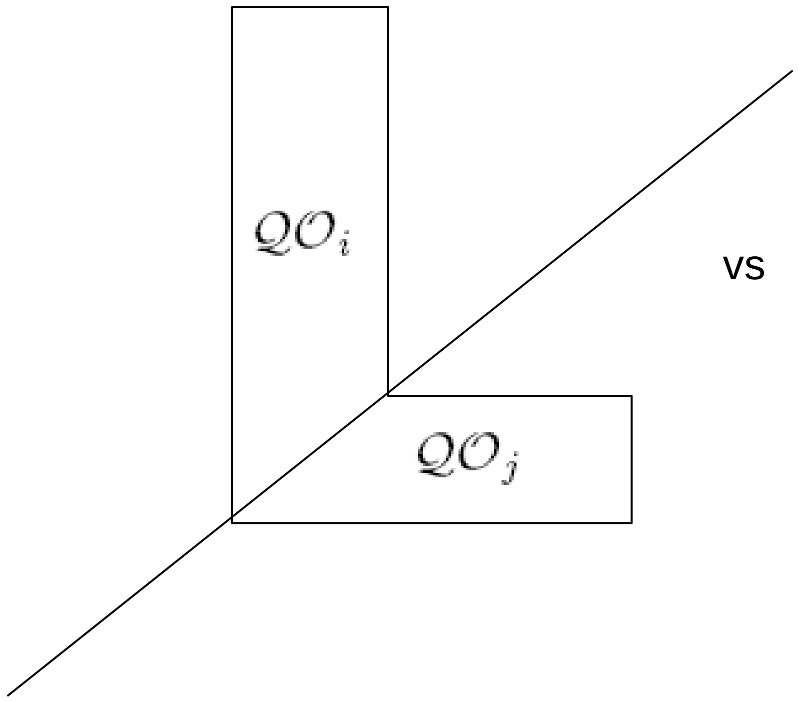
$$F_{ij} = \{x \in S_{ij} : d_i(x) = d_j(x) \leq d_h(x), \forall h \neq i, j\}$$

# General Voronoi Diagram

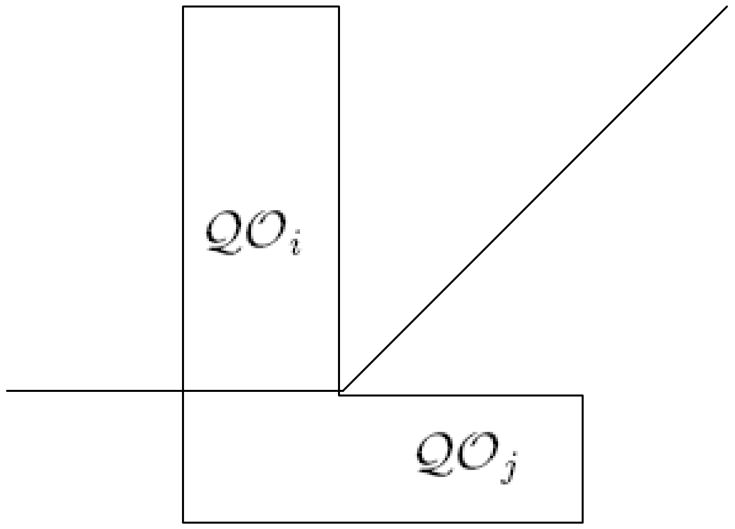
$$\mathrm{GVD} = \bigcup_{i=1}^{n-1} \bigcup_{j=i+1}^{n} F_{ij}$$



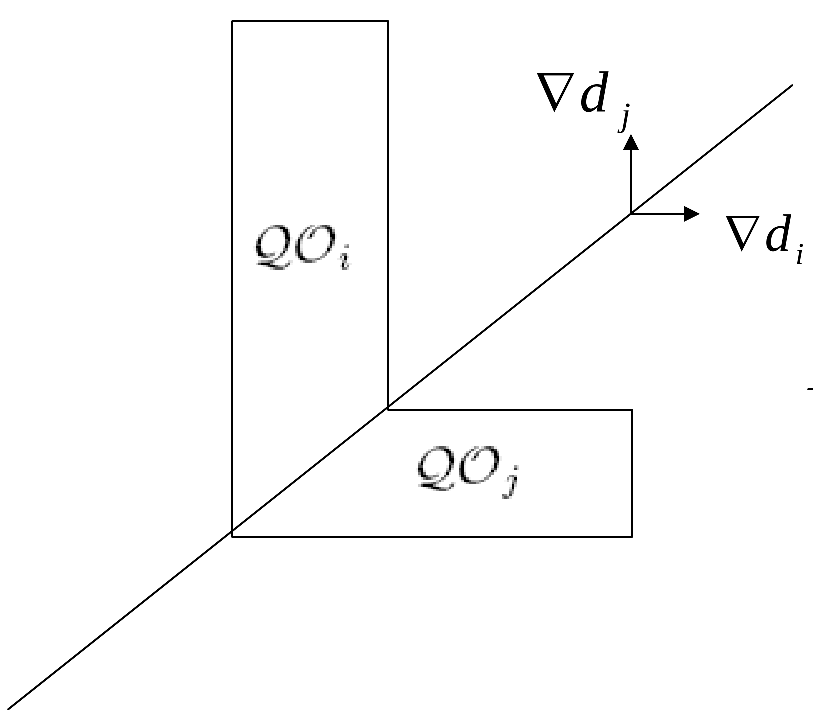
# What about concave obstacles?



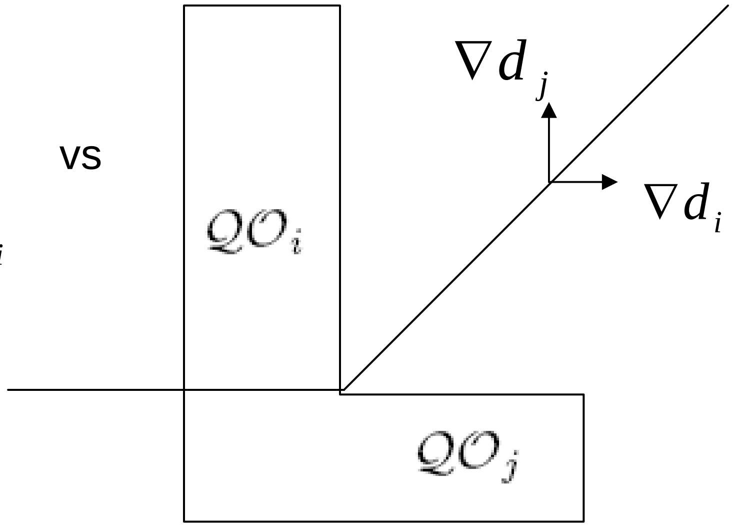
vs



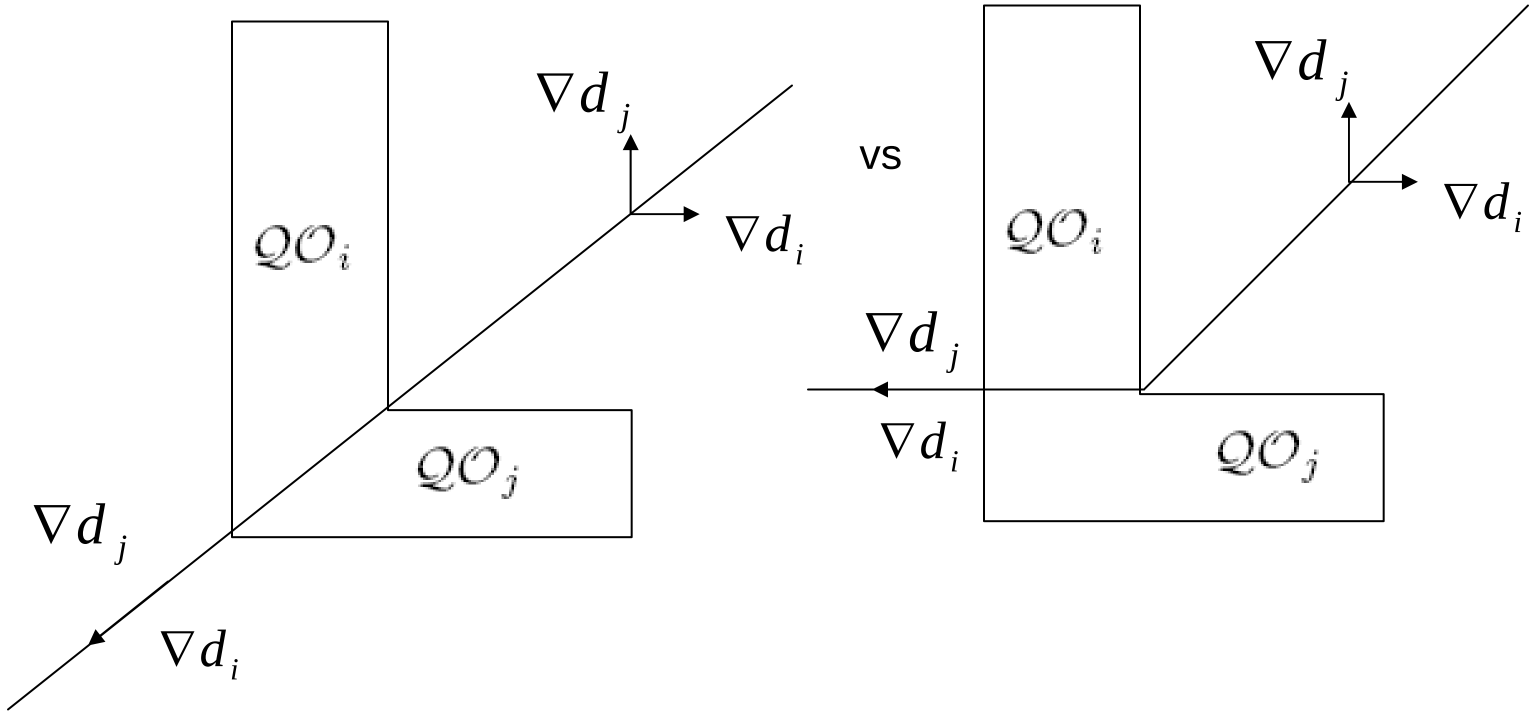
# What about concave obstacles?



vs



# What about concave obstacles?



## Two-Equidistant

- *Two-equidistant surface*

$$S_{ij} = \{x \in Q_{\text{free}} : d_i(x) - d_j(x) = 0\}$$

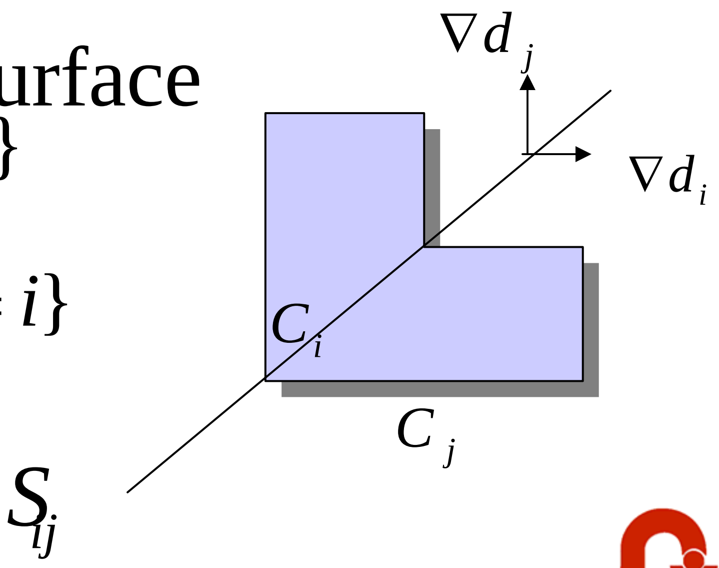
Two-equidistant surjective surface

$$SS_{ij} = \{x \in S_{ij} : \nabla d_i(x) \neq \nabla d_j(x)\}$$

$$F_{ij} = \{x \in SS_{ij} : d_i(x) \leq d_h(x), \forall h \neq i\}$$

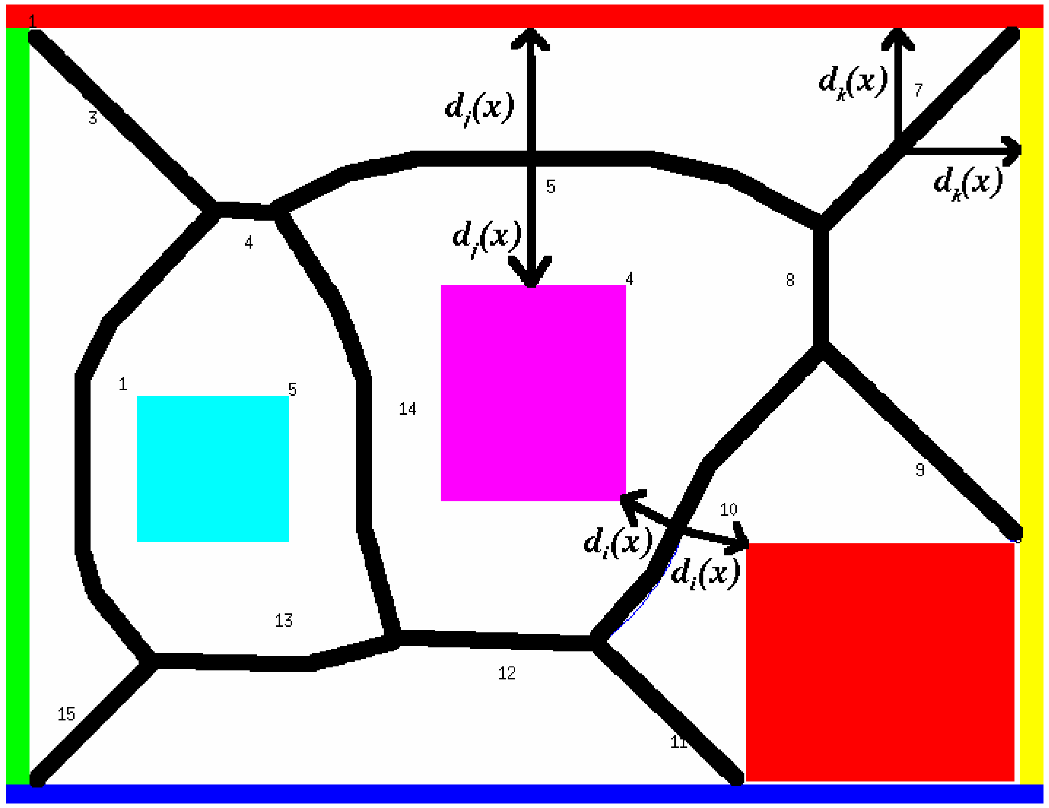
$$\text{GVD} = \bigcup_{i=1}^{n-1} \bigcup_{j=i+1}^{n} F_{ij}$$

