Midterm Exam

16-281: General Robotics

2026

This exam is being administered in person. You should write all of your work and solutions by hand. We will collect your answers when the test is finished.

- You will have 1 hour and 10 minutes to complete this exam
- There are 5 sections on 19 pages. Make sure you have all of the pages. There may be blank pages throughout the sections.
- When making drawings, please be precise. Rounded edges should look rounded, sharp edges should look sharp, sizes should be close to scale. Neatness counts.
- Show your work. Partial credit may apply. Where applicable, algebraically justify your work and show intermediate steps to ensure full credit.
- It should be very clear what your final answer is. Circling your final answer is recommended.
- You may need to make certain assumptions to answer a problem. State them (e.g. what is optimal?).
- You are allowed one handwritten two-sided reference sheet for the exam. No cell phones, laptops, neighbors, etc. allowed.
- Good luck and you can do it.

Contents

1 Vision [20 points]

Alan and Rahul have taken a trip to the nation of Suriname to look at some blue poison dart frogs. Since these frogs are poisonous, Alan and Rahul use their 16-281 knowledge to make a robot to get up close and take pictures of the frogs.

Figure 1: A blue poison dart frog

In this question, you will help our explorers use computer vision to take some pictures of these amazing amphibians.

1. We want to create a program that thresholds the frog from the forest background. What color representation would be the best for this? How would you tune this threshold for varying light conditions? [5 points]

2. Due to poor lighting conditions, after thresholding you are tasked with highlighting the edges of the frog. Explain how you would do this using convolution. Write out the matrix of the kernel/filter that you would use. [5 points]

3. Oh no! It started to rain! After using your proposed method above, you realize that there are many false edge detections due to image noise. What filtering method can be used to resolve this? Why does this reduce noise/improve edge detection? [5 points]

4. Our explorers have found a frog! Using a LIDAR, the explorers find that the frog is 3 meters away. They decide to take pictures of the frog using a camera with these specifications:

   - Focal length: 30 mm
   - Camera resolution: 5000 x 3500 pixels
   - Pixel size: 8 μm

   In the first picture, the center of the frog is 55 pixels to the left of the center of the image. The robot moves to the right and the second picture. The center of the frog is now 35 pixels to the right of center. The slippery rainforest terrain means that odometry can't be trusted. Use the information provided to calculate how far the robot moved between the first and second pictures. The robot moves while maintaining the same camera orientation. [5 points]

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You can work out problem 1 here

2 Control [20 points]

A telescope must precisely track planets as they move across the sky. The telescope rotates about a single axis with angular position θ. Due to disturbances, the telescope's pointing angle drifts over time, there will be steady-state error. To maintain accurate tracking, the system uses a PID controller to regulate the telescope's angular position.

The control system measures the error

$$e(t) = \theta_{desired}(t) - \theta(t)$$

and computes the control input (a torque)

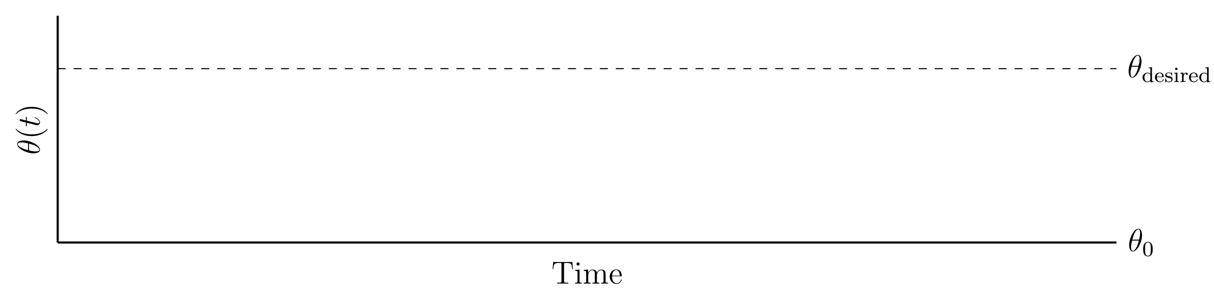
$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt}$$

where K_p, K_i, and K_d are the proportional, integral, and derivative gains respectively.

1. Suppose the telescope is commanded to rotate to a new angle using a step input. At first, the controller only uses proportional control ($K_p > 0$, $K_i = 0$, $K_d = 0$).

