# Intro. to Inertial Navigation

Special Delivery to Introduction to Robotics

By Alonzo Kelly

# Outline

- **Intro**
- Implementation
- Performance
- Real INSes
- Summary

# History

- 1940s Germany:
  - V2 program, gyroscopic guidance
- 1950s Draper Labs, MIT:
  - Shuler tuned INS
  - Floated rate integrating gyros (0.01 deg/hr)
- 1960s DTGs
  - not floated or temp compensated
- 1970s RLGs, USA
- 1980s Strapdown INS
- 1990s GPS

**V2**


**RLG Experiment**

# Concept

- Use Inertial Properties of Matter
  - Accelerometers
  - Gyros
- Do "Dead Reckoning"
  - Integrate acceleration twice

**IMU**

$$\vec{v}(t) = \vec{v}(0) + \int_0^t \vec{a}\,dt$$

$$\vec{r}(t) = \vec{r}(0) + \int_0^t \vec{v}\,dt$$

**Computation**

# Characteristics

| Attribute | Dead Reckoning (e.g. INS) | Triangulation (e.g. GPS) |
|---|---|---|
| Process | Integrate Differential Equations | Solve Nonlinear Algebraic Equations |
| Initial Conditions | Required | Not required |
| Errors | Time Dependent | Position Dependent |
| Error Propagation | Depends on history of errors. | Does not depend on history of errors. |
| Requires Map* | No | Yes |

* INS needs a map of gravity

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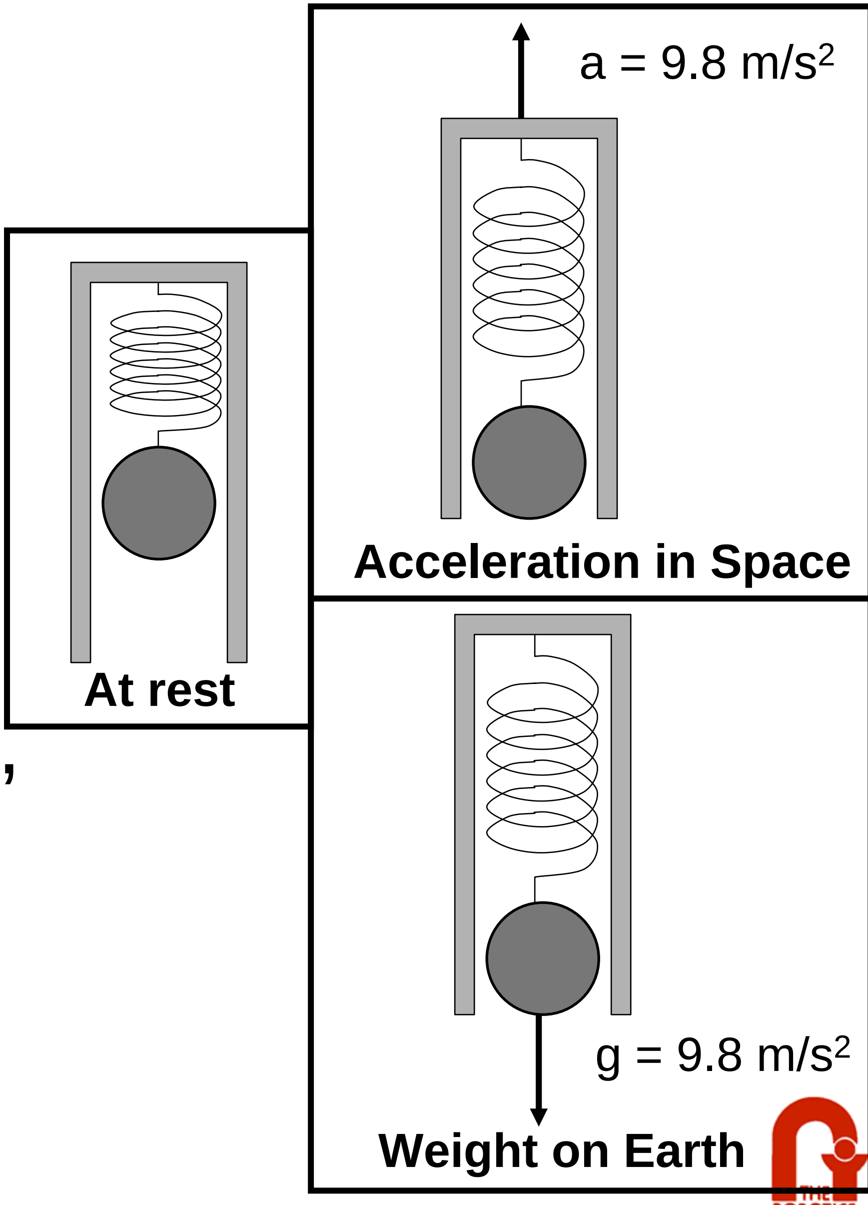
# Problem 1: Equivalence