Two-equidistant Face



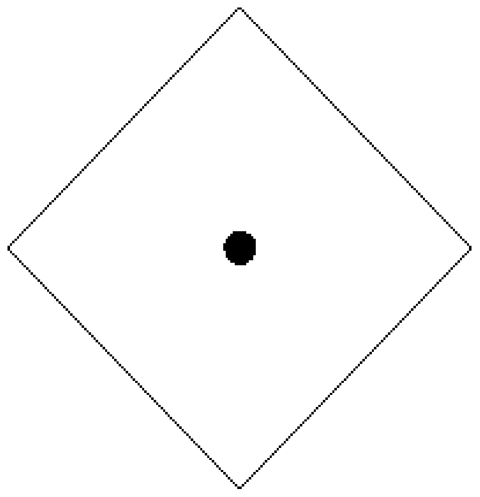
## Roadmap: GVG

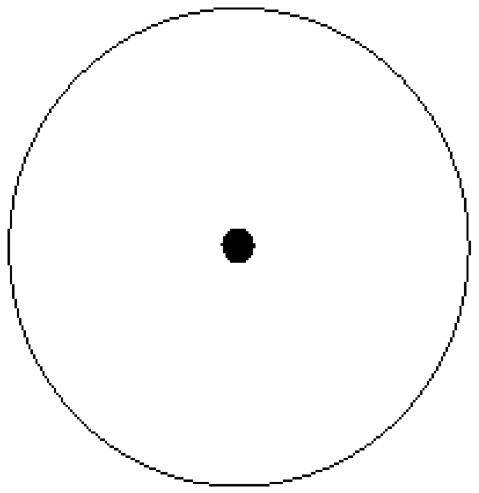
- A GVG is formed by paths equidistant from the two closest objects
- *Remember “spokes”, start and goal*



- This generates a very safe roadmap which avoids obstacles as much as possible

# Voronoi Diagram: Metrics





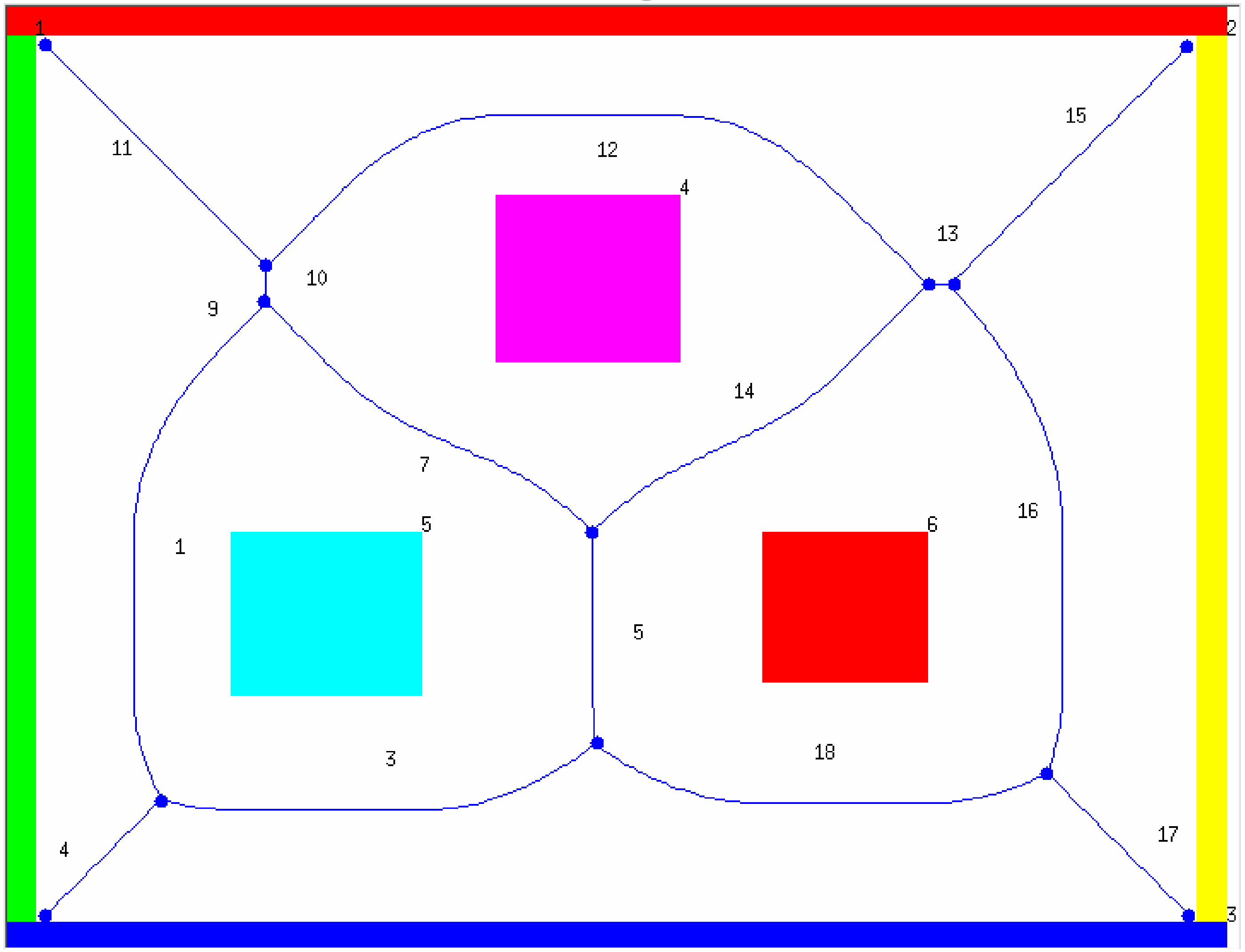
$\{(x,y) : |x| + |y| = \text{const}\}$

L1

$\{(x,y) : x^2 + y^2 = \text{const}\}$

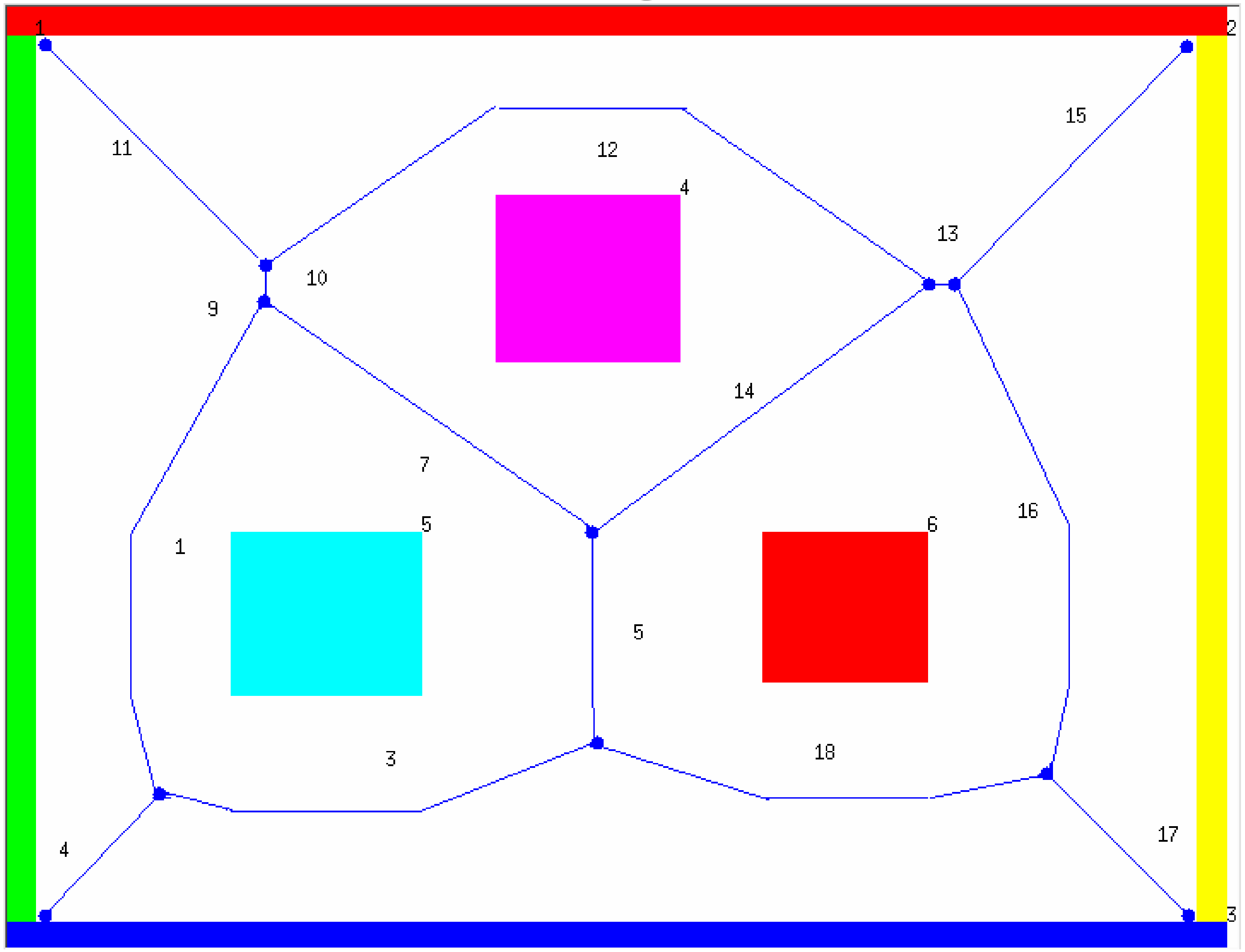
L2

# Voronoi Diagram (L2)



Note the curved edges

# Voronoi Diagram (L1)



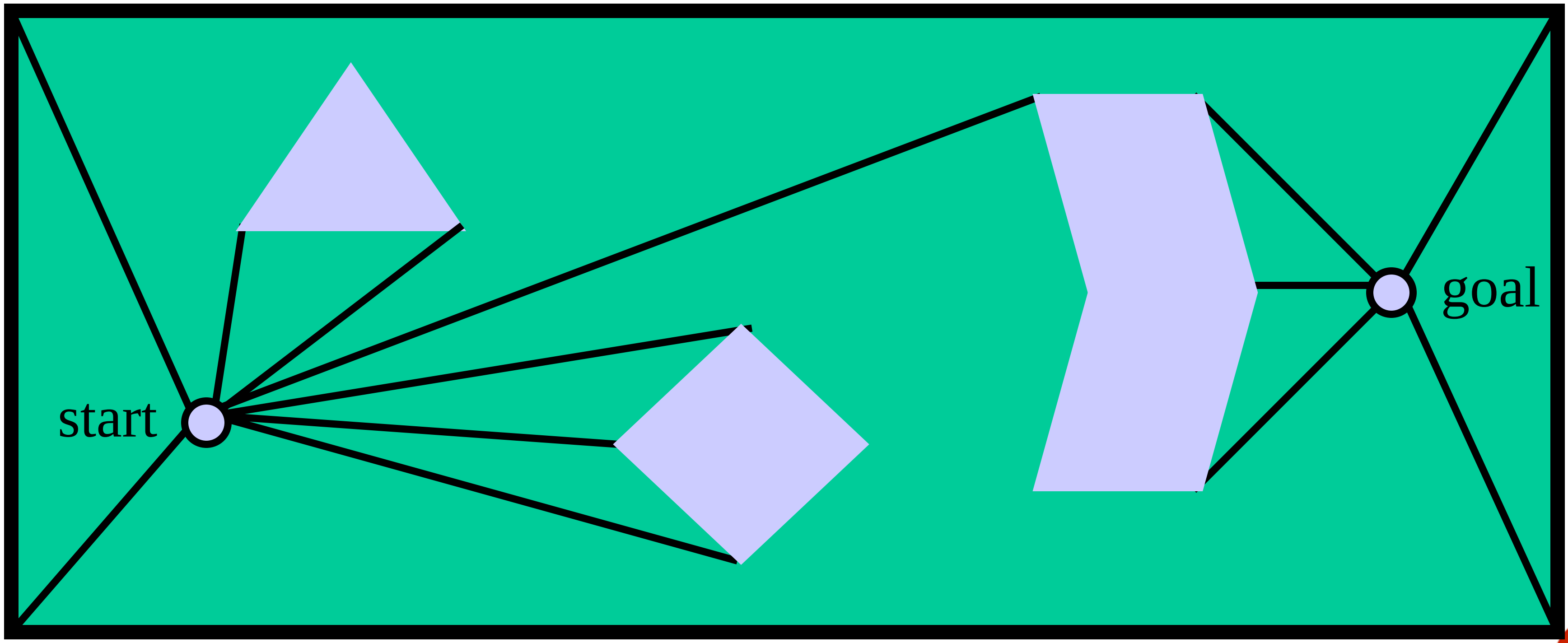
Note the lack of curved edges

## Roadmap: Visibility Graph

- Formed by connecting all “visible” vertices, the start point and the end point, to each other
- For two points to be “visible” no obstacle can exist between them
  - Paths exist on the perimeter of obstacles
- In our example, this produces the shortest path with respect to the L2 metric.  However, the close proximity of paths to obstacles makes it dangerous