   Sketch the expected response of the telescope angle $\theta(t)$ over time. Note, there will be some steady-state error. [4 points]

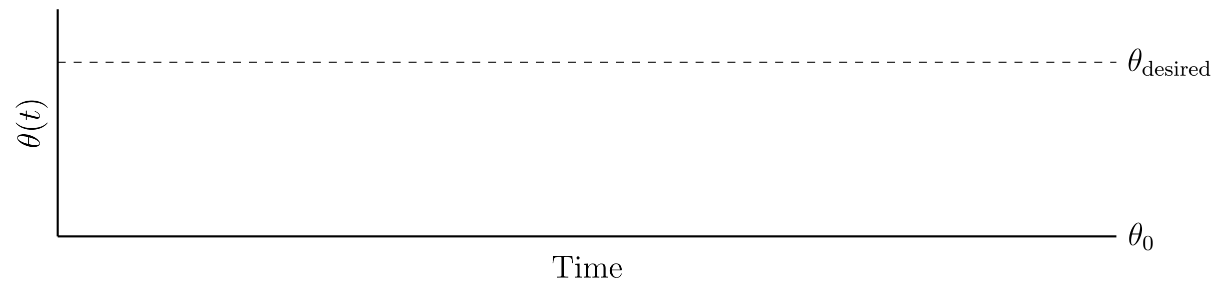
System Response with Proportional Control



2. Engineers notice that the telescope overshoots the target angle significantly when the proportional gains are large and decide to add a derivative term ($K_d > 0$).

   Explain what effect the derivative term has on the response and briefly sketch the new response. [4 points]

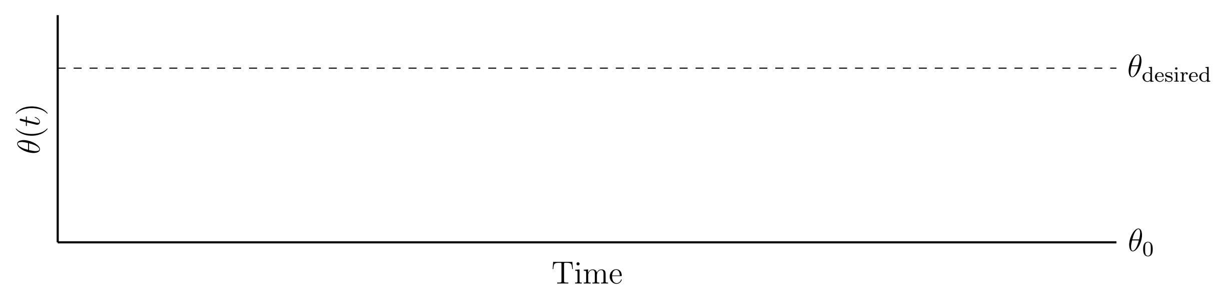
System Response with Proportional and Derivative Control



3. The steady-state error is due to a constant external disturbance torque.

   Which term(s) in the PID controller help reject this disturbance and why? Briefly sketch the new response after adding your selected term(s). [4 points]

System Response with Steady-State Error Mitigation



4. Suppose the derivative gain K_d is increased too much.

   Describe one drawback of very high derivative gain in a real robotic system with noisy sensors. [4 points]

5. The telescope must follow a predictable trajectory when tracking a moving planet. Engineers add a feedforward control term based on the desired motion.

   Explain the purpose of feedforward control and how it differs from feedback control. [4 points]

3 Path Planning [25 points]

The following map below has a robot (green circle) and a goal marker:

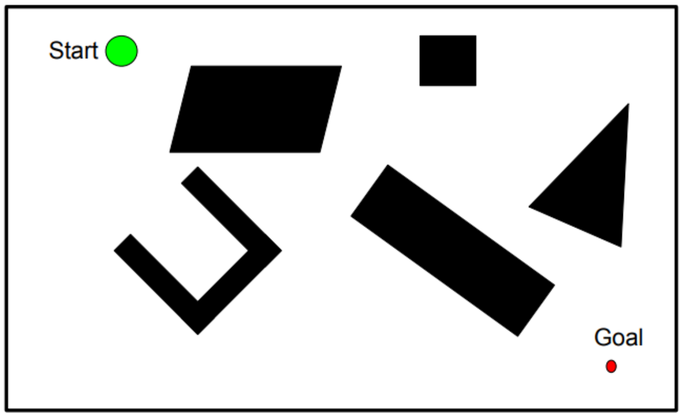


Figure 2: Map with start and goal positions

1. Your friend wants the robot to traverse this map to the goal using a potential function. Is this a good idea? Why or why not? [3 points]

