Bug Catching: Automated Program Verification
15414/15614 Spring 2025
Lecture 1: Introduction

Matt Fredrikson

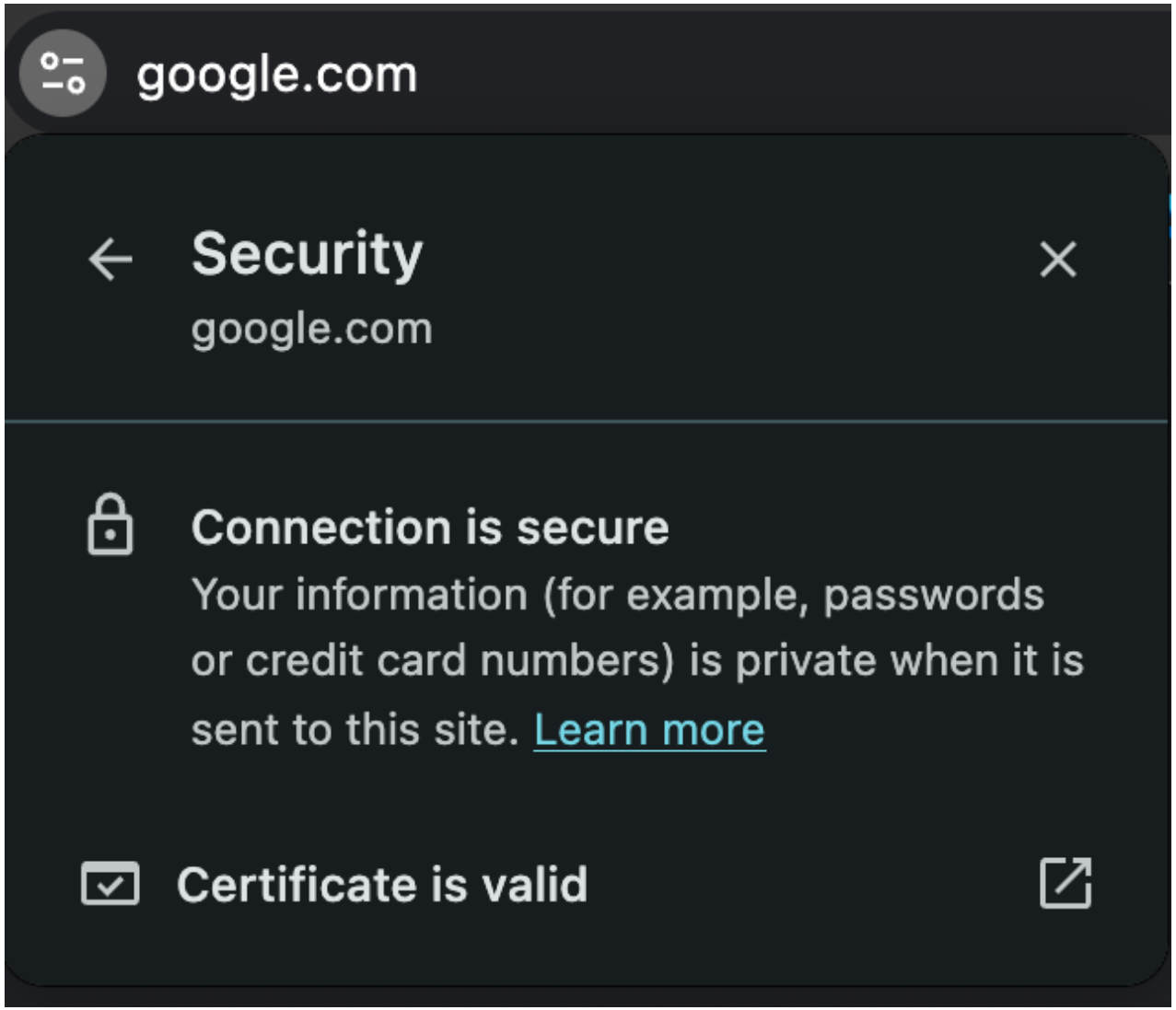
January 14, 2025

Learning objectives

For this lecture

- What is this course about?
- What are the learning objectives for the course?
- How does it fit into the curriculum?
- How does the course work?
- Remember ...

Critical Infrastucture



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  - Most widely deployed security protocol on the Internet.

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  - Most widely deployed security protocol on the Internet.
- The most widely used *implementation* is OpenSSL.

