SSA (2 of 2)

15-411/15-611 Compiler Design

Ben L. Titzer and Seth Goldstein

January 30, 2025

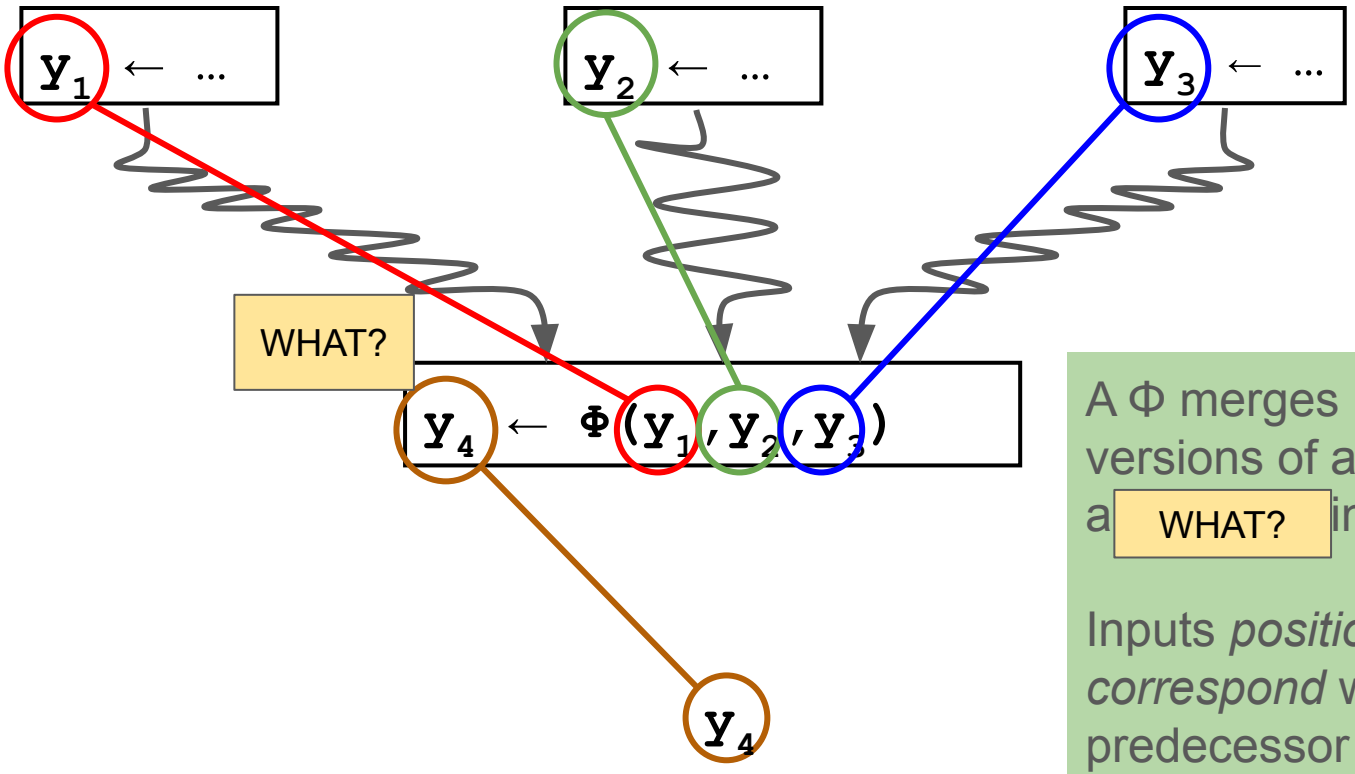
Today

- Thinking in SSA: Local reasoning
- Maintaining SSA properties during optimization
- Deconstructing SSA
  - Two issues: critical edges and ordering moves
- Syntax-directed SSA generation
- Bonus: Virgil compiler's SSA CFG

Recap From Last Time

- SSA is an IR for analysis and optimization
- Every modern optimizing compiler uses it!
- Invariant: every variable defined statically once
- Φ-functions are a notational fiction for merging dataflow at joins
- Can build SSA using the dominator tree
- Iterated dominance frontier guides Φ placement
- Renaming step walks down dominator tree
- General algorithm handles all possible CFGs

What is a Φ anyway?



A Φ merges multiple versions of a variable at a WHAT? in the CFG.

Inputs *positionally correspond* with predecessor edges.

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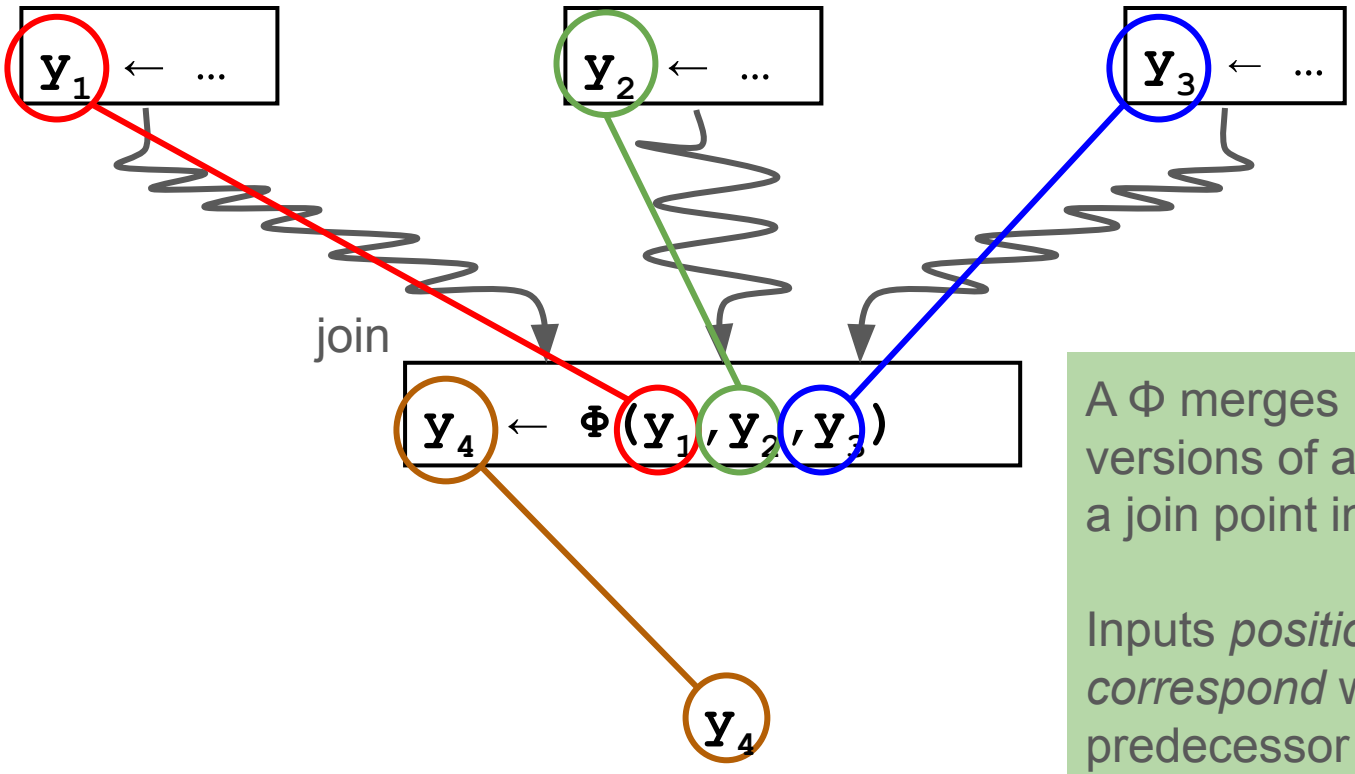

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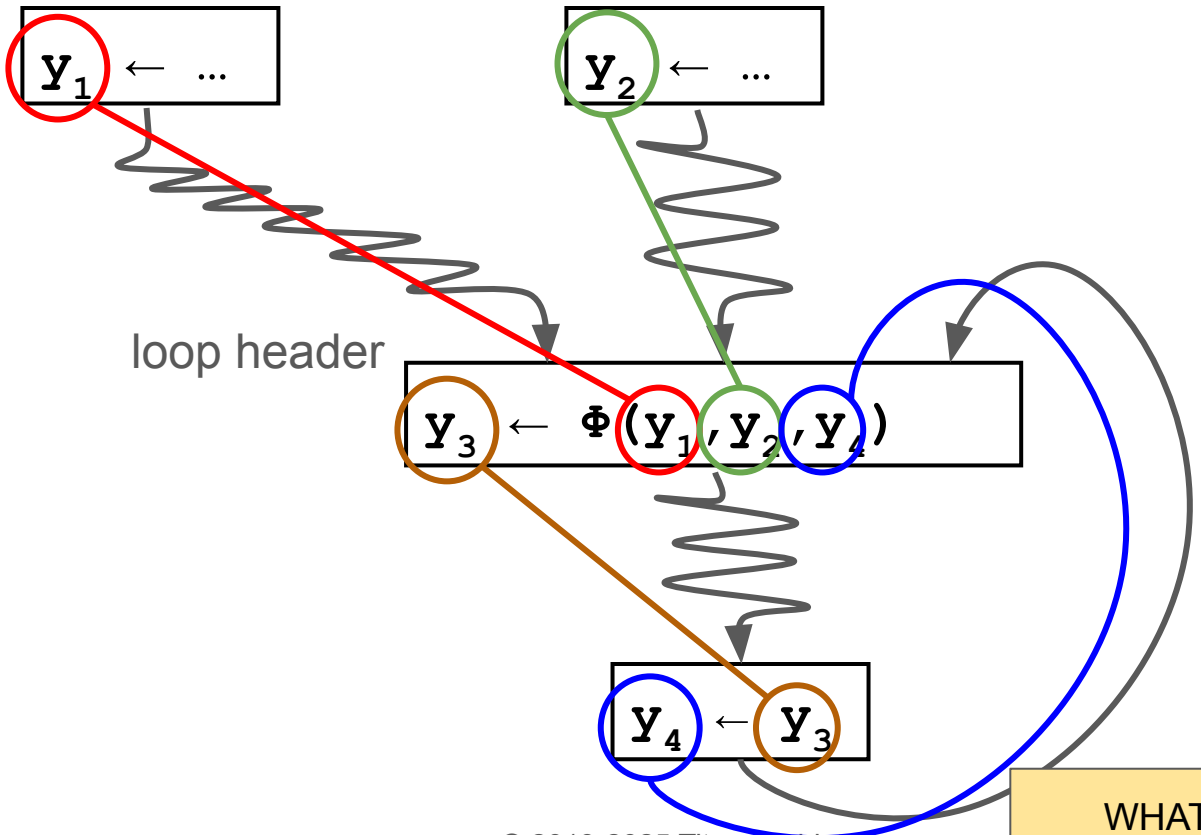
What is a Φ (for a loop) anyway?



Φs at loop headers relate the dataflow on a lo WHAT? with the control flow.

Allows finding induction variables really easily.

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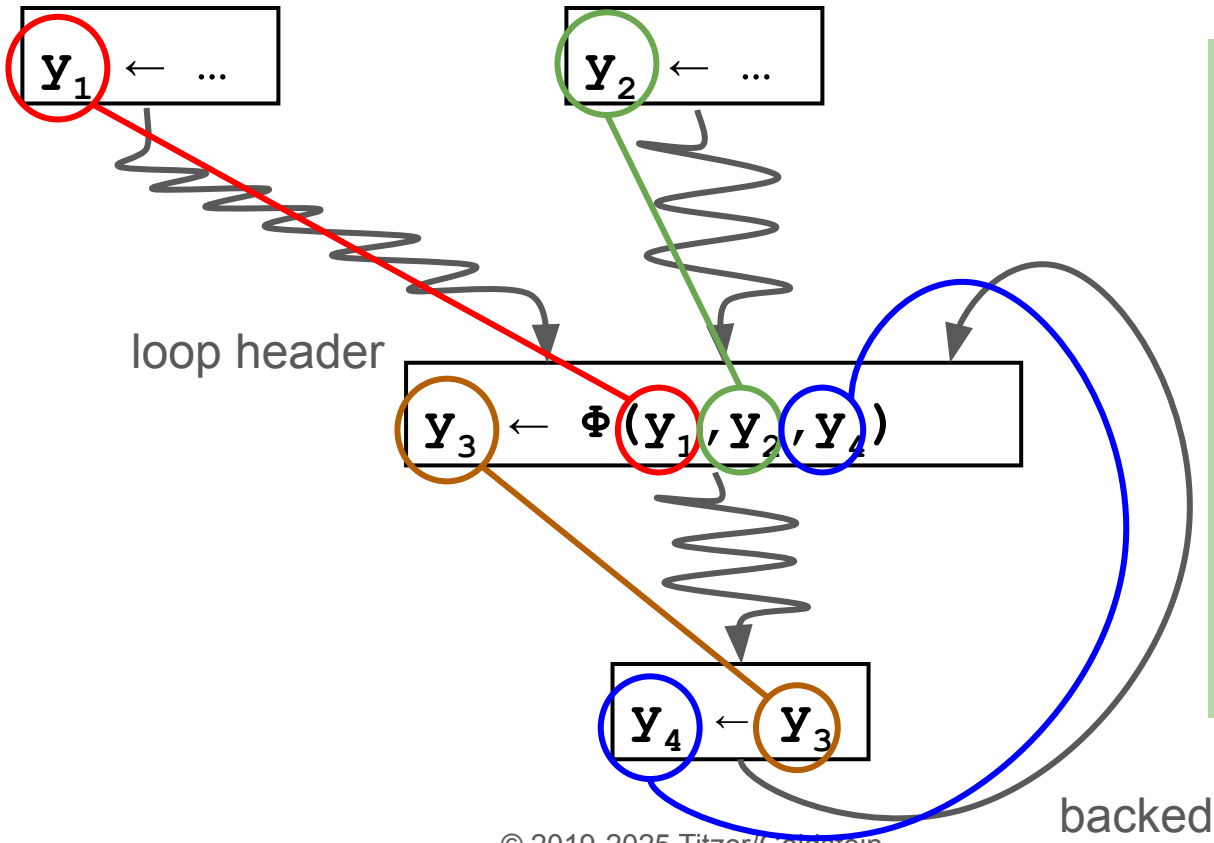
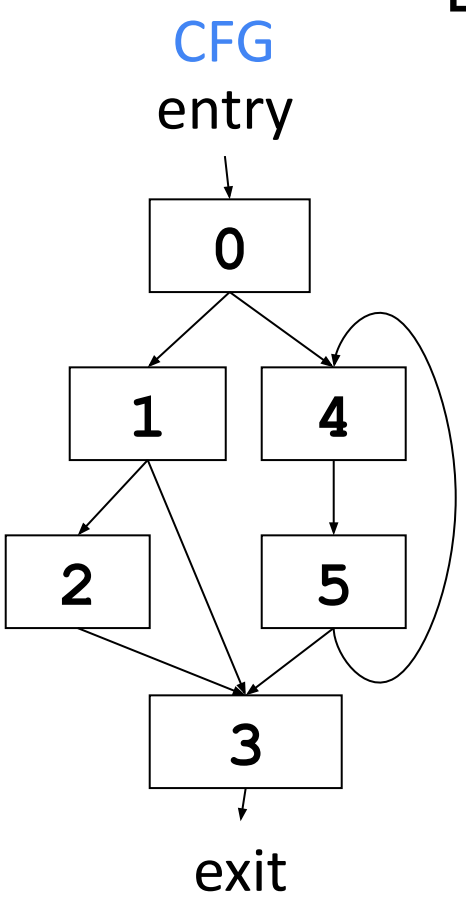


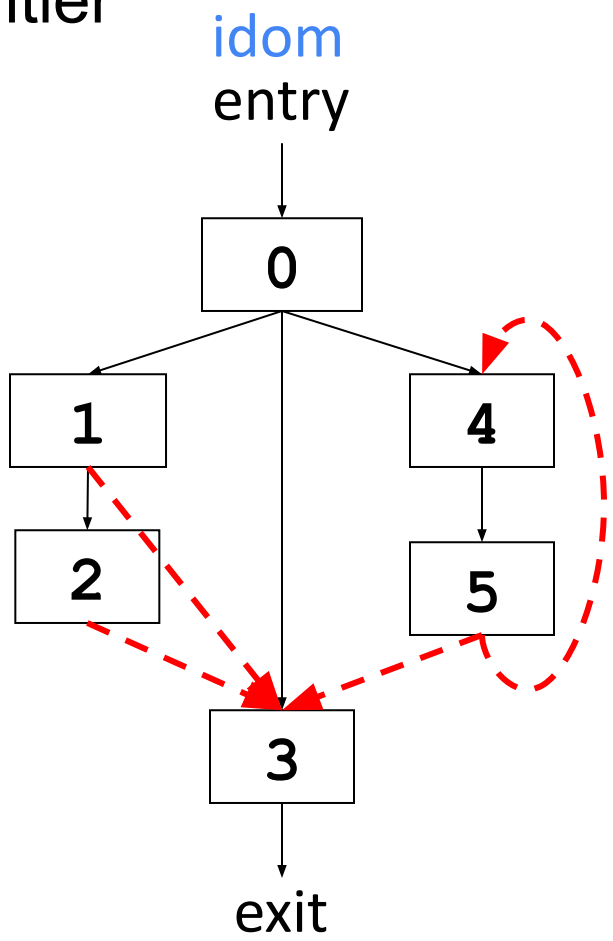
Φs at loop headers relate the dataflow on a loop backedge with the control flow.

