Optimization 2

15-411/15-611 Compiler Design

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March 25, 2025

Common loop optimizations

- Hoisting of loop-invariant computations
  - pre-compute before entering the loop
- Elimination of induction variables
  - change p=i*w+b to p=b,p+=w, when w,b invariant
- Loop unrolling
  - to improve scheduling of the loop body

Scalar opts,
DF analysis,
Control flow analysis

- Software pipelining
  - To improve scheduling of the loop body
- Loop permutation
  - to improve cache memory performance

Requires understanding data dependencies

Loop Terminology

Loop: Strongly Connected Component of CFG

Entry Edge: tail not in loop, head in loop.

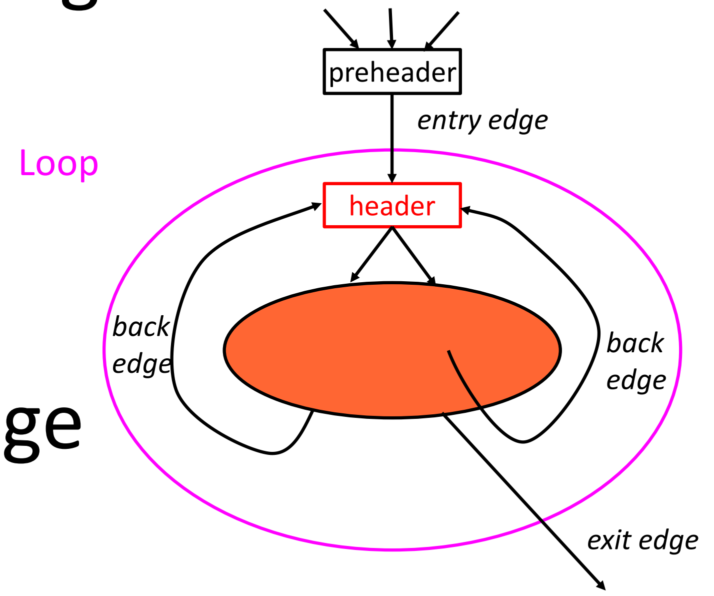
Exit edge: tail in loop, head not in loop

Loop Header: target of entry edge

Back Edge: target is header,
source is in loop

Preheader:
Source of the only entry edge

Natural Loop:
A Loop with only a single loop header



Loop optimizations: Hoisting of loop-invariant computations

Loop-invariant computations

- A definition

  t = x op y

  in a loop is (conservatively) loop-invariant if
  - x and y are constants, or
  - all reaching definitions of x and y are outside the loop, or
  - only one definition reaches x (and y), and that definition is loop-invariant
    - so keep marking iteratively

Loop-invariant computations

- If not in SSA Be careful

```
t = expr;
for () {
    s = t * 2;
    t = loop_invari
    x = t + 2;
    …
}
```

Of course, not an issue in SSA

```
     t1 = expr;
L1:
     brc L2;
     t2 = phi(t1, t3);
     s = t2 * 2;
     t3 = loop_invariant_expr;
     x1 = t3 * 2;
     …
     jmp L1;
L2:
```

- Even though t's two reaching expressions are each invariant, s is not invariant…

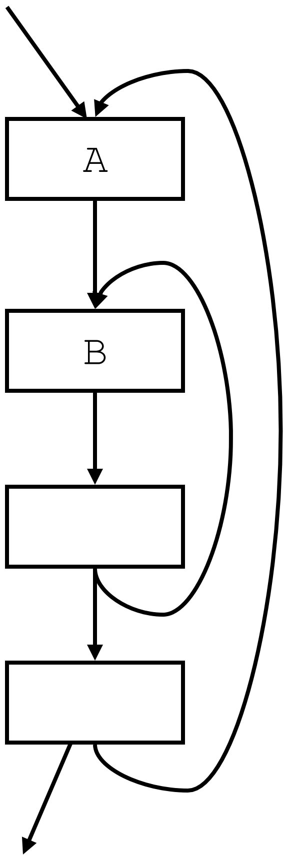
Hoisting

- In order to “hoist” a loop-invariant computation out of a loop, we need a place to put it