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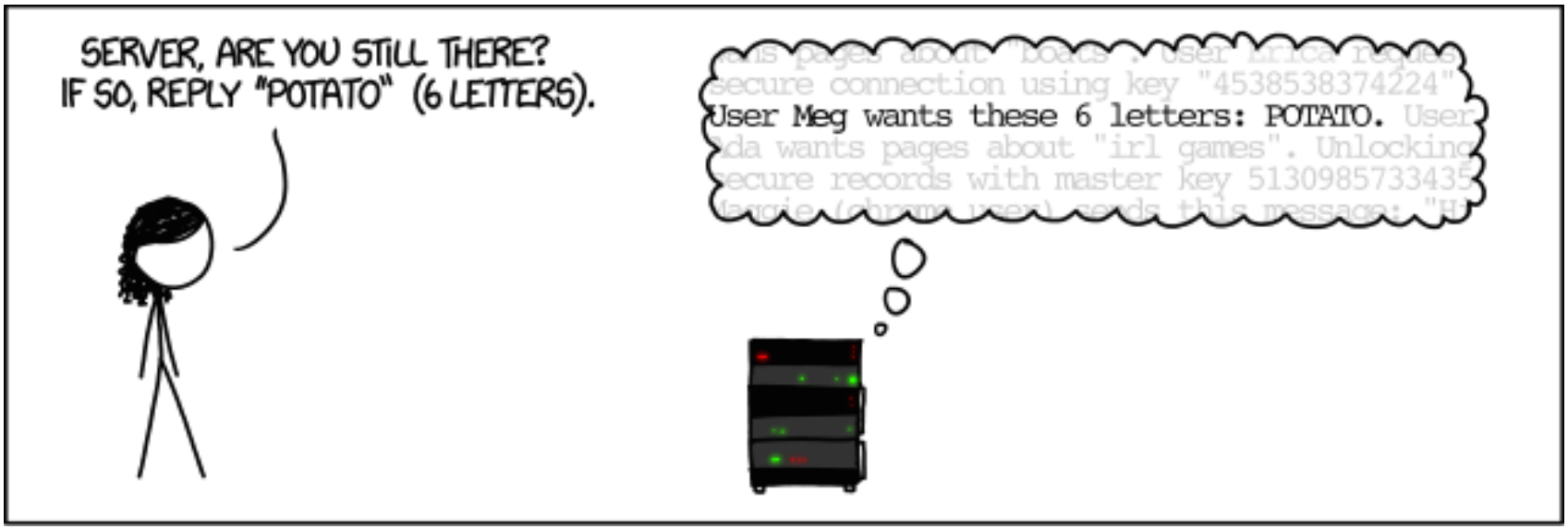
- "The Heartbleed bug allows anyone on the Internet to read the memory of the systems protected by the vulnerable versions of the OpenSSL software."
- "...this allows attackers to eavesdrop on communications, steal data directly from the services and users and to impersonate services and users."

Heartbleed, explained



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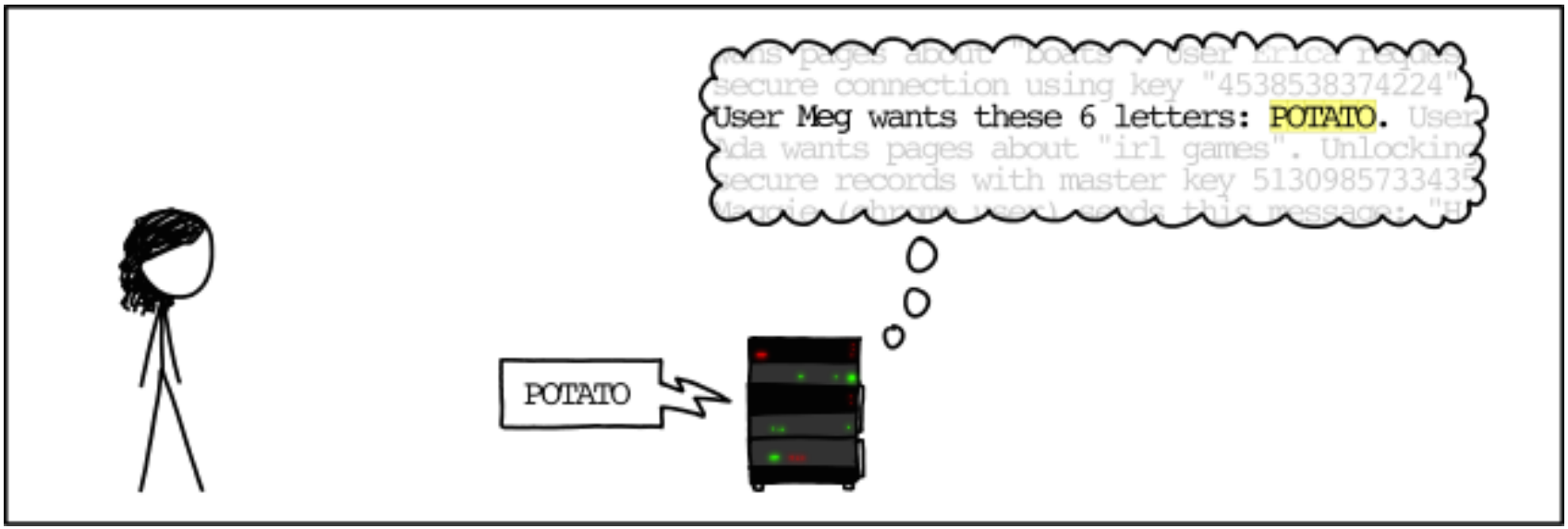