Allows finding induction variables really easily.

Dominance Frontier



The *dominance frontier* can be determined by looking at CFG edges that ***leave a dominator subtree*** to non-dominated nodes.



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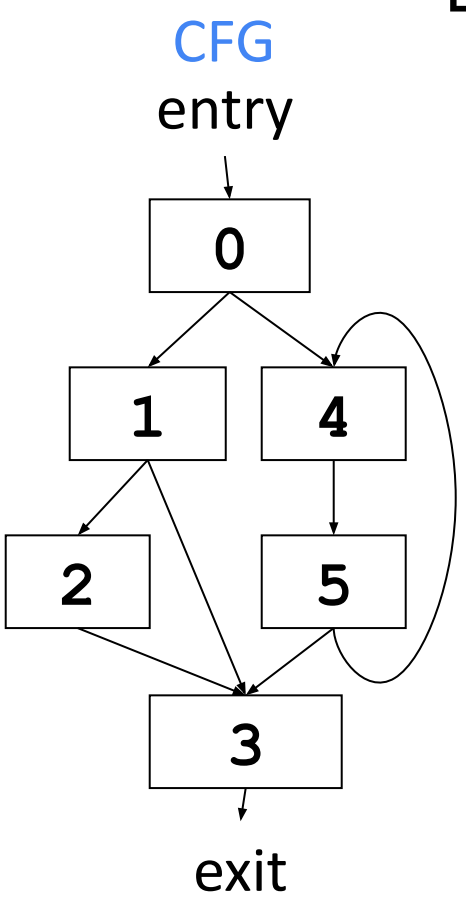

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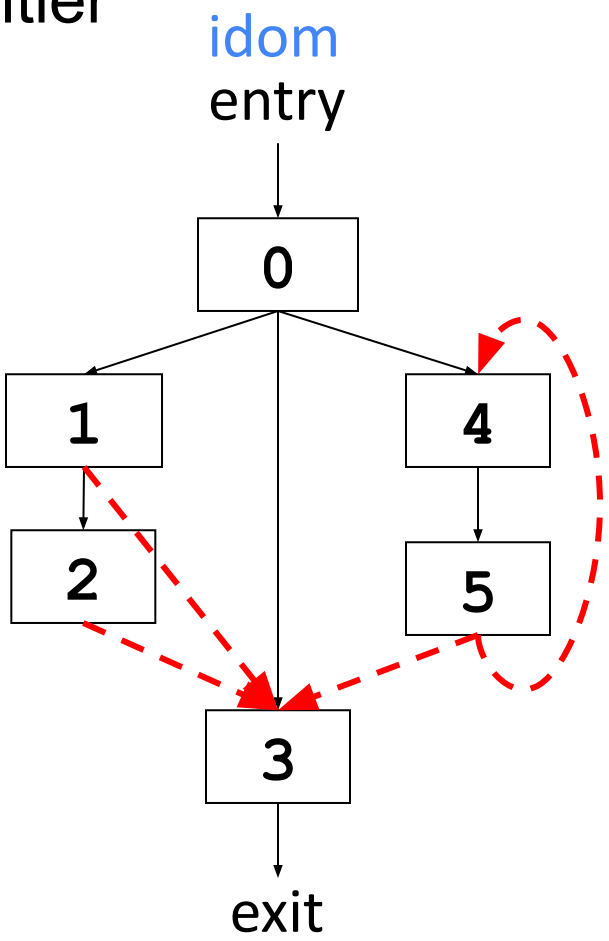

Thinking in SSA: Local Reasoning

- Local Φ simplifications
  - y = Φ(x, x) ↠ x WHAT?
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Thinking in SSA: Local Reasoning

- Local Φ simplifications
  - y = Φ(x, x) ⇝ x
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A Φ can be simplified if all inputs are the same, or all inputs are the same or the Φ itself.

Thinking in SSA: Local Reasoning

- Constant propagation
  - x = 13;
    y = x + 2;
    z = func(x); ⇝
    w = x - x;

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After SSA construction, any variable which is assigned a constant can be substituted ***everywhere it occurs***.

Thinking in SSA: Local Reasoning

- Copy propagation
  - x = a;
    y = x + 2;
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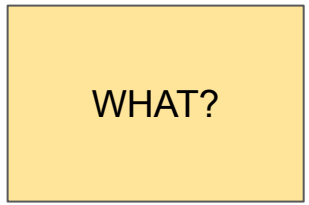
    w = x - x;



Thinking in SSA: Local Reasoning

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Copies are vestigial.

After SSA construction, any variable which is assigned a simple copy can be substituted ***everywhere it occurs***.

Thinking in SSA: Local Reasoning

- Reasoning about value ranges of Φs
  - x = Φ(3, 7);
    y = x > 0;
    z = x == 12; ⇝
    w = 33 / x;

WHAT?

Thinking in SSA: Local Reasoning

- Reasoning about value ranges of Φs
  - x = Φ(3, 7);
    y = x > 0;
    z = x == 12;
    w = 33 / x;

    ⇝

    y = true;
    z = false;
    w = 33 / x (safe);

Φ inputs represent all the possible values the phi could take on across all control flow. Therefore some conditions are statically decidable.

Thinking in SSA: Local Reasoning

- Dominating conditions
  - if (x == 0) return x; ⇝ if (x == 0) return WHAT?
  - if (x != null) return x.f; ⇝ if (x != null) return WHAT?
  - if (x != 0) return 1000 / x; ⇝ if (x != 0) return WHAT?

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Thinking in SSA: Local Reasoning

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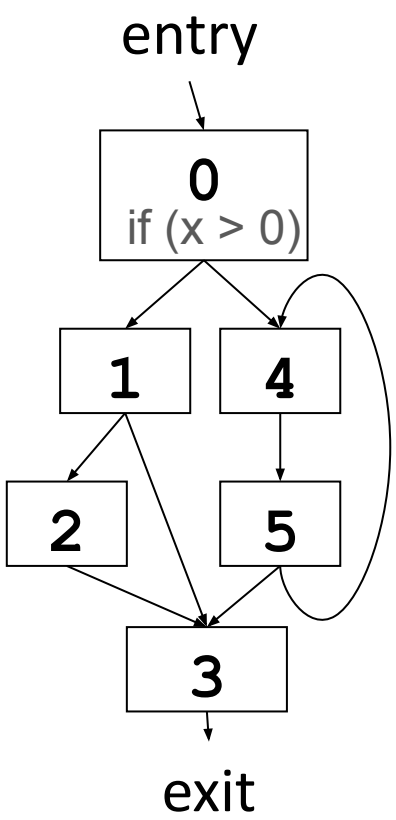
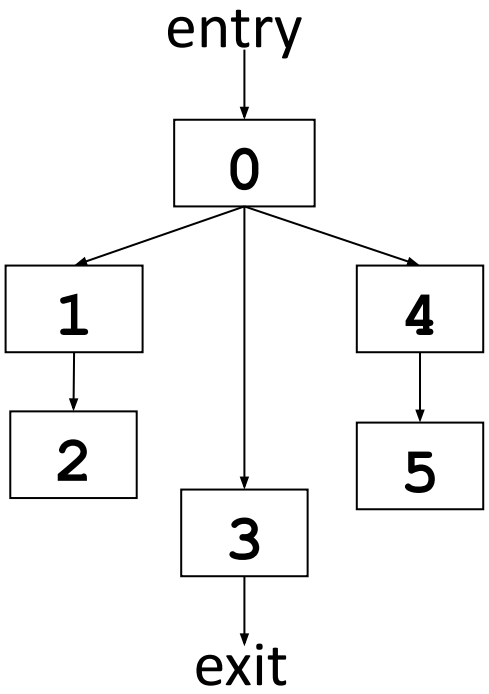


Dominator Tree



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Thinking in SSA: Local Reasoning

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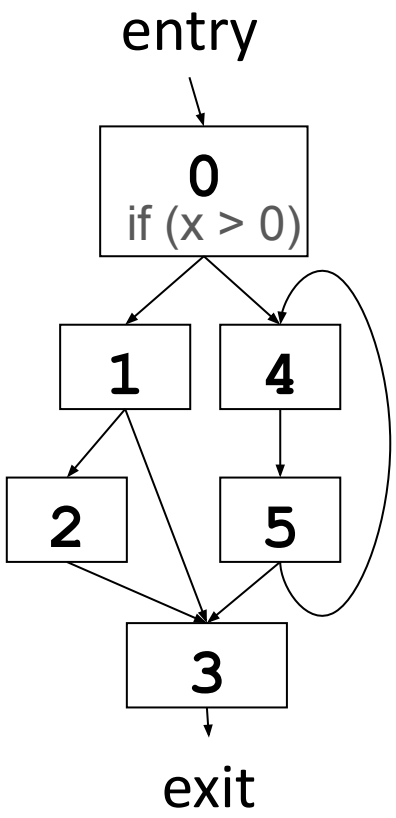

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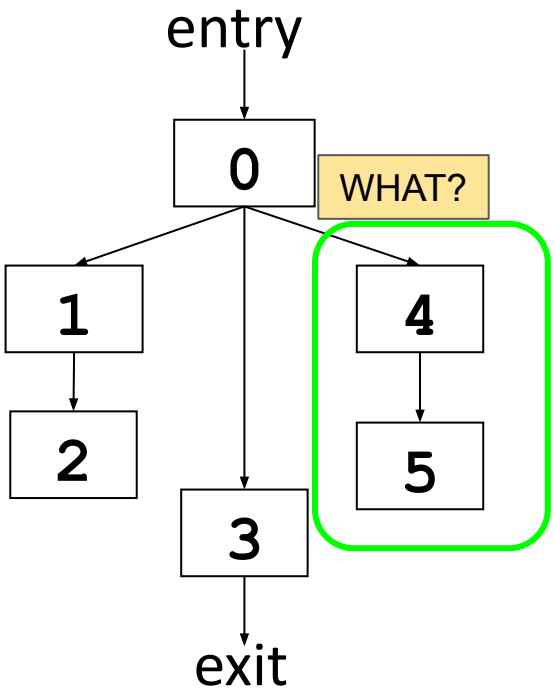

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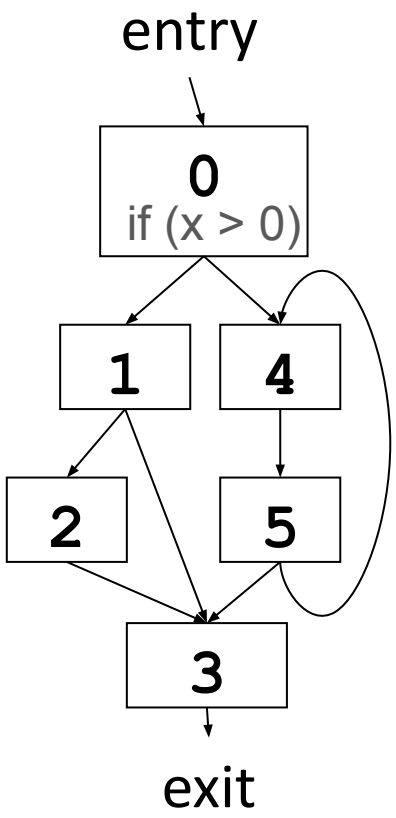

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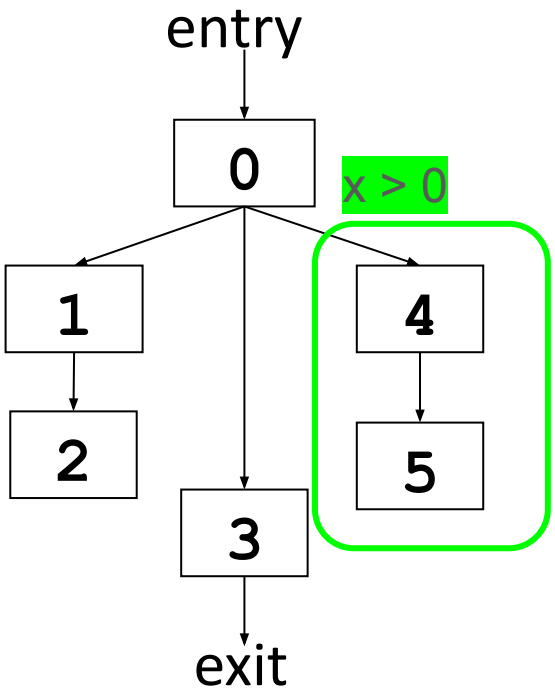

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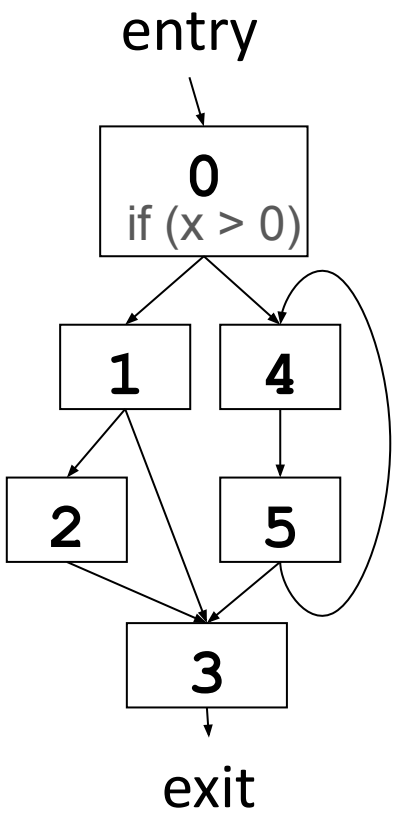

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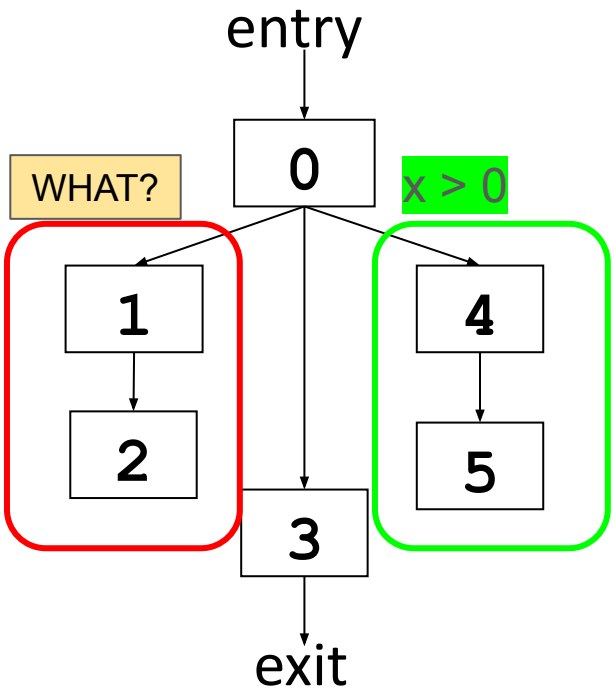

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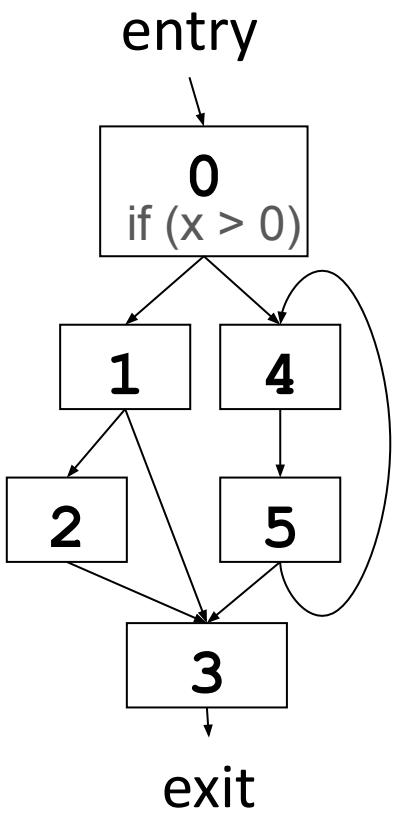

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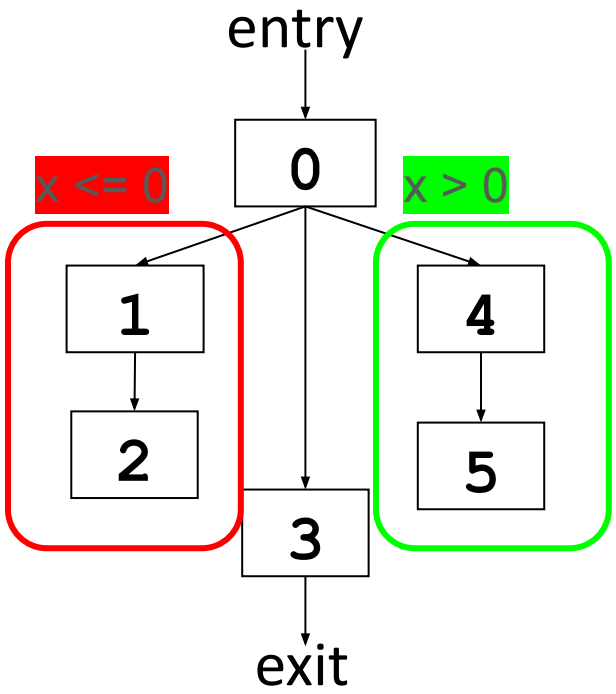

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Thinking in SSA: Local Reasoning

- Branch folding
  - if (x == x) {
    y_1 = a + 3;
    } else {
    y_2 = a + 5;
    }
    $y_3 = \Phi(y_1, y_2)$

WHAT?

