$\boxed{= 1/0}$

can parallel park

inputs
steer + drive

step

1) determine constraints

2) determine allowable motions, g

3) lie bracket

touch sensor

manual input - no

camera

parallel park out

To do

1) make sure we can speak

$\boxed{=}$ $\quad w = [-\sin\theta \quad \cos\theta$

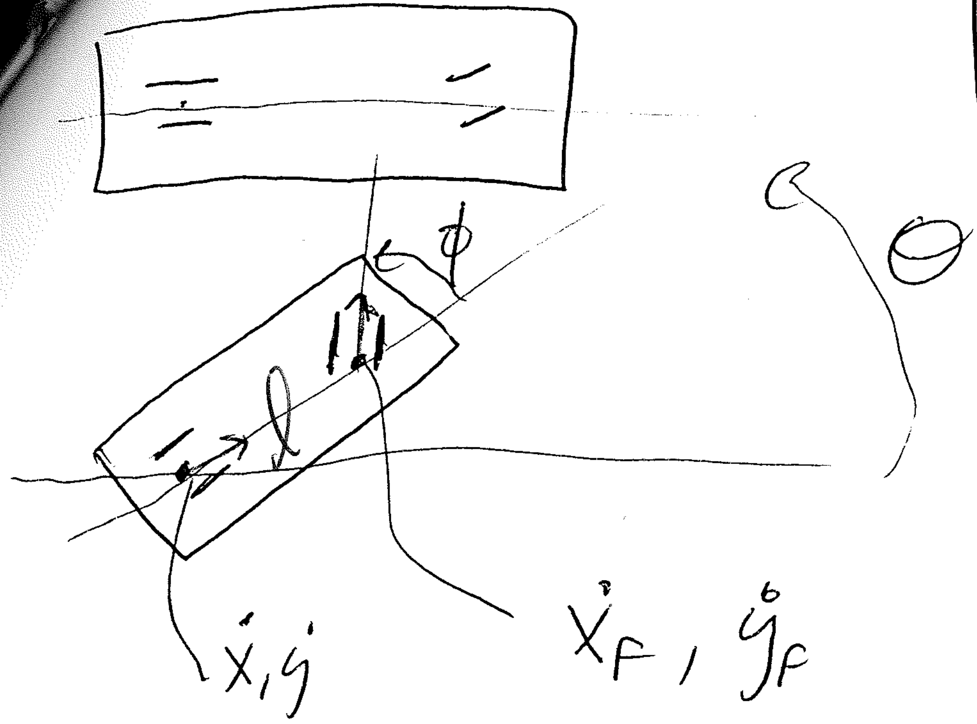
We can control x, y, θ lie 3 thr in
$3-1=2$

$g_1 = \begin{matrix} \cos\theta \\ \sin\theta \\ 0 \end{matrix} \qquad g_2 = \begin{matrix} 0 \\ 0 \\ 1 \end{matrix}$

$$\begin{matrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{matrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2$$

<u>Q:</u> We control each wheel indep
why do we seem to con
dir & steer?

lect 11
9/29/98



Constraint 1) back wheels can't slide sideways

$$\cos\theta\,\dot{y} - \sin\theta\,\dot{x} = 0$$

2) front wheels can't slide sideways

$$\cos(\theta+\phi)\,\dot{y}_F - \sin(\theta+\phi)\,\dot{x}_F = 0$$

$$x_F = x + l\cos\theta \Rightarrow \dot{x}_F = \dot{x} - l\sin\theta\,\dot{\theta}$$

$$y_F = y + l\sin\theta \Rightarrow \dot{y}_F = \dot{y} + l\cos\theta\,\dot{\theta}$$

(go to next pg)

$$\cos(\theta+\phi)(\dot{y}+l\cos\theta\,\dot{\theta})$$

$$-\sin(\theta+\phi)(\dot{x}-l\sin\theta\,\dot{\theta})=0$$

$$\dot{y}\cos(\theta+\phi)-\dot{x}\sin(\theta+\phi)$$

$$+l(\cos(\theta+\phi)\cos\theta+\sin(\theta+\phi)\sin\theta)$$

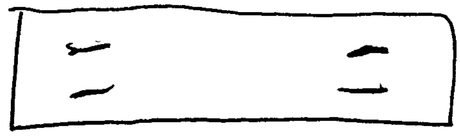
$$=0$$

$$\cos a\cos b-\sin a\sin b=(\cos(a+b))$$

$$w_1=\begin{bmatrix}-\sin\theta & \cos\theta & 0 & 0\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\\ \dot{\phi}\end{bmatrix}$$

$$w_2=\begin{bmatrix}-\sin(\theta+\phi) & \cos(\theta+\phi) & l\cos\phi & 0\end{bmatrix}$$