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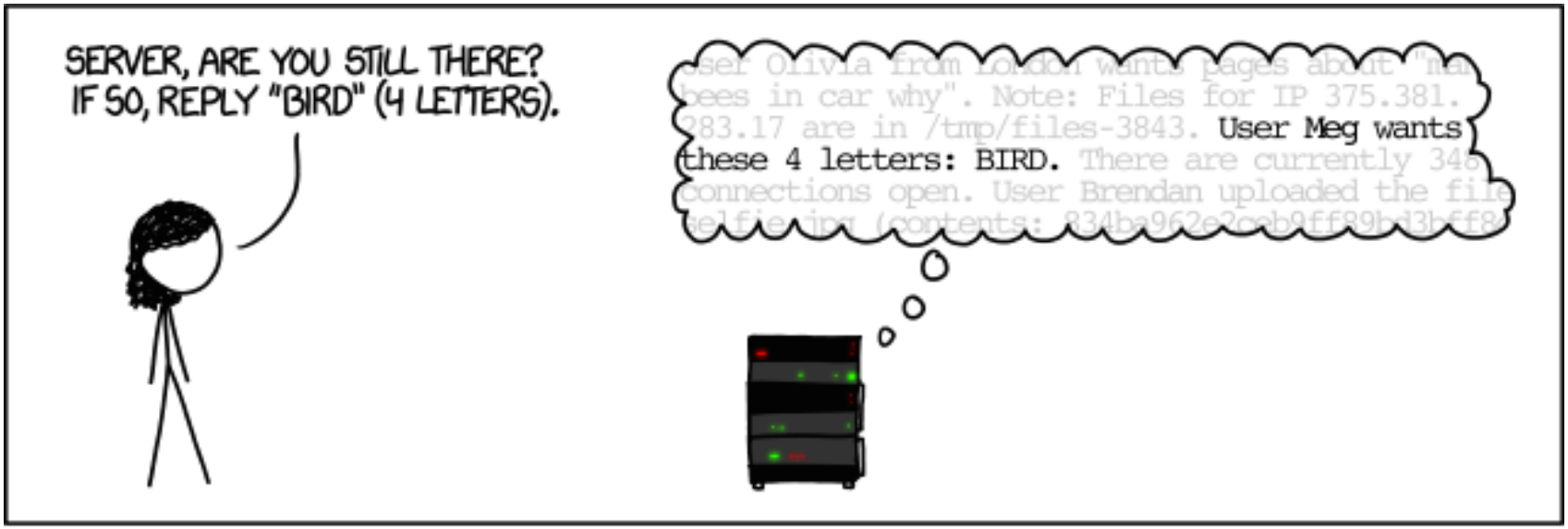

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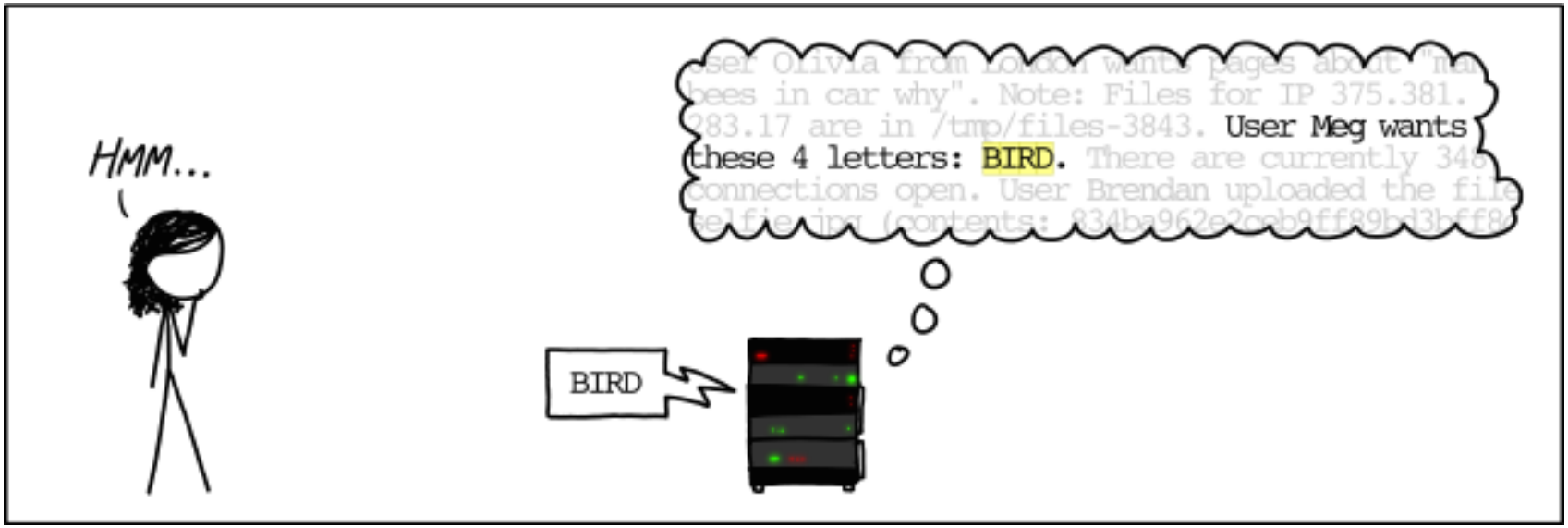