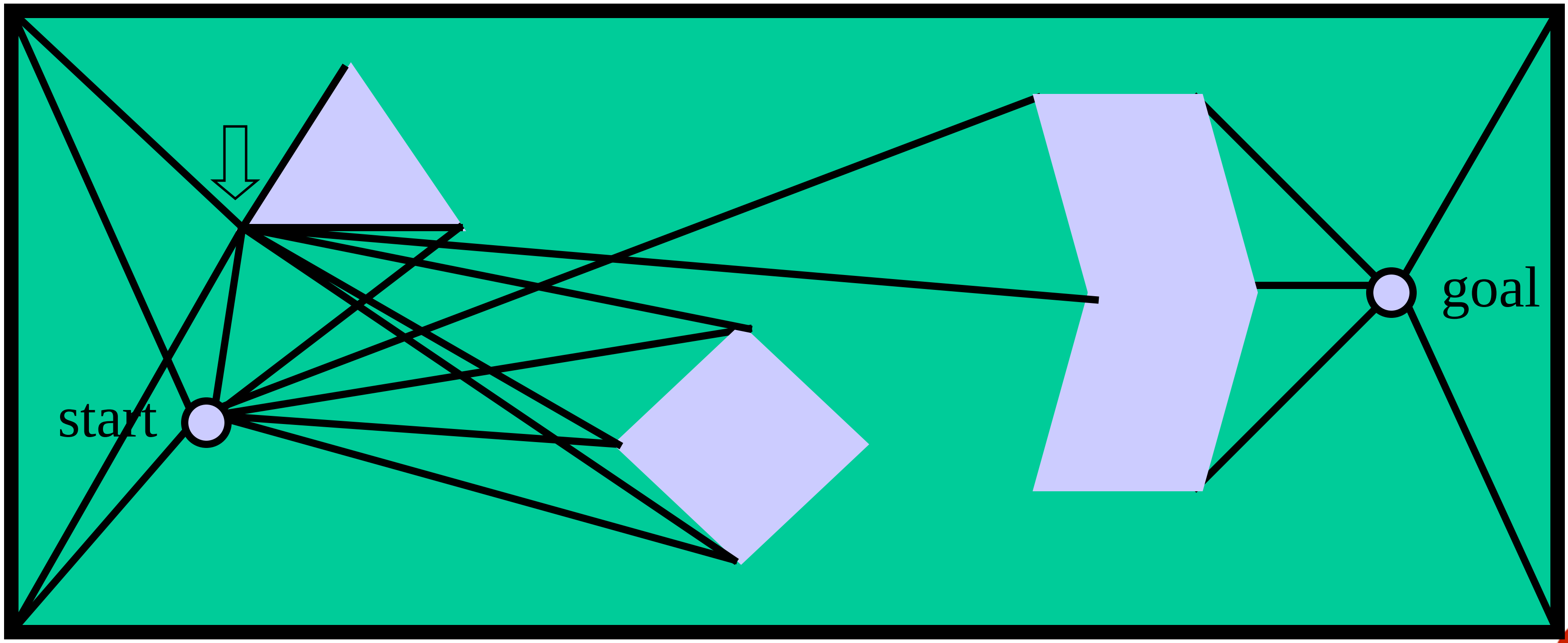
# The Visibility Graph in Action (Part 1)

- First, draw lines of sight from the start and goal to all "visible" vertices and corners of the world.



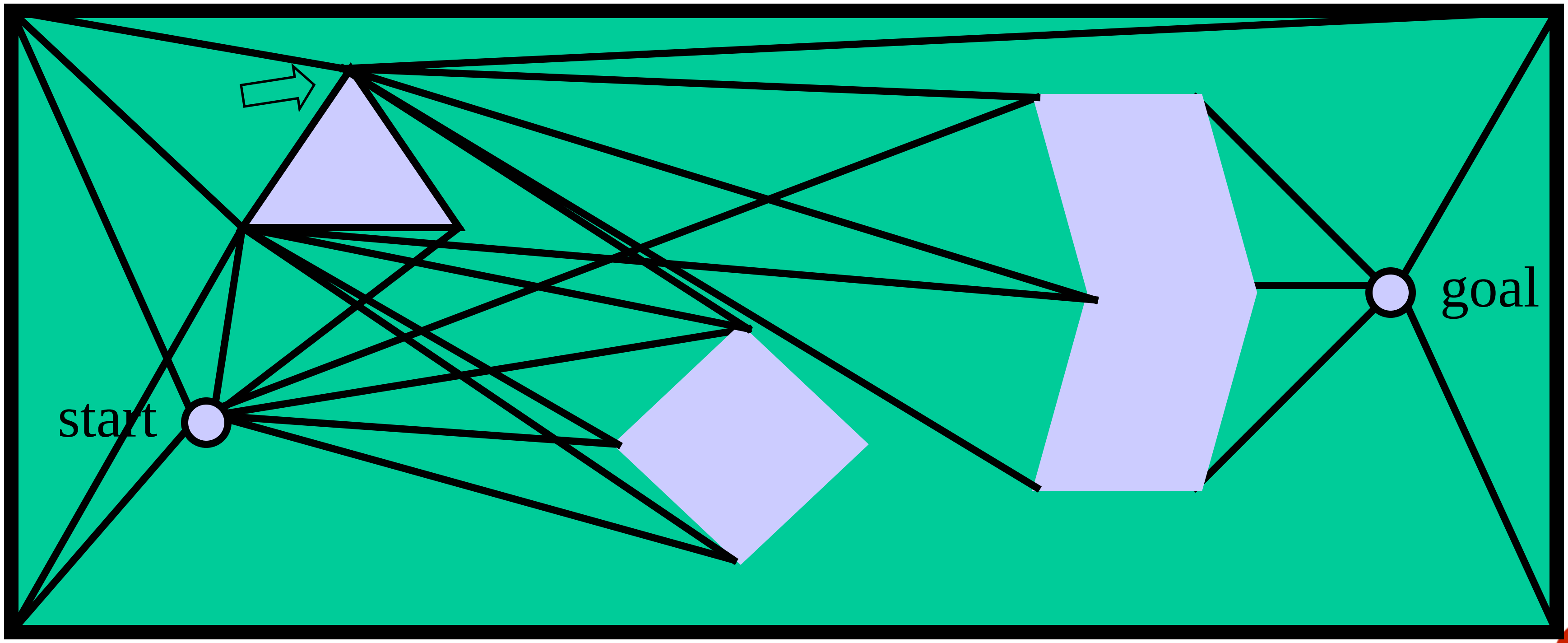
# The Visibility Graph in Action (Part 2)

- Second, draw lines of sight from every vertex of every obstacle like before. Remember lines along edges are also lines of sight.



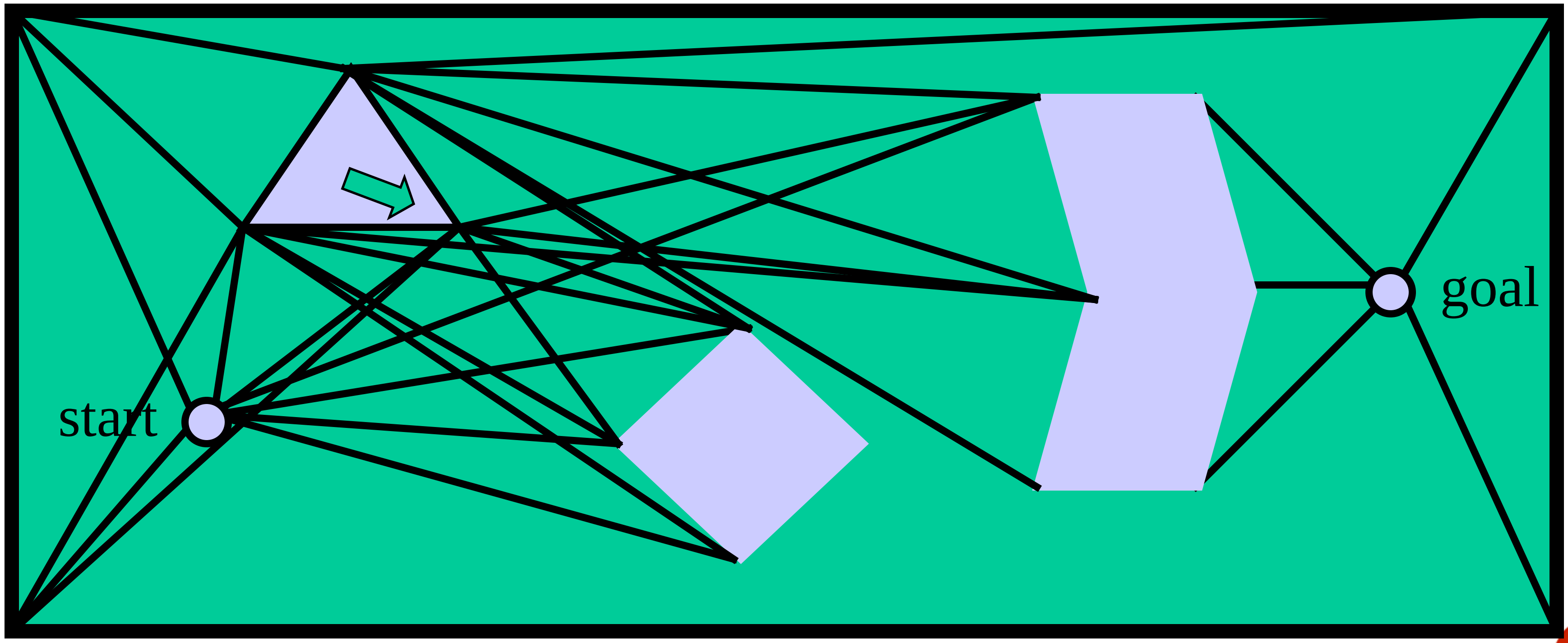
# The Visibility Graph in Action (Part 3)

- Second, draw lines of sight from every vertex of every obstacle like before. Remember lines along edges are also lines of sight.



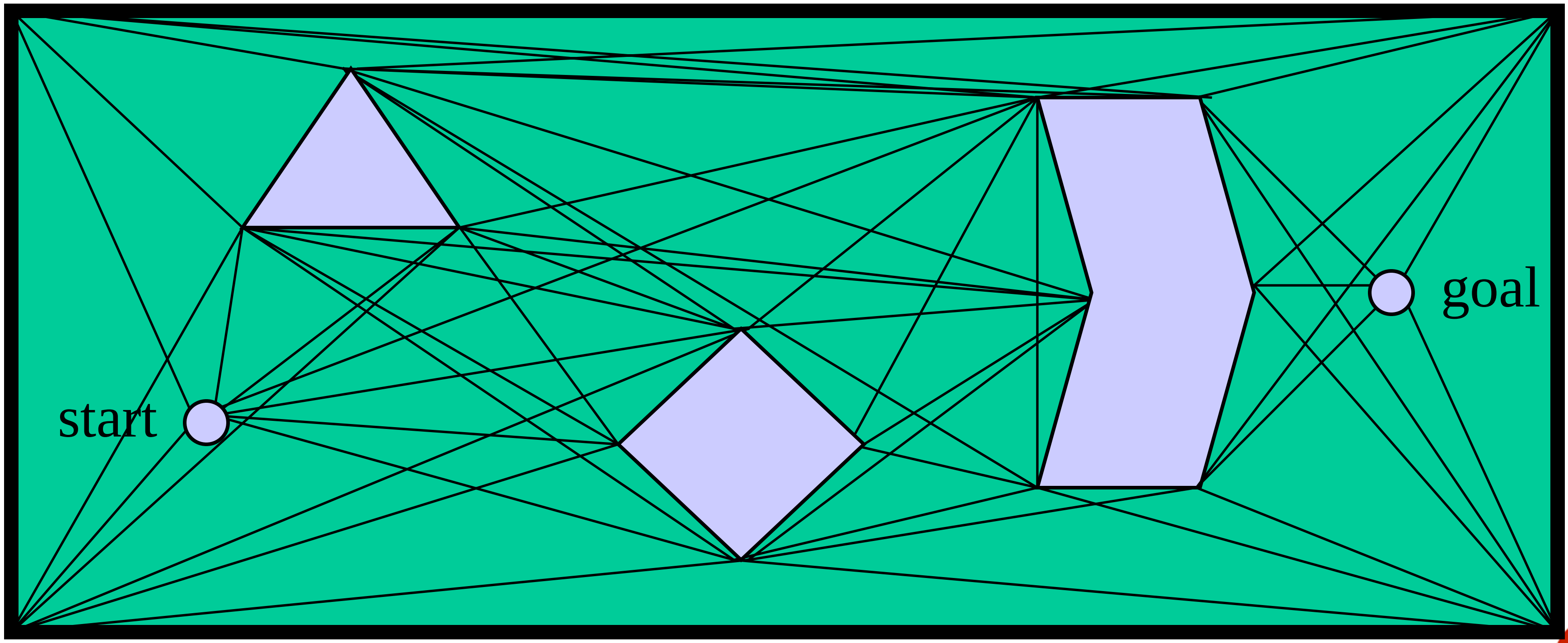
## The Visibility Graph in Action (Part 4)

- Second, draw lines of sight from every vertex of every obstacle like before. Remember lines along edges are also lines of sight.



# The Visibility Graph (Done)

- Repeat until you're done.

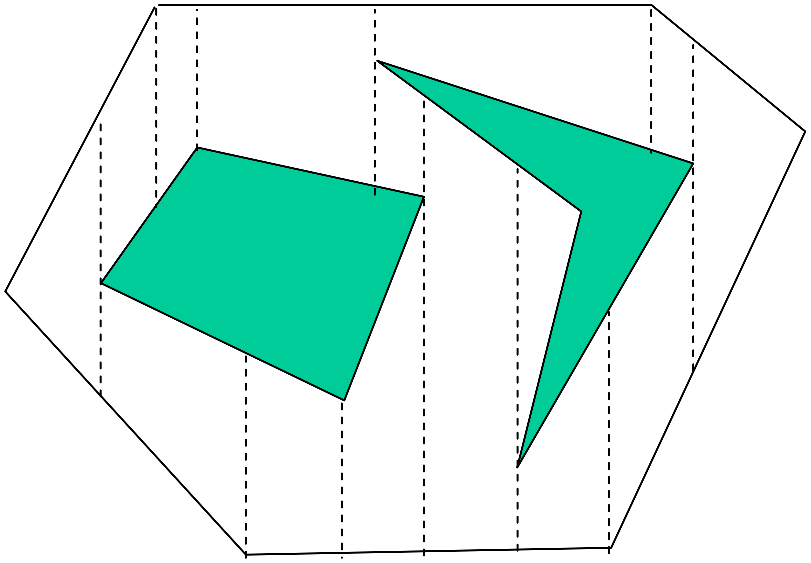


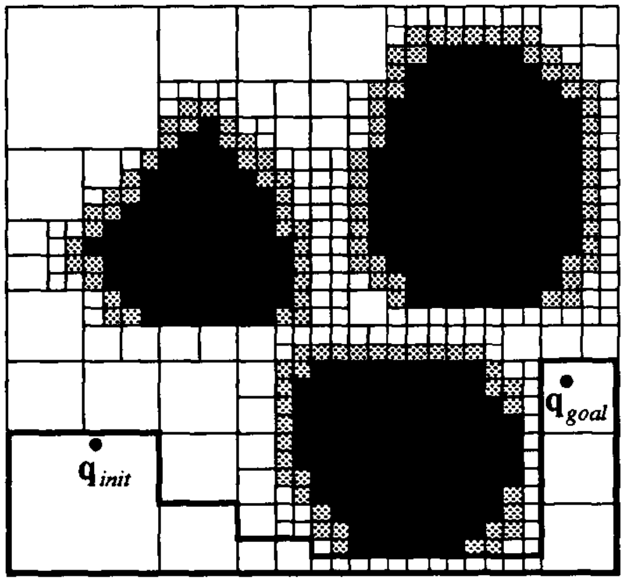
## Visibility Graph Overview

- Start with a map of the world, draw lines of sight from the start and goal to every “corner” of the world and vertex of the obstacles, not cutting through any obstacles.
- Draw lines of sight from every vertex of every obstacle like above.  Lines along edges of obstacles are lines of sight too, since they don’t pass through the obstacles.
- If the map was in Configuration space, each line potentially represents part of a path from the start to the goal.