- Accelerometers don't measure acceleration.
- Specific force is:

$$\vec{f} = \vec{a} - \vec{g}$$

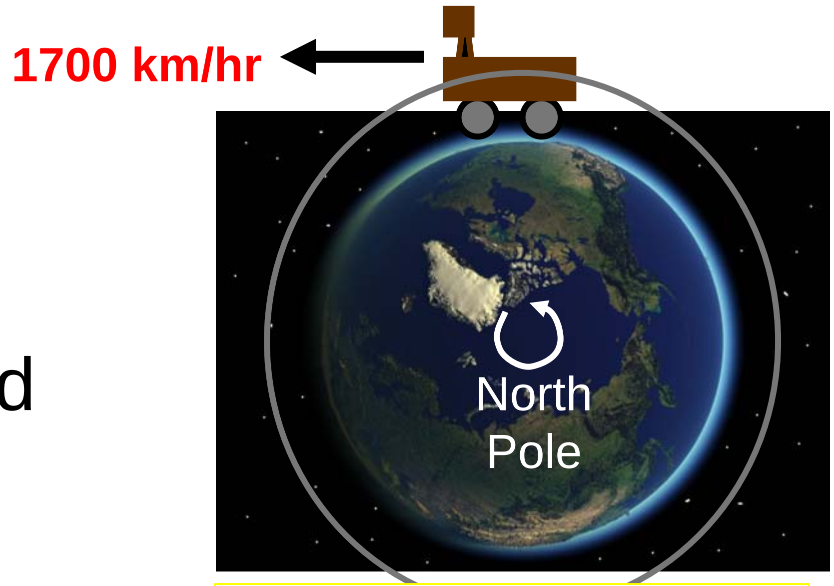
- Fix: must know gravity, then:

