H. Choset Fall, 1997

ME 24-701: Mathematical Techniques in Mechanical Engineering I
1 Exam

Date Handed Out: October 13, 1998
Time Allotted: 1 hour and 15 minutes

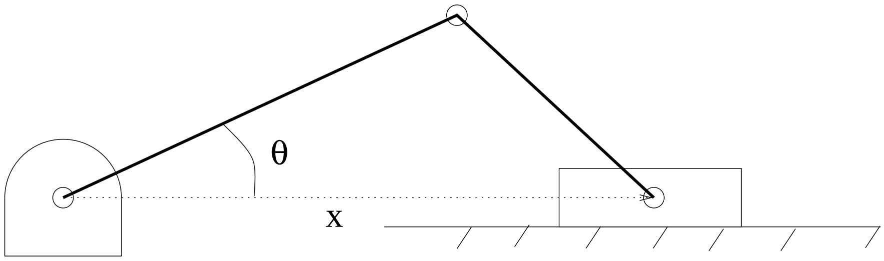
- Please show all work.
- You can use one crib sheet.
- You must attempt all problems.
- GOOD LUCK!!!

P1. [***Homogenous Represenation***] Let

$$T = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

be a homogenous transformation. Demonstrate that T is either a translation, followed by a rotation *or* a rotation followed by a translation.

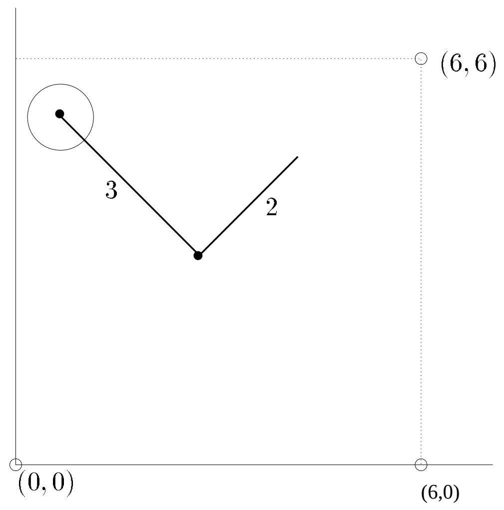
P2. Consider the slider-crank system in Figure 0. The box is constrained to only slide along the floor, as if it were on a track. Given θ_1, find x. Is this a forward or inverse kinematics problem? Why?



P3. A mobile robot has a planar two-link manipulator, where link 1 is 3 units and link 2 is 2 units in length.

(a) Determine an (x, y) coordinate pair for the mobile base such that once it is placed at (x, y), the two-link manipulator can reach all three points $(0, 0), (6, 0)$, and $(6, 6)$ (the mobile base cannot move once it is placed).

(b) With the mobile base fixed at (x, y), perform the inverse kinematics for all three pairs of points for the two-link manipulator.

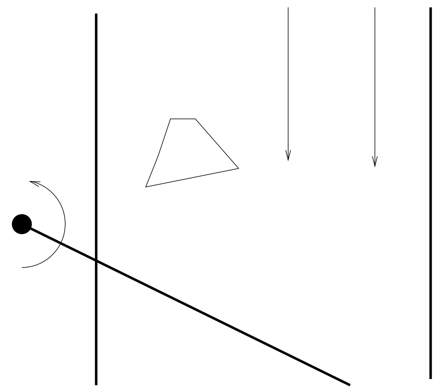


P4. Positioning a planar object with Lie Brackets.

(a) How many degrees of freedom does an arbitrarily planar shaped object have? (No explanation required)

(b) How many degrees of freedom are necessary for a serial linkage, i.e., a sequence of links connected by revolute joints, to arbitrarily place an arbitrarily shaped object in the plane? (No explanation required)

(c) Consider a conveyor belt system on which arbitarily shaped objects are placed. Naturally, these objects move with the conveyor. Downstream along the conveyor, there is a one-degree-of-freedom fence which can only rotate about a pivot point.



In class, when dealing with non-holonomic constraint problems, we first derived the constraints ($w_i(q)$), and from the constraints, we derived the initial set of allowable motions ($g_i(q)$). We then performed

Lie bracket operations on the original set of motions to obtain the full set of motion achievable by the robot.

Sometimes, it is easier to directly start with the initial set of allowable motions: $g_1(x, y, \theta) = (0, 1, 0)^T$ for the belt and $g_2(x, y, \theta) = (-y, x, 1)^T$ for the fence.

Can the fence-conveyor system arbitarily position and orient a planar part? Show all work (hint: use Lie Brackets) and explain your result.