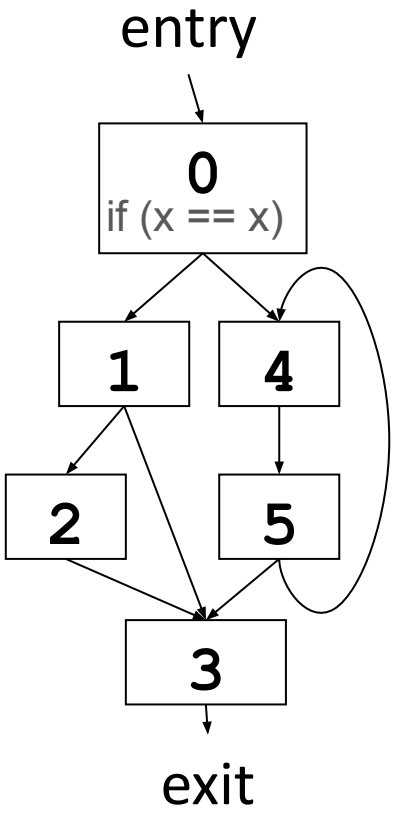
Thinking in SSA: Local Reasoning

- Branch folding
  - if (x == x) {
  y_1 = a + 3;
} else {
  y_2 = a + 5;
}
$y_3 = \Phi(y_1, y_2)$

if (true) {
  y_1 = a + 3;
} else {
  y_2 = a + 5;
}
$y_3 = y_1$

Any branch with a constant condition can be folded and edges removed from the CFG.

Thinking in SSA: Local Reasoning



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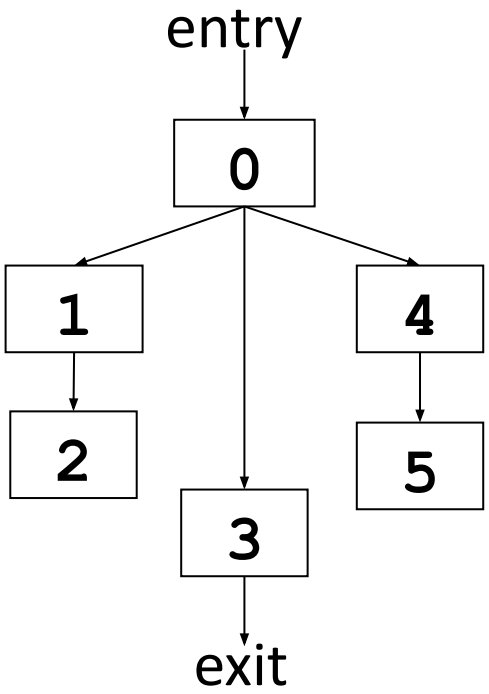

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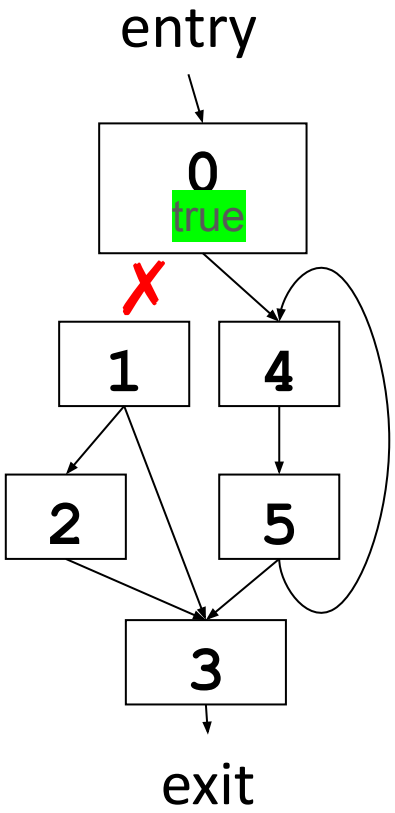

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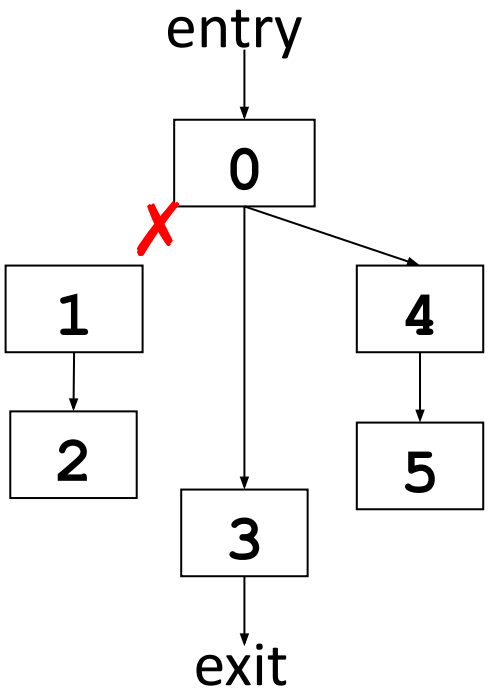

The entire dominator subtree will become dead and can be removed.

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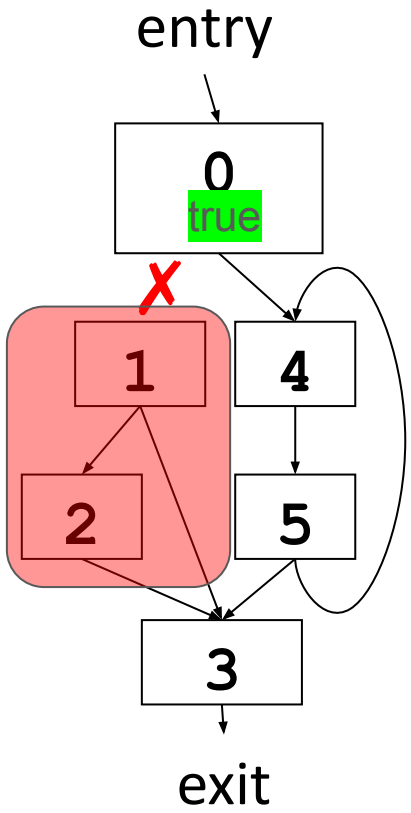

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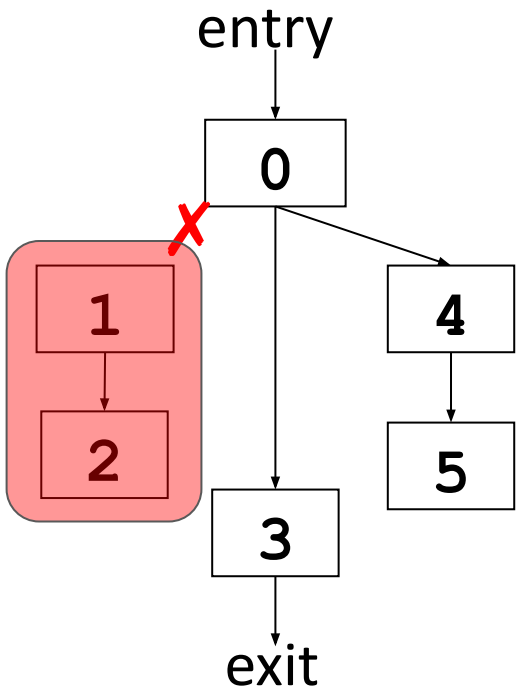

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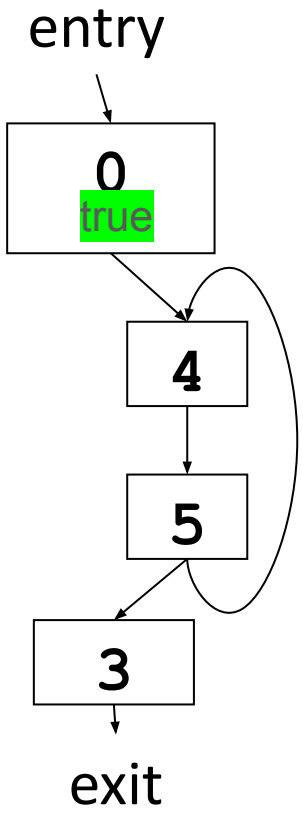

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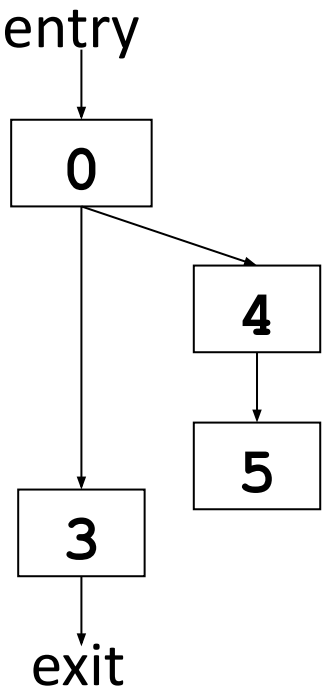

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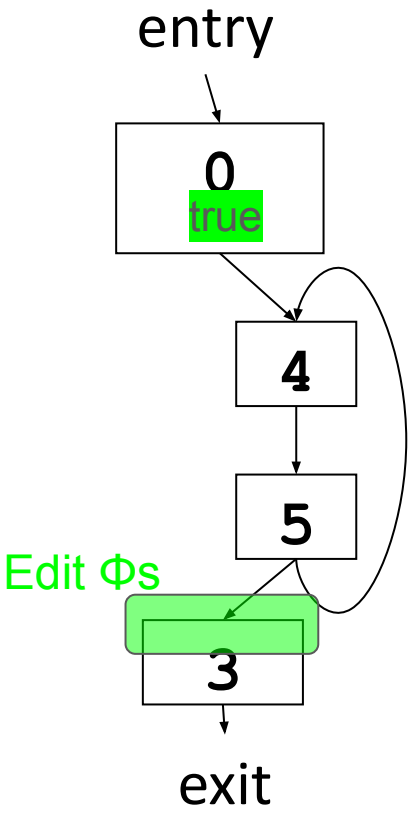

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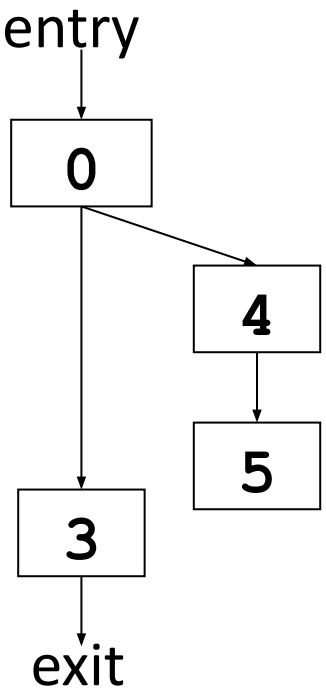
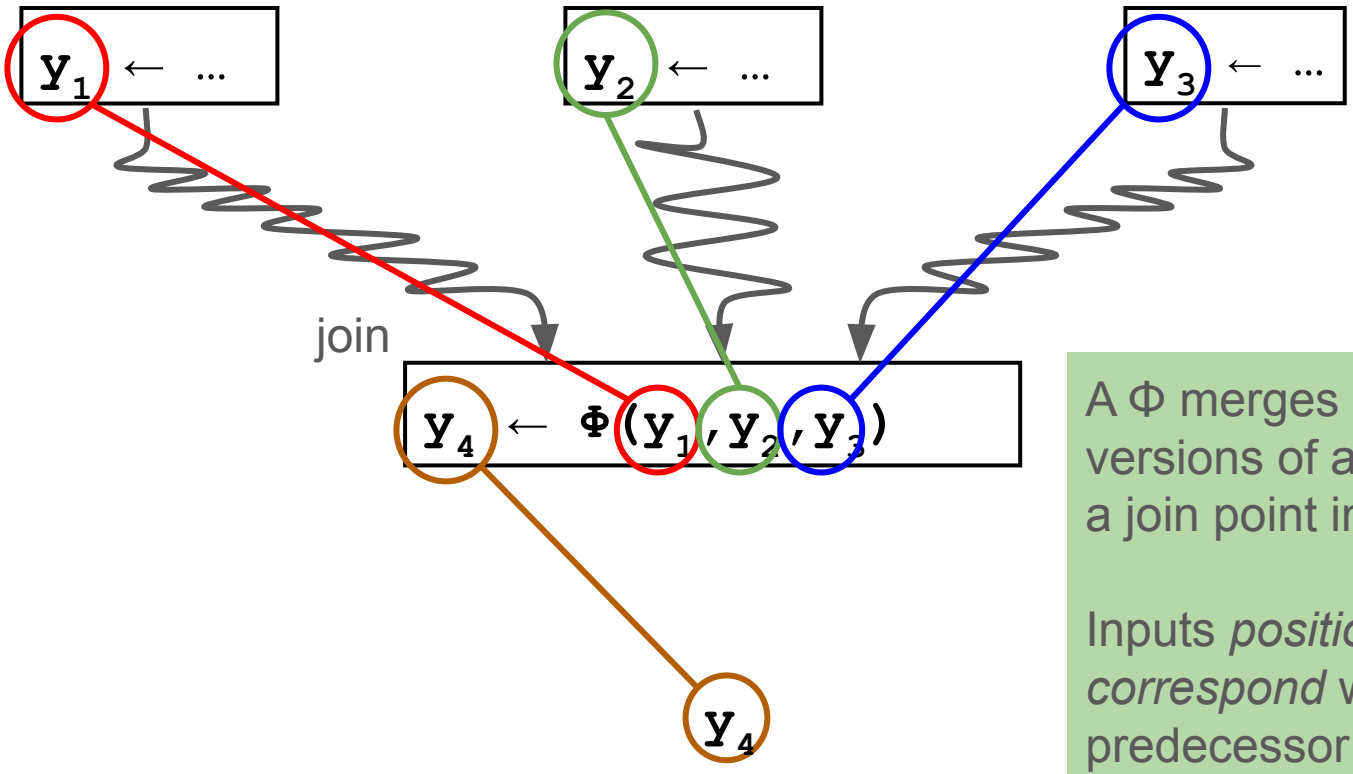


To maintain SSA, any Φs at join points with dead predecessors need to be edited and simplified.

Editing and simplifying Φs



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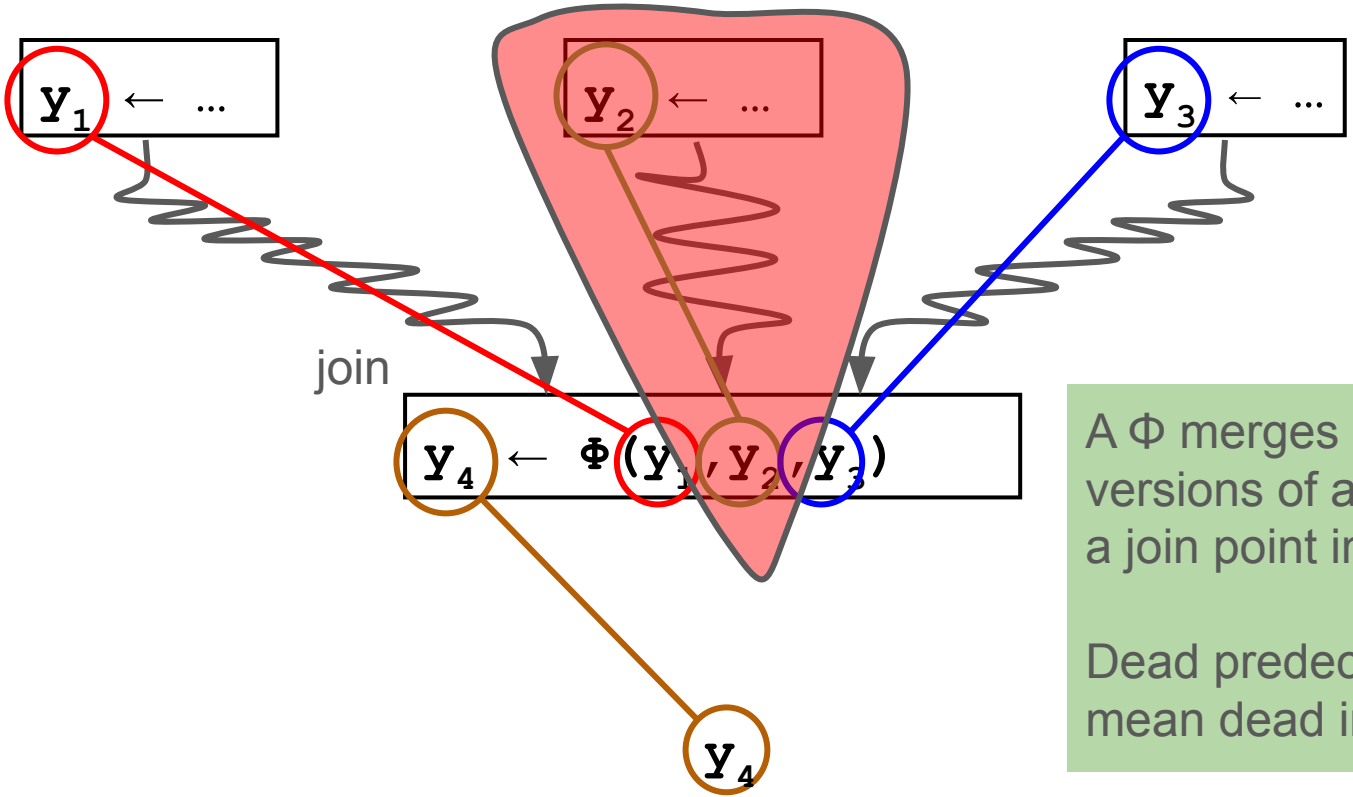