# Exact Cell vs. Approximate Cell

- Cell: simple region



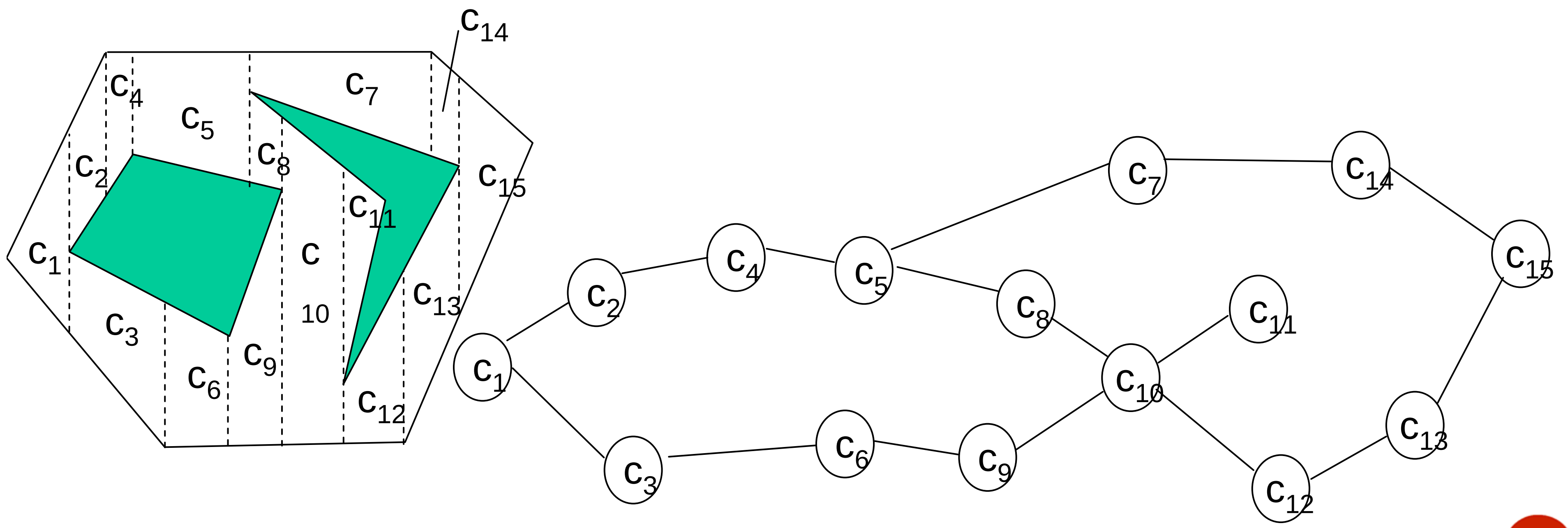


# Adjacency Graph

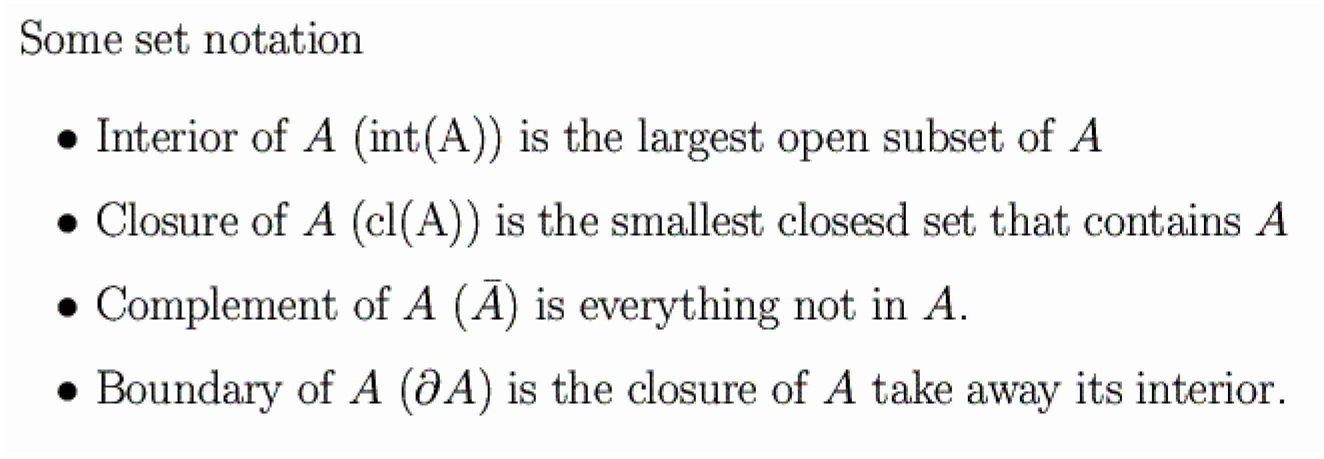
– Node correspond to a cell

– Edge connects nodes of adjacent cells

- Two cells are *adjacent* if they share a common boundary



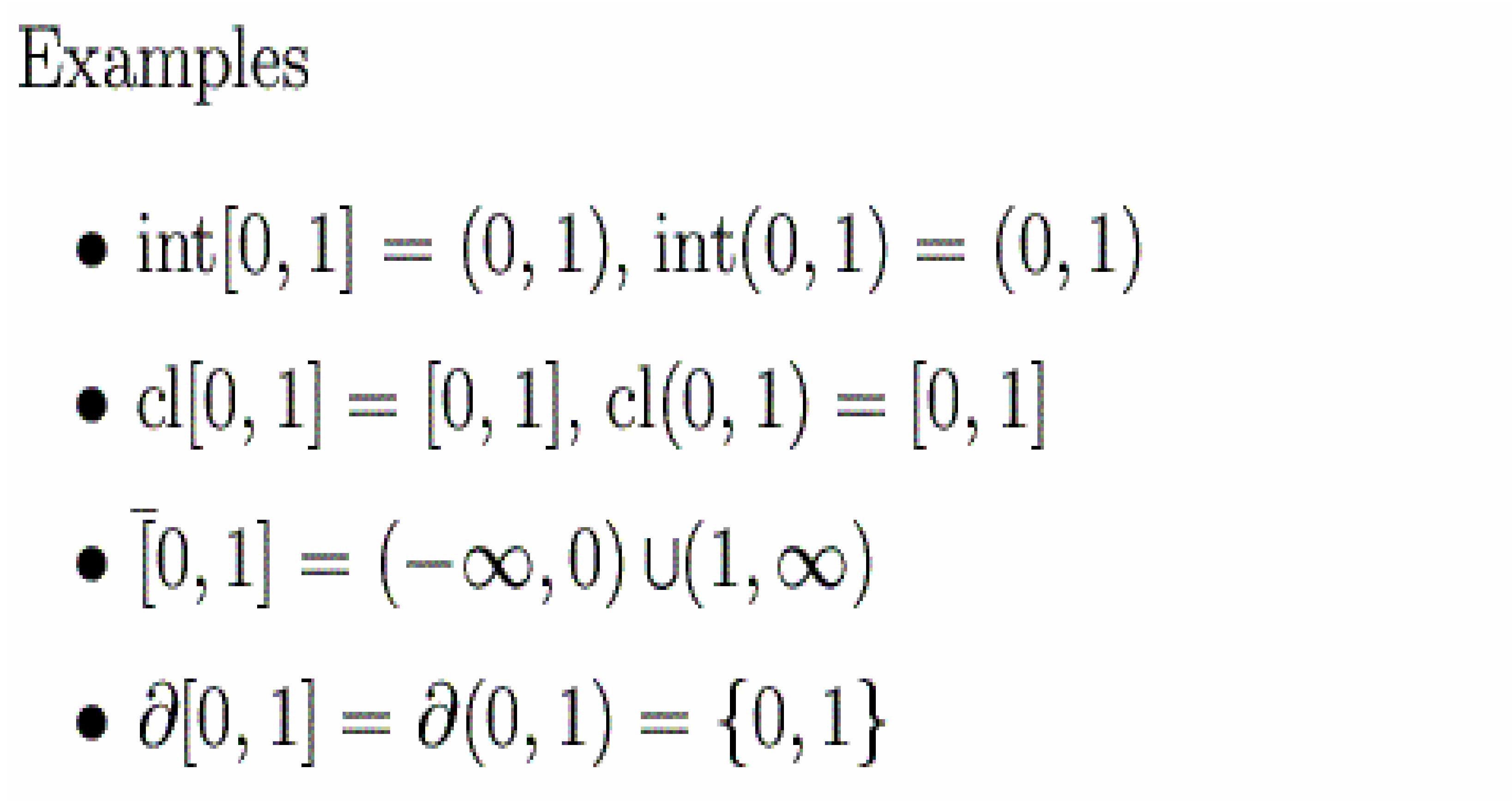
## Set Notation



Some set notation

- Interior of $A$ (int(A)) is the largest open subset of $A$
- Closure of $A$ (cl(A)) is the smallest closesd set that contains $A$
- Complement of $A$ ($\bar{A}$) is everything not in $A$.
- Boundary of $A$ ($\partial A$) is the closure of $A$ take away its interior.

## Examples



Examples

- $\operatorname{int}[0,1] = (0,1)$, $\operatorname{int}(0,1) = (0,1)$
- $\operatorname{cl}[0,1] = [0,1]$, $\operatorname{cl}(0,1) = [0,1]$
- $\overline{[0,1]} = (-\infty,0) \cup (1,\infty)$
- $\partial[0,1] = \partial(0,1) = \{0,1\}$

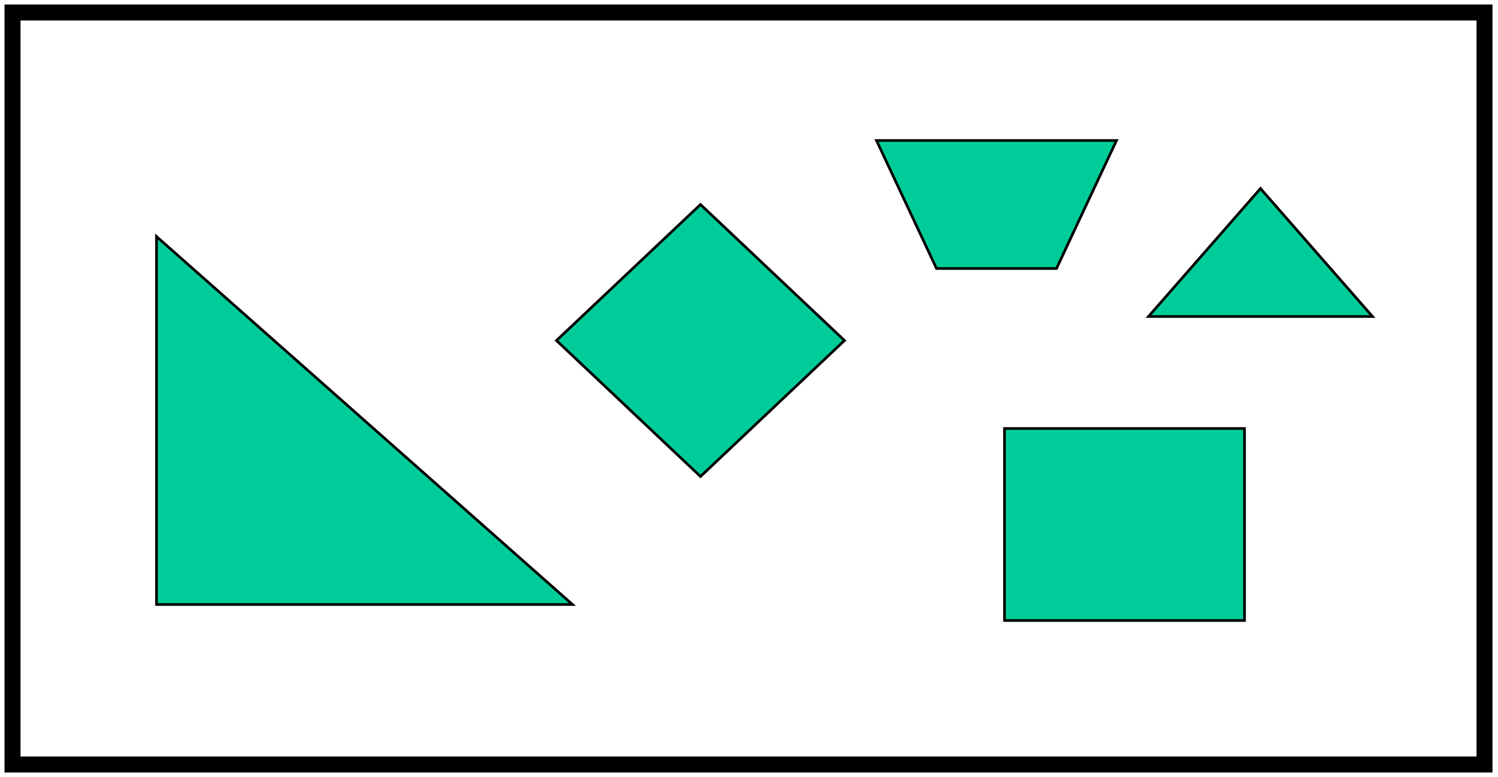
## Definition

Exact Cellular Decomposition (as opposed to approximate)