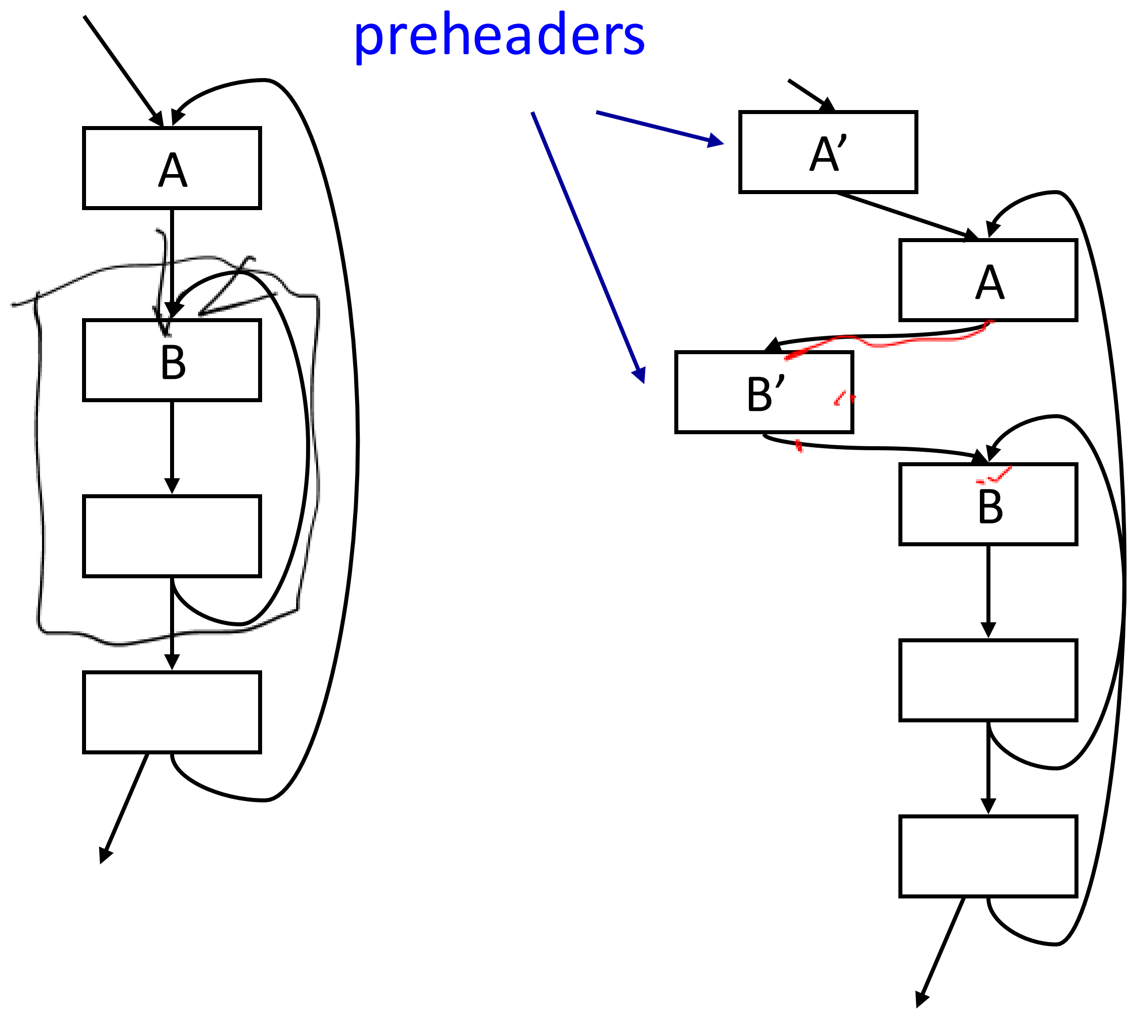
- We could copy it to all immediate predecessors (except along the back-edge) of the loop header...

- ...But we can avoid code duplication by ensuring there is a pre-header

Hoisting Uses Pre-Headers



Hoisting Uses Pre-Headers



General Hoisting conditions

- For a loop-invariant definition

  d: t = x op y

- we can hoist d into the loop's pre-header only if
  1. d's block dominates all loop exits at which t is live-out, and
  2. d is the only definition of t in the loop, and
  3. t is not live-out of the pre-header

We need to be careful...