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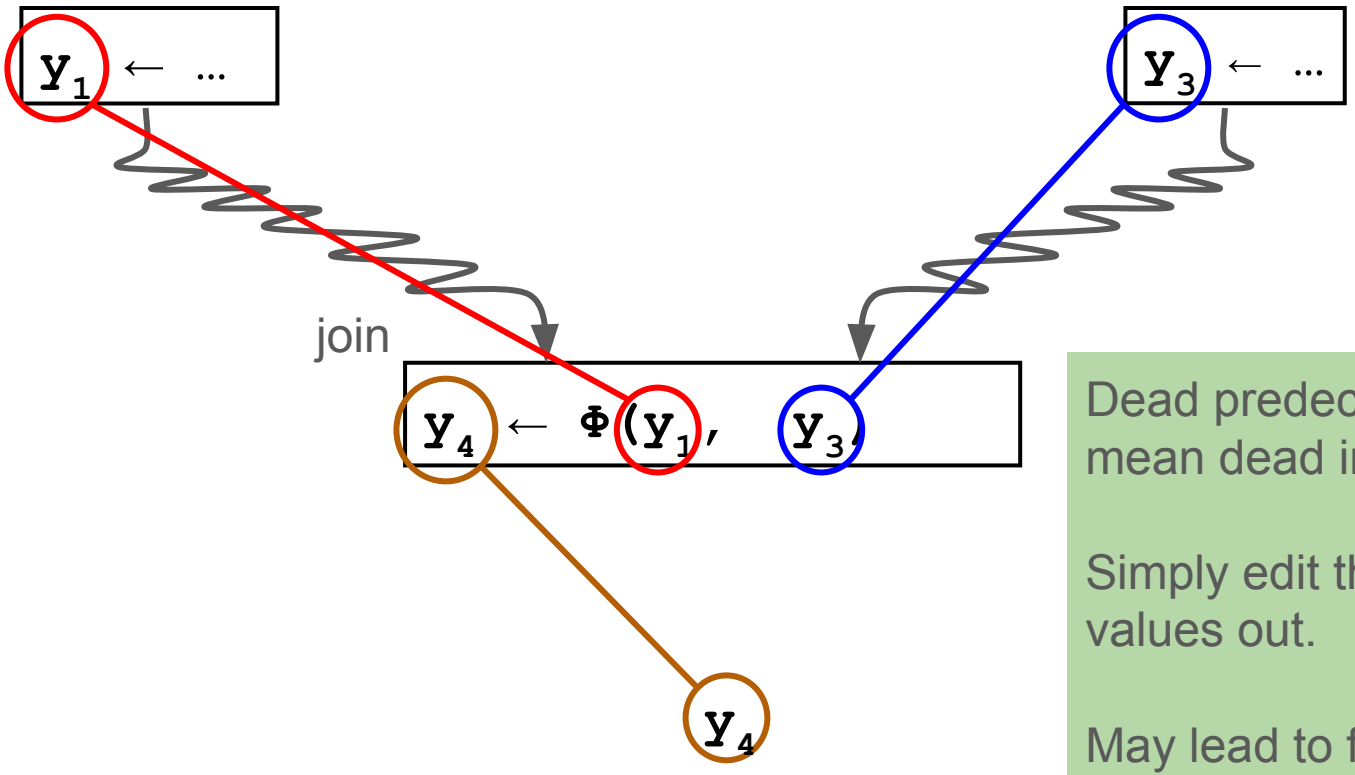


Dead predecessors mean dead input values.

Simply edit the input values out.

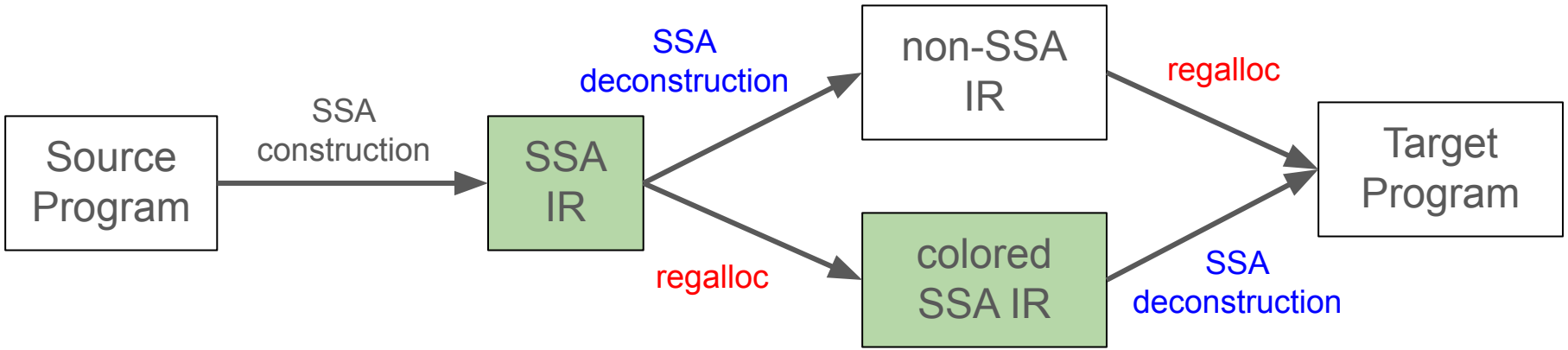
May lead to further simplification.

Deconstructing SSA

- Real machines don’t have Φ functions.
- Have to insert moves at predecessors.
- Mentioned earlier, but with huge caveats.
- We resolve those caveats today.

Deconstructing SSA

- When during compilation to deconstruct SSA?
- There are two common choices: before or after regalloc.
- Regalloc before deconstruction is relatively new (2010s).



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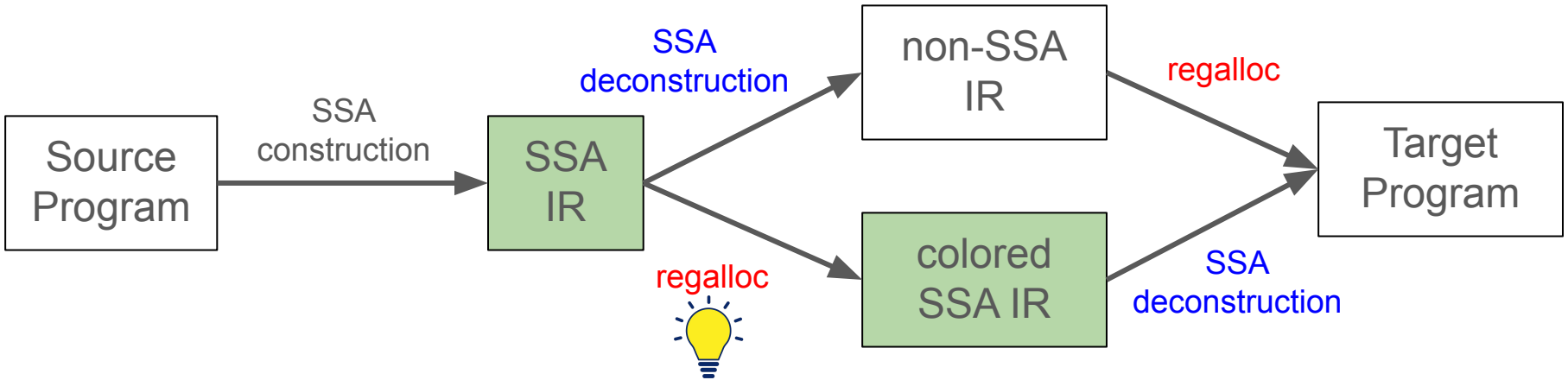

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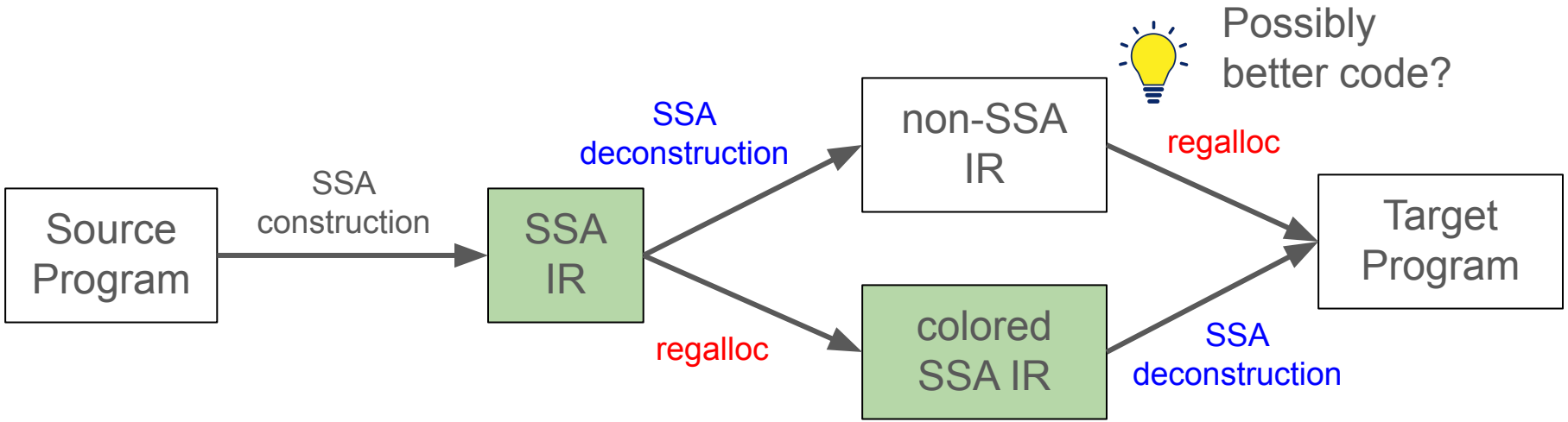

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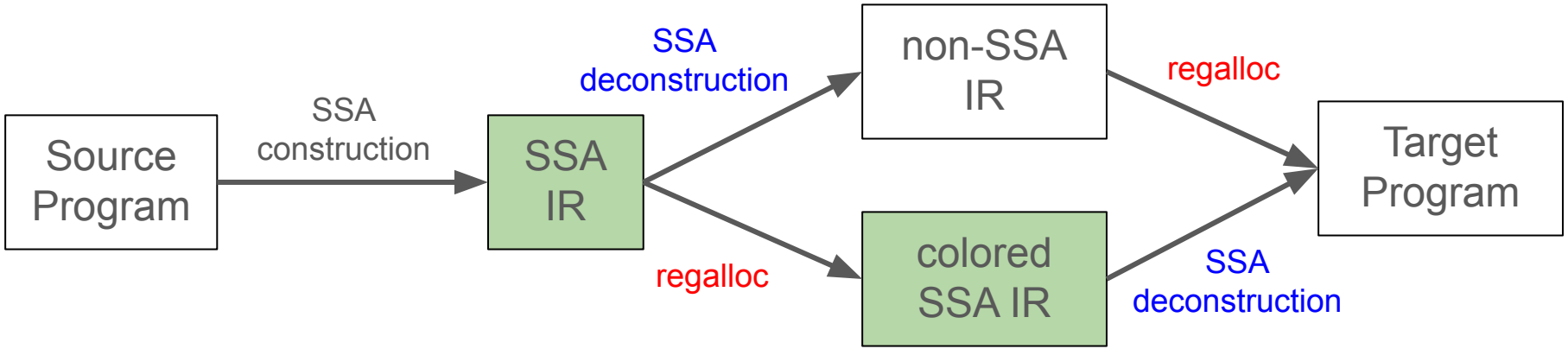
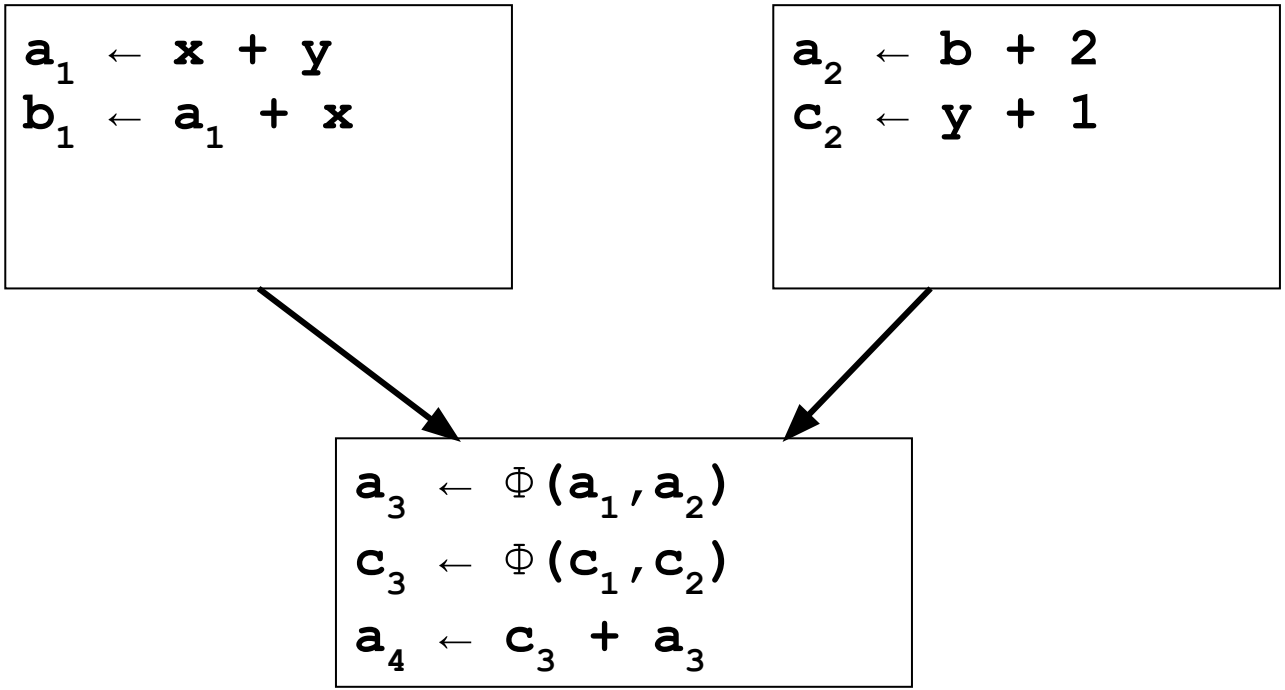


Deconstruction is more or less the same either way.

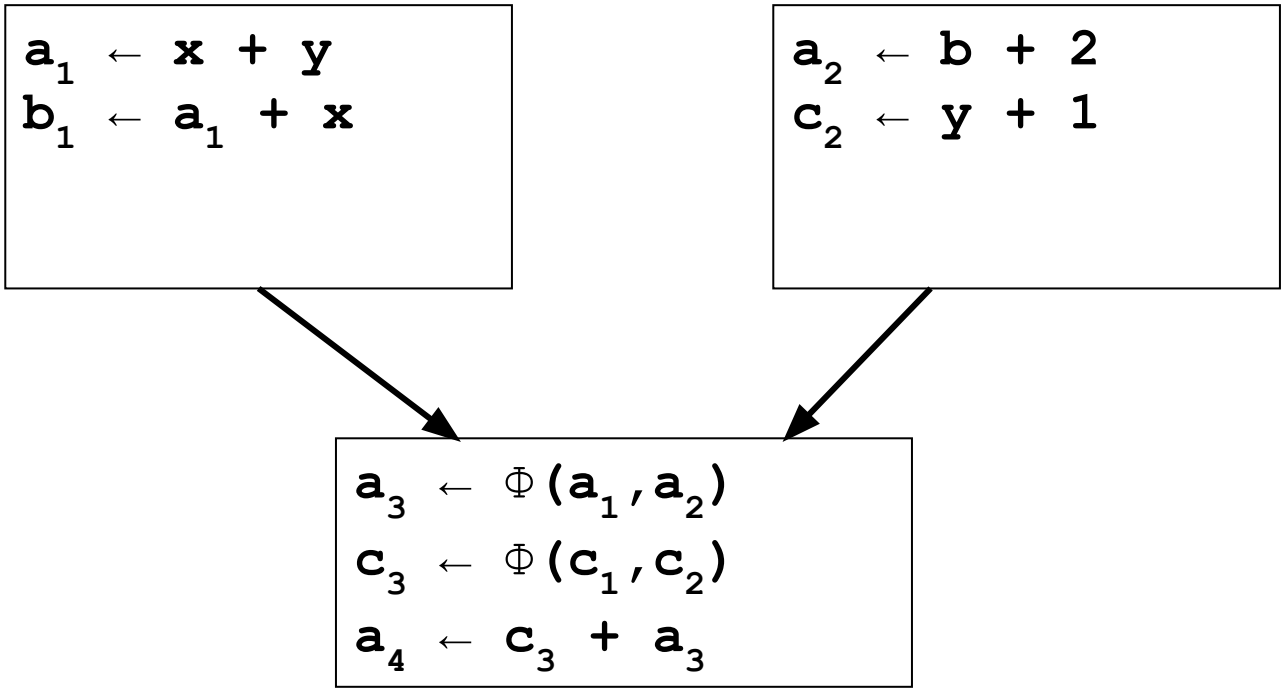
Deconstructing SSA

- Insert moves according to the positional correspondence of inputs.



