Max Flow via Electric Flow

15-859N
11/6/13

1) $G=(V,E,s,t)$

2) F is a flow value

3) B^n_∞ = unit L_∞ ball.

4) $e_{ts} = \begin{array}{c} \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ -1 \\ 0 \\ \vdots \end{pmatrix} \end{array} \begin{array}{l} \\ \\ s \\ \\ t \\ \\ \\ \end{array}$

Goal: find $f \in R^m$ st

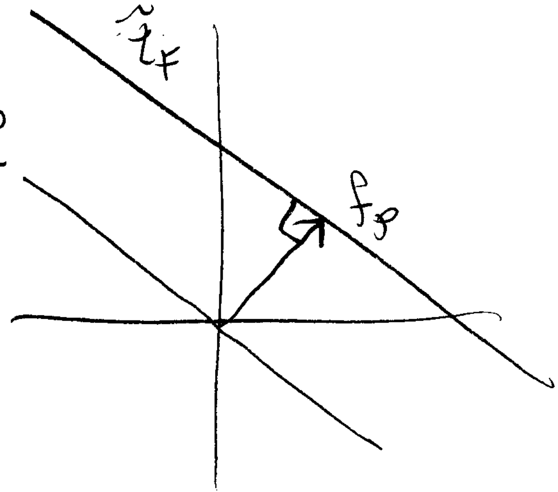
$$\partial_1 f = F e_{ts} \ \& \ f \in B^m_\infty$$

$C = \{ f \in R^m \mid \partial_1 f = 0 \}$ circulation

$Z_F = \{ f : \partial_1 f = F e_{ts} \}$ flows meet the demand.

$$\underset{f_p \in Z_F}{\operatorname{argmin}} \|f_p\|_2$$

Fact: $\partial_1 \partial_1^T v = F e_{ts}$ then $f_p = \partial_1^T v$.



$$\tilde{z}_F = C + f_p$$

Potential flow $\equiv \{ f \mid f = \partial_1^T v \} = \text{Pot}$

$$\Pi = \text{Proj}_{\text{Pot}} \equiv \partial_1^T (\partial_1 \partial_1^T)^{+} \partial_1$$

$$\text{Proj}_C \equiv I - \Pi$$

POCS Alg

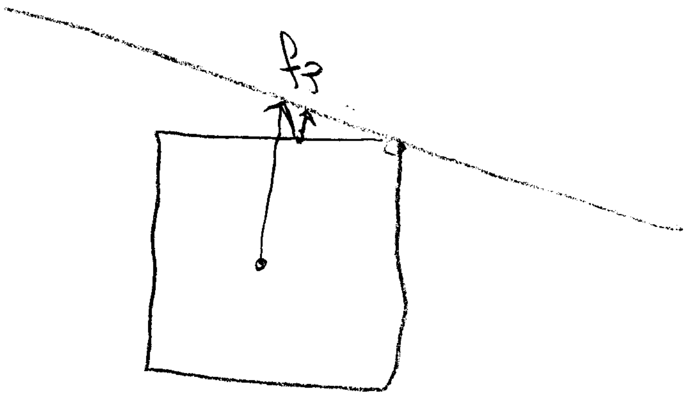
Input G, s, t, F

Init: $f_p = F\,\pi\, e_{ts}$; $f = f_p$

Repeat

$\hat{f} = \mathrm{Proj}_{B_\infty^m}(f)$

$f = \mathrm{Proj}_{\tilde{Z}_F}(\hat{f})$



We now think of 2-Proj as one op.

Map: $\tilde{Z}_F \to \tilde{Z}_F$

Objective fcn

$$\Phi(y) = \frac{1}{2}\| y - \mathrm{Proj}_B(y)\|_2^2 = \frac{1}{2}\|\mathrm{overflow}(y)\|_2^2$$

<u>Fact</u> $\nabla \Phi(f) = y - \mathrm{Proj}_B(y) = \mathrm{over}(f)$

Let $\nabla_{\tilde{Z}_F} \equiv$ gradient restricted to $\tilde{Z}_F$

$$\nabla_{\tilde{Z}_F}(f) = \mathrm{Proj}_{\tilde{Z}_F} \nabla \Phi f = \Pi^{\perp} \nabla \Phi f.$$

$$= (I - \Pi)\,\mathrm{over}(y) =$$

Gradient step

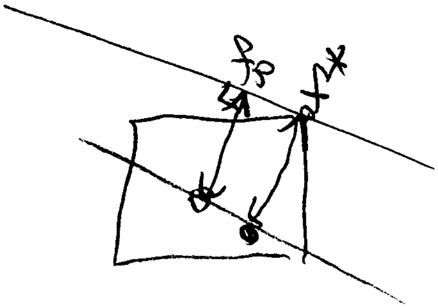
$$y - (I - \Pi)\,\mathrm{over}(y) = y - \mathrm{over}(y) + \partial_1^T L^+ \partial_1 (y)$$

<u>Idea:</u> Find a 2-Term recurrance for swap

$\mathrm{Proj}_{Z_F} \circ \mathrm{Proj}_{B_\infty^m}(f)$ with better convergence!

Init guess f_p let f^* feasible flow of value F,

$$\|f_p - f^*\|_2$$



$$= \|\mathrm{Proj}_{Z_F}(0) - \mathrm{Proj}_{Z_F}(f^*)\|_2$$

$$= \|0 - f^*\|_2$$

$$= \sqrt{\sum f_i^{*2}} \le \sqrt{m}$$