- All hoisting conditions must be satisfied!

```
L0:
 t = 0
L1:
 i = i + 1
 t = a * b
 M[i] = t
 if i<N goto L1
L2:
 x = t
```

```
L0:
 t = 0
L1:
 if i>=N goto L2
 i = i + 1
 t = a * b
 M[i] = t
 goto L1
L2:
 x = t
```

```
L0:
 t = 0
L1:
 i = i + 1
 t = a * b
 M[i] = t
 t = 0
 M[j] = t
 if i<N goto L1
L2:
```

OK

violates 1,3

violates 2

We need to be careful...

- All hoisting conditions must be satisfied!

```
L0:
 t = 0
L1:
 i = i + 1
 t = a * b
 M[i] = t
 if i<N goto L1
L2:
 x = t
```

OK

```
L0:
 t = 0
L1:
 if i>=N goto L2
 i = i + 1
 t = a * b
 M[i] = t
 goto L1
L2:
 x = t
```

violates 1,3

```
L0:
 t = 0
L1:
 i = i + 1
 t = a * b
 M[i] = t
 t = 0
 M[j] = t
 if i<N goto L1
L2:
```

violates 2

SSA Hoisting conditions

- For a loop-invariant definition

  d: t = x op y

- we can hoist d into the loop's pre-header only if

1. d's block dominates all loop exits at which t is live-out, and
2. d is the only definition of t in the loop, and
3. t is not live-out of the pre-header

easy
trivial
easy

Condition 1:

- Can be violated if?
- Why would you?

t = a * b

Enabling Transformations

- Convert while into repeat-until

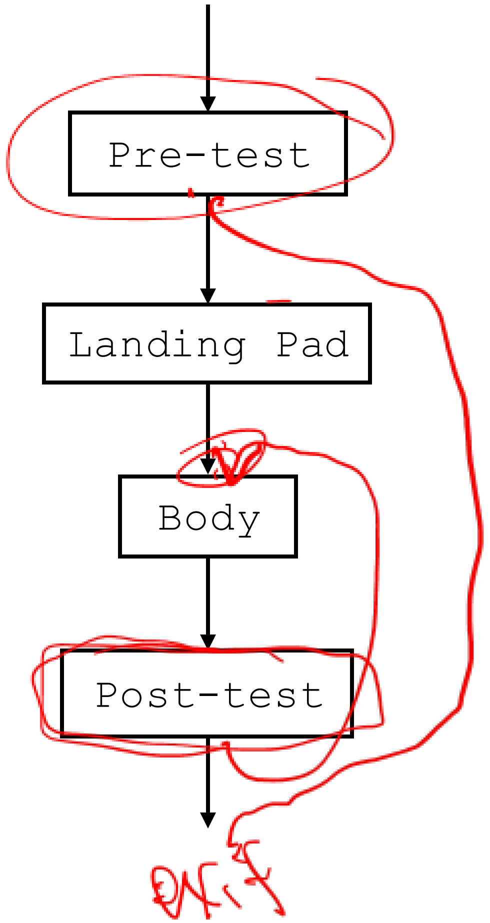
```
while (e) {
  T
  j = loopinv // does not dominate all loop exits
  S
}
```

⇓

```
If (e) {
  repeat {
    T
    j = loopinv
    S
  } until (!e)
}
```

Enabling Transformations

- More Generally, add landing pad
  - For any speculative code:
    add test before pre-header



Should You?

- Does Loop Body always execute?
- Do we speculate?
  - Use profiling information?
- Register Pressure?

LICM subsumed by PRE

- Don't have to implement Loop invariant code motion if you implement PRE, since PRE subsumes it anyway!
- (But, PRE is difficult)

Loop optimizations:
Induction-variable Elimination
Strength reduction

The basic idea of IVE

- Suppose we have a loop variable
  - i initially 0; each iteration i = i + 1
- and a variable that linearly depends on it:

  x = i * c1 + c2

- In such cases, we can try to
  - initialize $x = i_o * c1 + c2$ (execute once)
  - increment x by c1 each iteration

Induction Variable

- Basic Induction Variable has the form:

  $$\boxed{X = X \pm C}$$

  where C is constant or loop-invariant

- Derived Induction Variable has form:

  $$X = C_1 * Y \pm C_2$$

  where
  - Y is a Basic induction variable
  - C_1 and C_2 are constants

Simple Example of IVE

```
for i = 0 to n
    a[i] = 0
```

```
        i <- 0
H:
        if i >= n goto exit
        j <- i * 4
        k <- j + a
        M[k] <- 0
        i <- i + 1
        goto H
```

Clearly, j & k do not need to be computed anew each time since they are related to i and i changes linearly.