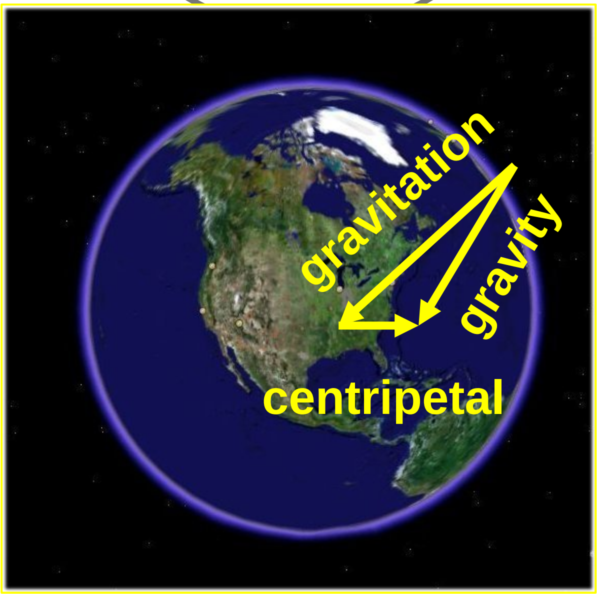
$$\vec{a} = \vec{f} + \vec{g}$$



# Problem 2: Inertial Frame of Reference

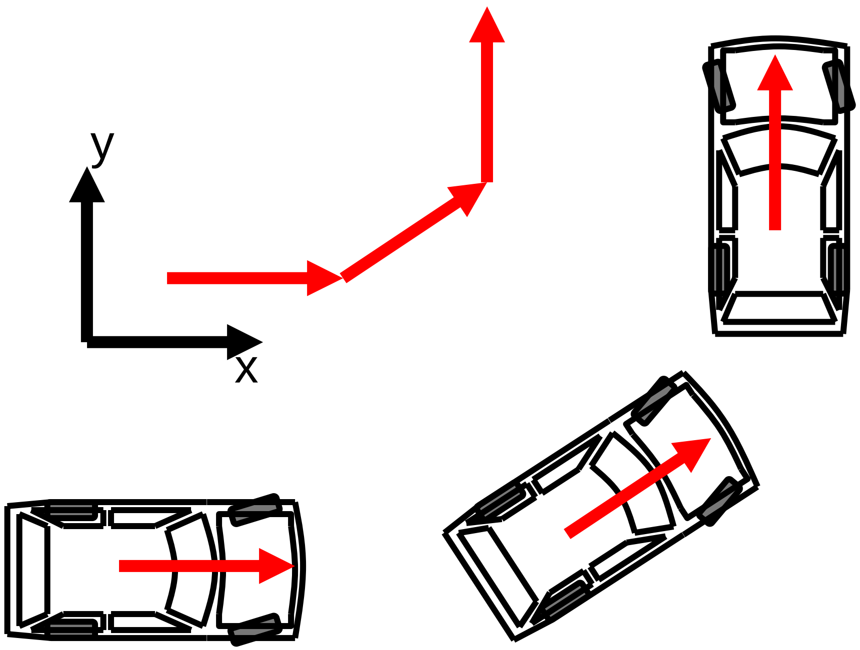
- Now have inertial acceleration.
  - Want earth-referenced acceleration.
- Fix: account for earth angular velocity:
  - “Apparent forces”.
  - “gravity”, not gravitation



# Problem 3: Body Coordinates

- Accelerometers are fixed to vehicle.
- Need to know instantaneous heading.




- Track orientation
  - Use gyros.

$$\theta(t) = \int_0^t \omega(t)dt$$

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# Error Explosion

- Centripetal term is tiny:
  - $1.5 \times 10^{-4}$ g

$$\vec{a}_{cent}(t) = \vec{\omega}(t) \times \vec{\omega}(t) \times \vec{r}(t)$$

- Consider an error of this magnitude…

$$\vec{a}_{meas}(t) = \vec{a}_{true}(t) + \vec{a}_{err}(t)$$

- In one hour:
  - t2 = $(3600)^2$ = 13 million !!

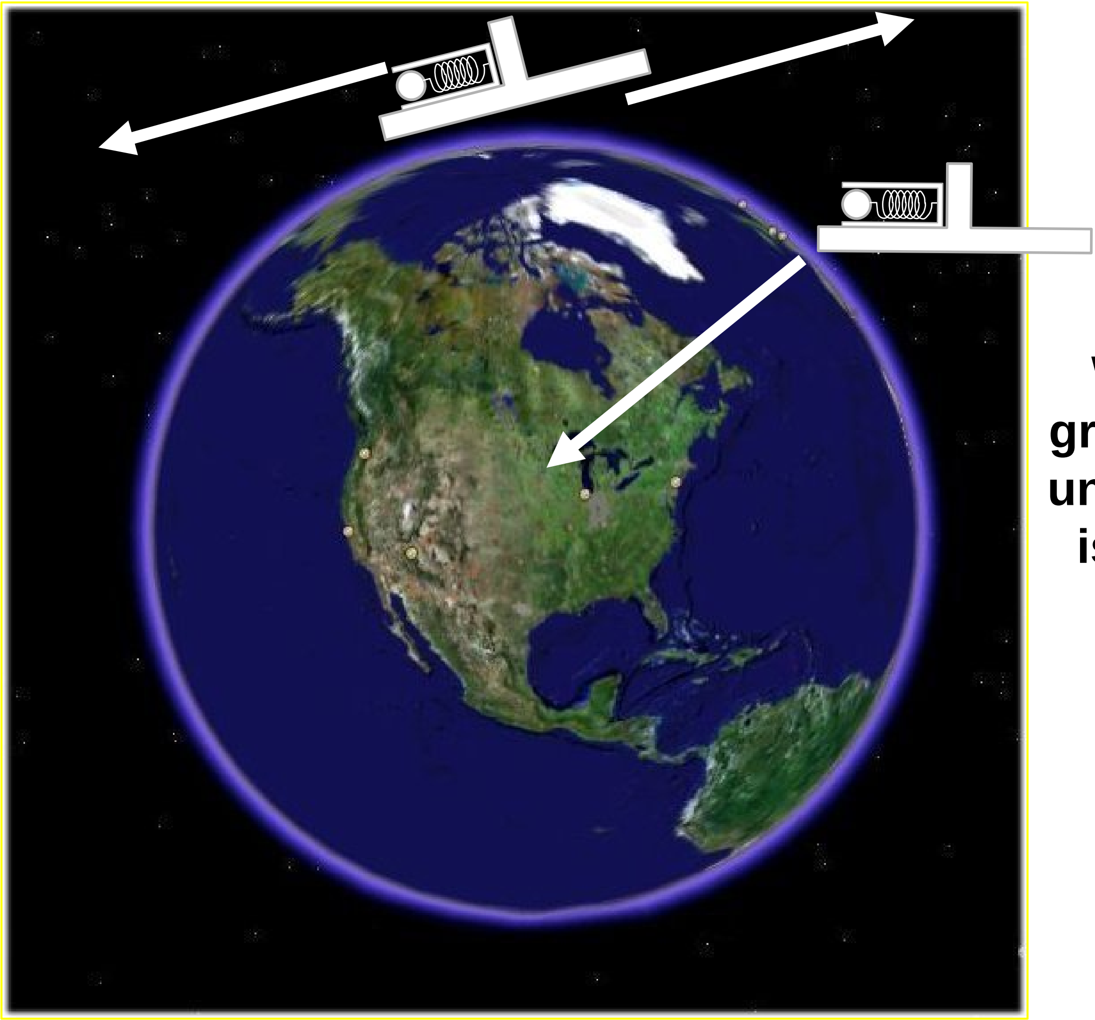
$$\vec{r}_{err}(t) = \int_0^t \int_0^t \vec{a}_{err}(t)\,dt = \frac{\vec{a}_{err} t^2}{2}$$

