Final Exam

16-281: General Robotics

2026

This exam is being administered in person. You should write all of your work and solutions by hand. We will collect your answers when the test is finished.

- You will have 2 hours to complete this exam
- There are 6 sections on 15 pages. Make sure you have all of the pages. There may be blank pages throughout the sections.
- When making drawings, please be precise. Rounded edges should look rounded, sharp edges should look sharp, sizes should be close to scale. Neatness counts.
- Show your work. Partial credit may apply. Where applicable, algebraically justify your work and show intermediate steps to ensure full credit.
- It should be very clear what your final answer is. Circling your final answer is recommended.
- You may need to make certain assumptions to answer a problem. State them (e.g. what is optimal?).
- You are allowed one handwritten two-sided reference sheet for the exam. No cell phones, laptops, neighbors, etc. allowed.
- Good luck and you can do it.

Contents

1 Vision [15 points]

The Carnegie Optics team takes expensive photos for high school yearbooks. They take photos of students in front of green screens and are asking 16-281 students for help to process their images. The photo room has a window so the lighting can vary day to day.

1. The team wants to create a program that thresholds out the green screen. What color representation would be the best for this? In one sentence, how would you tune this threshold for varying light conditions? [3 points]

2. Now, the team wants to explore an alternate approach that involves highlighting the edges of each student's profile, but they aren't sure how to proceed. You remember from class that you can convolve images with certain filters to achieve this result. Write out the matrix of the kernel/filter that you would use and explain why you would use it. [4 points]

Answer the following two questions with the two approaches you suggested above in mind.

3. List two benefits and two drawbacks of each implementation approach. Which is optimal? [6 points]

4. If the student were wearing a green shirt, would the color-based method from problem 1 still work reliably? In one sentence, explain why or why not. [2 points]

2 Control [15 points]

Engineers at a startup are building a self-balancing robot. For their robot, the control system measures the error

$$e(t) = R - \theta(t)$$

and computes the control input (a torque)

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt} + u_{\text{ff}}(t)$$

where K_p, K_i, and K_d are the proportional, integral, and derivative gains, and $u_{\text{ff}}(t)$ is the feed forward term computed from the desired motion. The block diagram in Figure 1 describes how the control input to the plant is calculated.

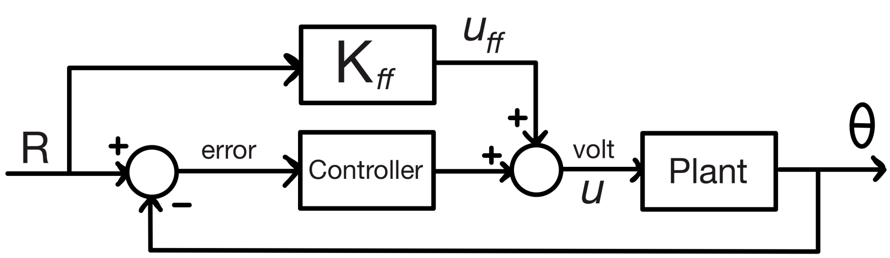


Figure 1: Block Diagram for Balance Bot

1. At one instant in time, suppose: $R = 10, \theta = 8, \int e(t)dt = 1, \frac{de}{dt} = 0$, and the PID gains are $K_p = 2, K_i = 1, K_d = 0.5$. The desired voltage input to the plant at this instant is $u = 10$. What feed forward gain K_{ff} would achieve this? [5 points]

2. In practice, the robot dynamics are not perfectly known and are modeled imperfectly which in turn affects the performance of the feedforward term. Explain how the PID feedback term compensates for these errors. [3 points]

While testing, the engineers find that the robot is able to balance near upright, but there is a persistent small tilt error when an external disturbance force is applied. One engineer proposes increasing only the derivative gain K_d to eliminate this error.