Simple Example of IVE

```
      i <- 0
H:
      if i >= n goto exit
      j <- i * 4
      k <- j + a
      M[k] <- 0
      i <- i + 1
      goto H
```

```
      i <- 0
      j' <- 0
      k' <- a
H:
      if i >= n goto exit
      j <- j'
      k <- k'
      M[k] <- 0
      i <- i + 1
      j' <- j' + 4
      k' <- k' + 4
      goto H
```

But, then we don't even need j (or j')

Simple Example of IVE

```
      i <- 0
      j' <- 0
      k' <- a
H:
      if i >= n goto exit
      j <- j'
      k <- k'
      M[k] <- 0
      i <- i + 1
      j' <- j' + 4
      k' <- k' + 4
      goto H
```

```
      i <- 0
      k' <- a
H:
      if i >= n goto exit
      k <- k'
      M[k] <- 0
      i <- i + 1
      k' <- k' + 4
      goto H
```

Do we need i?

Simple Example of IVE

Rewrite comparison

```
      i <- 0
      k' <- a
H:
      if i >= n goto exit
      k <- k'
      M[k] <- 0
      i <- i + 1
      k' <- k' + 4
      goto H
```

```
      i <- 0
      k' <- a
H:
      if k' >= a+(n*4) goto exi
      k <- k'
      M[k] <- 0
      k' <- k' + 4
      goto H
```

But, a+(n*4) is loop invariant

Simple Example of IVE

Invariant code motion on a+(n*4)

```
      i <- 0
      k' <- a
H:
      if k' >= a+(n*4)goto exit
      k <- k'
      M[k] <- 0
      k' <- k' + 4
      goto H
```

```
      k' <- a
      n' <- a + (n * 4)
H:
      if k' >= n' goto exit
      k <- k'
      M[k] <- 0
      k' <- k' + 4
      goto H
```

now, we do copy propagation and eliminate k

Simple Example of IVE

Copy propagation

```
     k' <- a
     n' <- a + (n * 4)
H:
     if k' >= n' goto exit
     k <- k'
     M[k] <- 0
     k' <- k' + 4
     goto H
```

```
     k' <- a
     n' <- a + (n * 4)
H:
     if k' >= n' goto exit
     M[k'] <- 0
     k' <- k' + 4
     goto H
```

Voila!

Simple Example of IVE

Compare original and result of IVE

```
    i <- 0
H:
    if i >= n goto exit
    j <- i * 4
    k <- j + a
    M[k] <- 0
    i <- i + 1
    goto H
```

```
    k' <- a
    n' <- a + (n * 4)
H:
    if k' >= n' goto exit
    M[k'] <- 0
    k' <- k' + 4
    goto H
```

Voila!

What we did

- identified induction variables (i,j,k)
- strength reduction (changed * into +)
- dead-code elimination (j <- j')
- useless-variable elimination (j' <- j' + 4)
  (This can also be done with ADCE)
- loop invariant identification & code-motion
- almost useless-variable elimination (i)
- copy propagation

Is it faster?

- On some hardware, adds are much faster than multiplies
- Fewer instructions (better $ behavior)
- Furthermore, one fewer value is computed,
  - thus potentially saving a register
  - and decreasing the possibility of spilling
- Can be used to eliminate bounds checking in loop

Loop preparation

- Before attempting IVE, it is best to first perform :
  - constant propagation & constant folding
  - copy propagation
  - loop-invariant hoisting

How to do it, step 1

- First, find the basic IVs
  - scan loop body for defs of the form

    x = x + c or x = x – c

    where c is loop-invariant
  - record these basic IVs as

    x = (x, 0, c)
  - this represents the IV: x = x * c

$x = \phi(x, x')$

$x' = x + c$

Representing IVs

- Characterize all induction variables by:

  (base-variable, offset, multiple)

  – where the offset and multiple are loop-invariant

- IOW, after an induction variable is defined it equals:

  offset + multiple * base-variable

How to do it, step 2

- Scan for derived IVs of the form

  k = i * c1 + c2

  – where i is a basic IV,
  this is the only def of k in the loop, and
  c1 and c2 are loop invariant

- We say k is in the family of i
- Record as k = (i, c2, c1)