2. Draw a Voronoi diagram using the map above [4 points]

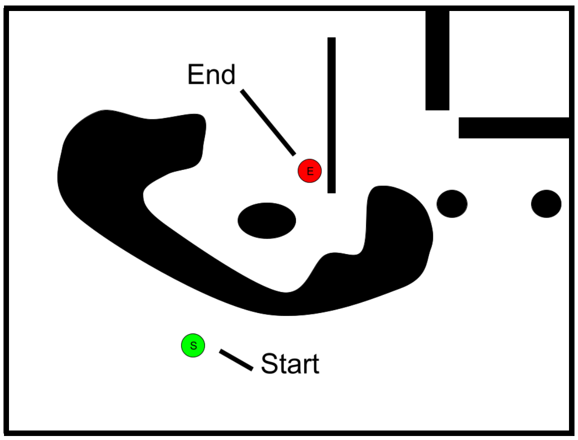
3. Using path on your Voronoi diagram that connects the start and goal, comment on whether or not this path would be considered optimal. State your assumptions and explain your answer. [4 points]

4. What is an alternative method to Voronoi that could perform better by some metric of your choosing? Explain your answer. [4 points]

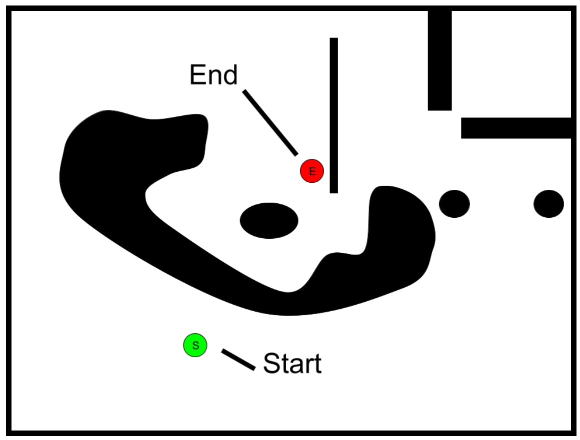
For this portion of the exam, we will be using the following map. Assume the "position" of the circular robot is defined by its center on the map and the size of the robot is proportional to the rest of the map (e.g. it cannot travel through to the upper right corner from its start position). The robot is 1 meter in diameter.

5. Indicate (draw) the configuration space and the free space of the robot for this map. Consider the entire map. Make it clear what is and is not included in each space. [4 points]

Free Space



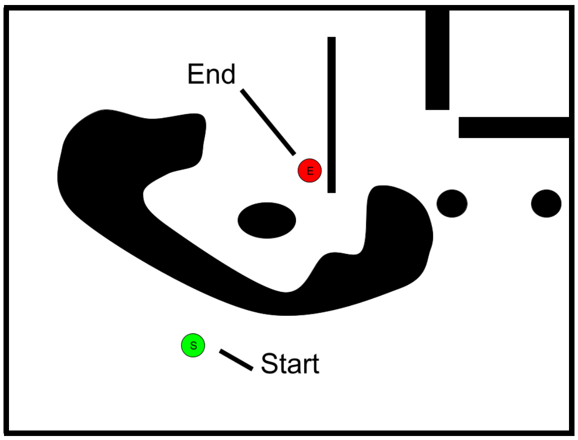
Configuration Space



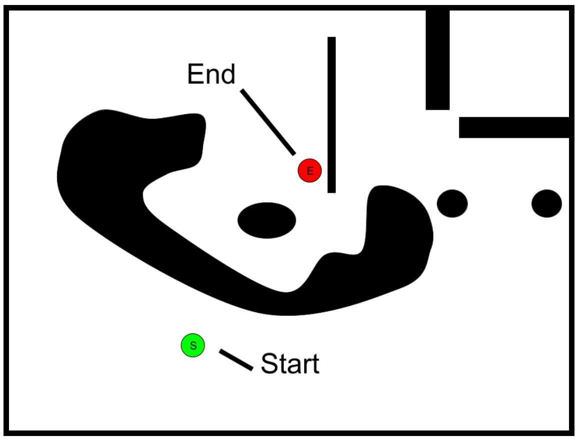
6. How does the configuration space compare to the free space of this map (i.e. is one a subset of the other)? [3 points]