- Position Error:
  - 3 Kilometers!!!

# Gravity Feedback

Step 2:
Spring is deflected this way.

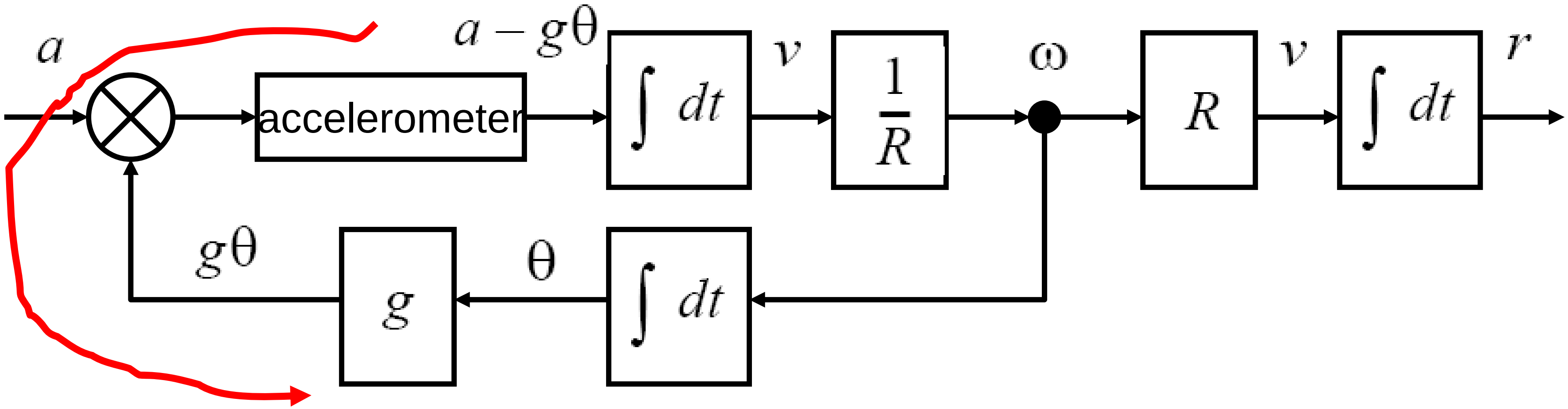
Step 1:
Orientation error, system thinks it is level.



Step 3:
Interpret as motion this way.

Step 4:
Which rotates gravity prediction until more motion is unnecessary

# Error Dynamics




$$\cancel{a} - g\theta = a - g\int \omega\, dt = a - \frac{g}{R}\int v\, dt = \cancel{a} - \frac{g}{R}\int\int [a - g\theta]\, dt$$

$$\cancel{g}\ddot{\theta} = \frac{\cancel{g}}{R}[a - g\theta]$$

Simple Harmonic Motion !!

$$\ddot{\theta} + \frac{g}{R}\theta = \frac{a}{R}$$

# Schuler Period

- Once again:

$$\ddot{\theta} + \frac{g}{R}\theta = \frac{a}{R}$$

- Period of oscillation:

$$T = 2\pi\sqrt{\frac{R}{g}} = 84 \text{ minutes}$$

Period of a pendulum whose length equals the radius of the earth!!