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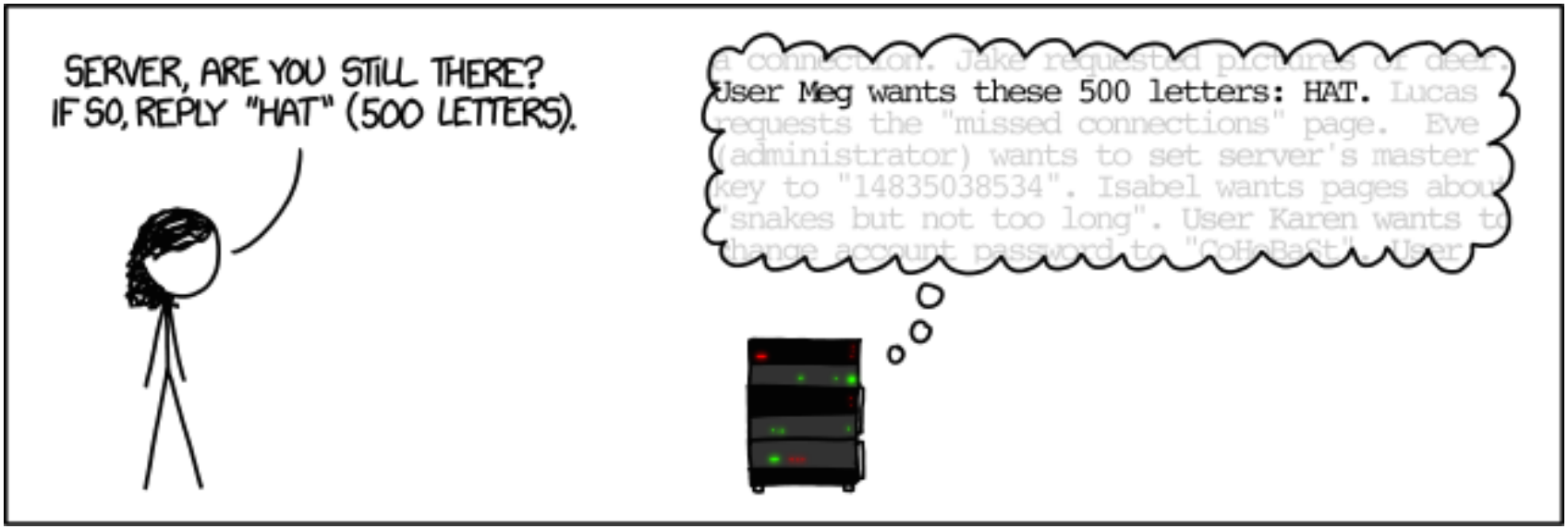

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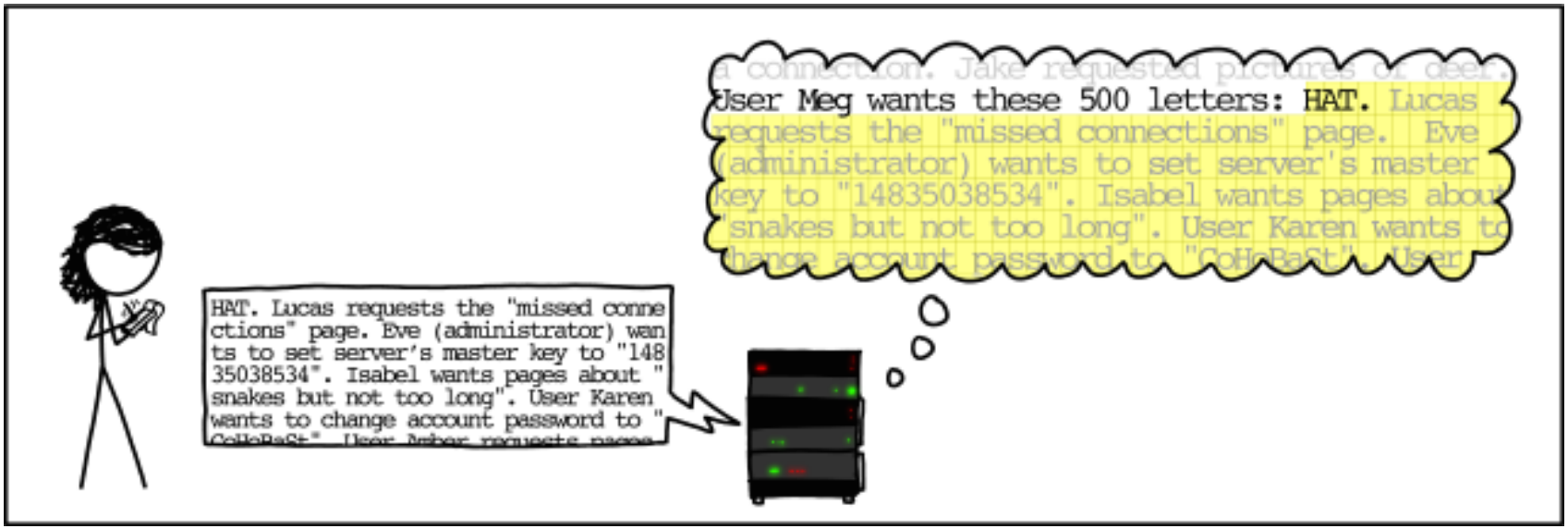

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Tech giants spend millions to stop another Heartbleed

25 April 2014

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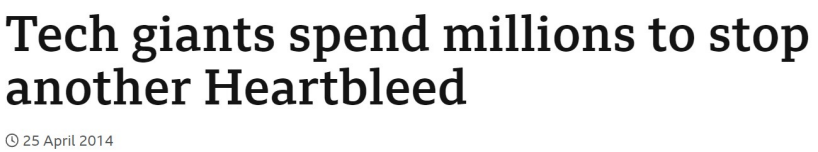

Algorithms vs. code

```
1  int binarySearch(int key, int[] a, int n) {
2      int low = 0;
3      int high = n;
4
5      while (low < high) {
6          int mid = (low + high) / 2;
7
8          if(a[mid] == key) return mid; // key found
9          else if(a[mid] < key) {
10             low = mid + 1;
11         } else {
12             high = mid;
13         }
14     }
15     return -1;  // key not found.
16 }
```

Code matters

This is a correct binary search algorithm

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But what if $\texttt{low} + \texttt{high} > 2^{31} - 1$?