3. Is increasing K_d alone an effective way to remove steady-state error? Explain why or why not. [2 points]

4. Which PID term is directly responsible for eliminating steady-state error caused by a constant disturbance, and why? [2 points]

5. Another engineer suggests that instead of adding a feedforward term, they could simply increase K_i to reduce steady-state error. Compare how increasing K_i and adding a feedforward term affect steady-state error. Which approach guarantees elimination of steady-state error, and why? [3 points]

3 Path Planning [15 points]

1. Label the grid in Fig. 2 with the numbers that would be produced by a wavefront planner using 8-point connectivity. When doing so, label the goal with 0. Draw a path from start to goal that could be returned by this wavefront planner. [4 points]

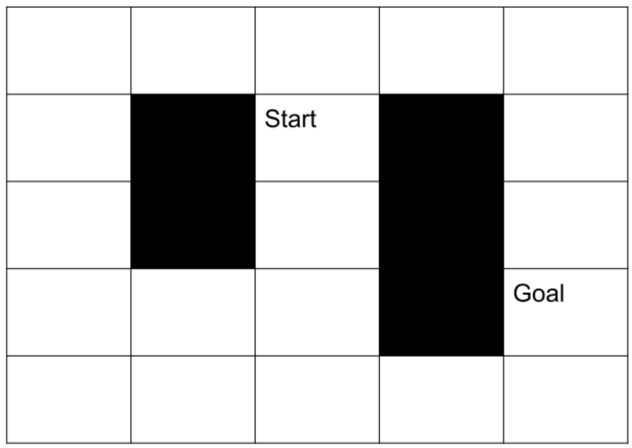


Figure 2: Grid for wavefront question. Black cells are obstacles.

2. Draw the configuration space for the triangular robot in Fig. 3. The robot is only capable of *translational* movement. Also take note of the reference point (bottom left of the triangle)! Treat the environment boundary as an obstacle. [4 points]

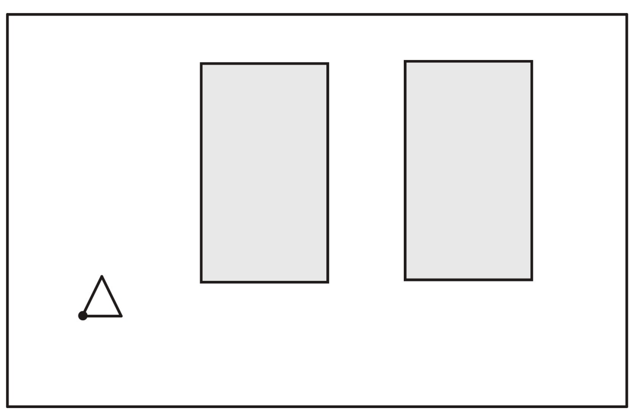


Figure 3: Triangle robot exploring a fixed environment

3. Draw the c-space with the robot centroid as the reference in Fig. 4. Treat the environment boundary as an obstacle. [4 points]

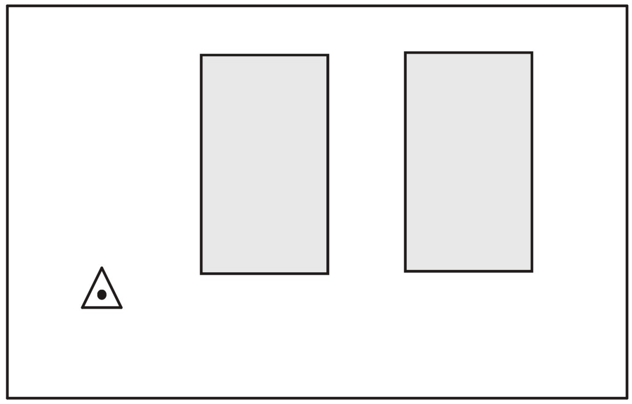


Figure 4: Triangle robot exploring a fixed environment

4. How would the dimensionality of the configuration space change if the triangle was capable of rotational movement as well? [3 points]