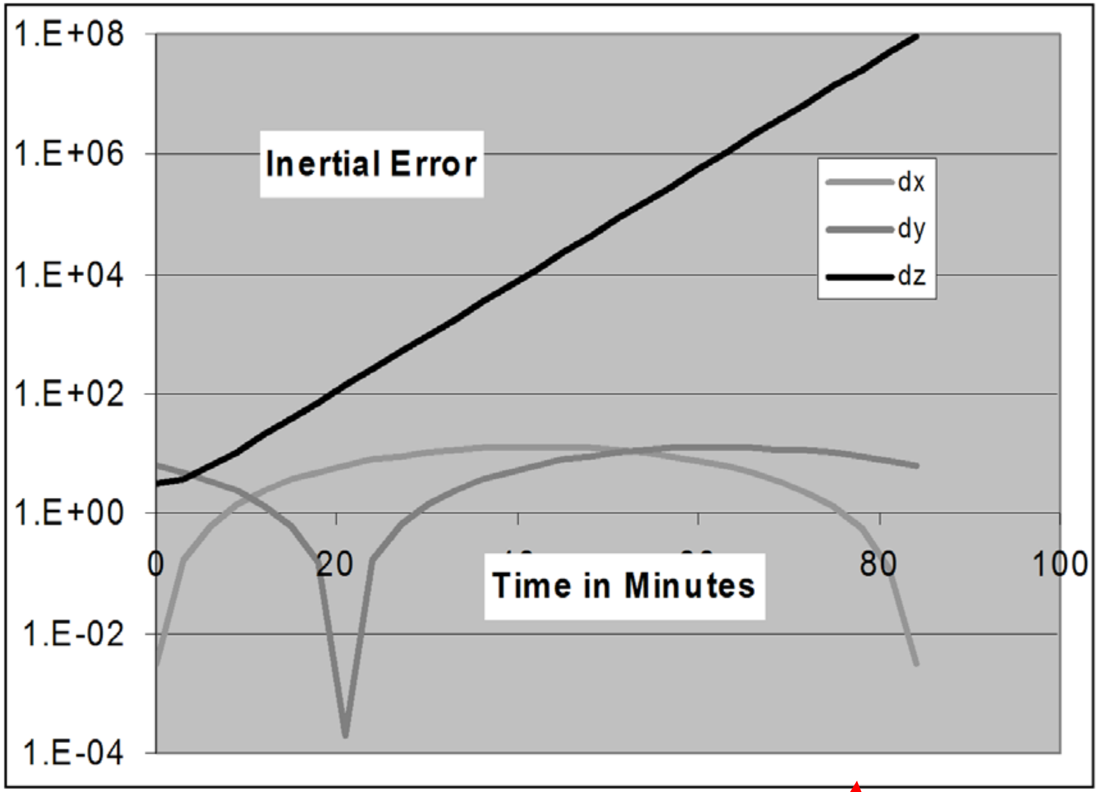
- Most INS errors oscillate with this period.

# Horizontal vs Vertical Error

$$\delta x = \frac{\delta t_x}{g/R}\left[1 - \cos\sqrt{\frac{R}{g}}\, t\right]$$

$$\delta y = \frac{\delta t_y}{g/R}\left[1 - \sin\sqrt{\frac{R}{g}}\, t\right]$$

$$\delta z = \frac{\delta t_z}{2g/R}\left[\cosh\sqrt{\frac{2R}{g}}\, t\right]$$




- Gravity field is a mixed blessing !!

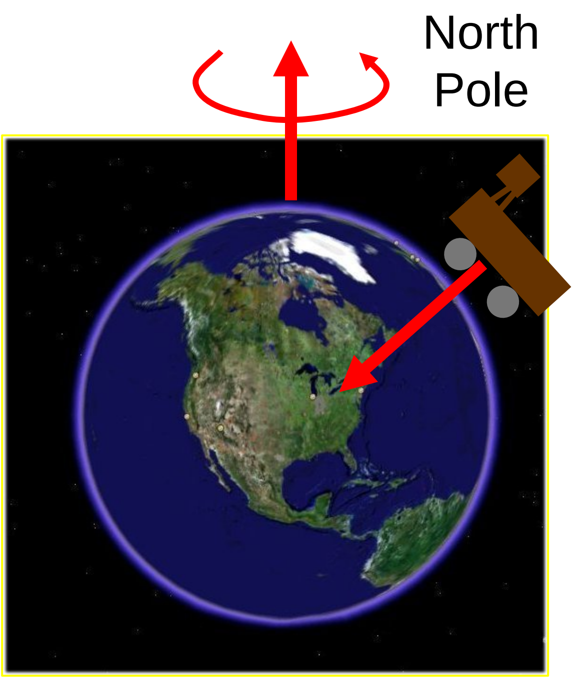
Error magnitudes for 1 micro-g biases.