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Then `mid = (low + high) / 2` becomes negative

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Algorithm may be correct—but we run code, not algorithms.

How do we fix it?

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Solution: `mid = low + (high - low)/2`

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2  //@requires 0 <= n && n <= \length(a);
3  /*@ensures (\result == -1 && !is_in(key, A, 0, n))
4    @      || (0 <= \result && \result < n
5    @           && A[\result] == key); @*/
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The fix

```
1  int binarySearch(int key, int[] a, int n)
2  //@requires 0 <= n && n <= \length(a);
3  //@requires is_sorted(a, 0, n);
4  /*@ensures (\result == -1 && !is_in(key, A, 0, n))
5    @      || (0 <= \result && \result < n
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$$\textit{Specification} \iff \textit{Implementation}$$

- Specification must be **precise**
- Meaning of code must be **comprehensive**
- Reasoning must be **sound**

Course objectives

- Identify and formalize program correctness
- Understand language semantics
- Apply mathematical reasoning to program correctness
- Learn how to write correct software, from beginning to end
- Use automated tools that assist verifying your code
- Understand how verification tools work

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- Make you better programmers

Course outline

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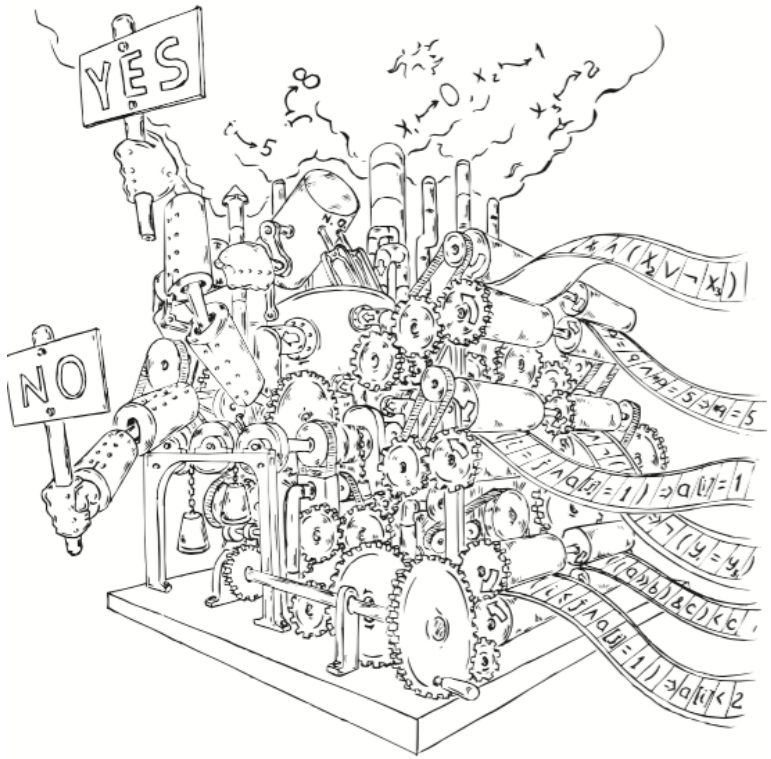
Part III: Mechanized reasoning

- Techniques for automated proving

Algorithmic approaches

Formal proofs are tedious

Automatic methods can:



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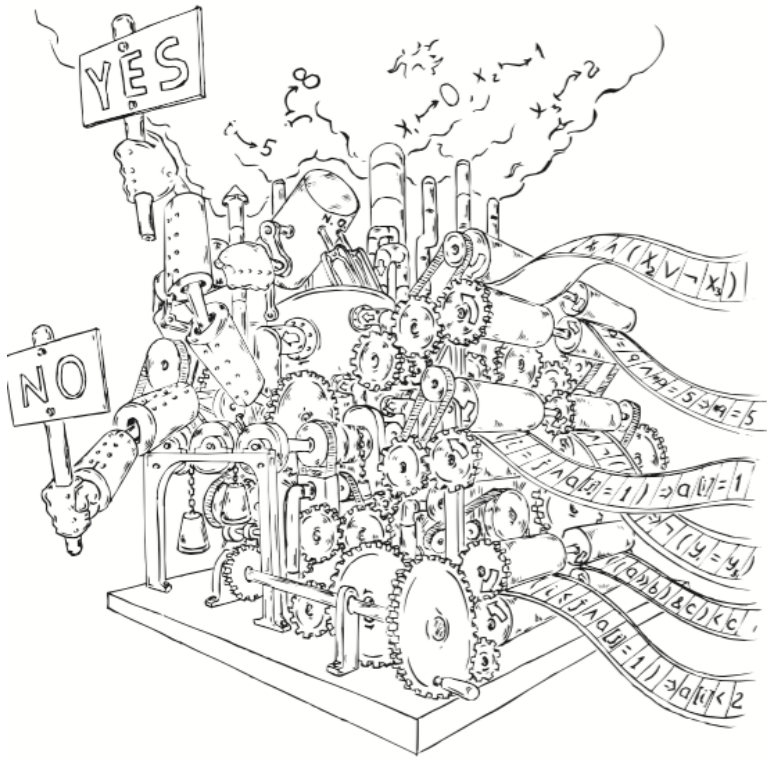

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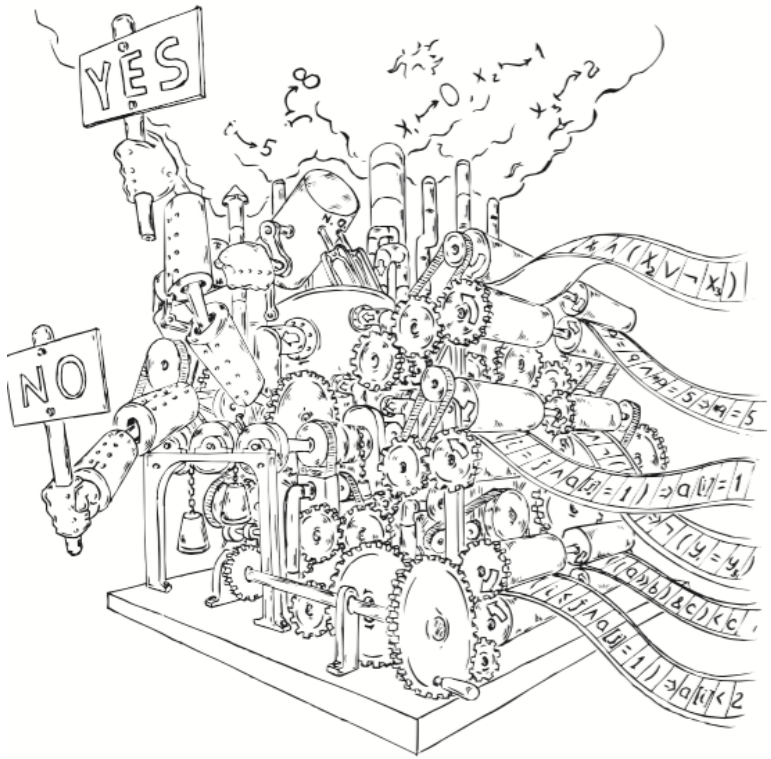

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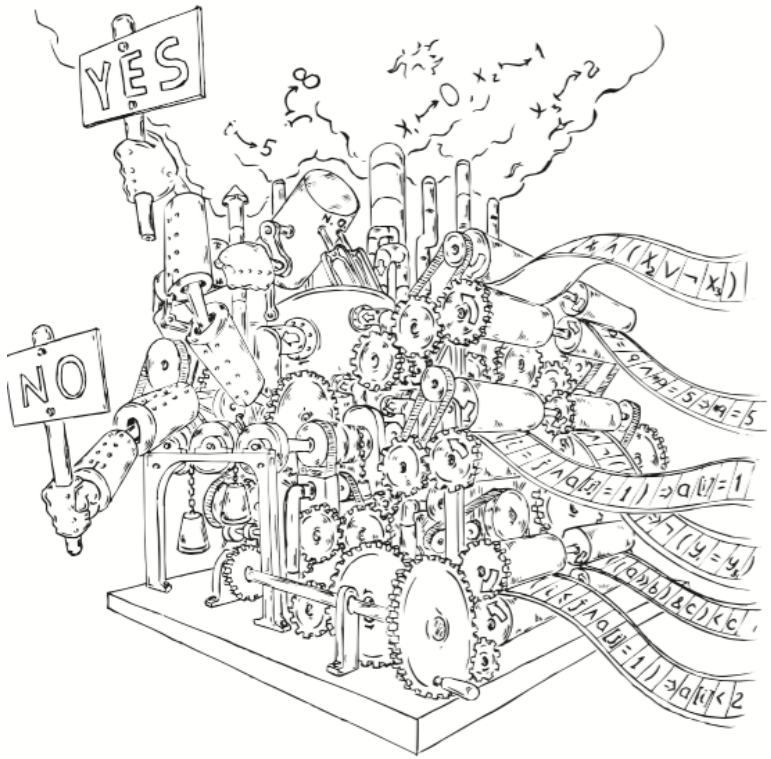

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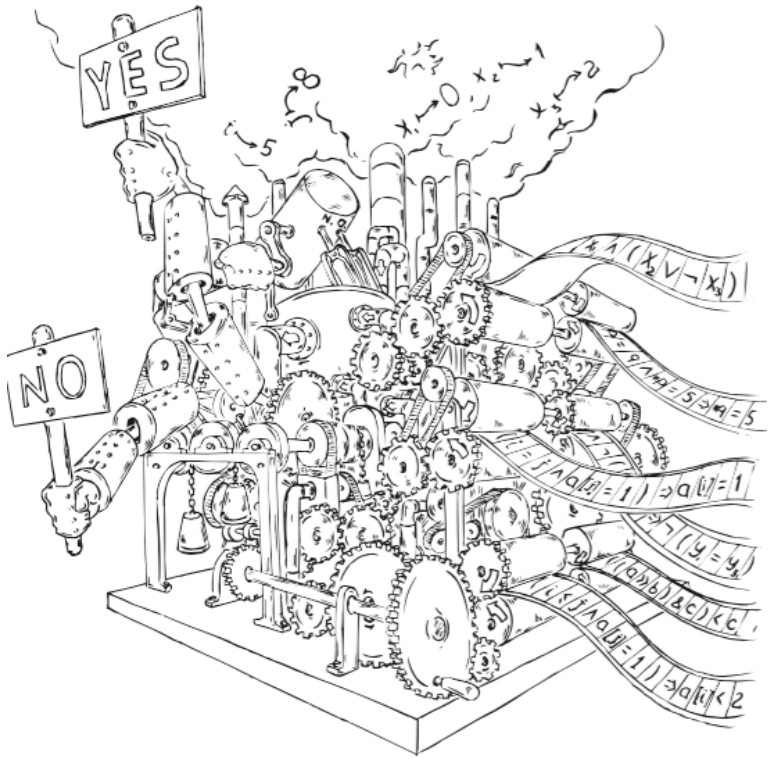
This is what you will learn!

- Make use of these methods
- How (and when) they work



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Why3 (this course) supports both!

- Functional and imperative code in WhyML
- Automated provers for VC (Z3, CVC, alt-ergo, ...)
- Interactive provers for VC (Coq)

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- Specification
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3  //@ensures \length(\result) == n;
4  {
5    int[] B = alloc_array(int, n);
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7    for (int i = 0; i < n; i++)
8    //@loop_invariant 0 <= i;
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But . . .

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When it works! (It's not quite like a type-checker . . .)

Automated verifiers

Systems that prove that programs match their specifications

Basic idea:

1. Translate programs into *proof obligations*
2. Encode proof obligations as satisfiability
3. Solve using a *decision procedure*

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Verifiers are complex systems

- We'll deep-dive into selected components
- Understand "big picture" for the rest

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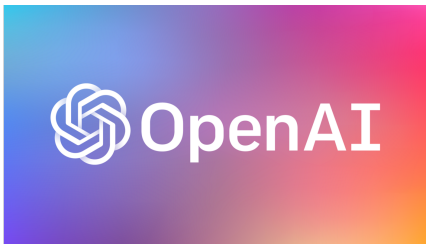
Recent developments



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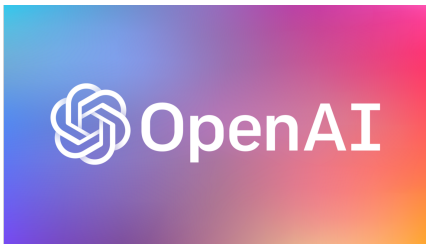