4 Graph Search [15 points]

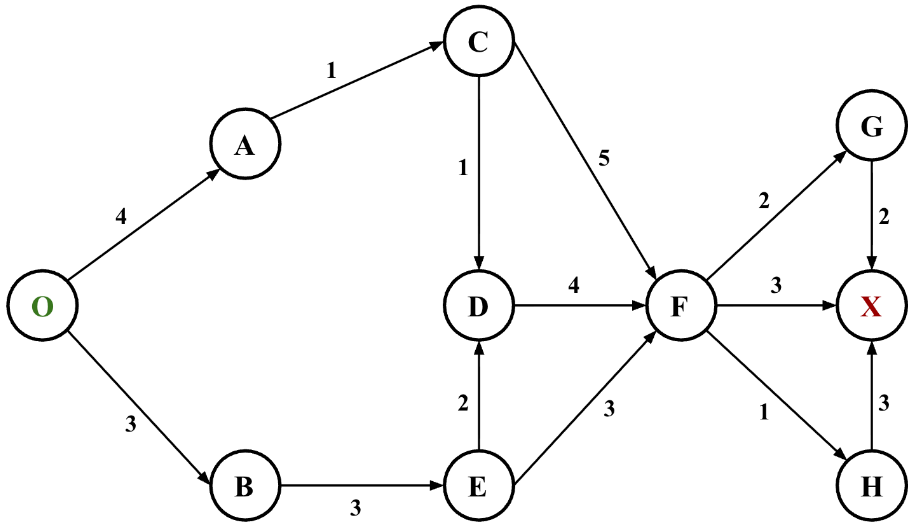


Figure 5: Graph to Search

1. Perform Dijkstra's search on the graph shown above. O is the starting node and X is the goal node. Break ties alphabetically. List the nodes that are EXPANDED and added to the closed list in the order in which they are expanded. [6 points]:

   ____

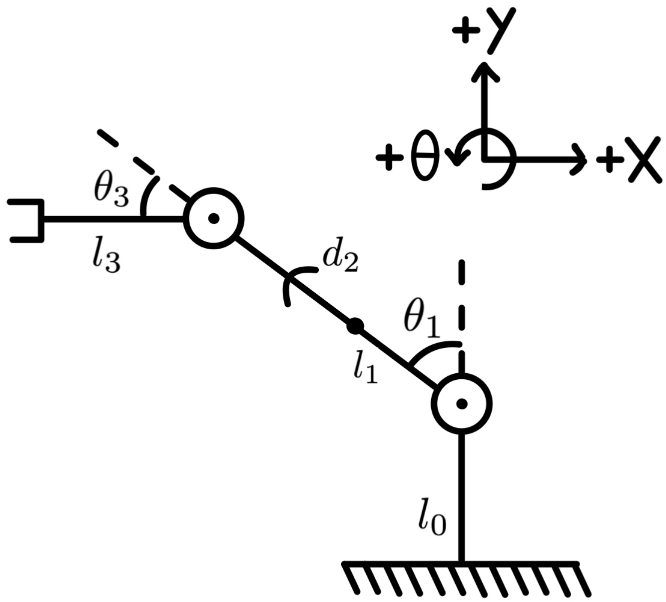
2. What is the path found by Dijkstra's search, and how long is it? [6 points]:

   ____

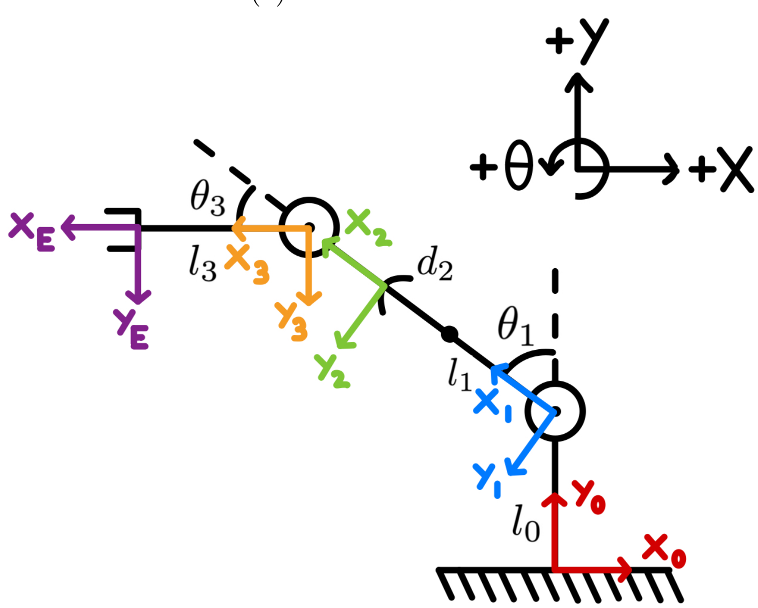
3. If this were to be switched to an A* problem, give an example of an admissible versus inadmissible heuristic value for node E. [3 points]:

