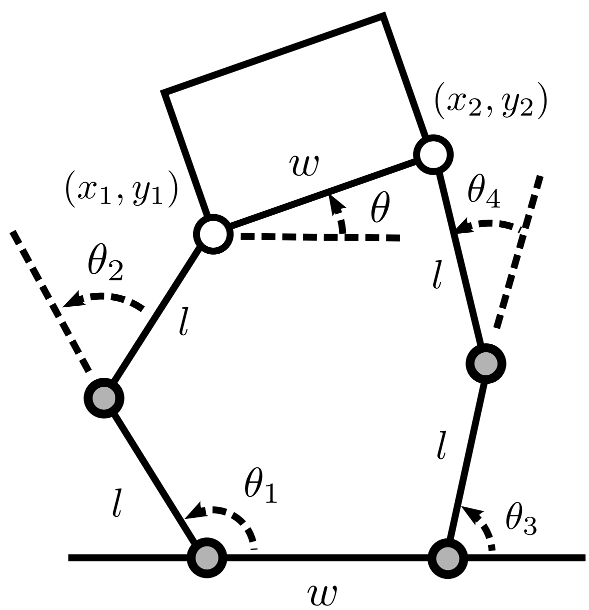
Final Exam 16-311 Intro to Robotics

Name:____________________ Team: ____________________

- You will have 1 hour and 30 minutes to complete this exam
- There are 4 questions on 13 pages. Make sure you have all of them.
- When making drawings - be precise. Rounded edges should look rounded, sharp edges should look sharp, sizes should be close to scale. Neatness counts.
- Show your work. Partial credit may apply. Likewise, justify algerbraically your work to ensure full credit, where applicable.
- It should be *very* clear what your final answer is, circle it if necessary.
- You may need to make certain assumptions to answer a problem. State them (e.g. what is optimal).
- You are allowed one *handwritten* two-sided reference sheet for the exam. No cell phones, laptops, neighbors, etc. allowed.
- Good Luck!

1 Redundant Manipulation - 35 pts

In this problem you will explore multi-arm manipulation. Consider the following scenario where two rotary-rotary (RR) arms are collaboratively manipulating a box in 2D. Each arm is fixed to the box with a free rotating bearing. Arm 1 has joint angles θ_1 and θ_2 and is depicted on the left, and arm 2 has joint angles θ_3 and θ_4 and is depicted on the right.



Hint: The inverse kinematics for arm 1 are:

$$\theta_2 = \arccos\left(\frac{x_1^2 + y_1^2 - 2l^2}{2l^2}\right)$$

$$\theta_1 = \arcsin\left(\frac{l \sin\theta_2}{\sqrt{x_1^2 + y_1^2}}\right) + \arctan\left(\frac{y_1}{x_1}\right)$$

a) How many degrees of freedom for the box can the two arms achieve? How many of the arm joints are redundant? (5 pts)

b) Pick a point p on the box to represent the origin of the box. Draw the workspace corresponding to the box's (p_x, p_y) position, i.e. all the positions where the box's origin can be moved to at any orientation. Clearly label any important dimensions. Hint: A good choice of p might be $p_x = x_1$, $p_y = y_1$. (10 pts)

c) Write the forward kinematics expressions for the box's position and orientation (p_x, p_y, θ) as a function of the arm tip positions (x_1, y_1) and (x_2, y_2). (5 pts)

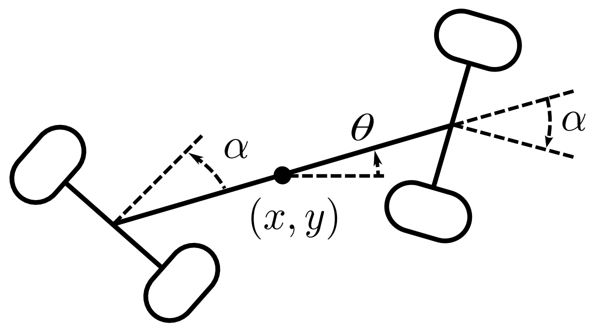
d) Write (x_2, y_2) in terms of (x_1, y_1, θ)) to satisfy the box's physical constraints (avoid tearing the box apart). (5 pts)

e) Using your previous answer and the inverse kinematics for a RR arm, determine the inverse kinematics for θ_3 and θ_4 as a function of (x, y, θ). (5 pts)

f) What are the joint angles to place the box origin at $(0, 2l)$ with orientation $\theta = \pi/6$? If it is not feasible, show why. (5 pts)

2 Snakeboard Kinematics - 15 Points

Consider the mechanism shown below, commonly known as a *snakeboard*, consisting of a rigid board supported by two caster wheels constrained to steer in opposite directions. Let the mechanism have a 4-dimensional state of (x, y, θ, α). Note that α steers both wheels, but in opposite directions.



a) What is the state space, i.e., what is the q vector (1 pts)?

b) Write out the non-holonomic constraints for this sytem if the wheels are not allowed to slip sideways (4 pts).

c) Verify whether or not the following 5 vectors are initial allowable motions. Show your work (1 pt each).