Deconstructing SSA

- Insert *Φ-resolution moves* and remove Φs.



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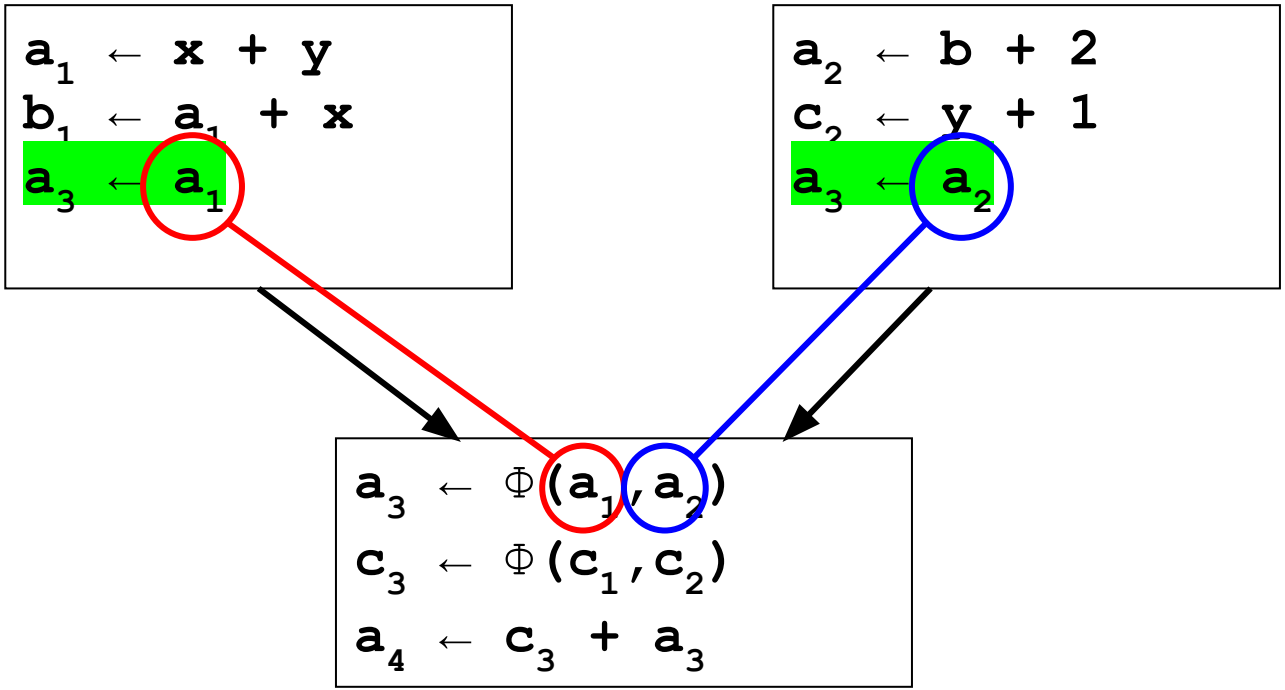

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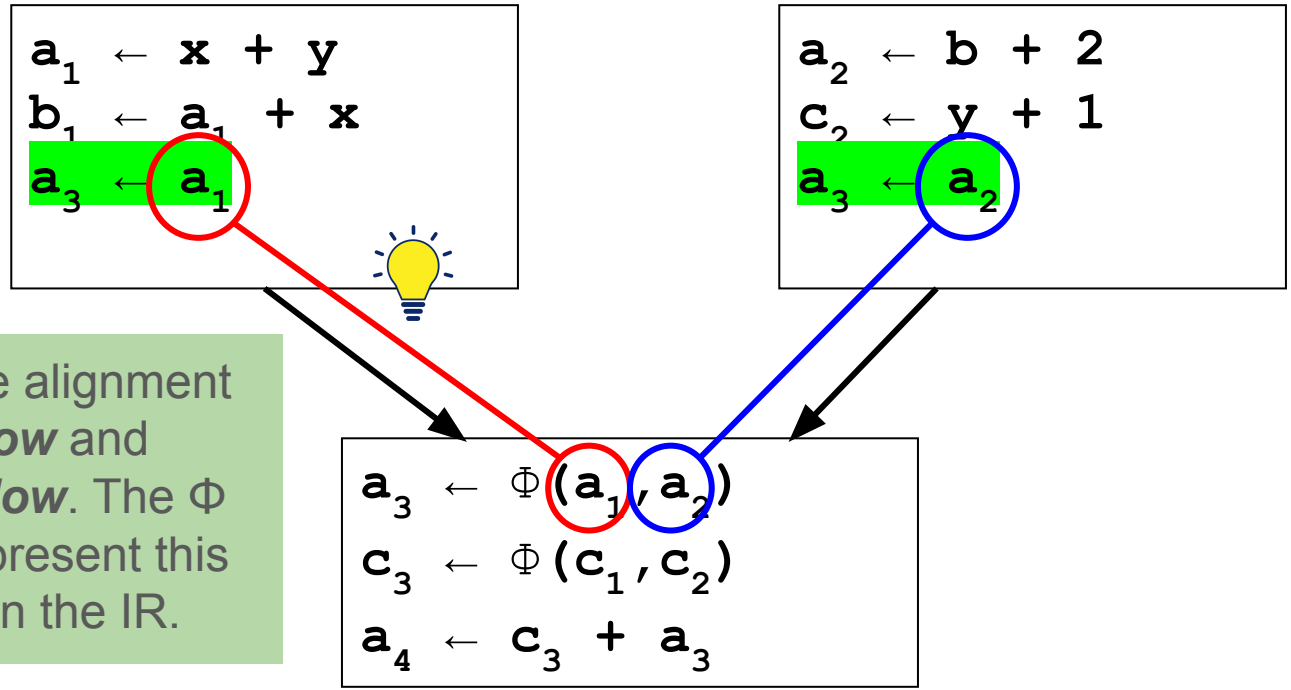

Notice the alignment of ***data flow*** and ***control flow***. The Φ nodes represent this explicitly in the IR.

Deconstructing SSA

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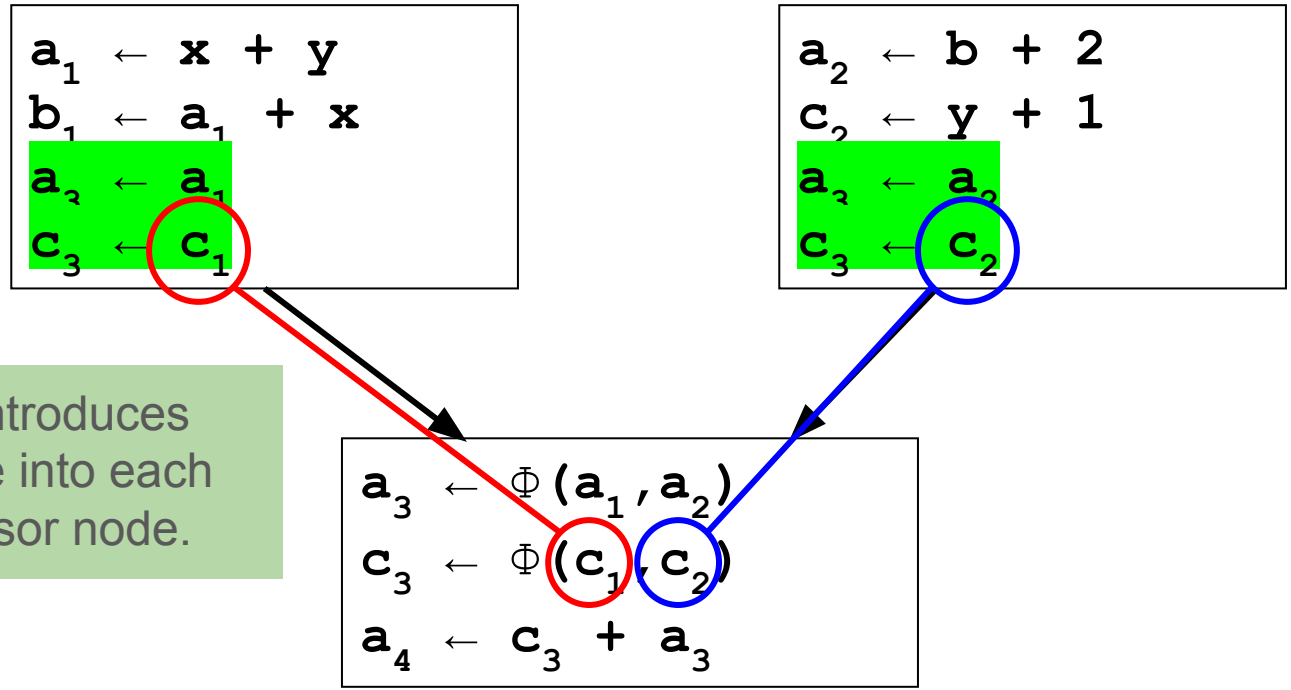

Each Φ introduces one move into each predecessor node.

Deconstructing SSA

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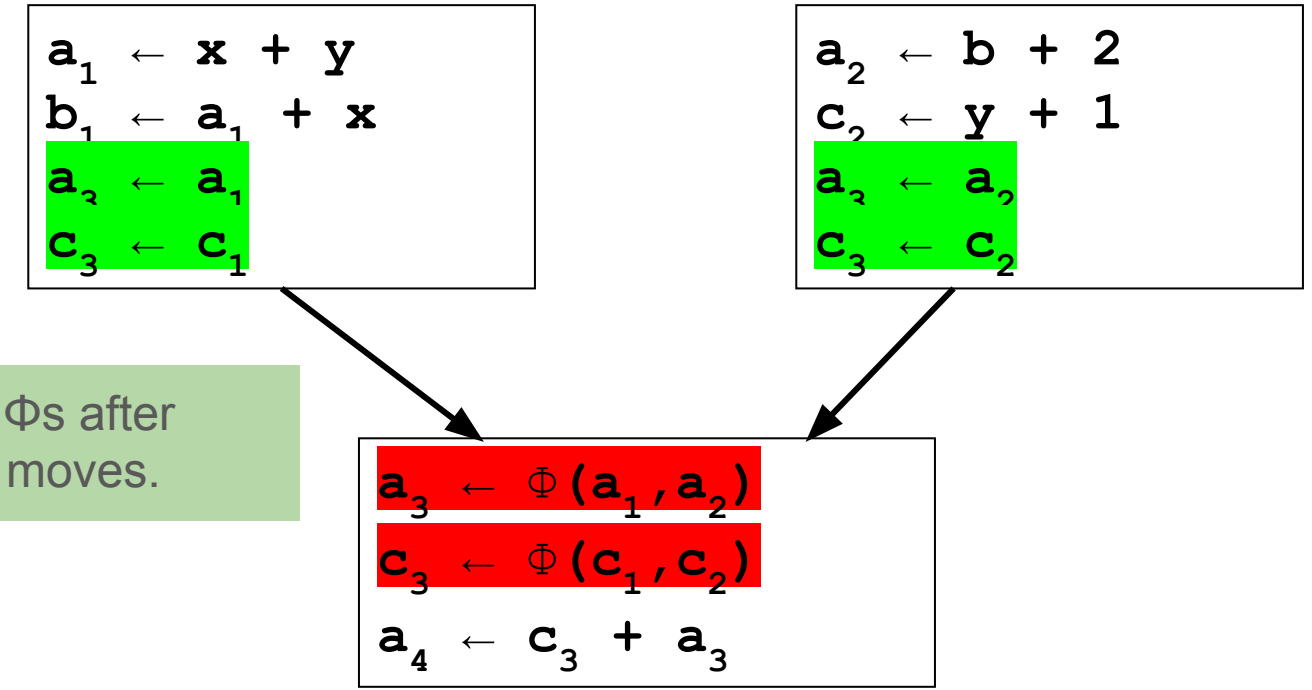
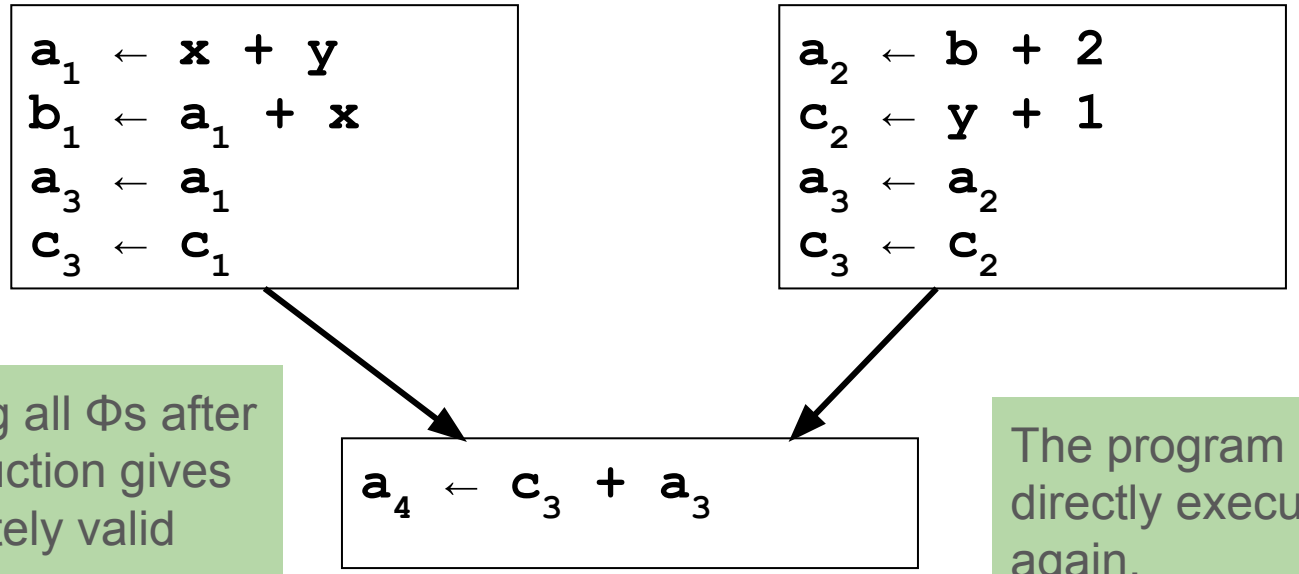


Remove Φs after inserting moves.

Deconstructing SSA

- Insert moves according to the positional correspondence of inputs.

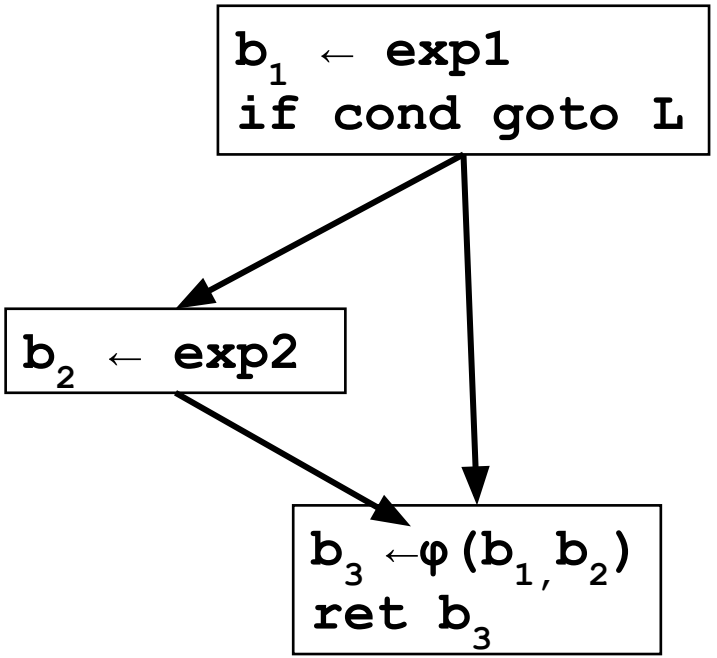


Removing all Φs after deconstruction gives a completely valid non-SSA program.

The program is now directly executable again.

Issue 1: Critical Edges

- Consider a simple triangle CFG.



