

# 15-451/651 Algorithms, Spring 2019

## Recitation #13 Worksheet

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### VCG and Pricing Advertisements

We saw the VCG mechanism for incentive-compatible auctions in Lecture. Let's use this for pricing online advertising slots. There are 2 ad slots that ElGogo wants to sell on a page, the first slot has a clickthru rate of 0.5, the second has a clickthru rate of 0.3. *Each bidder can get at most one slot.* There are 4 bidders, with the following valuations:

- A: \$10 per click (so, e.g., this bidder values the first slot at  $10 \cdot 0.5 = 5$ , and the second slot at  $10 \cdot 0.3 = 3$ .)
- B: \$8 per click
- C: \$7 per click
- D: \$2 per click

1. What is the social-welfare maximizing allocation?

2. What are the VCG payments?

### Combinatorial Auctions

VCG can be used even with complicated preferences. Suppose we have two identical hotel rooms in Las Vegas, a flight ticket  $f$  from PIT to LAS, and a concert ticket  $c$  in Vegas to auction off. In the following, a generic hotel room is denoted by  $h$ , and none of the people want two rooms.

- Buyer A: values  $\{h\}$  at \$100,  $\{f\}$  at \$200,  $\{h, f\}$  at \$450,  $\{h, f, c\}$  at \$440. (He hates the band in question so much, he gets *negative value* from getting  $c$  along with  $h, f$ .) All other sets are valued at \$0.
- Buyer B (doesn't care for the concert): values  $\{h\}$  at \$50,  $\{f\}$  at \$400,  $\{h, f\}$  at \$500, and  $\{h, f, c\}$  at \$501. All other sets are valued at \$0.
- Buyer C (lives in Vegas): values  $\{c\}$  (and all sets containing  $c$ ) at \$200.

What is the social-welfare maximizing allocation, and what are the VCG payments?

## Polynomials and Interpolation

Find a polynomial  $P(x)$  of degree ~~at most 3~~ **at most 2** that satisfies

$$P(1) = 3, P(59) = 81, P(10000) = 1.$$

You must explicitly write down the polynomial, but need not simplify it.

## Recap and Practice Final

Please ask us questions about topics in the course, and try to solve problems on the practice final.