7. Draw the paths that would be found by the Bug 1 and Bug 2 algorithms **for this robot.** Assume both algorithms prefer to travel right. You should be drawing the path of the center of the robot according to your configuration space. [3 points]

Bug 1



Bug 2



4 Graph Search [15 points]

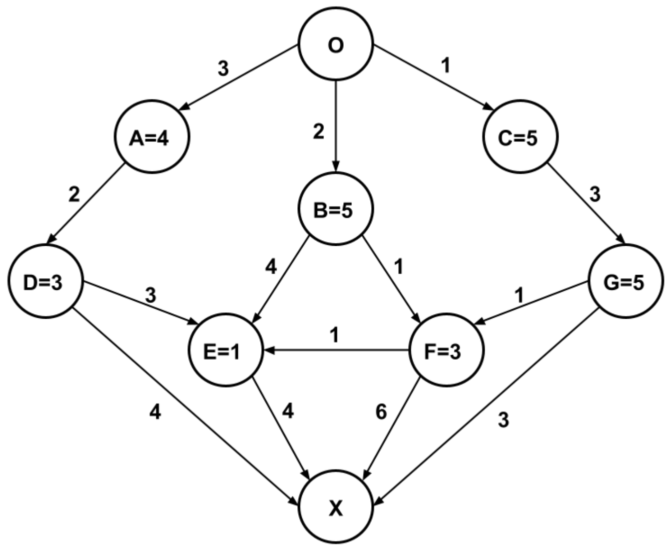


Figure 3: Graph for the A^* search

1. Perform an A^* search on the graph shown in Figure 4. O is the starting node, and X is the goal node. Break ties alphabetically. List the nodes that are EXPANDED and added to the closed list in the A^* algorithm above in the order in which they are expanded: [10 points]

2. What is the path found by the A^* algorithm and how long is it? [5 points]

3. Is the heuristic admissible? How is that related to finding the shortest path? [BONUS 5 points]

5 Localization [20 points]

You're on a spaceship many years into the future that has traveled to another universe in which the planets are stationary around its star. You begin your journey orbiting the star when suddenly you travel through a meteor field and damage your GPS (Galactic Positioning System) system. Now you don't know where you are! You must rely on odometry and a sensor that can detect the planets around you, and apply probabilistic localization methods (Bayesian) to determine your most likely position in the orbit.

At time $t - 1$, the spaceship's position x is represented as a probability distribution $p(x_{t-1})$ using samples. When the spaceship moves at time t based on a control signal u_t, its new position follows an updated probability distribution $p(x_t)$.

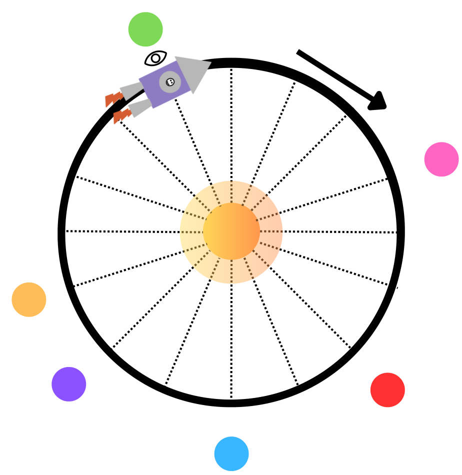
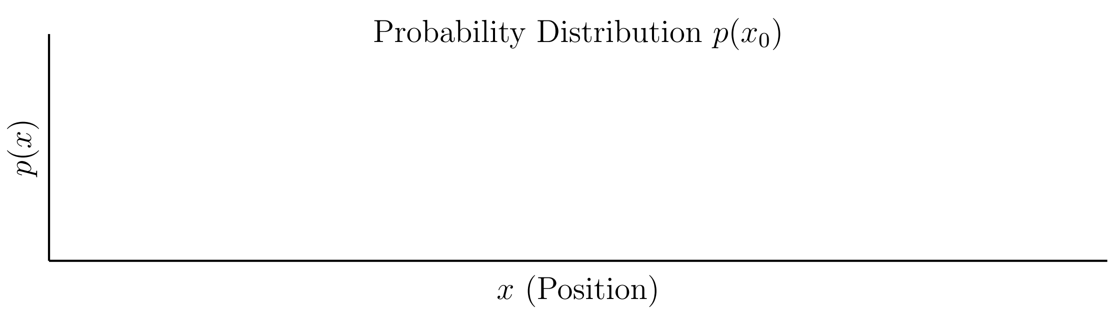
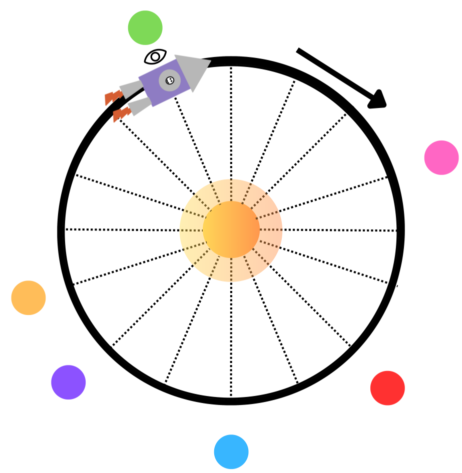


