15-451/651 Algorithm Design & Analysis, Spring 2025

Recitation #13

Objectives

- Provide practice reducing problems to properties and applications of polynomials.
- Review the gradient descent algorithm

Recitation Problems

1. **(Polynomials and fields review)**

   (a) Write down a polynomial that interpolates the points $(1,1),(4,1),(6,2)$. You do not have to simplify your polynomial.

   (b) Let ω be a primitive eighth root of unity. What are the cardinalities of the following sets?

   - $\{\omega^0, \omega^1, \omega^2, \omega^3, \omega^4, \omega^5, \omega^6, \omega^7\}$
   - $\{\omega^1, \omega^3, \omega^9, \omega^{16}, \omega^{27}, \omega^{45}\}$
   - $\{\omega^{-8}, \omega^0, \omega^8, \omega^{8888}\}$

2. **(2-SUM and 3-SUM)**

   You are given an array of n integers A, each element of which is at most $O(n)$ in size. Your goal is to determine the possible sums that you can make by taking integers from this array.

   (a) Solve the 2-SUM problem. That is, output a list of all of the integers that you can possibly make by summing any pair of elements from A. You are allowed to use the same element twice.

   (b) Now solve 2-SUM but without allowing the same element to be used twice.

   (c) Solve the 3-SUM problem, That is, output a list of all of the integers that you can possibly make by summing any triple of elements from A. You are allowed to use the same element multiple times.

   (d) Finally, solve 3-SUM but without allowing the same element to be used multiple times.

3. **(Gradient Decent)** Suppose you would like to minimize a (differentiable) convex function $f : \mathbb{R} \to \mathbb{R}$ using gradient descent for T iterations. Let $x^* = 5$ be the minimum and suppose we run gradient descent starting at $x_0 = 3$. Suppose $\|f'(x)\| \leq 10/(|x|+1)$ for all $x \in \mathbb{R}$. We would like to output an x for which $f(x) - f(x^*) \leq 1/2$. What should we set the learning rate η and the number T of iterations to be so that the gradient descent algorithm we learned in class outputs an x with the above property?