- $\nu_i$ is a cell
- $\text{int}(\nu_i) \cap \text{int}(\nu_j) = \emptyset$ if and only if $i \neq j$
- $Fs \cap (\text{cl}(\nu_i) \cap \text{cl}(\nu_j)) \neq \emptyset$ if $\nu_i$ and $\nu_j$ are adjacent cells
- $Fs = \cup_i(\nu_i)$

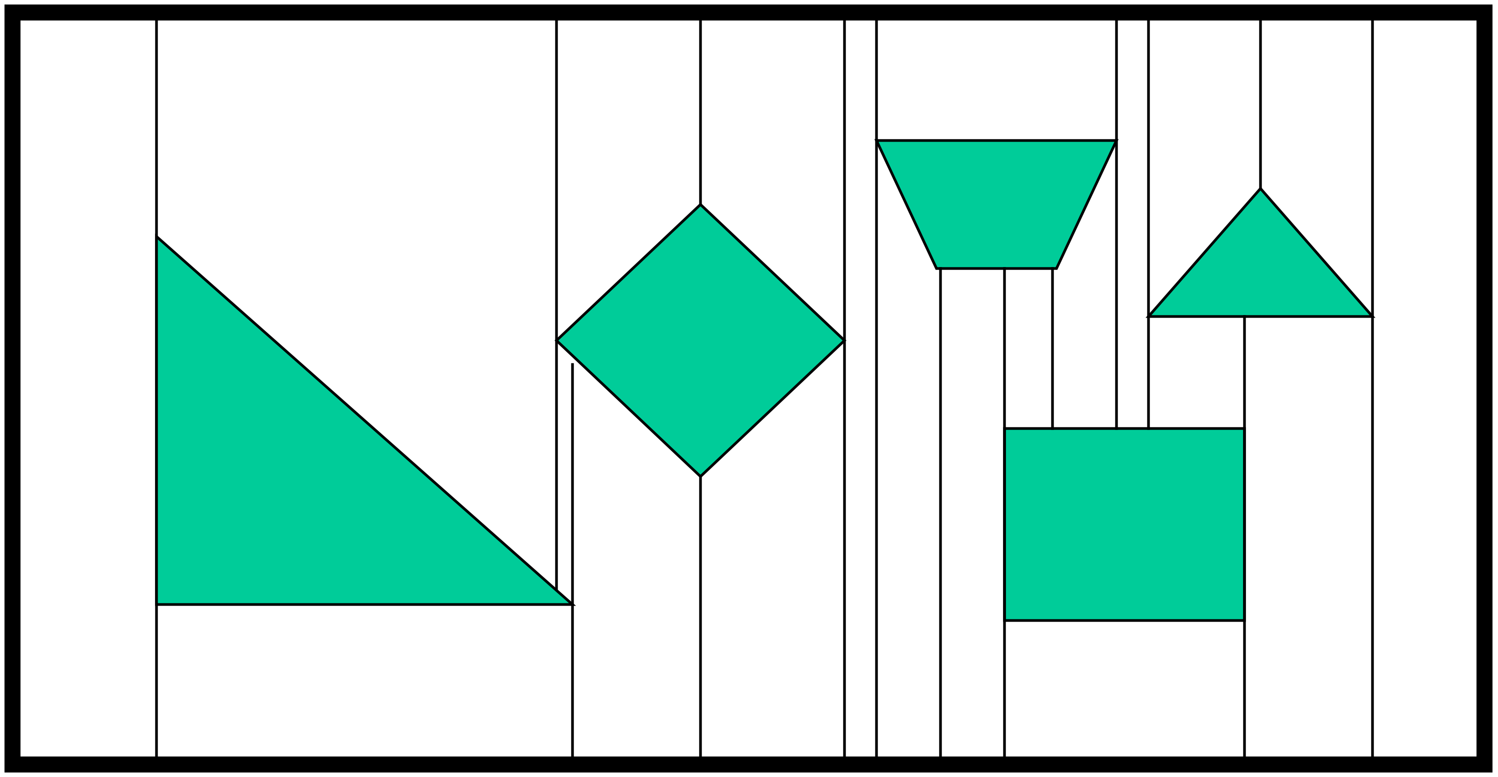
## Cell Decompositions: Trapezoidal Decomposition

- A way to divide the world into smaller regions
- Assume a polygonal world



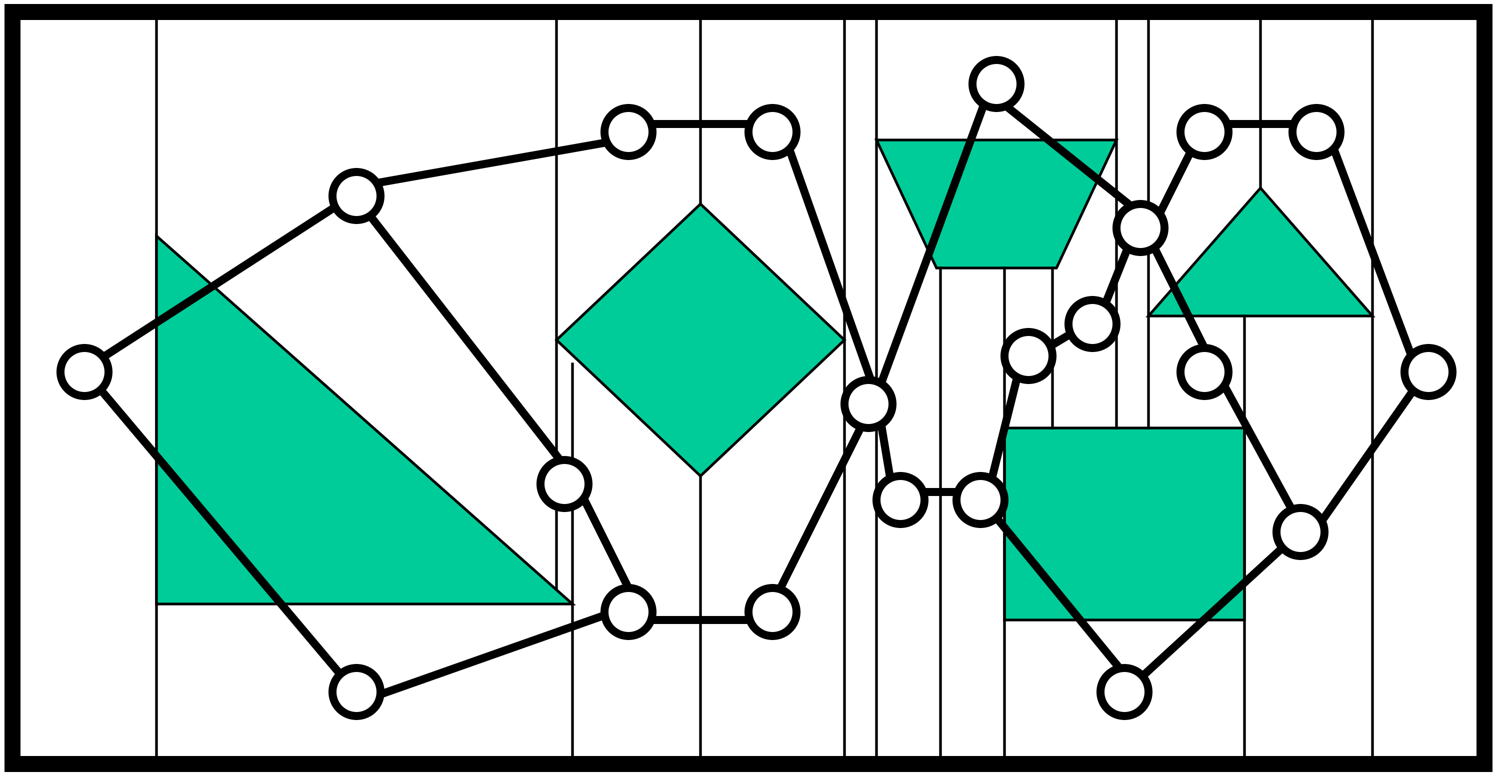
## Cell Decompositions: Trapezoidal Decomposition

- Simply draw a vertical line from each vertex until you hit an obstacle. This reduces the world to a union of trapezoid-shaped cells



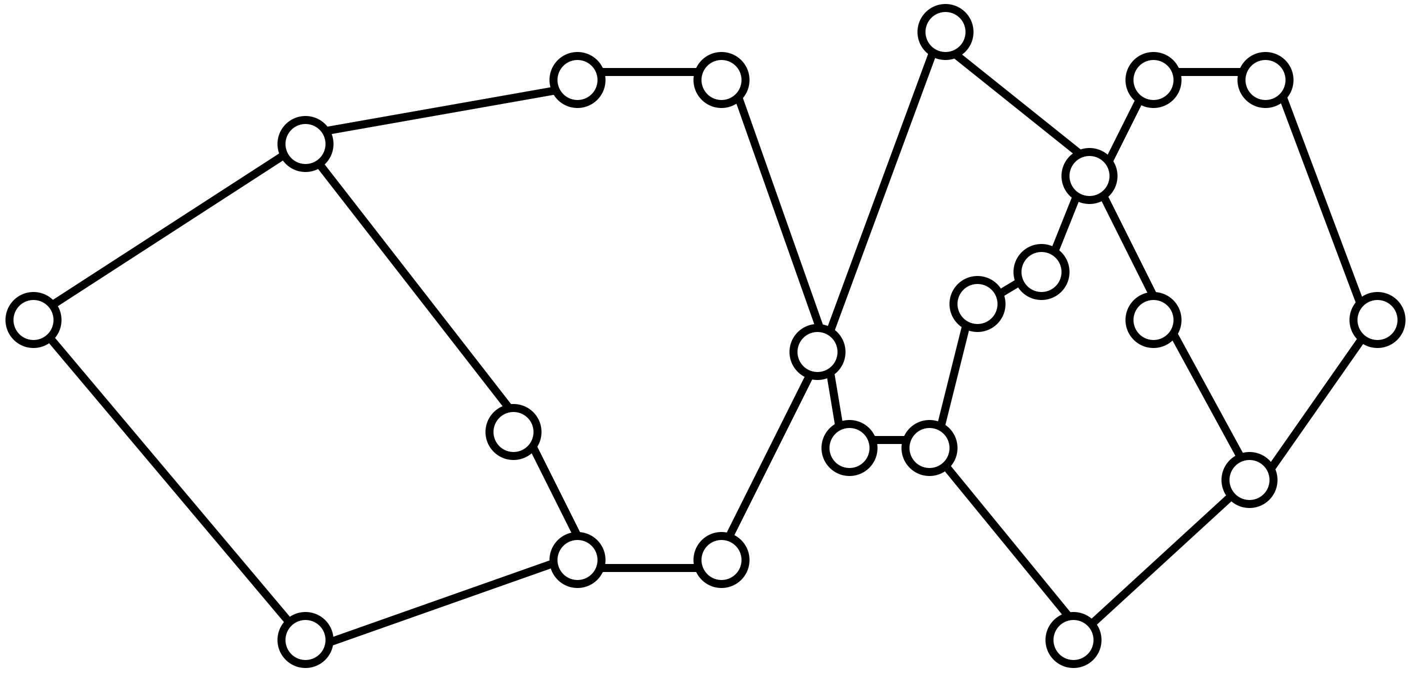
# Applications: Coverage

- By reducing the world to cells, we've essentially abstracted the world to a graph.



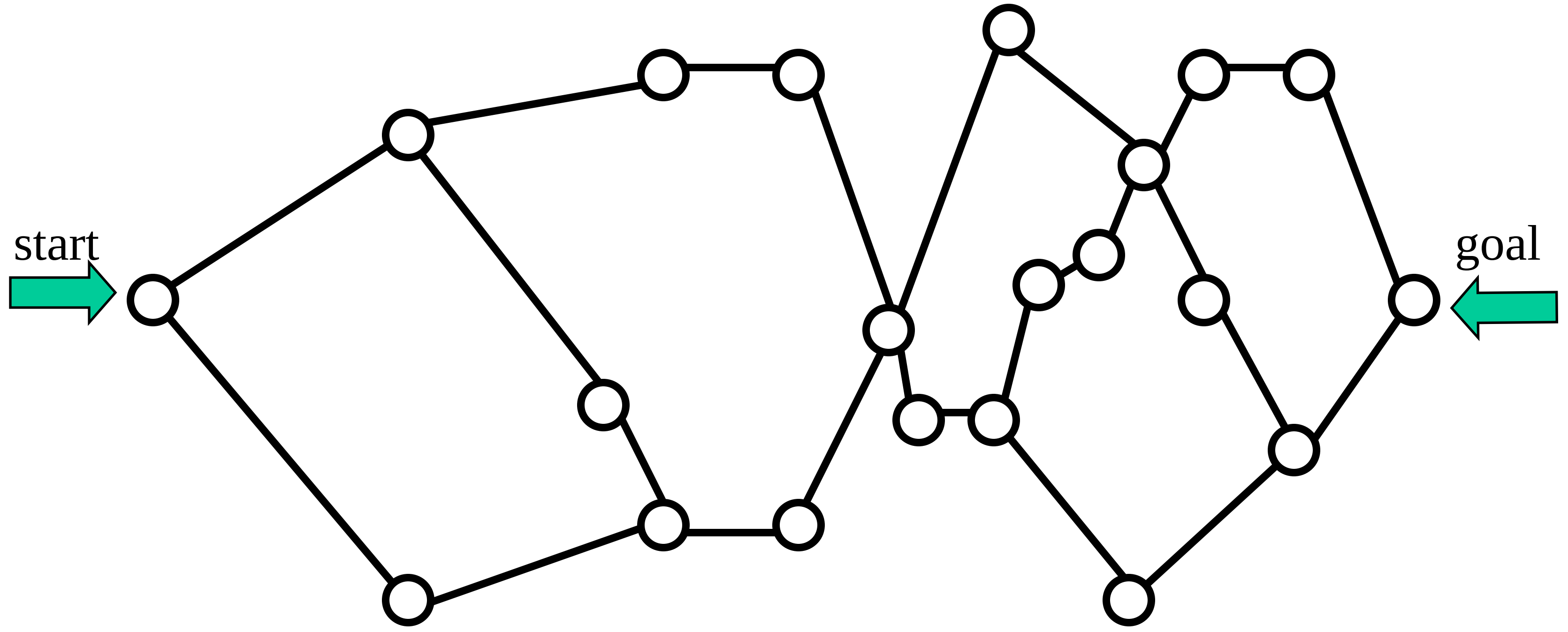
## Find a path

- By reducing the world to cells, we've essentially abstracted the world to a graph.