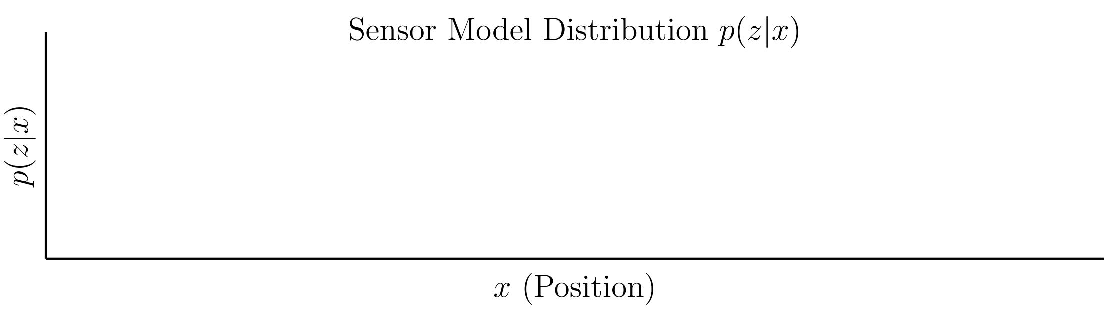
Figure 4: Orbit of our spaceship, *The Howie*

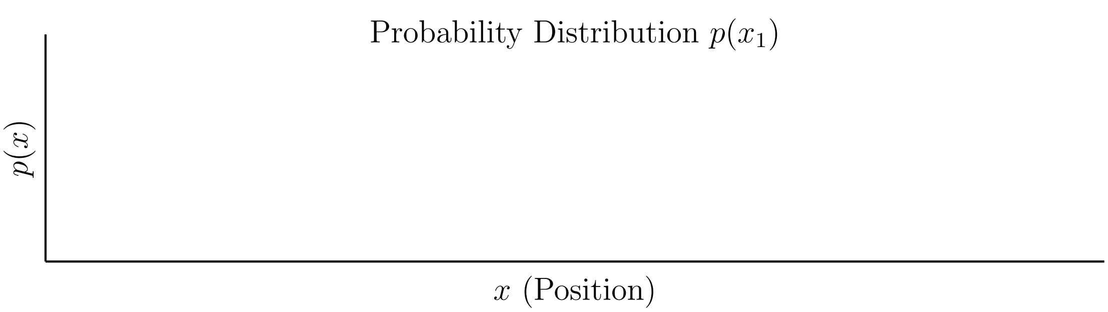
1. The spaceship is on a circular path around the star. Assuming that the spaceship can travel perfectly around this circular path and that we know the relative positioning of the planets, but not where the spaceship is on the circular path. **Sketch a continuous initial probability distribution $p(x_0)$ before any sensor readings.** [2 points]



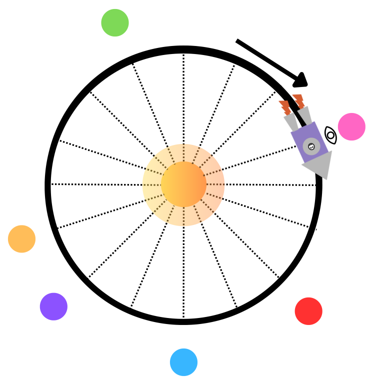
2. The spaceship takes an initial measurement and detects a planet in its surroundings! **Sketch both** $p(z|x)$ **(sensor model) and** $p(x_1)$ **(updated belief) after the measurement.** [6 points]