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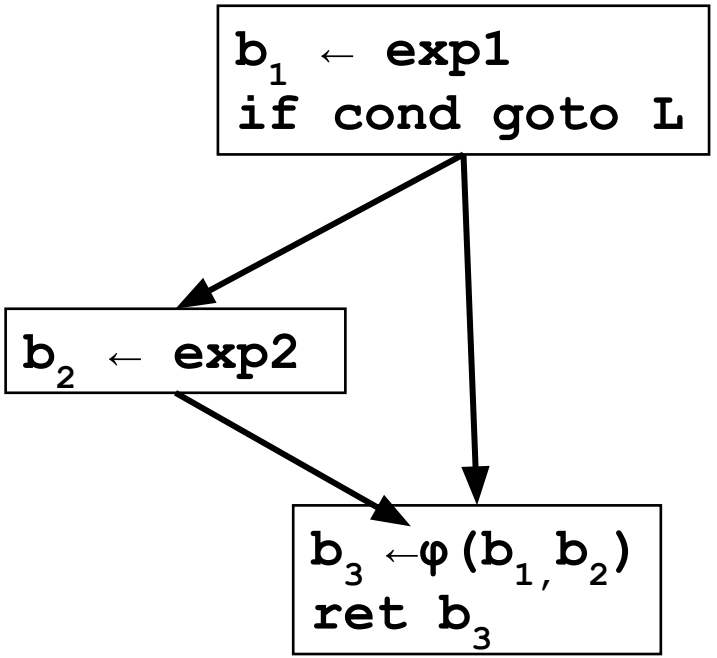

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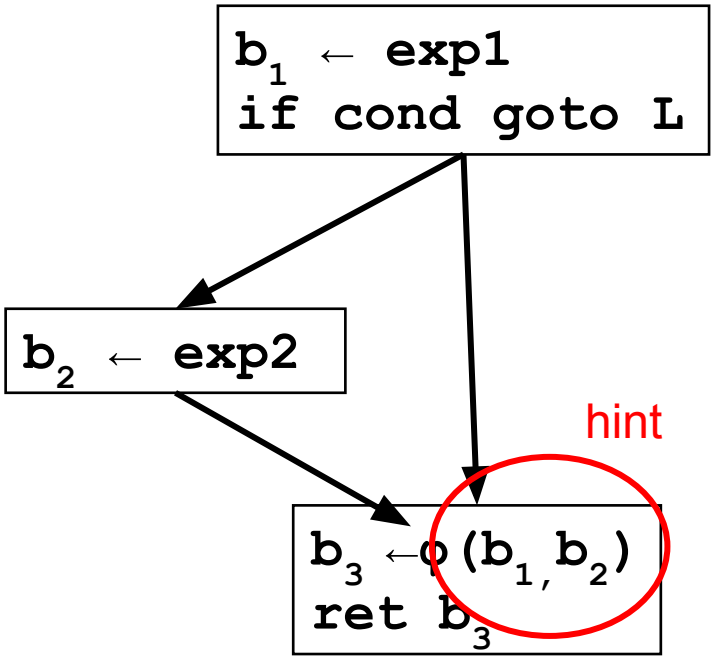

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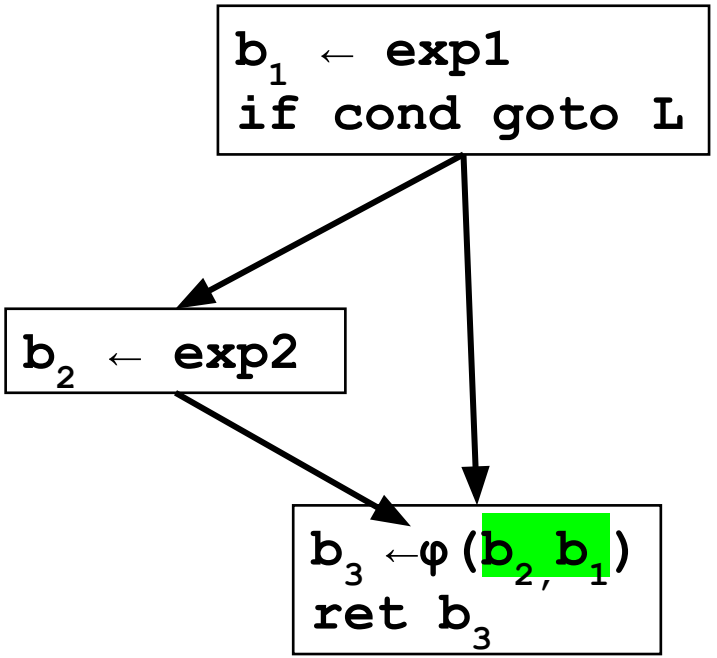
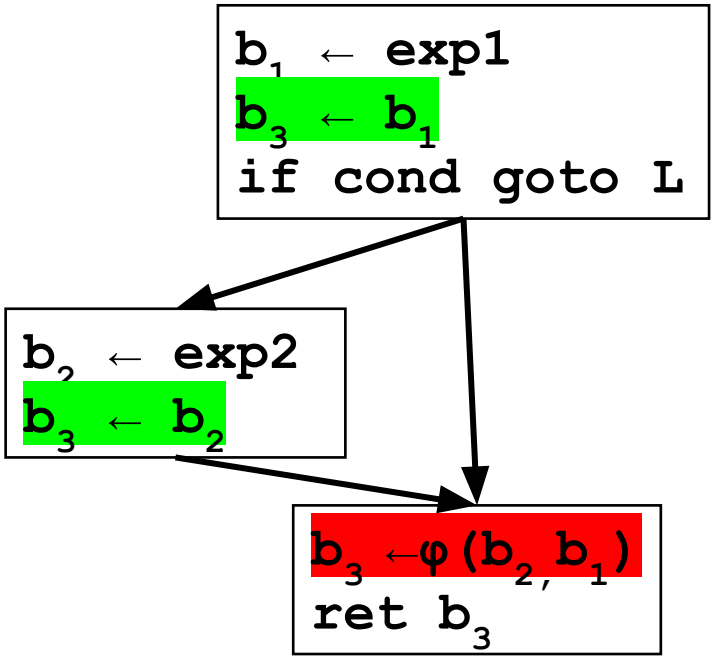


Issue 1: Critical Edges

- Consider a simple triangle CFG.
- We insert moves in both predecessors and remove the Φ.



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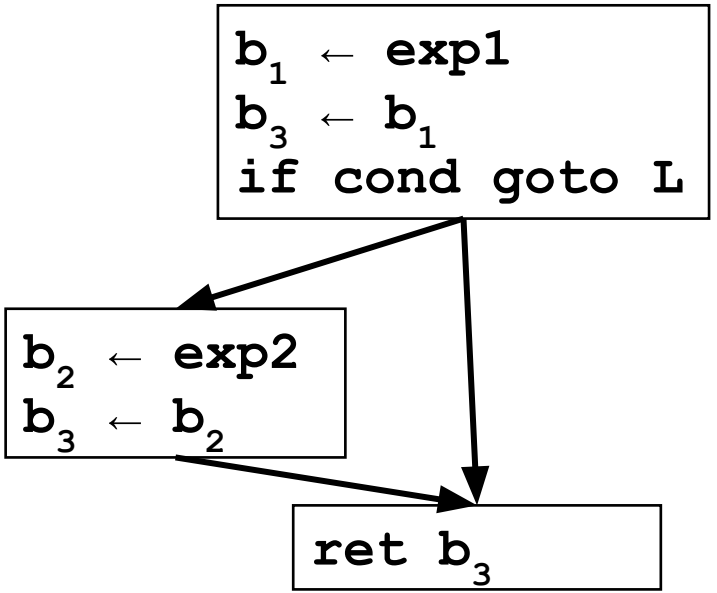

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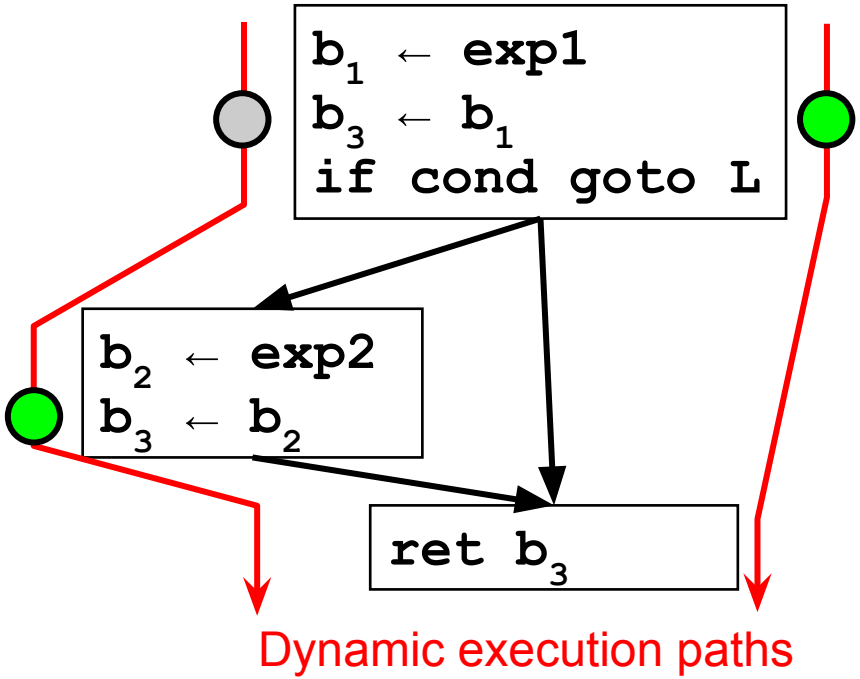

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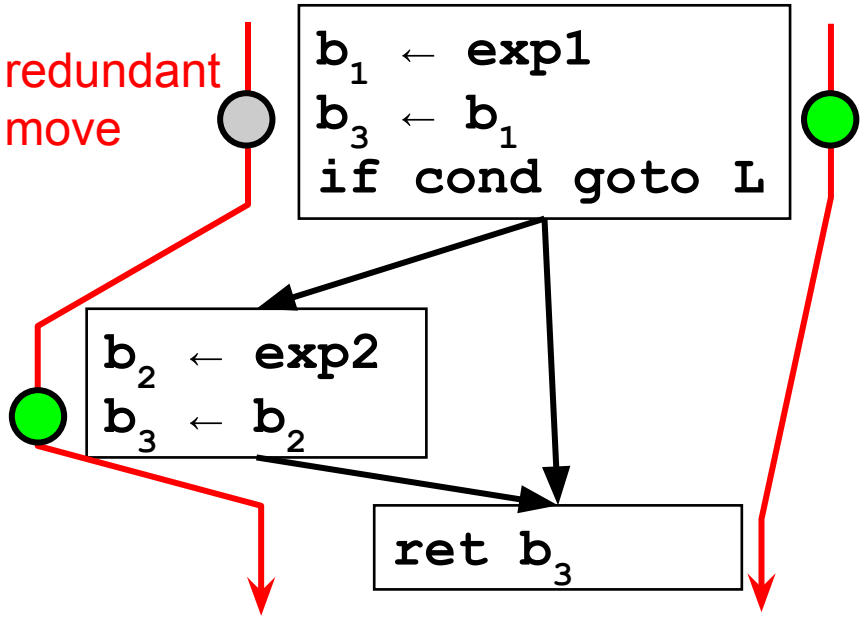
Naïve insertion can introduce redundant code on some execution paths.



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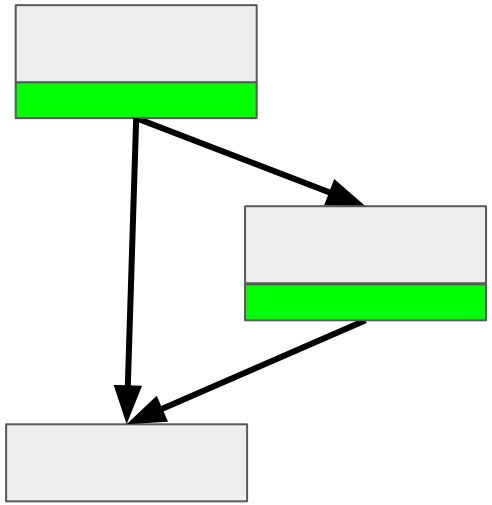

Issue 1: Critical Edges

- Consider a *more complicated* CFG.
- We insert moves in *all* predecessors and remove the Φ.

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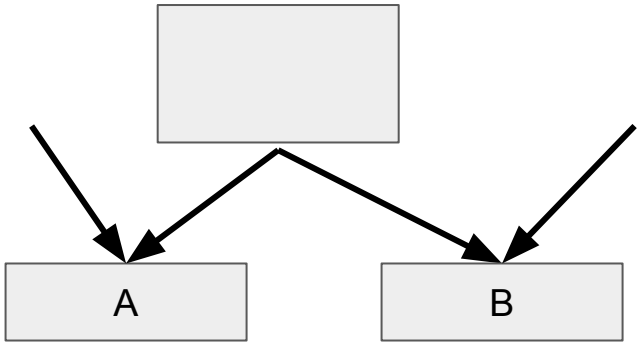

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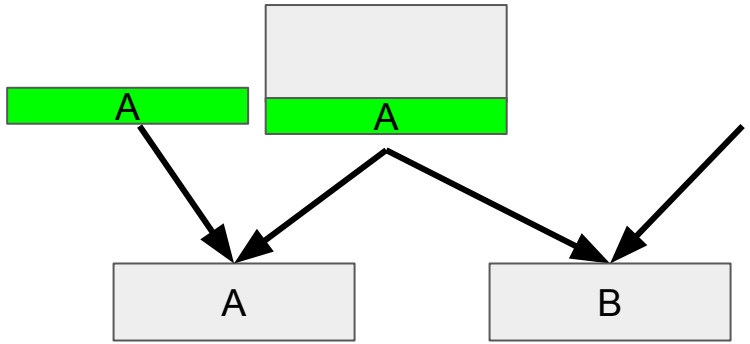

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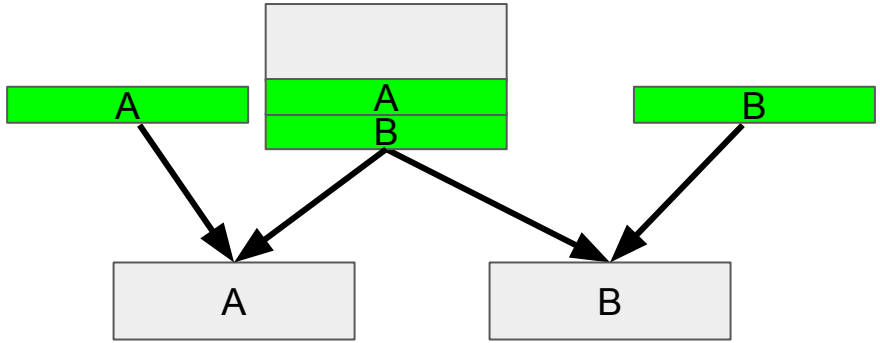
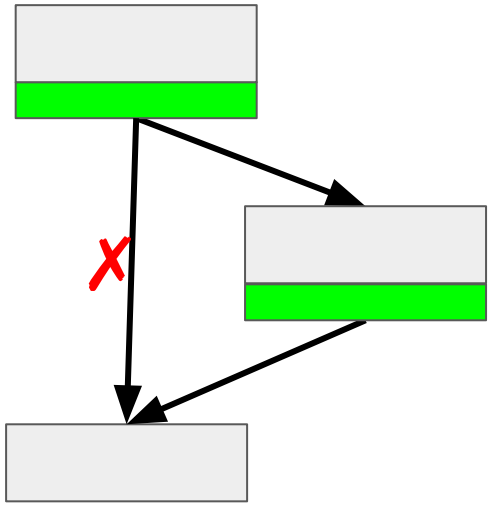
Naïve insertion can introduce redundant code on some execution paths.

Can actually get *really* bad.



Splitting Critical Edges

- To avoid redundant moves, split *critical edges* by inserting an empty block between.



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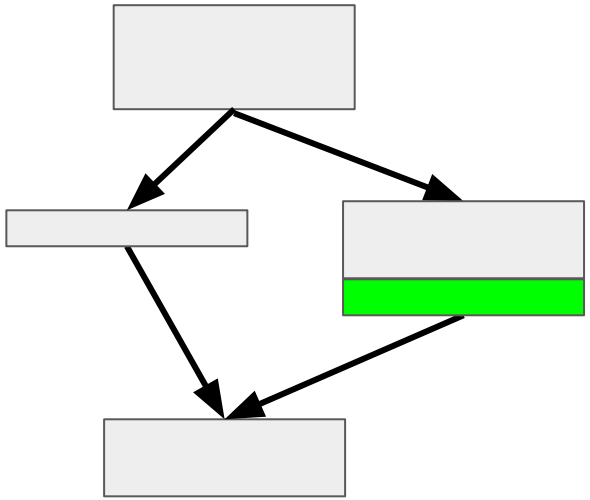

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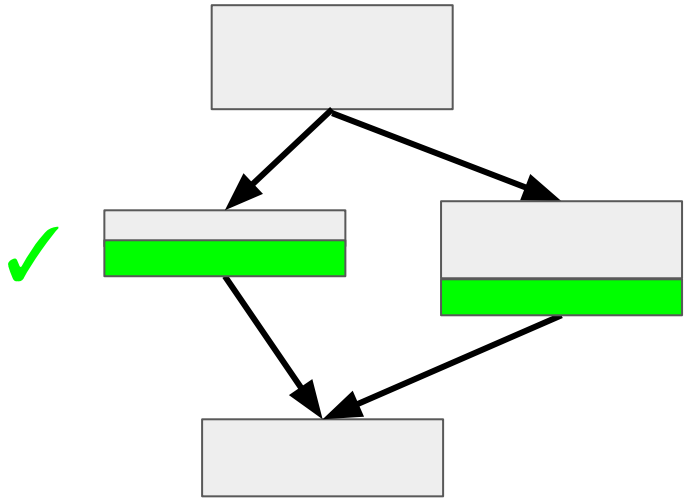
  - This block is the proper place for Φ-resolution moves.