# Aiding

- Used on mobile robots:
  - Odometry
  - GPS
  - Landmarks / Map matching
  - Zero velocity update
  - Magnetic heading
- Used more generally:
  - Barometric altitude
  - Radar altimeters
  - Doppler radar velocity

# Initialization

- Need to measure two non-collinear vectors.
- Earth conveniently has two:
  - Gravity - easy
  - Earth spin – takes time
- Can take several minutes.



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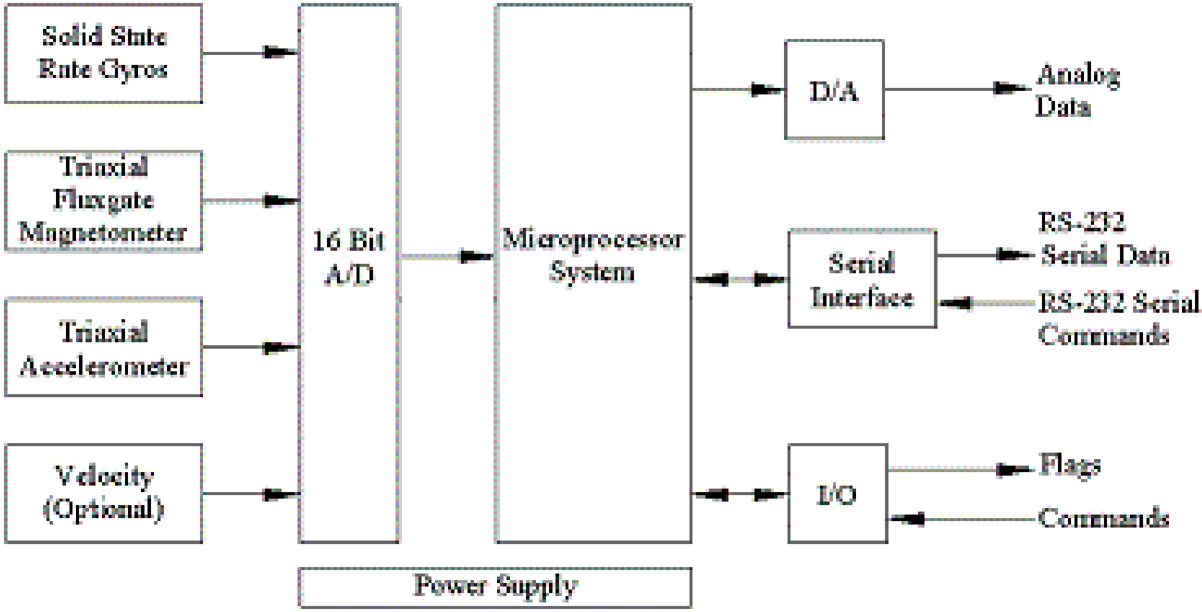
# Smiths Industries LNS

- Without GPS
  - Static Heading: <0.1 deg. rms
  - Position: <0.35% DT Horizontal
  - Altitude: <0.25% DT Vertical
- With GPS
  - Dynamic Heading: <0.1 deg. rms
  - Position: <10 meters CEP
  - Altitude Accuracy: <10 meters VEP
- Pitch and Roll Outputs: <0.05 deg. rms

- Initialization Time – Static: 3-5 minutes (gyrocompassing)
- Initialization Time – On-the-Move: 1-3 minutes

# Watson Industries AHRS E304

- Attitude:
  - 0.25% static, 2% dynamic
- Heading:
  - 1% static, 2% dynamic
- Angular Rate:
  - Scale factor 1%
  - Bias 0.02 deg/sec.
  - Bandwidth 25 Hz
- Acceleration:
  - Scale factor 1%
  - Bias 5 mg
  - Bandwidth 20 Hz



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# Summary

- Black magic ?
- Hard to do well.
  - Costs big bucks.
- Most accurate dead reckoning available.
  - Cruise: 0.2 nautical miles of error per hour of operation.
- Indispensable on outdoor mobile robots.
- Complementary technology to GPS.