   Admissible: ____________ Inadmissible: ____________

5 Forward Kinematics [20 points]



(a) Planar RPR Arm



(b) RPR Arm With Drawn Coordinate Frames

Figure 6: Design for a Planar RPR Arm

Use Figure 6 as a means to answer the following questions in this section. Note that when $d_2 = 0$, the distance between the two revolute joints is l_1.

1. If the robot is commanded to the joint configuration $\theta_1 = 90°$, $d_2 = 1$ cm, and $\theta_3 = -90°$, what are the exact (x, y, θ) coordinates of the end-effector in the base frame? Solve geometrically. [5 points]

For the following questions, feel free to plug in $\theta_1 = 90°, d_2 = 1 \text{ cm}, \theta_3 = -90°$ up front to simplify the math.

2. Find the homogeneous matrix H_1^0 that maps frame 1 to frame 0 [2 points]

3. Find the homogeneous matrix H_2^1 that maps frame 2 to frame 1 [2 points]

4. Find the homogeneous matrix H_3^2 that maps frame 3 to frame 2 [2 points]

5. Find the homogeneous matrix H_E^3 that maps frame E to frame 3 [2 points]

6. Compute the full forward-kinematics transform H_E^0 in terms of H_1^0, H_2^1 H_3^2, H_E^3, and any other necessary variables. Write out the full matrix. [3 points]

7. Based on your solution from H_E^0, what are the exact (x, y, θ) coordinates? Use the same numerical parameters from problem 1 when solving. Is the solution the same as in problem 1? [4 points]

6 Inverse Kinematics [20 points]

A planar robotic arm consists of three degrees of freedom: a prismatic joint followed by two revolute joints (PRR configuration). The configuration of the robot is given by:

$$q = (d, \theta_1, \theta_2)$$

The extension of the prismatic joint is constrained as so: $d \in \left[l_1 + l_2,\ 3(l_1 + l_2)\right]$
It is important to note that $l_1 > l_2$.

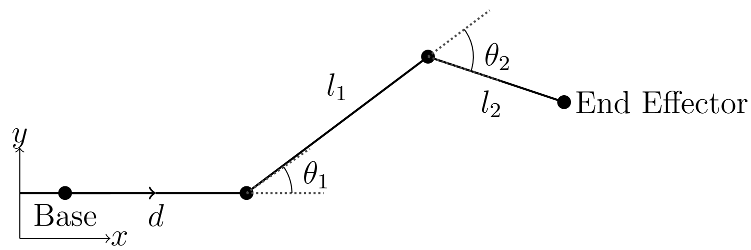


Figure 7: PRR planar manipulator

You are tasked with analyzing the inverse kinematics of this system.

1. Draw the workspace of the arm, i.e. the set of points where the end-effector can reach. Identify how many solutions exist in each region. Put dimensions in your drawing (in terms of the given variables). [10 points]

2. Assuming d is fixed, derive expressions for θ_1 and θ_2 using geometric inverse kinematics. You can assume that for the given d, the point will be achievable. [6 points]

3. For any fixed d, explain why multiple solutions may exist, and describe how you would select a preferred solution in practice. [4 points]

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This is the end of the test.