Splitting Critical Edges

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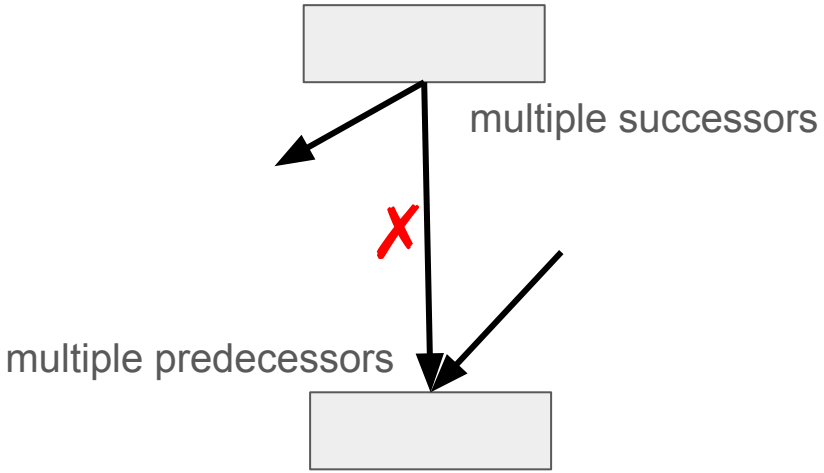
A *critical edge* is any edge that connects a block with multiple successors to a block with multiple predecessors.



Splitting Critical Edges

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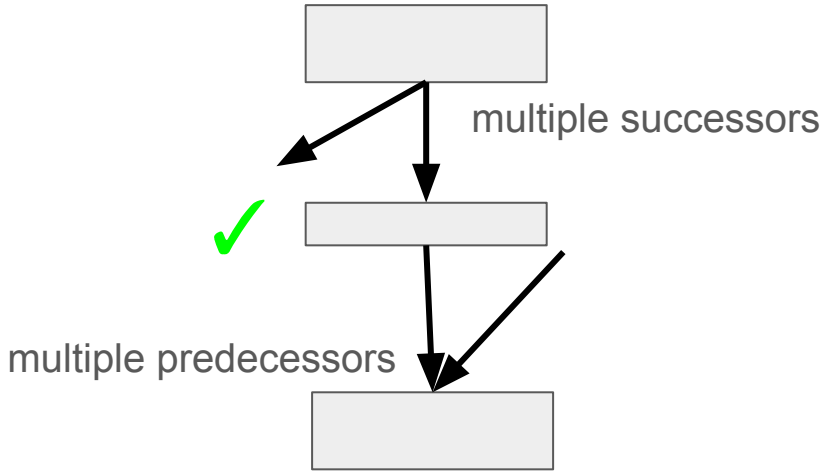
Splitting all critical edges prior to SSA deconstruction is easy.



Issue 2: Ordering Moves

- Does the order of Φ-resolution moves matter?
- For CFGs without loops, *no*.
- Let’s convince ourselves.

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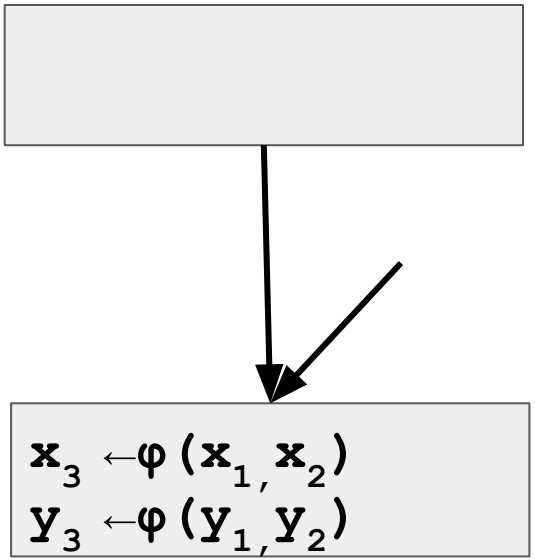

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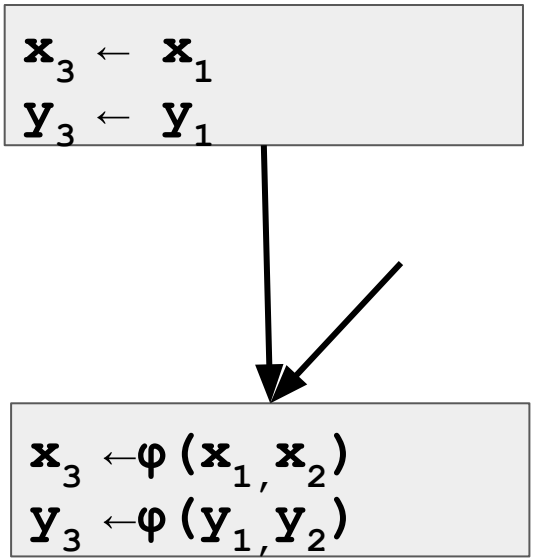

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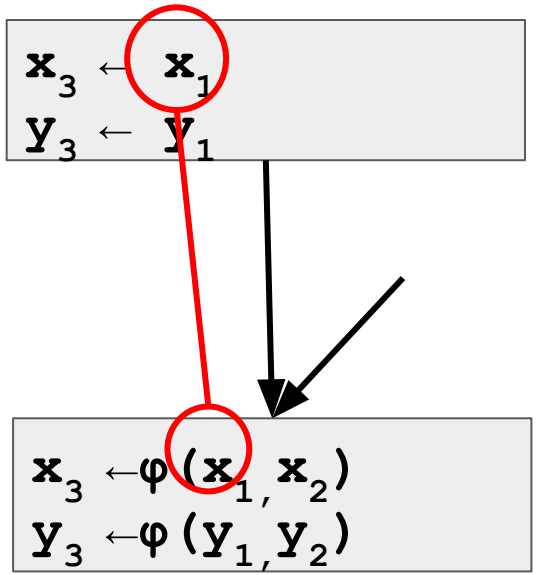

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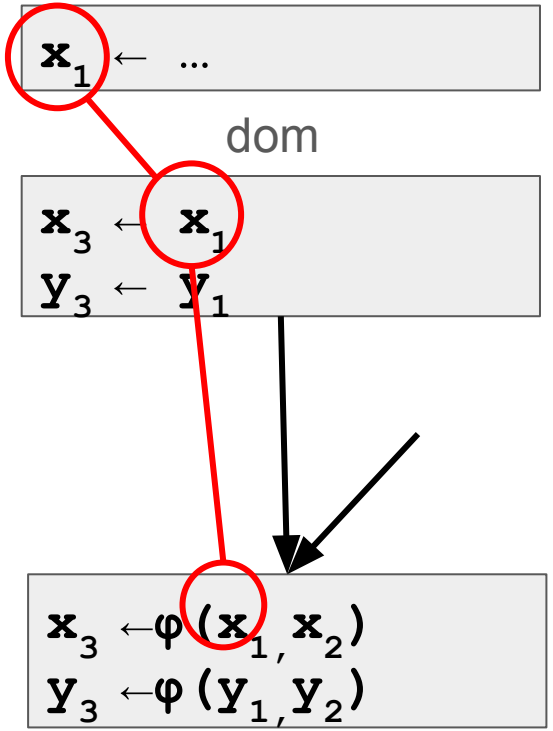

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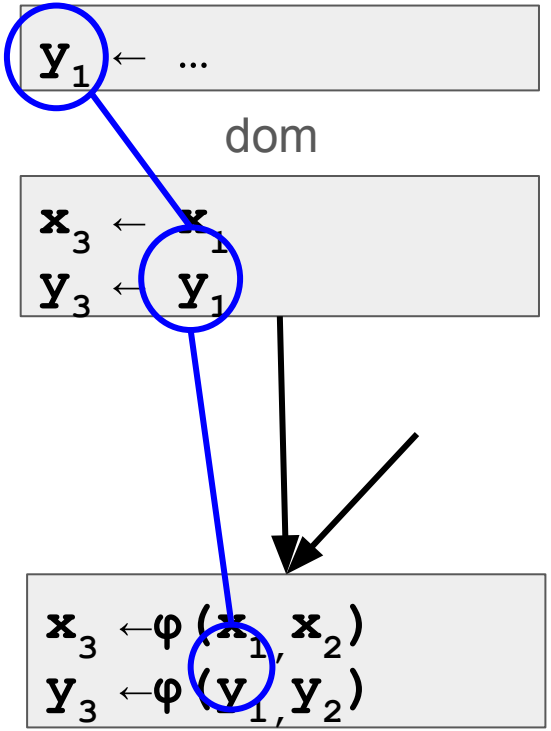

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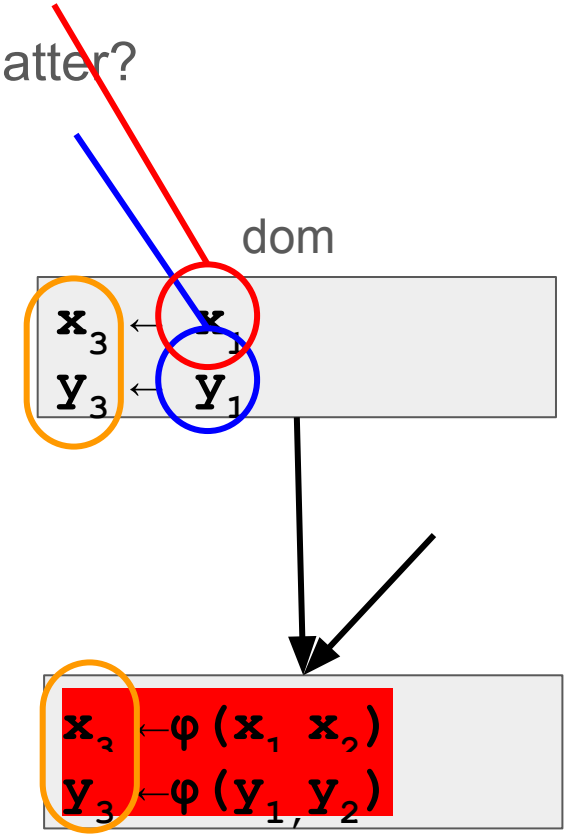

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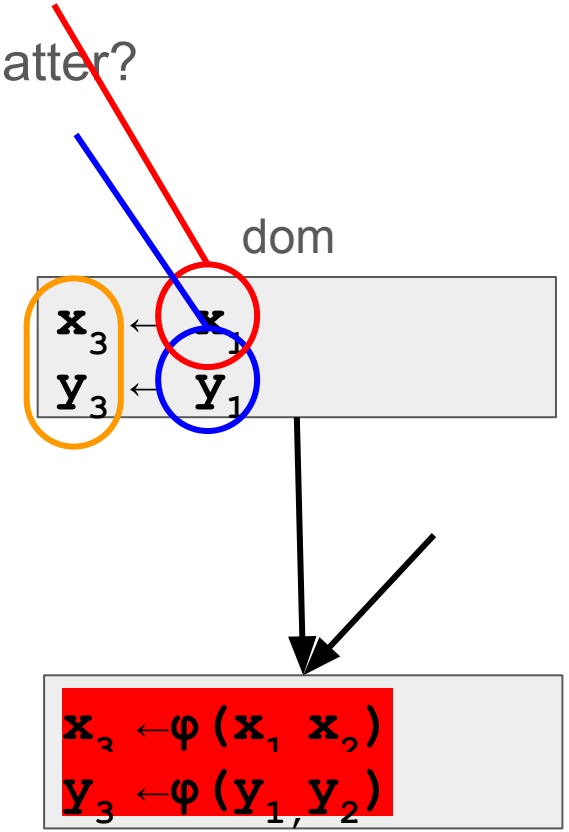

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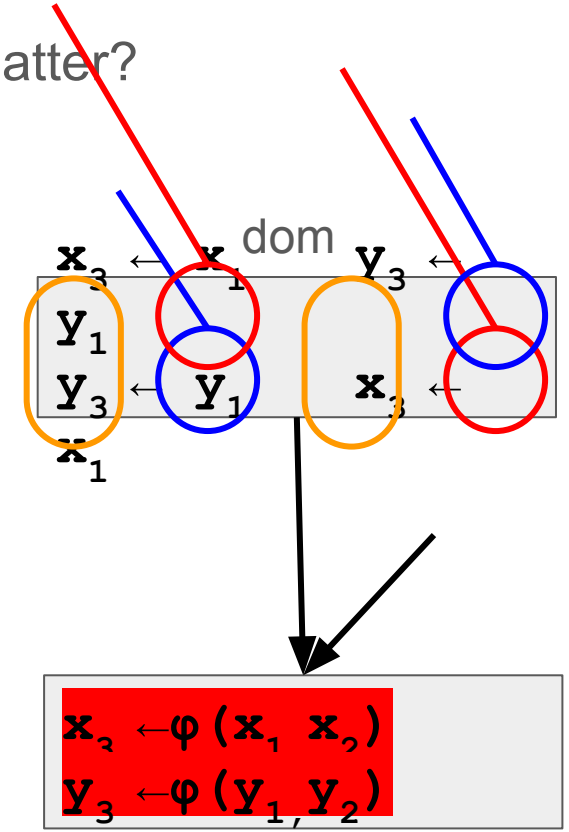

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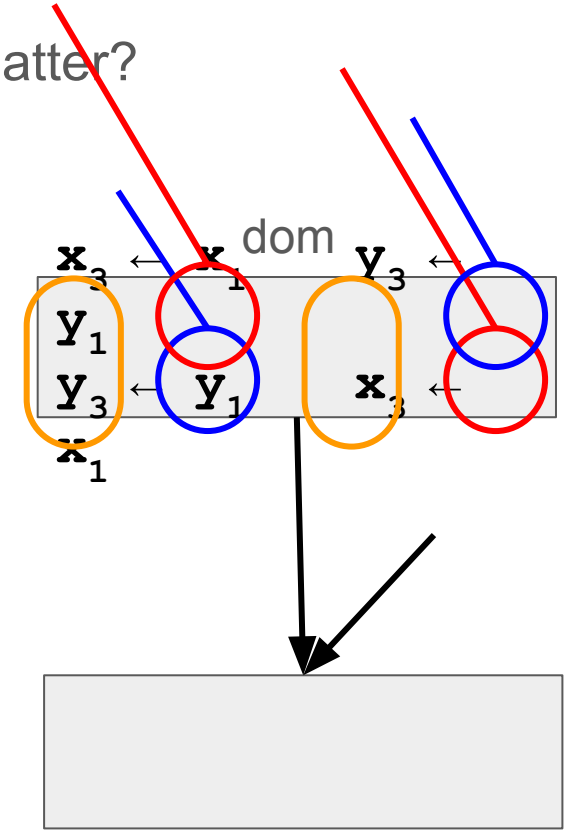
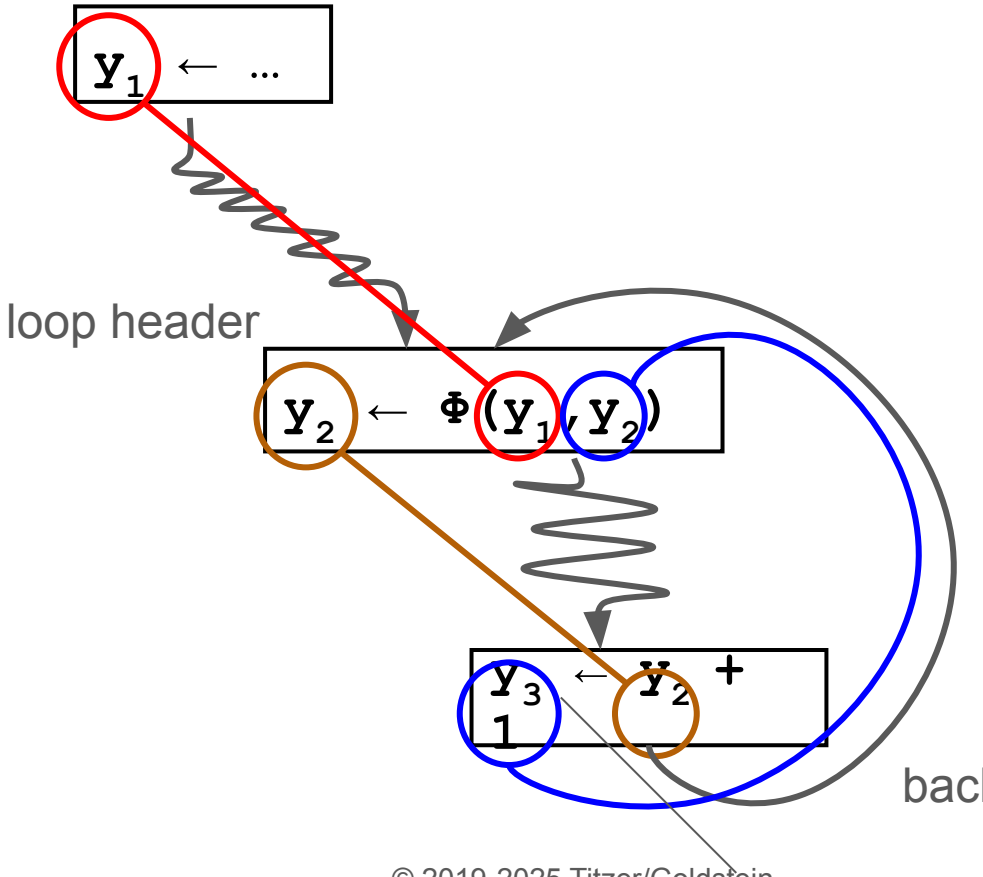


Issue 2: Ordering Moves

- Does the order of Φ-resolution moves matter?
- For CFGs without loops, *no*.
- But what about loops?