How to do it, step 3

- Iterate, looking for derived IVs of the form

  $$k = j * c1 + c2$$

  - where IV j =(i, a, b), and
  - this is the only def of k in the loop, and
  - there is no def of i between the def of j and the def of k
  - c1 and c2 are loop invariant
- Record as k = (i, a*c1, b*c1+c2)

Simple Example of IVE

```
      i <- 0
H:
      if i >= n goto exit
      j <- i * 4
      k <- j + a
      M[k] <- 0
      i <- i + 1
      goto H
```

i: (i, 0, 1) **i.e., i = 0 + 1 * i**

j: (i, 0, 4) **i.e., j = 0 + 4 * i**

k: (i, a, 4) **i.e., k = a + 4 * i**

So, j & k are in family of i

Identifying Induction Variables

- Two steps:
  - Find Basic IVs of form $i \leftarrow i \pm c$
  - Find Derived IVs of form $k \leftarrow j * c$ or $k \leftarrow j \pm c$

Finding Basic IVs

- Maintain two tables:
  - basic: Holds all vars that can be basic IV
  - other: Holds all vars that cannot be basic IV
- Scan stmts in loop:
  - if i $\leftarrow$ i $\pm$ c and I $\notin$ other, then put in basic
  - if i $\leftarrow$ anything else, then remove from basic and put in other

Finding Derived IVs

- Scan statements to create worklist W
  - if var defined more than once and var $\notin$ basic, then, put into other
  - if stmt uses any var $\in$ basic, insert into W
- Repeat until W is empty:
  - if s has form "$k \leftarrow j * x$" or "$k \leftarrow j \pm x$" AND
    $k \notin$ other AND
    x is loop invariant, then
    - if $j \in$ basic, then k is derived IV
      enter k into derivedTable
      put all stmts using k into W

Finding Derived IVs

- Repeat until W is empty:
  - if s has form "k $\leftarrow$ j * x" or "k $\leftarrow$ j $\pm$ x" AND
    k $\notin$ other AND
    x is loop invariant, then
    - if j $\in$ basic, then k is derived IV
      enter k into derivedTable
      put all stmts using k into W
    - else if j $\in$ derivedTable, then
      - if only def of j reaching k is in loop AND
        only 1 def reaches k AND
        no assignment to i between j & k, then
        put k in derivedTable
        put all stmts using k into W

Tracking tuples

- As we gather IVs we record:
  (base, offset, multiple) for each one

- For IV k:
  - if it is basic, the record: (k, 0, c)
  - else if defined as "k $\leftarrow$ j * x" AND j has (i, a, b)
    record: (i, a*x, b*x)
  - else if defined as "k $\leftarrow$ j $\pm$ x" AND j has (i,a,b)
    record: (i, a$\pm$x, b)

IV Optimizations

- Once we have identified all IVs and recorded their tuples, we perform 3 optimizations:
  - strength reduction
  - useless-variable elimination
  - Comparison rewriting

How to do it, step 4

- This is the strength reduction step

- For an induction variable k = (i, c1, c2)
  - initialize k = i * c2 + c1 in the preheader
  - replace k's def in the loop by

    k = k + c2
  - make sure to do this after i's def

How to do it, step 5

- This is the comparison rewriting step
- For an induction variable $k = (i, a_k, b_k)$
  - If k used only in definition and comparison
  - There exists another variable, j, in the same class and is not "useless" and $j=(i, a_j, b_j)$
- Rewrite $k < n$ as

  $$j < (b_j/b_k)(n-a_k)+a_j$$

- Note: since they are in same class:

  $$(j-a_j)/b_j = (k-a_k)/b_k$$

Notes

- Are the c1, c2 constant, or just invariant?
  - if constant, then you can keep folding them: they're always a constant even for derived IVs
  - otherwise, they can be expressions of loop-invariant variables

- But if constant, can find IVs of the type

$$x = i/b$$

and know that it's legal, if b evenly divides the stride…

Is it faster? (2)

- On some hardware, adds are much faster than multiplies
- But…not always a win!
  – Constant multiplies might otherwise be reduced to shifts/adds that result in even better code than IVE
  – Scaling of addresses (i*4) might come for free on your processor's address modes
- So maybe: only convert **`i*c1+c2`** when c1 is loop invariant but <u>not</u> a constant
- Or, can be used to eliminate bound check!