$\phi = 0$



$w_1 = -\sin\theta \quad \cos\theta \quad 0 \quad 0$

$w_2 = -\sin\theta \quad \cos\theta \quad l \quad 0$

$w_1 \dot{q} = 0$

$-w_2 \dot{q} = 0 \Rightarrow (w_1 - w_2)\dot{q} = 0$

$(w_1 - w_2) = 0 \quad 0 \quad -l \quad 0$

$(w_1 - w_2)\dot{q} = -l\dot{\theta} = 0 \Rightarrow \dot{\theta} = 0$

~~Next the Lie Bracket~~

~~Problem [illegible]~~

restate problem as a control problem

$$\begin{bmatrix} — w_1 — \\ — w_2 — \\ \vdots \\ — w_k — \end{bmatrix} \dot{q} = 0 \iff$$

$$\dot{q} = \begin{bmatrix} \uparrow & & \uparrow \\ g_1(q) & \cdots & g_{n-k} \\ \downarrow & & \downarrow \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_{n-} \end{bmatrix}$$

where $\forall g_i \quad g_i(q) \perp w_j(q)$ ⁝

→ take a break to discuss null space

$$\{x : Ax = 0 \;\&\; x \neq 0\} = \mathrm{Null}(A)$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \iff$$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 0$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 0$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = 0$$

let $\vec{a1} = \begin{matrix} a_{11} \\ a_{12} \\ a_{13} \end{matrix}$ $\quad \vec{a2} = \begin{matrix} a_{21} \\ a_{22} \\ a_{23} \end{matrix}$ $\quad \vec{a3} = \begin{matrix} a_{31} \\ a_{32} \\ a_{33} \end{matrix}$

so $\quad x \cdot a1 = 0 \quad x \cdot a2 = 0 \quad x \cdot a3 = 0$

$\quad x \perp a_1 \quad x \perp a_2 \quad x \perp a_3$

Span of all X's are vect which go to zero which is null span

$$\begin{array}{l} \#\text{ vars in state} \\ -\ \#\text{ constrain} \\ \hline \#\text{ allowable motion} \end{array}$$

...ect to Car $q = \begin{bmatrix} x \\ y \\ \theta \\ \phi \end{bmatrix}$

$w_1 = -\sin\theta \quad \cos\theta \quad 0 \quad 0$

$w_2 = -\sin(\theta+\phi) \quad \cos(\theta+\phi) \quad l\cos\phi \quad 0$

$g_1 = \begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{1}{l}\tan\phi \\ 0 \end{bmatrix} \qquad g_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$[g_1, g_2]$

$\frac{\partial g_2}{\partial x} \quad \frac{\partial g_2}{\partial y}$

$\frac{\partial g_2}{\partial q} g_1 - \frac{\partial g_1}{\partial q} g_2$

next $\frac{\partial g_1}{\partial \phi}$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{1}{l}\tan\phi \\ 0 \end{bmatrix} - \begin{bmatrix} x & x & x & 0 \\ x & x & & 0 \\ x & x & & \frac{1}{l}\sec^2\phi \\ x & x & x & 0 \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}$$

$= 0$

$\frac{\partial g_1}{\cdot}$

Car $\quad g = \begin{matrix} x \\ y \\ \theta \\ \phi \end{matrix} \quad g.$

$$\frac{\partial g_2}{\partial q} = \begin{bmatrix} \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial y} & \frac{\partial g_2}{\partial \theta} & \frac{\partial g_2}{\partial \phi} \\ | & | & | & \end{bmatrix}$$

$$g_2 = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \Rightarrow$$

$$\frac{\partial g_2}{\partial q} = \begin{bmatrix} \frac{\partial a}{\partial x} & \frac{\partial a}{\partial y} & \frac{\partial a}{\partial \theta} & \frac{\partial a}{\partial \phi} \\ \frac{\partial b}{\partial x} & \frac{\partial b}{\partial y} & \frac{\partial b}{\partial \theta} & \frac{\partial b}{\partial \phi} \\ \frac{\partial c}{\partial x} & \frac{\partial c}{\partial y} & \frac{\partial c}{\partial \theta} & \frac{\partial c}{\partial \phi} \\ \frac{\partial d}{\partial x} & \frac{\partial d}{\partial} & \frac{\partial d}{\partial} & \frac{\partial d}{} \end{bmatrix}$$

...t to Car $\quad q = \begin{bmatrix} x \\ y \\ \theta \\ \phi \end{bmatrix}$

$w_1 = -\sin\theta \quad \cos\theta \quad 0 \quad 0$

$w_2 = -\sin(\theta+\phi) \quad \cos(\theta+\phi) \quad l\cos\phi \quad 0$

$g_1 = \begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{1}{l}\tan\phi \\ 0 \end{bmatrix} \qquad g_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$\begin{array}{cc} \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial y} \end{array}$

