

15-451/651 Algorithm Design & Analysis, Fall 2026

Homework #5

Homework #5, Due: Wednesday, October 28th at 11:59 pm

This homework has a survey, a short-answer problem, followed by two written problems. The short-answer problems are graded on correctness only (we reserve the right to give a point penalty for providing a long answer to a short answer question). You **should not** provide long-winded justification or proofs unless explicitly asked for. Long-answer problems require you to justify your answers and provide proofs unless stated otherwise. On this homework, there is a SOLO problem and some GROUP problems. You may work in a group of up to 3 people on the GROUP problems. **In all cases the write-up you submit must be written by you alone. You may not share your written solutions with each other. You can discuss the problem as a group, but then must write up your final solutions independently in your own words.**

Submission: Solutions to the problems should be submitted as a PDF file using gradescope, with the answer to each problem starting on a new page. Each problem has a specified page limit that you should stick to. Your solutions **must** be typeset, **not handwritten**. You should use US letter-size paper, a font size no smaller than 10pt, margins no smaller than 1in. 10 points on this homework will be allocated to correctly following the style guidelines (see the Written HWs section of the [course policies](#)).

0 **Collaboration statement** Fill out the [collaboration form](#) to disclose your group members for this homework. If you worked solo, you must still fill out the form. Note that you will have to fill out this form separately for every written homework.

1. (SOLO, 15 pts) **Short-answer problems** (Limit: 1 page)

(a) **Mid-Semester Feedback Survey**

Please complete this [anonymous feedback survey](#) so that we can improve the course! After you have completed it, you will get instructions on how to receive credit.

(b) You are given a simple directed graph $G = (V, E)$ where each edge $e \in E$ has a non-negative weight w_e . The bottleneck of a path is defined to be the minimum weight of any edge in the path. Your goal is to find the bottleneck value of the maximum-bottleneck path in G (i.e., among all paths, the one whose minimum weight is as large as possible) between all pairs of vertices $u, v \in V$.

- i. You decide to use dynamic programming to solve the problem. You will define a set of subproblems $P(u, v, k)$ for $u, v \in V$ and $1 \leq k \leq n = |V|$. Give a good definition for what these subproblems will represent.
- ii. Give values for a suitable set of base cases
- iii. Complete the recurrence relation by giving the remaining (i.e., non-base) cases.
- iv. What is the runtime of the resulting dynamic programming algorithm?

2. (GROUP, 25 pts) **Pizza Shop Problem** (Limit: 2 pages)

Customers $1, 2, \dots, n$ walk into a pizza shop all at the same time. The shop has p slices of pizza numbered $1, 2, \dots, p$ (in a variety of different types), and also s drinks, again numbered $1, 2, \dots, s$ (in a variety of different types).

Each customer i likes $Z_i \subseteq \{1, 2, \dots, p\}$ of the pizza slices, and also likes $W_i \subseteq \{1, 2, \dots, s\}$ of the drinks. Customer i *will only be satisfied* if she gets one slice of pizza from Z_i and one drink from W_i .

Show how the shopkeeper can set this up as a maximum flow problem in a graph G , such that the maximum flow will equal the maximum number of customers that can be satisfied. Show how to use the maximum flow in G to allocate the pizza and drinks to the customers to achieve this. Your reduction should be polynomial time in the size of the input (n and p and s). Remember to prove that your reduction is correct!

3. (GROUP, 25 pts) **Course selection** (Limit: 2 pages)

You are a CMU student trying to figure out which courses to enroll in for the remainder of your degree. There are n courses to choose from in total, the course i costs C_i dollars in tuition. In this problem, you can enroll in as many or as few courses as you like (including all of them or none of them).

You do some research to determine which skills are most in demand in the job market, and you come up with a list of m key skills that you think will increase your future income! Now, you have to figure out which courses to take. You determine that for each key skill j , there is a subset of courses $S_j \subseteq \{1, 2, \dots, n\}$ that you would have to complete to master that skill. (Note that you must take **all** of the courses in the subset, not just any one of them, to master the skill). You then determine that mastering skill j is worth V_j to your future net worth. Your goal is to select a subset of courses that maximizes the total utility to you, where the utility is the total value of the skills you master, minus the cost of the courses you had to take to master them.

- (a) Explain how maximizing your utility is equivalent to minimizing the value of the skills you **do not** master plus the cost of the courses you take.
- (b) Show how to determine the maximum utility by reducing this problem to an instance of the s - t minimum cut problem in polynomial time. (**HINT:** Based on (a), the value of a cut in the network should correspond to a set of courses you pay to take, and the corresponding set of skills that you **do not** master.) Make sure to prove that your reduction is correct and has an appropriate running time.