Loop Unrolling

- For loops with a small body:
  - significant portion of time spent incrementing and testing induction variables
  - May be stalled due to dependencies (more on this later)
- Loop unrolling reduces overhead (and increases opportunity for superscalar to tolerate latencies) by copying body of loop

Unroll Mechanism

- A loop L with header h and backedges $s_i \rightarrow h$
  - copy L to a new loop L' with header h' and backedges $s'_i \rightarrow h'$
  - changes edges $s_i \rightarrow h$ in L to $s_i \rightarrow h'$
  - change backedges in L' from $s'_i \rightarrow h$
- Change IVs
- Must deal with potential left over iterations in an epilogue

IV changes for unrolling

- Eliminate IV in L
- create new IV, i' $\leftarrow$i+c that dominates all back edges of new loop
- Change uses of IV, i, to be proper offset
- change final test of IV to account for Δ unrolls.
- Finally, insert epilogue to deal with left overs.

Simple Example

```
i <- 0
H:
cmp i, n
jg exit
sum <- sum + a[i]
i <- i + 1
jmp H
exit:
```

```
i <- 0
H:
cmp i, n
jg exit
sum <- sum + a[i]
i <- i + 1
jmp H1:
H1:
cmp i, n
jg exit
sum <- sum + a[i]
i <- i + 1
jmp H:
```

Simple Example

```
i <- 0
H:
cmp i, n
jg exit
sum <- sum + a[i]
i <- i + 1
jmp H1:
H1:
cmp i, n
jg exit
sum <- sum + a[i]
i <- i + 1
jmp H:
```

```
i <- 0
H:
cmp i, n
jg exit
sum <- sum + a[i]


cmp i, n
jg exit
sum <- sum + a[i+1]
i <- i + 2
jmp H:
```

Simple Example

```
i <- 0
H:
cmp i, n
jg exit
sum <- sum + a[i]
cmp i, n
jg exit
sum <- sum + a[i+1]
i <- i + 2
jmp H:
```

```
i <- 0
H:
cmp i, n-1
jg exit
sum <- sum + a[i]
sum <- sum + a[i+1]
i <- i + 2
jmp H:
exit:
H1:
cmp i, n
jg exit1
sum <- sum + a[i]
i <- i + 1
jmp H1:
exit1:
```

Common loop optimizations

- Hoisting of loop-invariant computations
  - pre-compute before entering the loop
- Elimination of induction variables
  - change p=i*w+b to p=b,p+=w, when w,b invariant
- Loop unrolling
  - to to improve scheduling of the loop body
- Software pipelining
  - To improve scheduling of the loop body
- Loop permutation
  - to improve cache memory performance

Requires understanding data dependencies

Dependencies in Loops

- Loop independent data dependence occurs between accesses in the same loop iteration.
- Loop-carried data dependence occurs between accesses across different loop iterations.
- There is data dependence between

  access a at iteration i-k and   dependence distance

  access b at iteration i when:

  – a and b access the same memory location

  – There is a path from a to b

  – Either a or b is a write

```
for (i=0; i<n; i++)
    A[i] = 0
A[i] = A[i-2]*2
```

Defining Dependencies

• Flow Dependence	W ➔ R	δ^f	}	true
• Anti-Dependence	R ➔ W	δ^a	}	false
• Output Dependence	W ➔ W	δ^o		

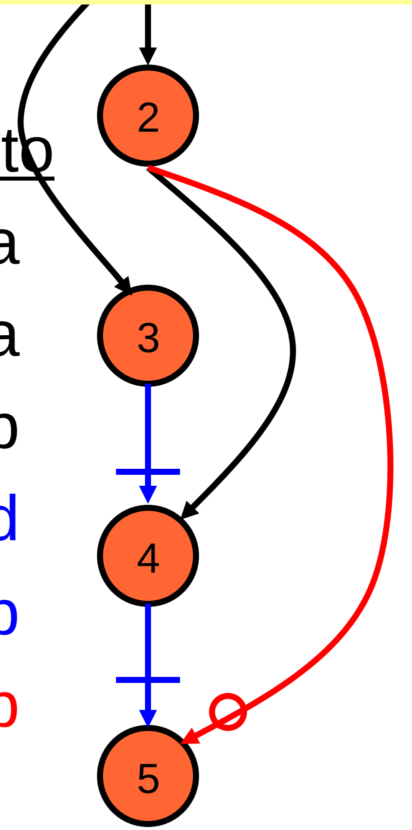
```
S1) a=0;
S2) b=a;
S3) c=a+d+e;
S4) d=b;
S5) b=5+e;
```

Example Dependencies

```
S1) a=0;
S2) b=a;
S3) c=a+d+e;
S4) d=b;
S5) b=5+e;
```

These are scalar dependencies. The same idea holds for memory accesses.

source	type	target	due to
S1	δ^f	S2	a
S1	δ^f	S3	a
S2	δ^f	S4	b
S3	δ^a	S4	d
S4	δ^a	S5	b
S2	δ^o	S5	b



- What can we do with this information?
- What are anti- and flow- called “false” dependences?