$[g_1, g_2]$

$\frac{\partial g_2}{\partial q} g_1 - \frac{\partial g_1}{\partial q} g_2$

$\frac{\partial g_1}{\partial \phi}$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \\ \tan\phi \\ 0 \end{bmatrix} - \begin{bmatrix} x & x & x & 0 \\ x & x & \checkmark & 0 \\ x & x & \checkmark & \frac{1}{l}\sec^2\phi \\ x & x & x & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$= 0$

$$g_3 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{l}\sec^2\phi \\ 0 \end{bmatrix}$$

so it looks like we can control θ

$$[g_1, g_3] =$$

$$\frac{\partial g_3}{\partial q} g_1 - \frac{\partial g_1}{\partial q} g_3$$

$$\underbrace{\begin{array}{cccc} x & y & \theta & \phi \end{array} \atop \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \begin{matrix} P \\ 0 \end{matrix} \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{1}{l}\tan\phi \\ 0 \end{bmatrix}}_{0} - \begin{bmatrix} x & x & -\sin\theta & x \\ x & x & \cos\theta & x \\ x & x & 0 & x \\ x & x & 0 & x \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \frac{1}{l} \\ 0 \end{bmatrix}$$

$\sec^2\phi$ scales x & y motion

$$= \begin{pmatrix} \frac{1}{l}\sec^2\phi\sin\theta \\ -\frac{1}{l}\sec^2\phi\cos\theta \\ 0 \end{pmatrix} = g_4$$

$$[g_2, g_3] = \begin{bmatrix} 0 \\ 0 \\ \dfrac{-2\tan\phi\sec^2\phi}{l} \\ 0 \end{bmatrix} = g_5$$

$$\begin{pmatrix} \cos\theta & 0 & 0 & +\dfrac{\sin\theta\sec^2\phi}{l} \\ \sin\theta & 0 & 0 & -\frac{1}{l}\cos\theta\sec^2\phi \\ \tan\phi & 0 & \frac{-1}{l}\sec^2\phi & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

full rank

g_5 doesn't

$$0 = \begin{bmatrix} -\sin(\theta+\phi) & \cos(\theta+\phi) & l\cos\phi & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix}$$

~~is the second constraint wheels place on car~~

<u>Step 1</u> : Determ constraints

<u>Step 2</u> : Determ allowable directions of motion from the constraints

<u>Step 3</u> : Lie Bracket

for $q = \begin{bmatrix} x \\ y \\ \theta \\ \phi \end{bmatrix}$

$$\frac{\partial g_2}{\partial q} = \begin{bmatrix} \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial y} & \frac{\partial g_2}{\partial \theta} & \frac{\partial g_2}{\partial \phi} \\ | & | & | & \end{bmatrix}$$

$$g_2 = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \Rightarrow$$

$$\frac{\partial g_2}{\partial q} = \begin{bmatrix} \frac{\partial a}{\partial x} & \frac{\partial a}{\partial y} & \frac{\partial a}{\partial \theta} & \frac{\partial a}{\partial \phi} \\ \frac{\partial b}{\partial x} & \frac{\partial b}{\partial y} & \frac{\partial b}{\partial \theta} & \frac{\partial b}{\partial \phi} \\ \frac{\partial c}{\partial x} & \frac{\partial c}{\partial y} & \frac{\partial c}{\partial \theta} & \frac{\partial c}{\partial \phi} \\ \frac{\partial d}{\partial x} & \frac{\partial d}{\partial y} & \frac{\partial d}{\partial \theta} & \frac{\partial d}{\partial \phi} \end{bmatrix}$$

: 1 Crib sheet for quiz
: save report graphs
see me after

Lect 12

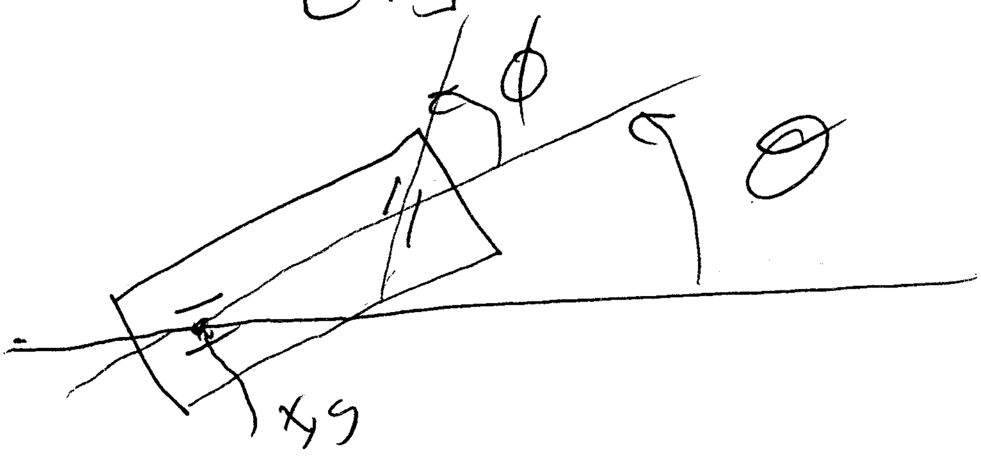
Using non-holonomic constraints to determine full DOF of system