$$\text{a)} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{b)} \begin{bmatrix} -L\cos(\theta)\cot(\alpha) \\ -L\sin(\theta)\cot(\alpha) \\ 1 \\ 0 \end{bmatrix} \quad \text{c)} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\text{d)} \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ \frac{1}{L}\tan(\alpha) \\ 0 \end{bmatrix} \quad \text{e)} \begin{bmatrix} -L\cos(\theta)\cot(\alpha) \\ -L\sin(\theta)\cot(\alpha) \\ 0 \\ 1 \end{bmatrix}$$

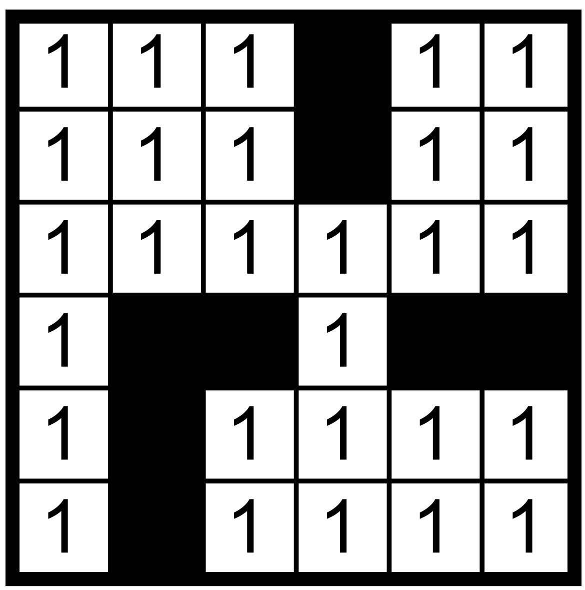
d) Calculate the Lie bracket for two of the initial allowable motions (5 pts)

3 Observation-Free Localization - 25 Points

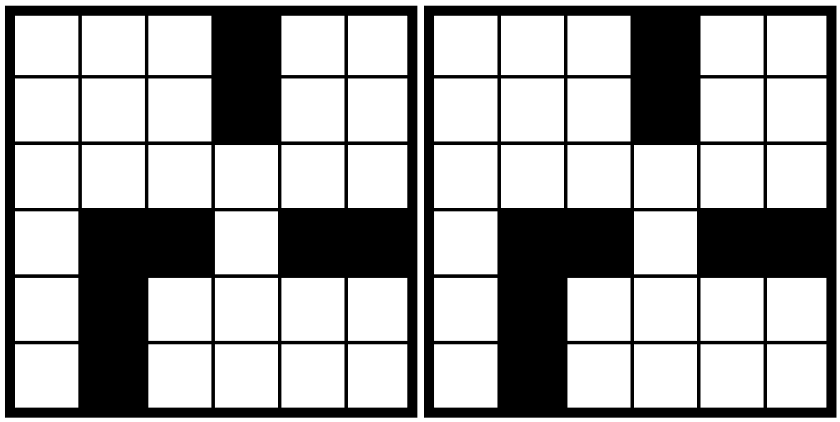
We have previously studied localization using transition and observation models. Now we will explore a situation where a robot has lost its sensors and cannot observe, but can still localize.

The robot in consideration has an omnidirectional base that allows it to move in any direction and begins knowing its orientation, but not position. The known map of the world is shown below, with black cells indicating obstacles and white cells indicating free space.

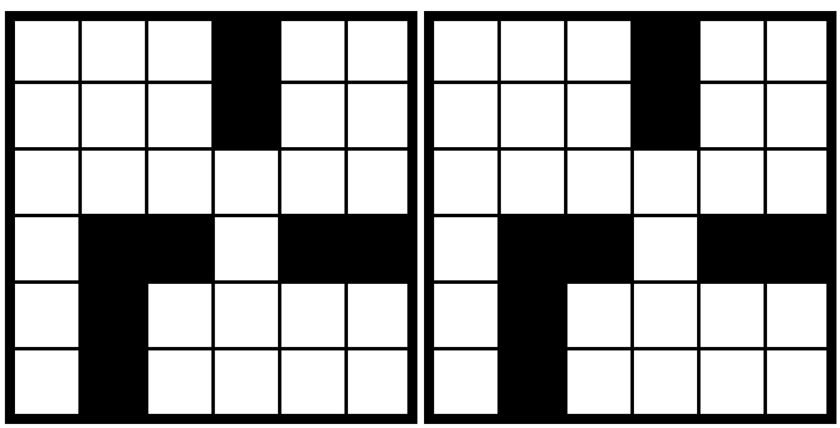
The robot begins with a uniform initial belief state shown below. A 1 in a cell indicates that the robot may exist in that cell, and a 0 means it cannot be there. On each time step, the robot executes a left-up movement at every step (1 unit left, 1 unit up) in the world coordinates. Assume that there is no transition noise. If there is an obstacle in the way of the robot, the robot will stay in place.