# Find a path

- With an adjacency graph, a path from start to goal can be found by simple traversal



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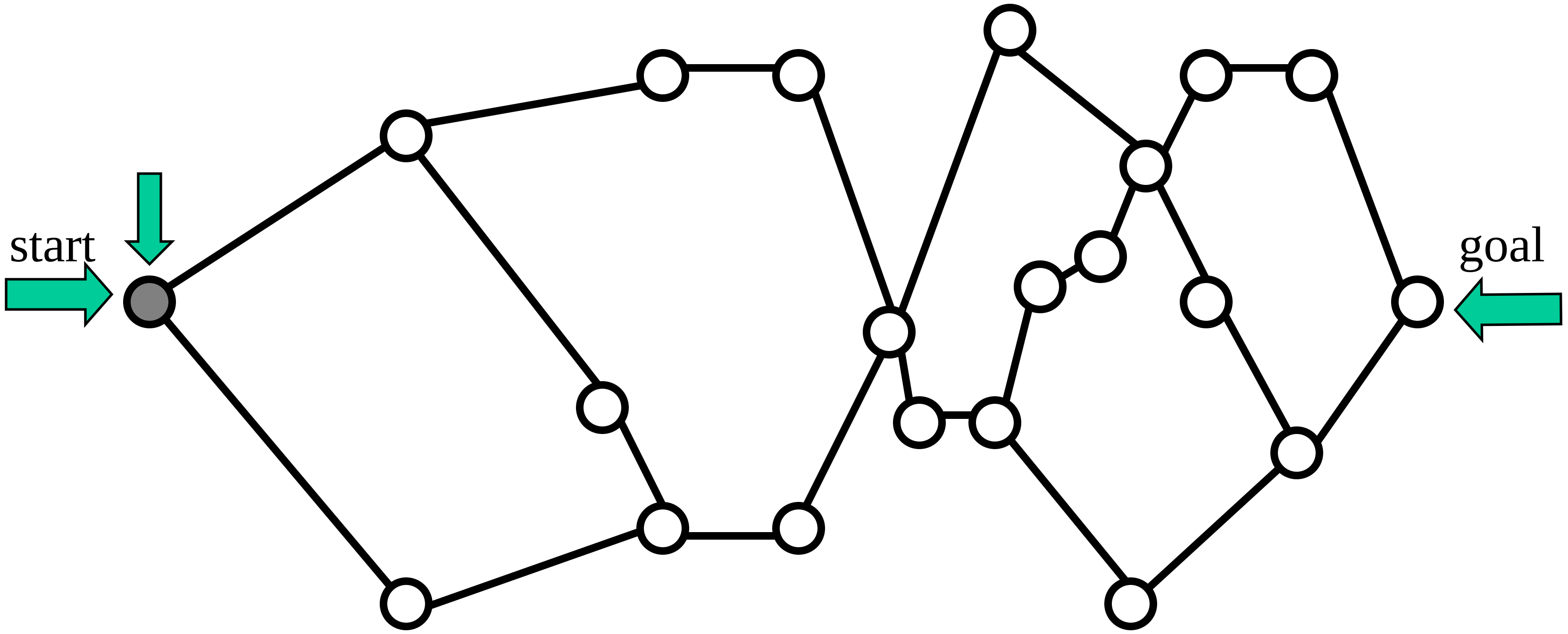

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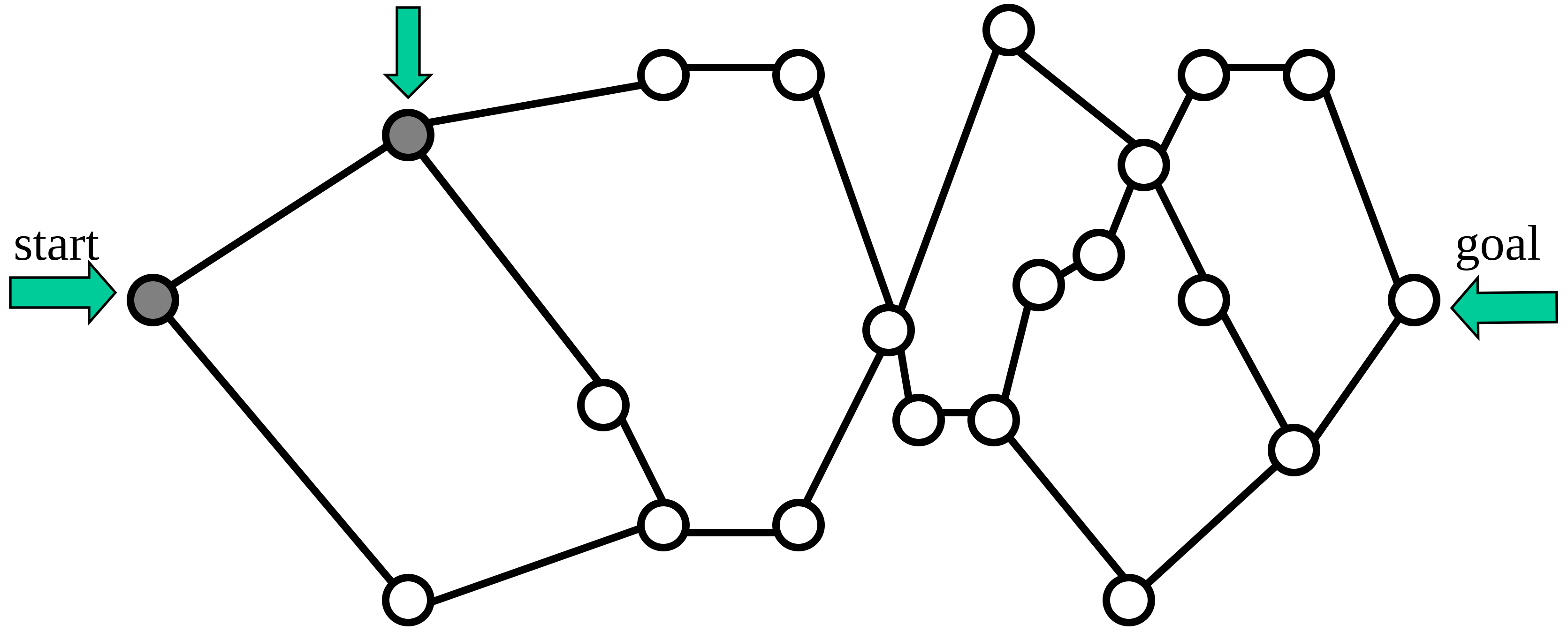

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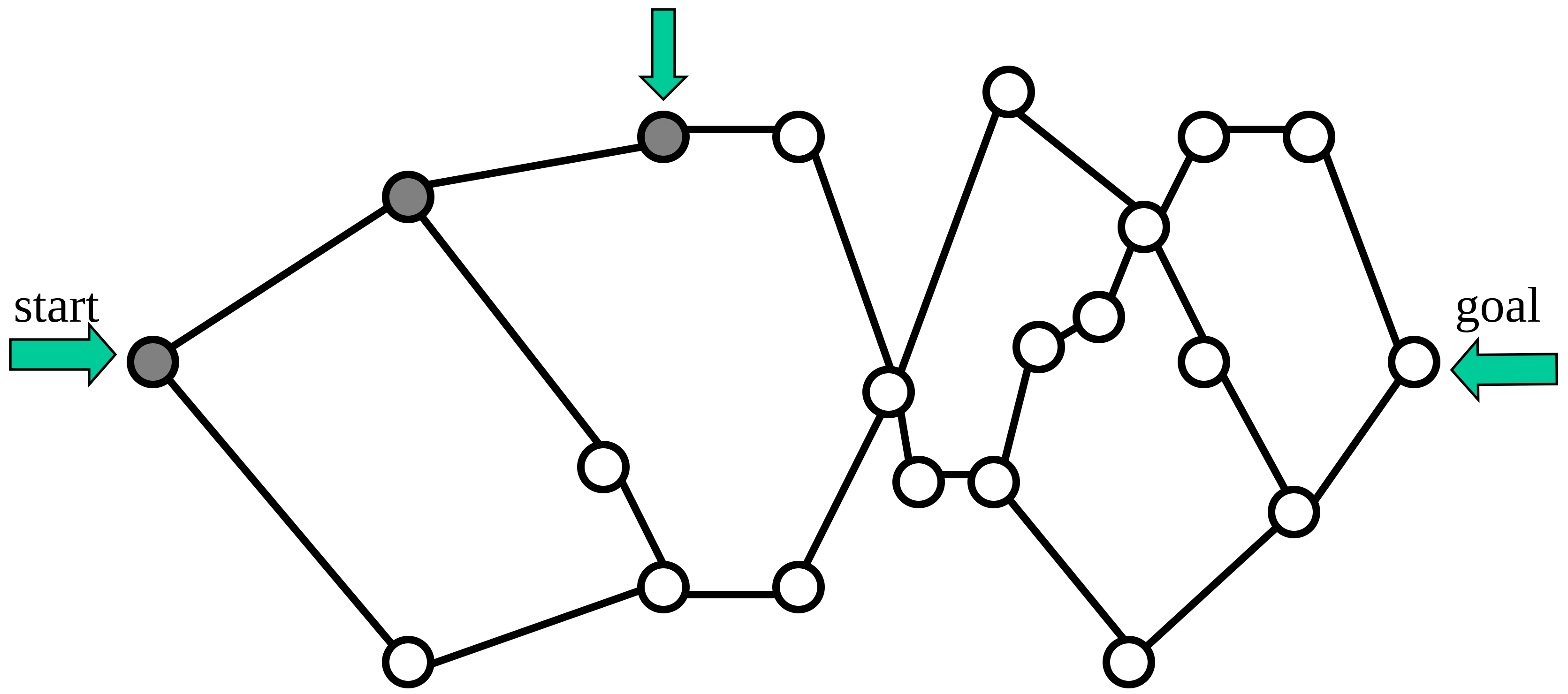

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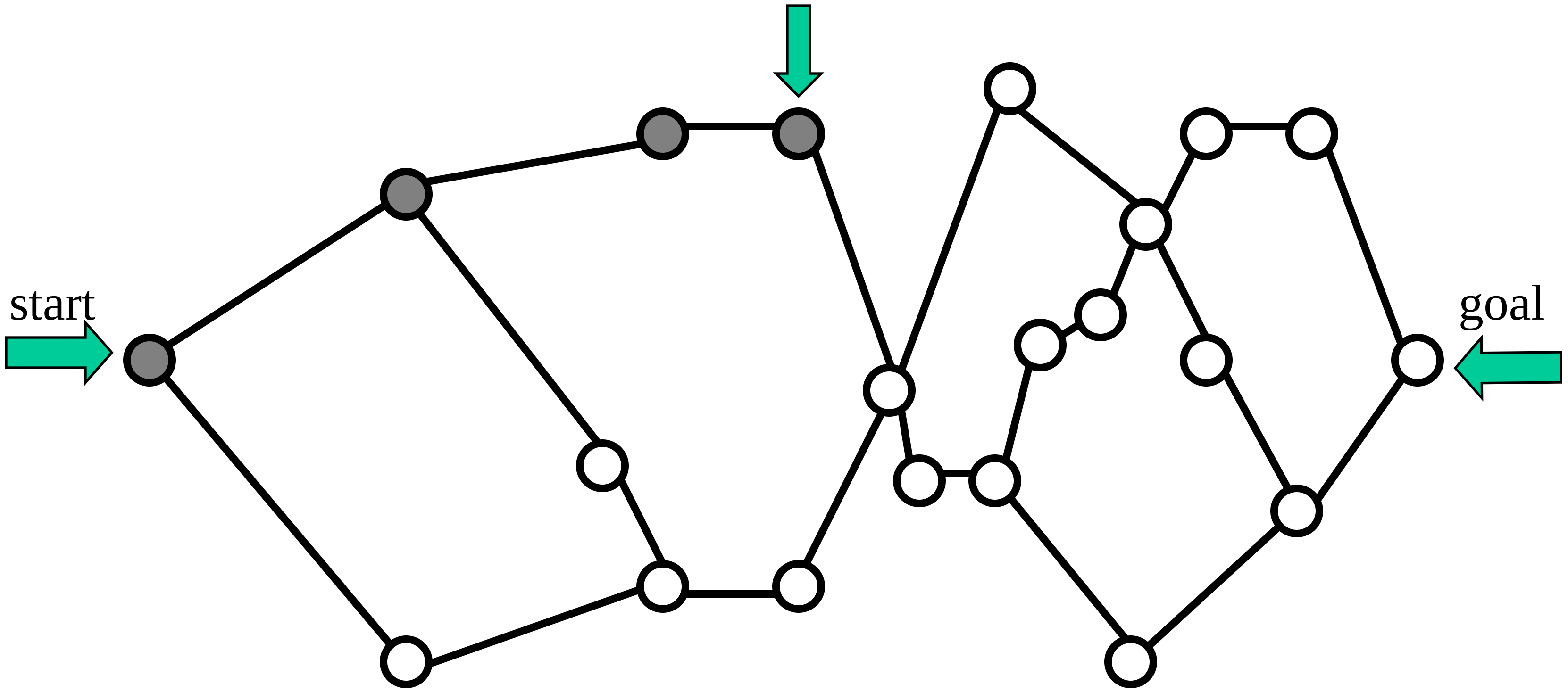

