

**Assignment 5: LTL Model Checking**  
**15-414/15-424 Bug Catching: Automated Program Verification**

Due: **11:59pm**, Friday 4/17/20

Total Points: 50

1. **Until, weakly (10 points).** Consider a temporal operator with the following semantics on traces  $\sigma$ :

$$\sigma \models PWQ \text{ iff, for all } i \geq 0, \text{ if } \sigma^i \models \neg P, \text{ then there exists } k \leq i \text{ such that } \sigma^k \models Q$$

This is a weaker version of the normal until operator, in that it doesn't require  $Q$  to eventually hold as long as  $P$  always does. Show that **W** can be expressed in terms of the temporal operators discussed in lecture 15 by writing an equivalence, and use the semantics of LTL to justify that your equivalence is correct.

2. **Distributing correctly (20 points).** Consider the following LTL equivalences that characterize distributive properties of temporal operators:

$$\begin{aligned} \diamond(P \vee Q) &\leftrightarrow \diamond P \vee \diamond Q \\ \diamond(P \wedge Q) &\leftrightarrow \diamond P \wedge \diamond Q \\ \square(P \vee Q) &\leftrightarrow \square P \vee \square Q \\ \square(P \wedge Q) &\leftrightarrow \square P \wedge \square Q \end{aligned}$$

First, identify which of those equivalences are correct and which are not. Then use the semantics of LTL to justify your answer with a proof. For the formulas that are not correct, describe an infinite trace that satisfies one side of the equivalence but not the other, i.e., provide a counterexample.

3. **Draw a picture (10 points).** Provide an NBA for the following LTL formulas.

$$\square(P \vee \neg \mathbf{X}Q) \quad \mathbf{X}\mathbf{X}(P \vee \diamond \square Q)$$

4. **Elevated correctness (10 points).** Your job is to design a specification for a basic elevator system that services four floors. There is a door at each floor, with a call button and an indicator light that indicates whether the elevator has been called to that floor. Describe a set of atomic propositions and LTL formulas to specify the following properties of the elevator.

- (a) A door never opens if the elevator is not present at the corresponding floor.
- (b) Pushing a call button results in the elevator eventually servicing the corresponding floor.
- (c) The elevator returns to floor 0 infinitely often.
- (d) When the call button on floor 4 is pressed, the elevator serves it without stopping at any other floors along the way.