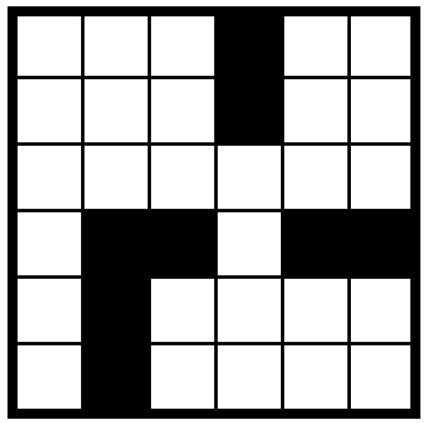
a) Assuming the robot cannot pass through obstacles, draw the robot's belief state after the first left step (left) followed by the first up step (right) (5 pts)



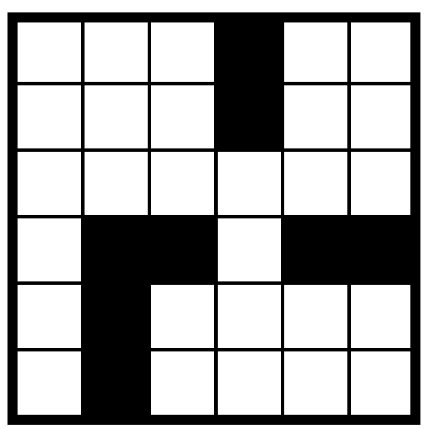
b) Draw the robot's belief state after another left and up step. (5 pts)



c) Finally, draw the belief state after another 50 time steps. (5 pts)



d) Has the robot determined its position? Give a new repeating movement sequence the robot can execute to finish localizing and indicate where it will converge to. (5 pts)



4 Smart Image Background Removal - 25 Points

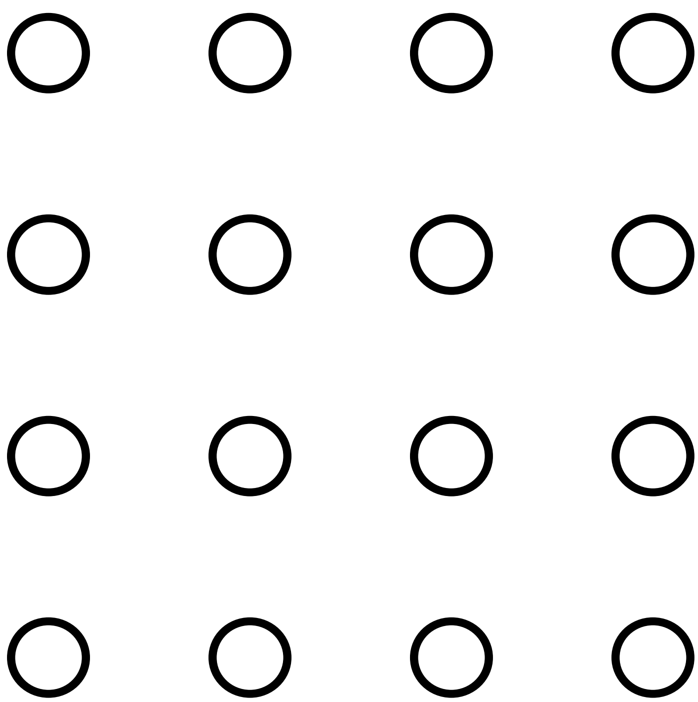
Most images have a wealth of information in grayscale or color information. In this problem, we will analyze a smart segmentation algorithm similar to those used in image processing programs.

Let the image red, green and blue values of the pixel at row i and column j be represented by the vector $p(i,j) = (r,g,b) \in \mathcal{R}^3$. We will slightly modify the A^* algorithm to perform segmentation for us.

a) Write expressions for the L1 and L2 RGB distances between two pixels x and y. Your answer should only involve the RGB components of x and y and not their image coordinate positions. (3 pts)

b) We can now form a graph from the image pixels, where each node is a pixel and edge costs are the L1 RGB distances between neighbors. Fill in the edges for the below 4 by 4 image using 4-connectivity. (10 pts)

$$\begin{bmatrix} (10,0,0) & (7,2,1) & (5,0,5) & (5,0,0) \\ (0,10,0) & (10,2,1) & (9,1,5) & (5,1,3) \\ (0,0,10) & (0,0,1) & (9,2,1) & (0,2,1) \\ (2,0,8) & (0,4,1) & (9,0,0) & (9,2,3) \end{bmatrix}$$



c) We will now search over the graph using A^*. Let our heuristic function be zero:

$$h(p) = 0$$

Consider the path produced by A^* for a start pixel x and goal pixel y. State if each of the following are true or false. (1 pt each)

i) The path has the least number of pixels among all paths from x to y.

ii) The path has the least total pixel-to-pixel change among all paths from x to y.

iii) The path traverses the least distance in image coordinates among all paths from x to y.

iv) The path has total cost strictly lower than all other paths from x to y.

v) The path has total cost lower or equal to all other paths from x to y.

d) Determine the lowest cost path starting from the upper left node to the bottom right node. (7 pts)