```
Implement a ‘sat‘ function in Why3 that matches this signature:

    sat (cnf : cnf) : option valuation

If ‘cnf‘ is satisfiable, then ‘sat‘ should return ‘Some rho‘,
where ‘rho‘ is a satisfying assignment for ‘cnf‘.
Otherwise, it should return ‘None‘.
```

Recent developments

```
let rec sat (cnf : cnf) : option valuation =
  ensures  {
    match result with
    | None -> unsat cnf
    | Some rho -> sat_with rho cnf
  }
  let rho = Array.make cnf.nvars False in
  let rec dfs (n : int) : bool =
    requires { valid_valuation rho cnf }
    requires { 0 <= n <= cnf.nvars }
    variant  { cnf.nvars - n }
    ensures  {
      if result == true then
        sat_with rho cnf
      else
        forall rho':valuation. array_eq_sub rho' rho 0 n -> not (sat_with rho' cnf)
    }
```

Recent developments

MF The `dfs` function needs additional specifications to prove that the array accesses are in bounds.

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The following is an updated version of the `dfs` function with added specifications:

Copy code

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let rec dfs (rho : valuation) (n : int) : bool =
  requires { valid_valuation rho cnf }
  requires { 0 <= n < cnf.nvars }
  variant  { n }
  ensures  {
    if result == true then
      sat_with rho cnf
    else
      forall i:int. 0 <= i <= n -> not (sat_with (set rho i true) cnf)
  }
```

Recent developments

MF

The `ensures` specification on `dfs` is not correct. Specifying `sat_with (set rho i true) cnf` for all `i` in the range from `0` to `n` is not relevant; what is relevant is that if `result <> true`, then `unsat cnf`.

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The following is an updated version of the **`dfs`** function with a corrected **`ensures`** specification:

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- Describe the relevant parts in your solution

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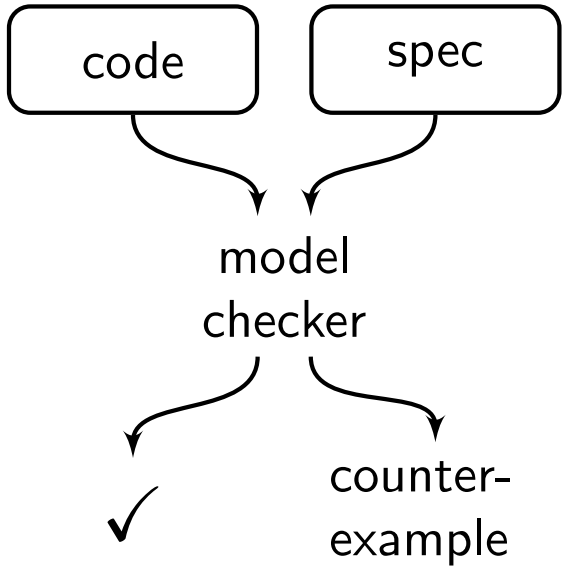
If you use a code model/assistant to complete a major task:

- Document your approach / save the transcript
- Describe the relevant parts in your solution
- Come talk to me about it

Model Checking

Fully-automatic techniques for finding bugs (or proving their absence)

- Specifications written in *propositional temporal logic*
- Verification by exhaustive state space search
- Diagnostic counterexamples