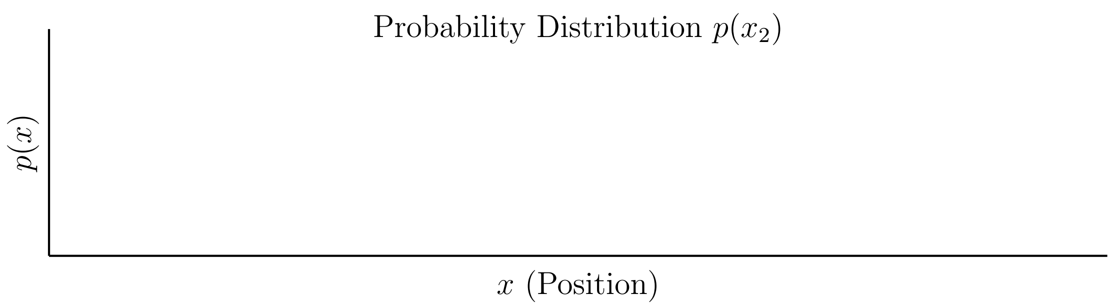






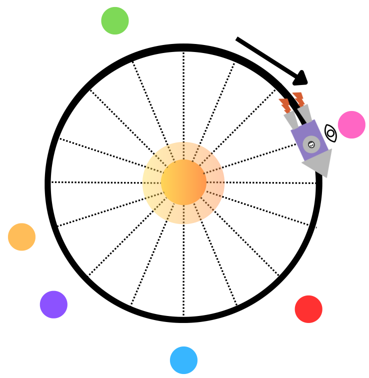
3. The spaceship moves forward around the orbit 4 sectors, as pictured. **Sketch the new continuous probability distribution $p(x_2)$ after the spaceship moves.** [4 points]

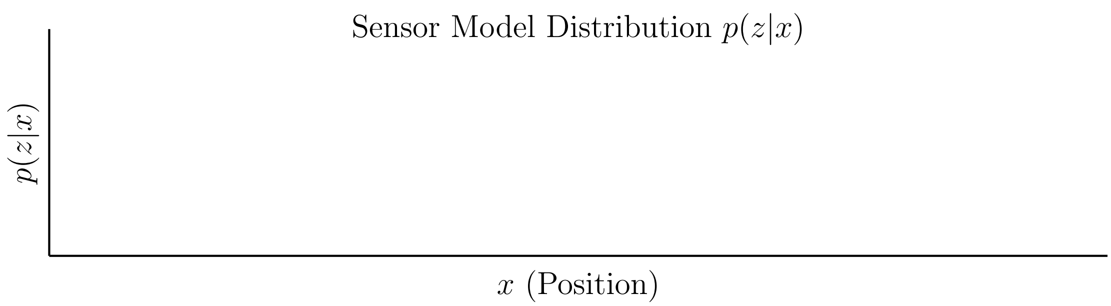


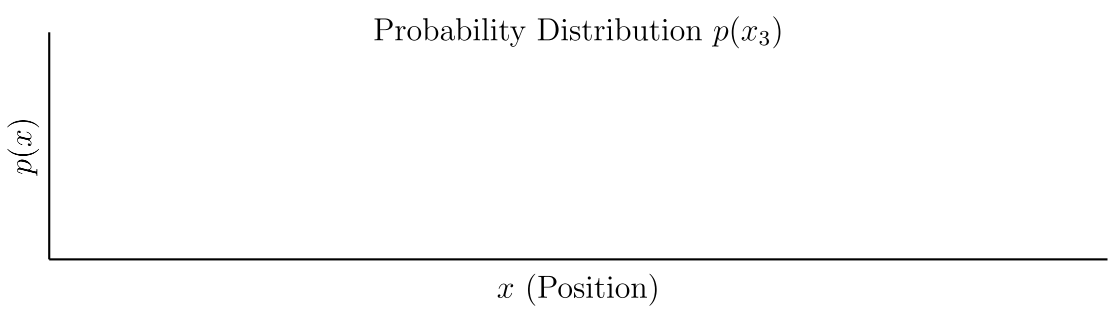


4. As you wait for the sensor to detect the next planet, you begin to think about other ways you could've localized. You remember from taking 16-281 that you can also represent probabilities discretely! If you used a particle filter, what would be one advantage and one drawback of increasing the amount of particles the algorithm uses? [4 points]

5. The spaceship takes another sensor measurement and detects another planet, helping refine its belief. **Sketch both $p(z|x)$ (sensor model) and $p(x_3)$ after incorporating the second sensor reading.** Did the spaceship localize to a single point? [4 points]







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