Data Dependence in Loops

- Dependence can flow across iterations of the loop.
- Dependence information is annotated with iteration information.
- If dependence is across iterations it is loop carried otherwise loop independent.

```
for (i=0; i<n; i++) {
     A[i] = B[i];
     B[i+1] = A[i];
}
```

Data Dependence in Loops

- Dependence can flow across iterations of the loop.
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- If dependence is across iterations it is loop carried otherwise loop independent.

δ^f loop carried

```
for (i=0; i<n; i++) {
    A[i] = B[i];
    B[i+1] = A[i];
}
```

δ^f loop independent

Unroll Loop to Find Dependencies

```
for (i=0; i<n; i++) {
    A[i] = B[i];
    B[i+1] = A[i];
}
```

δ^f loop carried

δ^f loop independent

```
A[0] = B[0];
B[1] = A[0];
```

i=0

```
A[1] = B[1];
B[2] = A[1];
```

i=1

```
A[2] = B[2];
B[3] = A[2];
```

i=2

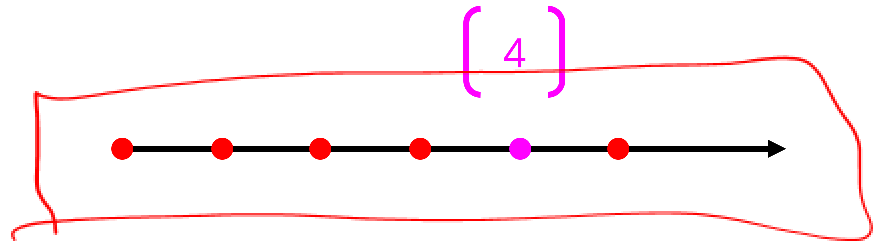
•
•
•

Distance/Direction of the dependence is also important.

Iteration Space

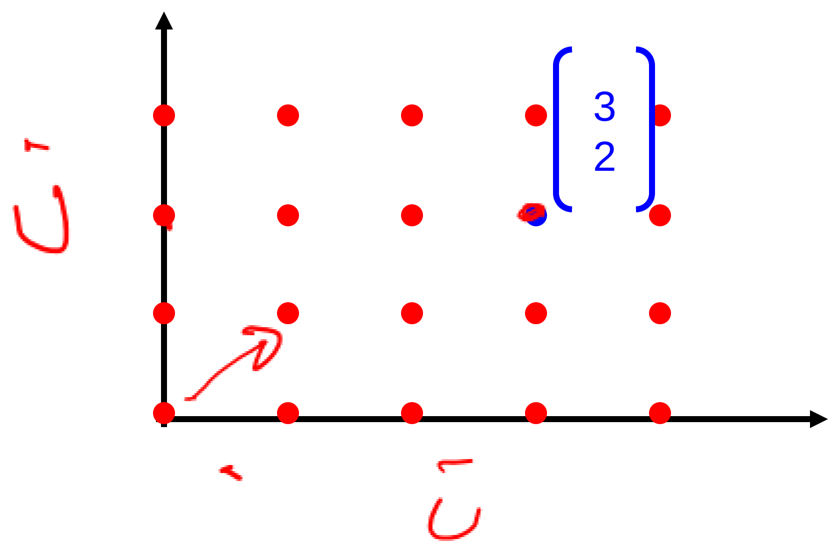
Every iteration generates a point in an n-dimensional space, where n is the depth of the loop nest.

```
for (i=0; i<n; i++) {
    •••
}
```




```
for (i=0; i<n; i++)
    for (j=0; j<4; j++) {
        •••
    }
```

(!)