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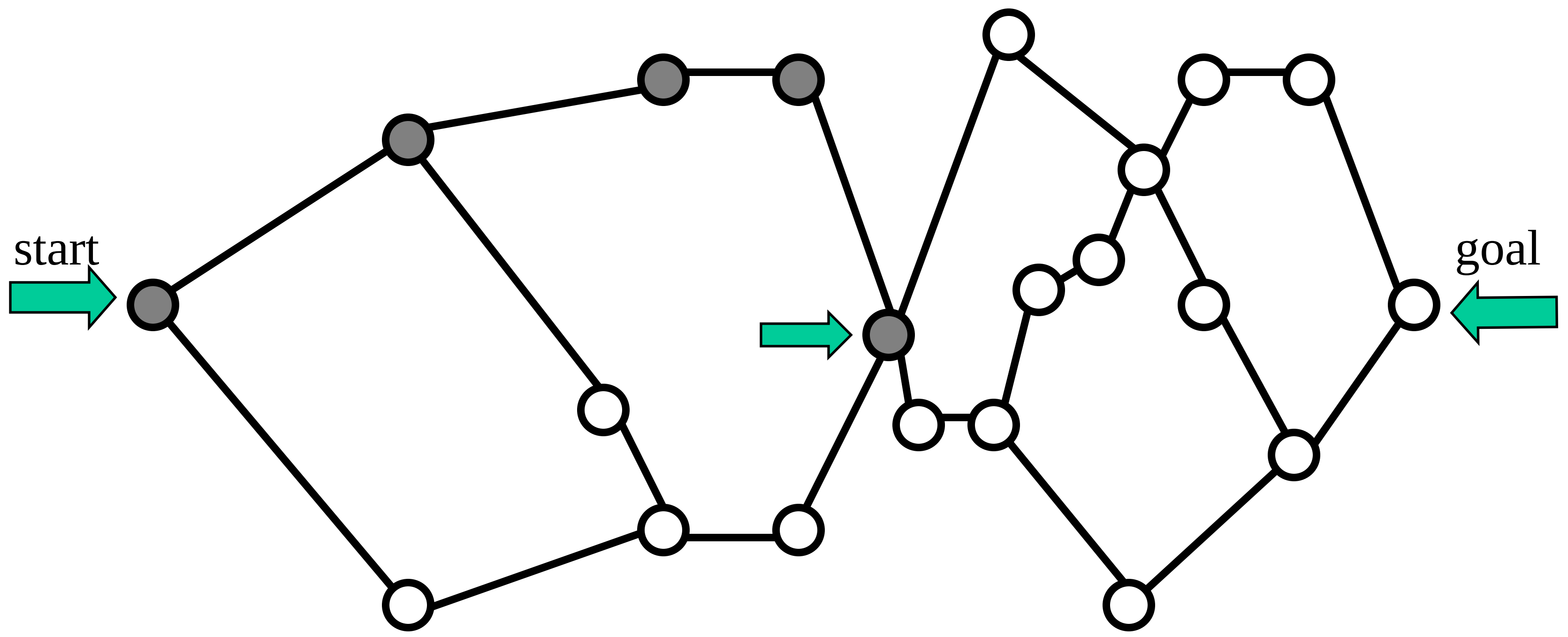

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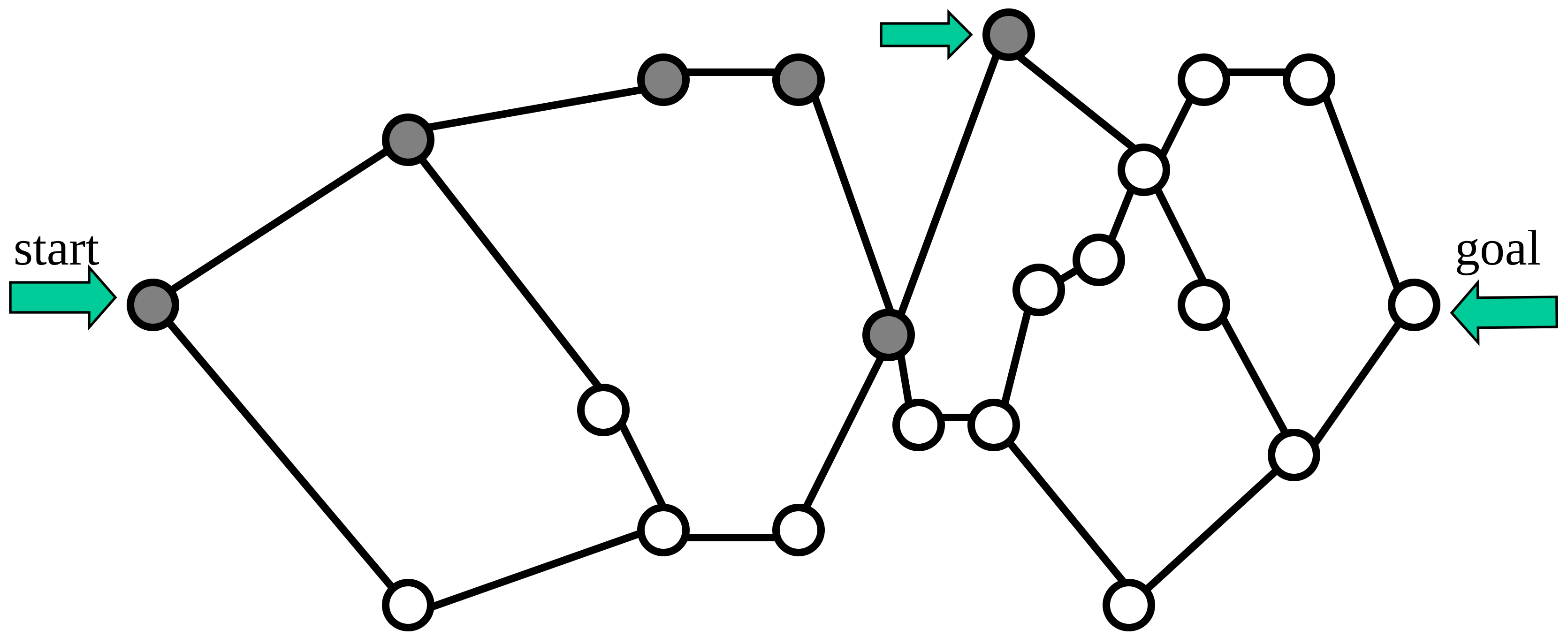

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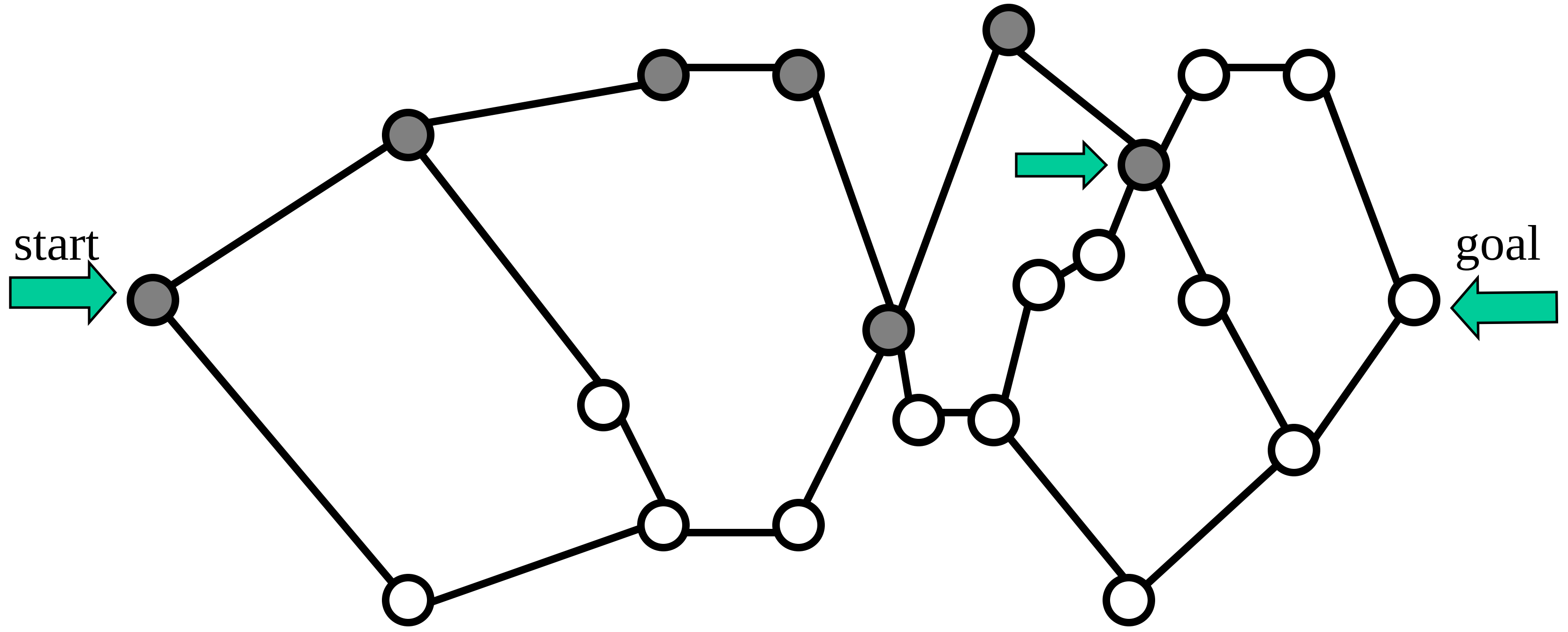

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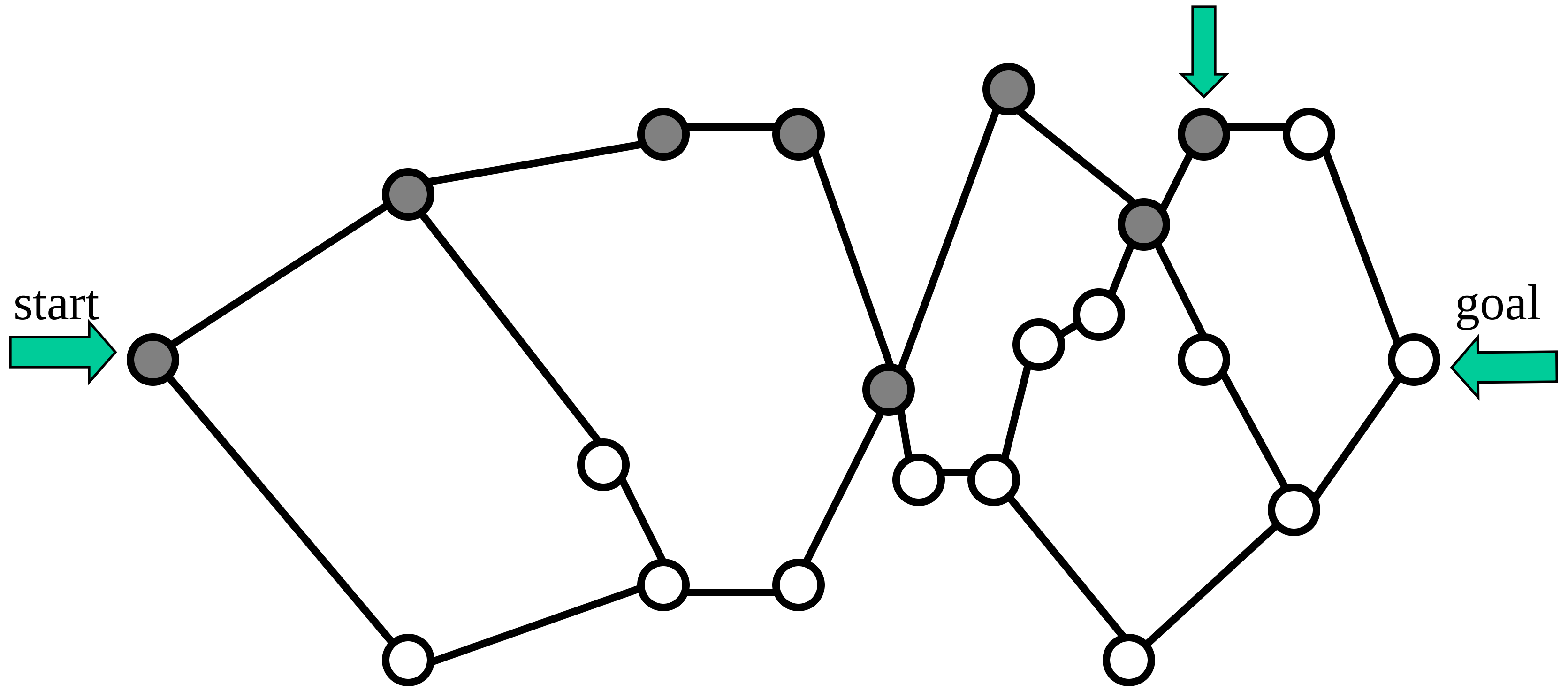

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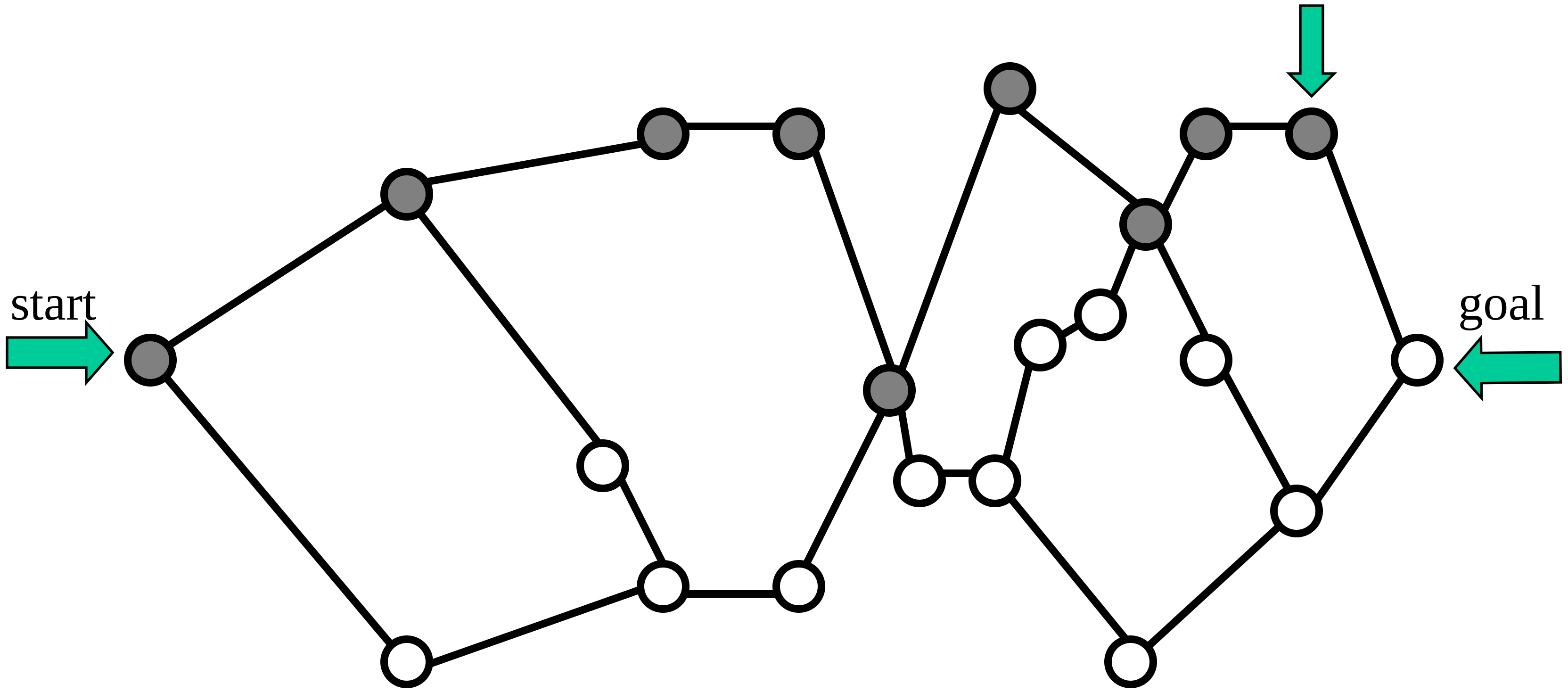