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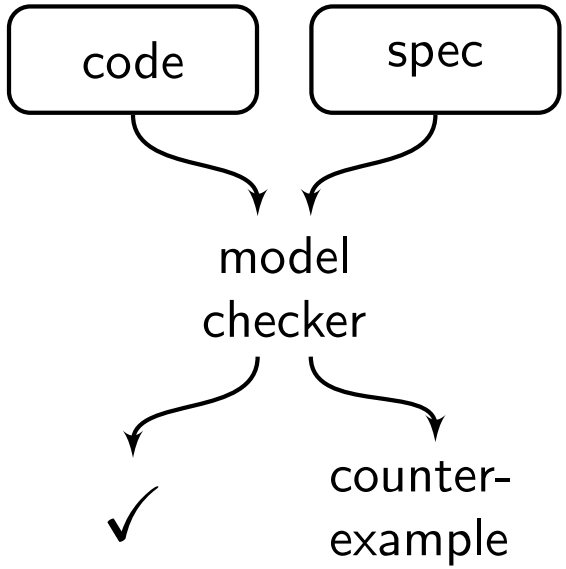
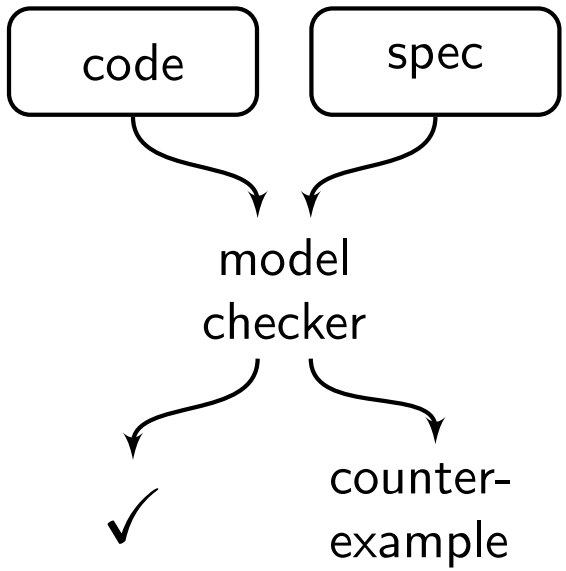
- Verification by exhaustive state space search
- Diagnostic counterexamples
- No proofs!



Model Checking

Fully-automatic techniques for finding bugs (or proving their absence)

- Specifications written in *propositional temporal logic*
- Verification by exhaustive state space search
- Diagnostic counterexamples
- No proofs!
- **Downside**: "State explosion"



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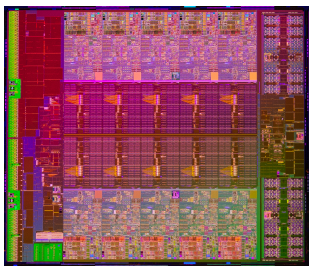
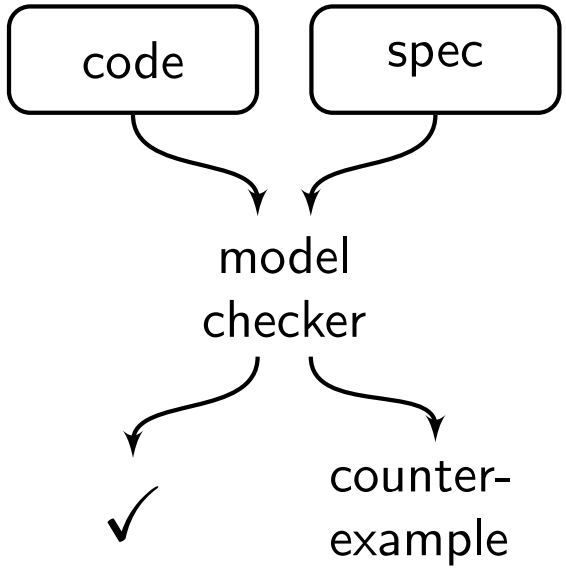
10^{70} atoms 10^{500000} states





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First developed this course!

Grading

Breakdown:

- 50% assignments
  (written + programming)
- 15% mini-project 1
- 15% mini-project 2
- 20% final exam

6 assignments
done individually

2 mini-projects
pick from small menu
can work with a partner

Participation:

- Come to lecture
- Answer questions
  (in class and on Piazza!)
- Contribute to discussion

Written parts of assignments

Written homeworks focus on theory and fundamental skills

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Grades are based on:

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- How you present your reasoning

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Strive for **clarity & conciseness**

- Show each step of your reasoning
- State your assumptions
- Answers without these $\longrightarrow$ no points

Programming parts of assignments

For the programming, you will:

- Implement some functionality (data structure or algorithm)
- Specify correctness for that functionality
- Use Why3 to prove it correct

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- Correct implementation
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- Correct specification
- Correct contracts
- Sufficient contracts for verification

Partial credit depending on how many of these you achieve

Clarity & conciseness is necessary for partial credit!

Mini-Projects

Mini-projects are intended to build proficiency in:

- Writing good specifications
- Applying course principles to practice
- Making effective use of automated tools
- **Writing useful & correct code**

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Gradual progression to sophistication:

1. Familiarize yourself with Why3
2. Implement and prove something
3. Work with more complex data structures
4. Implement and prove something really interesting
5. Optimize your implementation, still verified

Late Policy

Late days

- 5 late days to use throughout the semester
- No more than 2 late days on any assignment
- Late days do not apply to mini-projects!

Logistics

Website: `http://www.cs.cmu.edu/~15414`

Course staff contact: Piazza

Lecture: Tuesdays & Thursdays, 12:20-1:40pm

Office Hours: TBD, schedule on website and course calendar soon

Assignments: Gradescope