Distance Vector

```
for (i=0; i<n; i++) {
      A[i] = B[i];
      B[i+1] = A[i];
}
```

```
A[0] = B[0];
B[1] = A[0];
```
i=0

```
A[1] = B[1];
B[2] = A[1];
```
i=1

```
A[2] = B[2];
B[3] = A[2];
```
i=2

•
•
•

Distance vector is the difference between the target and source iterations.

$$\mathbf{d} = \mathbf{I}_t - \mathbf{I}_s$$

Exactly the distance of the dependence, i.e.,

$$\mathbf{I}_s + \mathbf{d} = \mathbf{I}_t$$

Example of Distance Vectors

```
for (i=0; i<n; i++)
   for (j=0; j<m; j++){
      A[i,j] =    ;
           = A[i,j];
      B[i,j+1] =    ;
           = B[i,j];
      C[i+1,j] =    ;
           = C[i,j+1] ;
   }
```

j			
	$A_{0,2}$= =$A_{0,2}$ $B_{0,3}$= =$B_{0,2}$ $C_{1,2}$= =$C_{0,3}$	$A_{1,2}$= =$A_{1,2}$ $B_{1,3}$= =$B_{1,2}$ $C_{2,2}$= =$C_{1,3}$	$A_{2,2}$= =$A_{2,2}$ $B_{2,3}$= =$B_{2,2}$ $C_{3,2}$= =$C_{2,3}$
	$A_{0,1}$= =$A_{0,1}$ $B_{0,2}$= =$B_{0,1}$ $C_{1,1}$= =$C_{0,2}$	$A_{1,1}$= =$A_{1,1}$ $B_{1,2}$= =$B_{1,1}$ $C_{2,1}$= =$C_{1,2}$	$A_{2,1}$= =$A_{2,1}$ $B_{2,2}$= =$B_{2,1}$ $C_{3,1}$= =$C_{2,2}$
	$A_{0,0}$= =$A_{0,0}$ $B_{0,1}$= =$B_{0,0}$ $C_{1,0}$= =$C_{0,1}$	$A_{1,0}$= =$A_{1,0}$ $B_{1,1}$= =$B_{1,0}$ $C_{2,0}$= =$C_{1,1}$	$A_{2,0}$= =$A_{2,0}$ $B_{2,1}$= =$B_{2,0}$ $C_{3,0}$= =$C_{2,1}$
		i	

$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Example of Distance Vectors

```
for (i=0; i<n; i++)
   for (j=0; j<m; j++){
      A[i,j] =    ;
         = A[i,j];
      B[i,j+1] =    ;
         = B[i,j];
      C[i+1,j] =    ;
         = C[i,j+1] ;
   }
```

j			
	$A_{0,2}=\ \ =A_{0,2}$ $B_{0,3}=\ \ =B_{0,2}$ $C_{1,2}=\ \ =C_{0,3}$	$A_{1,2}=\ \ =A_{1,2}$ $B_{1,3}=\ \ =B_{1,2}$ $C_{2,2}=\ \ =C_{1,3}$	$A_{2,2}=\ \ =A_{2,2}$ $B_{2,3}=\ \ =B_{2,2}$ $C_{3,2}=\ \ =C_{2,3}$
	$A_{0,1}=\ \ =A_{0,1}$ $B_{0,2}=\ \ =B_{0,1}$ $C_{1,1}=\ \ =C_{0,2}$	$A_{1,1}=\ \ =A_{1,1}$ $B_{1,2}=\ \ =B_{1,1}$ $C_{2,1}=\ \ =C_{1,2}$	$A_{2,1}=\ \ =A_{2,1}$ $B_{2,2}=\ \ =B_{2,1}$ $C_{3,1}=\ \ =C_{2,2}$
	$A_{0,0}=\ \ =A_{0,0}$ $B_{0,1}=\ \ =B_{0,0}$ $C_{1,0}=\ \ =C_{0,1}$	$A_{1,0}=\ \ =A_{1,0}$ $B_{1,1}=\ \ =B_{1,0}$ $C_{2,0}=\ \ =C_{1,1}$	$A_{2,0}=\ \ =A_{2,0}$ $B_{2,1}=\ \ =B_{2,0}$ $C_{3,0}=\ \ =C_{2,1}$

i

A yields: $\begin{pmatrix}0\\0\end{pmatrix}$ B yields: $\begin{pmatrix}0\\1\end{pmatrix}$ C yields: $\begin{pmatrix}1\\-1\end{pmatrix}$