10-1-98

outline
1) finish HA
2) AI - summ

w/ car we found

$$q = \begin{bmatrix} x \\ y \\ \theta \\ \phi \end{bmatrix}$$ to be a state vector



$$\begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = 0$$

let $\varepsilon g_1(q)$ be a small rotation

$\varepsilon g_2(q)$ be a small xlation

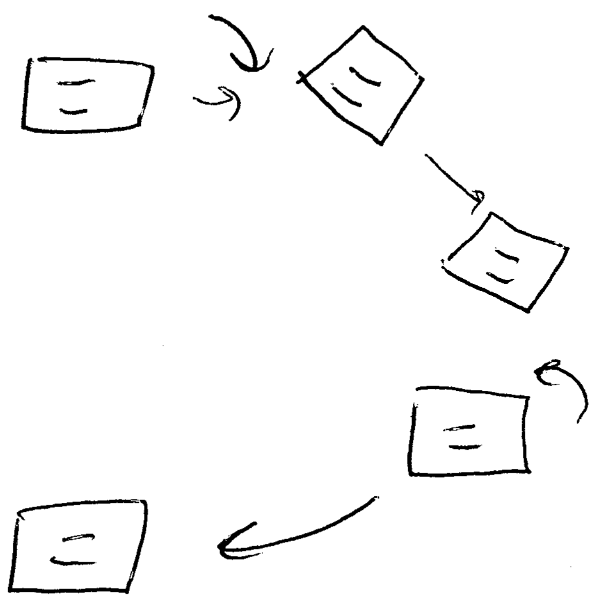
Follow the follow sequence

$\varepsilon g_1(q)$

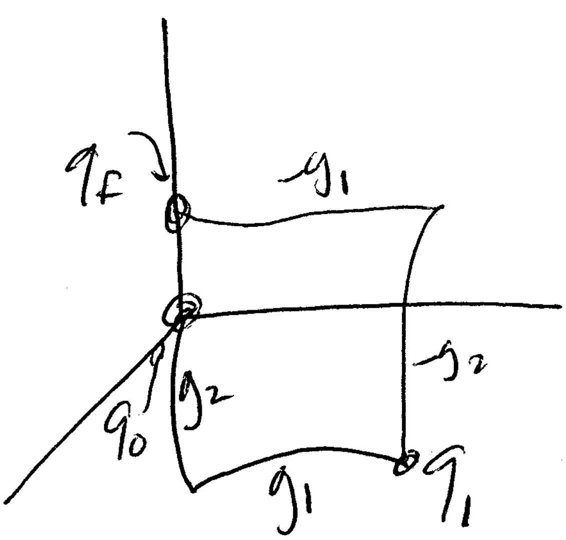
$\varepsilon g_2(q)$

$-\varepsilon g_1(q)$

$-\varepsilon g_2(q)$



Configuration is different. Moved some x-y amount.



$q_F = q_1 - \varepsilon g_1(q_1) + \varepsilon g_2(\varepsilon$

$- \varepsilon g_2(q_1 - \varepsilon g_1(q_1$

$q_0 = q_1 - \varepsilon g_2(q_1) - \varepsilon g_1(q_1 - \varepsilon$

recall

$$F(q_1+\varepsilon v) = f(q_1) + \varepsilon \frac{\partial f}{\partial q} v + HOT$$

$$q_f - q_0 = q_1 - \varepsilon g_1(q_1) - \varepsilon g_2(q_2) + \varepsilon^2 \frac{\partial g_2}{\partial q} g_1$$

$$- q_1 + \varepsilon g_2(q_1) + \varepsilon g_1(q_1) - \varepsilon^2 \frac{\partial g_1}{\partial q} g_2(q_1)$$

$$+ HOT$$

$$q_f - q_0 = \varepsilon^2 \left(\frac{\partial g_2}{\partial q} g_1 - \frac{\partial g_1}{\partial q} g_2 \right)$$

$n \times n$ $\quad$ $1 \times n$

$$\lim_{\varepsilon \to 0} \frac{q_f - q_0}{\varepsilon} = [g_1, g_2] = \frac{\partial g_2}{\partial q} g_1 - \frac{\partial g_1}{\partial q} g_2$$

lie bracket → think of it as a span operator

lie bracket of lie brackets