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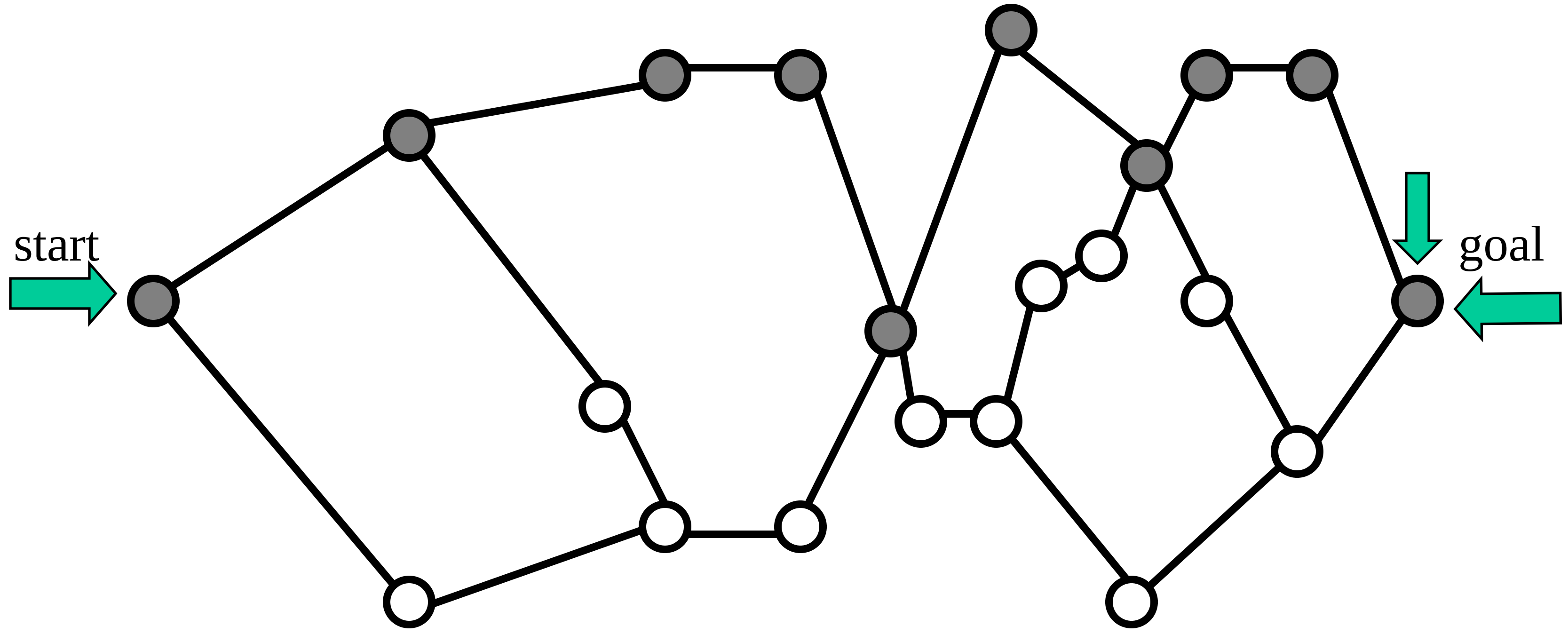


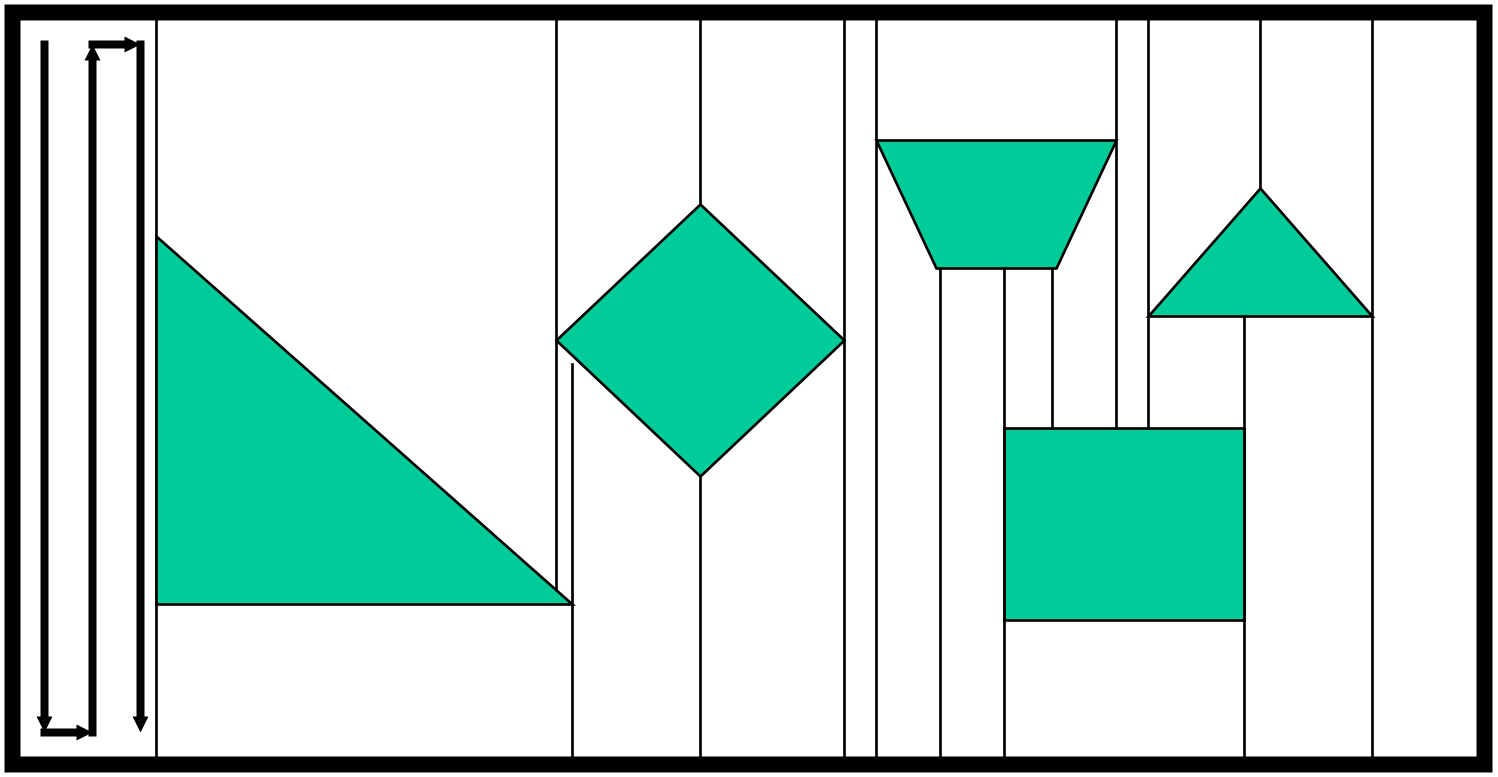
## Connect Midpoints of Traps

## Applications: Coverage

- First, a distinction between sensor and detector must be made
- *Sensor*: Senses obstacles
- *Detector*: What actually does the coverage
- We'll be observing the simple case of having an omniscient sensor and having the detector's footprint equal to the robot's footprint

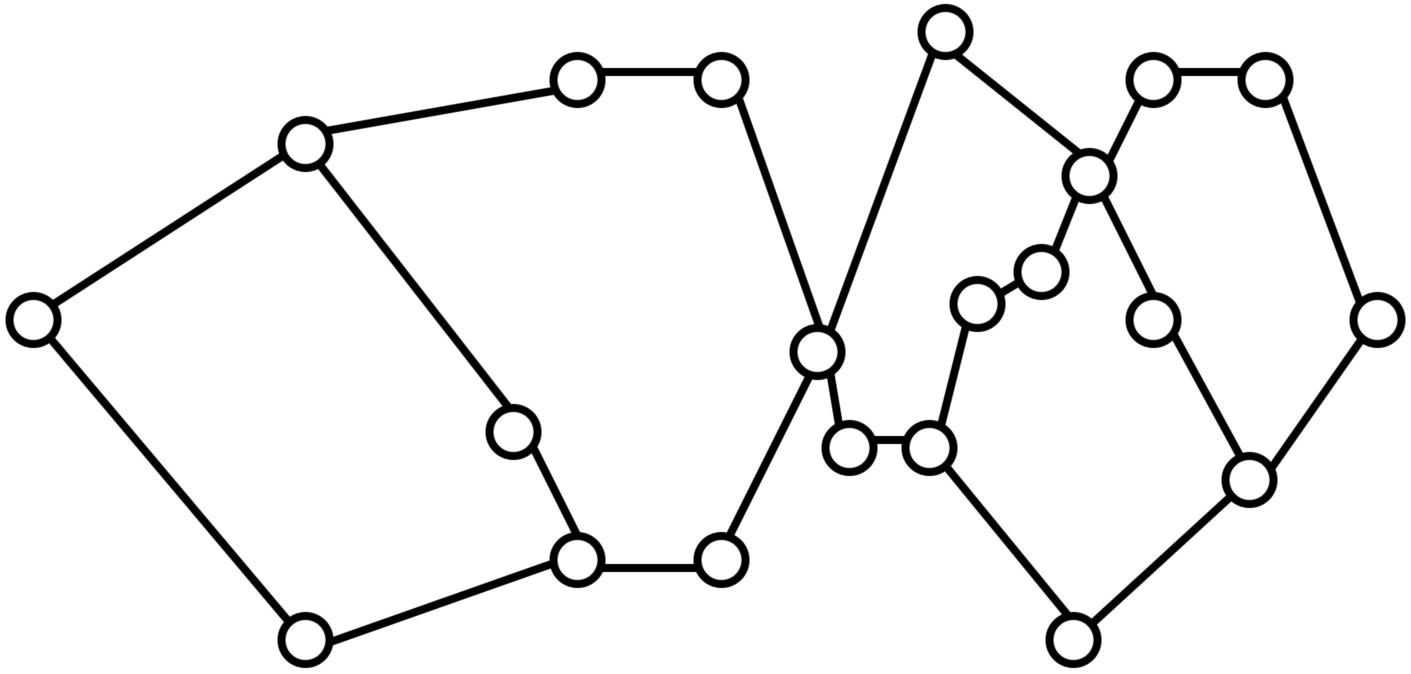
## Cell Decompositions: Trapezoidal Decomposition

- How is this useful? Well, trapezoids can easily be covered with simple back-and-forth sweeping motions. If we cover all the trapezoids, we can effectively cover the entire “reachable” world.

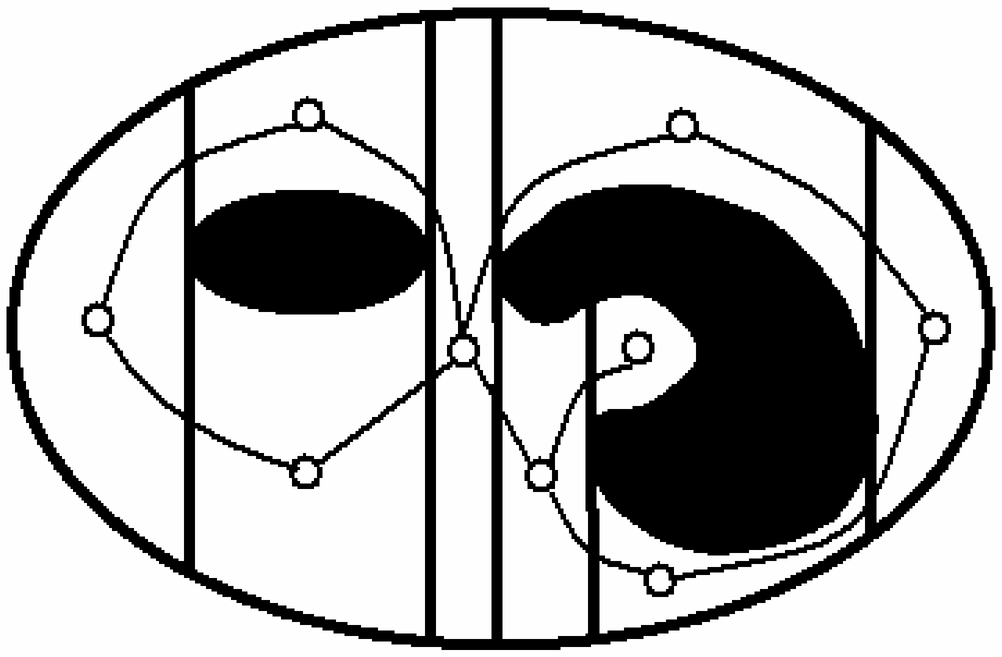


# Applications: Coverage

- Simply visit all the nodes, performing a sweeping motion in each, and you’re done.



# Boustrophedon Decomposition



# Conclusion: Complete Overview

- ✓ The Basics
  - Motion Planning Statement
  - The World and Robot
  - Configuration Space
  - Metrics
- ✓ Path Planning Algorithms
  - Start-Goal Methods
    - Lumelsky Bug Algorithms
    - Potential Charge Functions
    - The Wavefront Planner
  - Map-Based Approaches
    - Generalized Voronoi Graphs
    - Visibility Graphs
  - Cellular Decompositions => Coverage
- ✓ *Done with Motion Planning!*