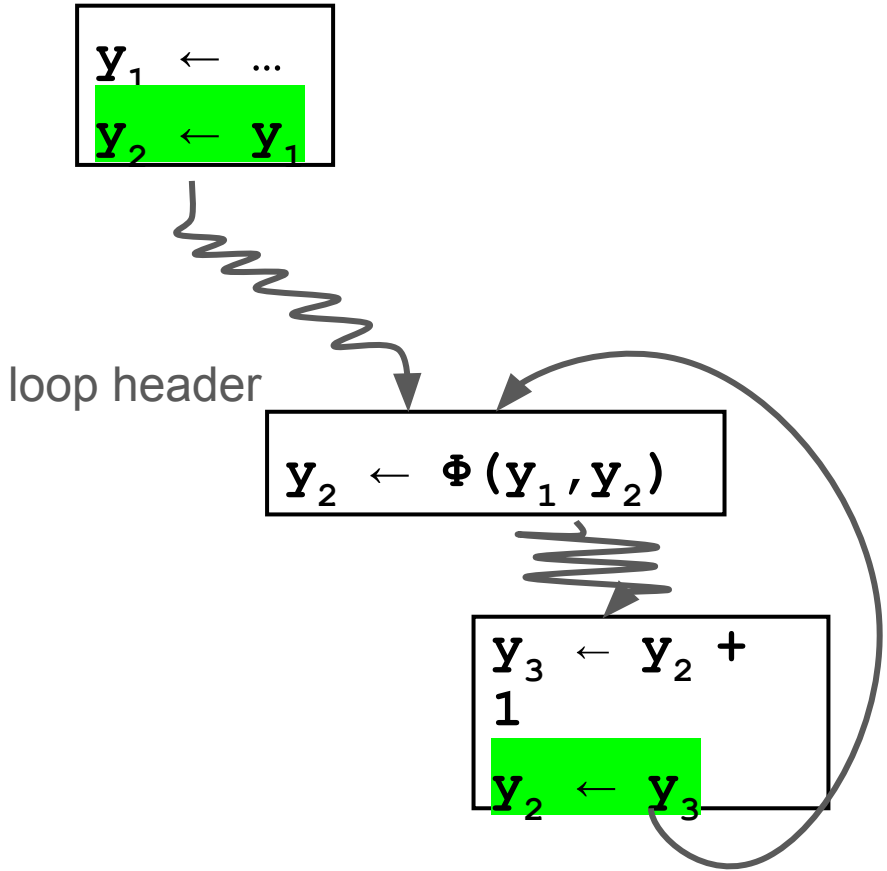
Issue 2: Ordering Moves



Φs at loop headers relate the dataflow on a loop backedge with the control flow.

A loop Φ can be defined in terms of itself.

Issue 2: Ordering Moves



Like any other join, we insert Φ-resolution moves at predecessors.

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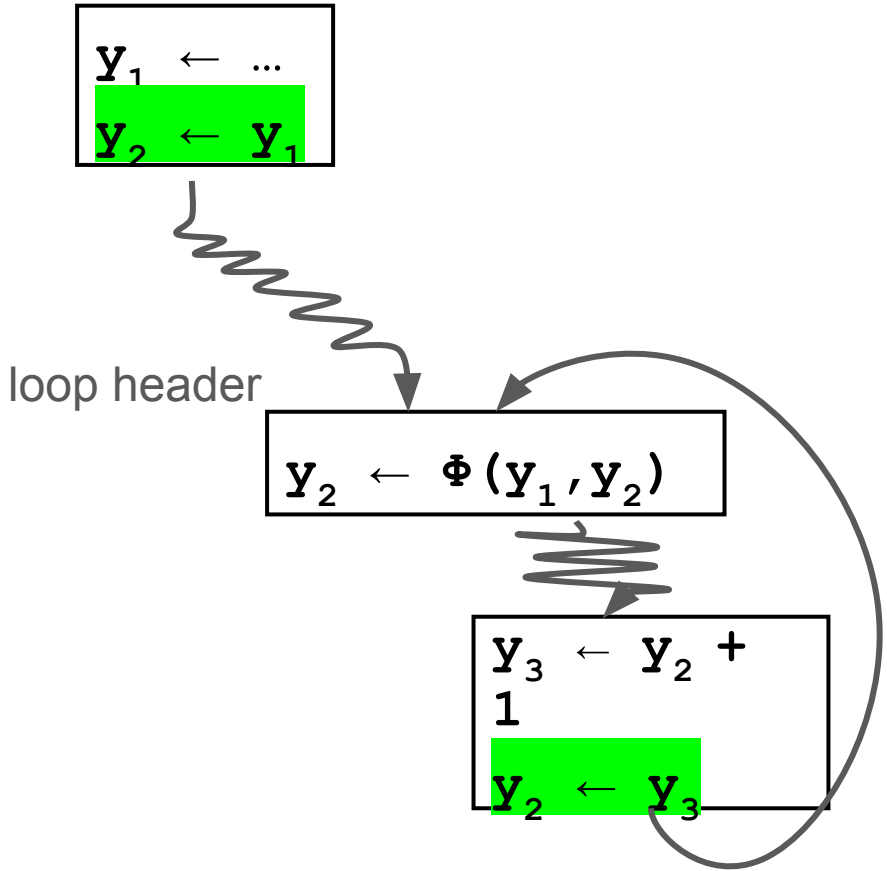

Like any other join, we insert Φ-resolution moves at predecessors.

With only one Φ, there is no problem yet.

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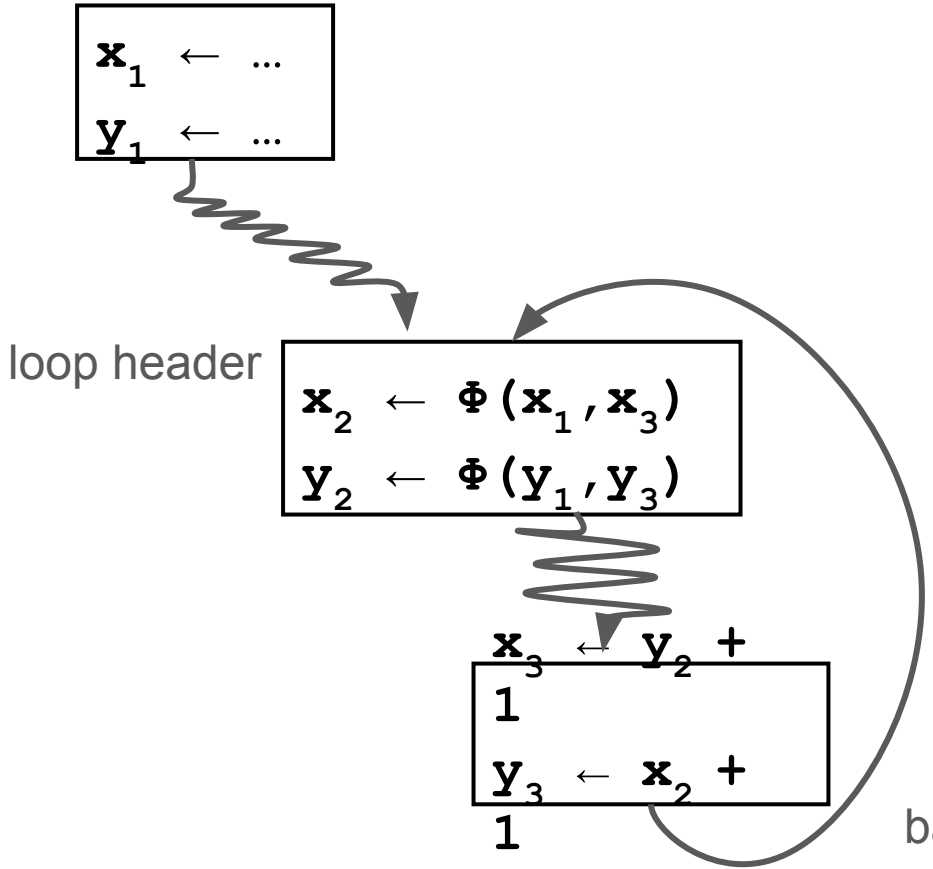
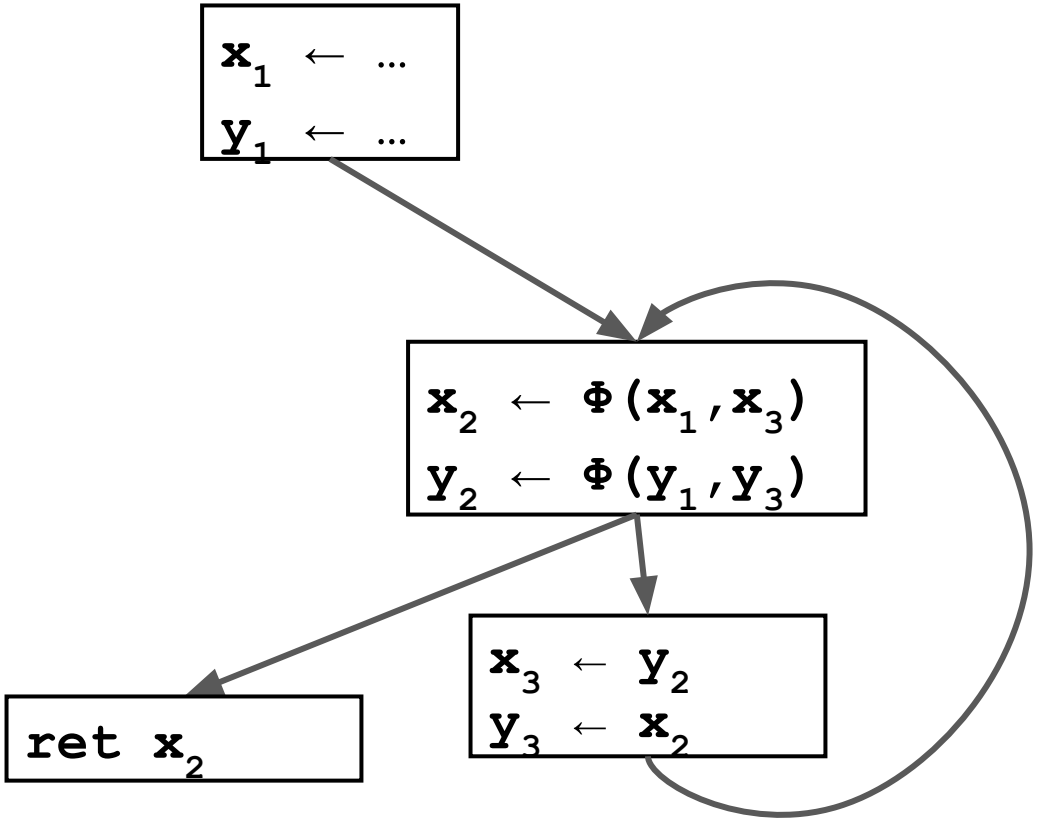


Like any join, a loop header can have multiple Φs.

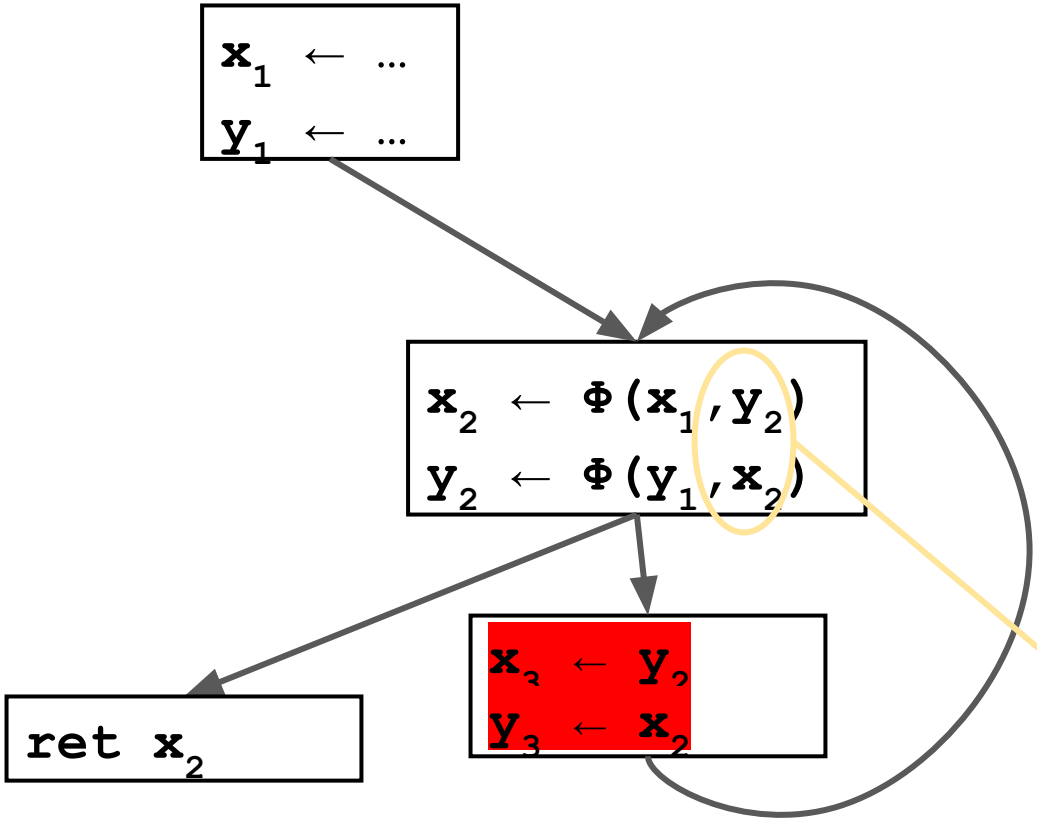
Because Φs can use inductively defined versions of themselves, they can be recursive or even ***mutually recursive***.

Issue 2: Ordering Moves



A simple example: swap of variables in a loop.

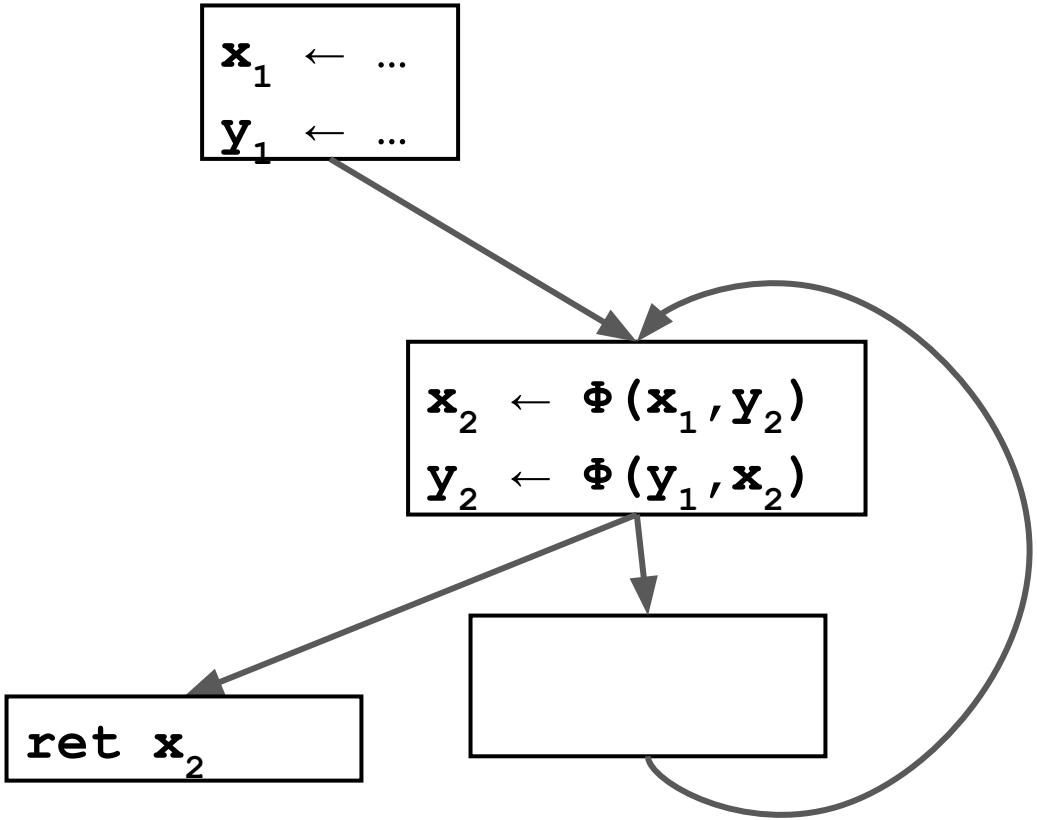
Issue 2: Ordering Moves



After optimizations such as copy propagation, the Φs can be mutually recursive.

Replace
x_3 with y_2
y_3 with x_2

Issue 2: Ordering Moves



After optimizations such as copy propagation, the Φs can be mutually recursive.

This is totally legal and cool.

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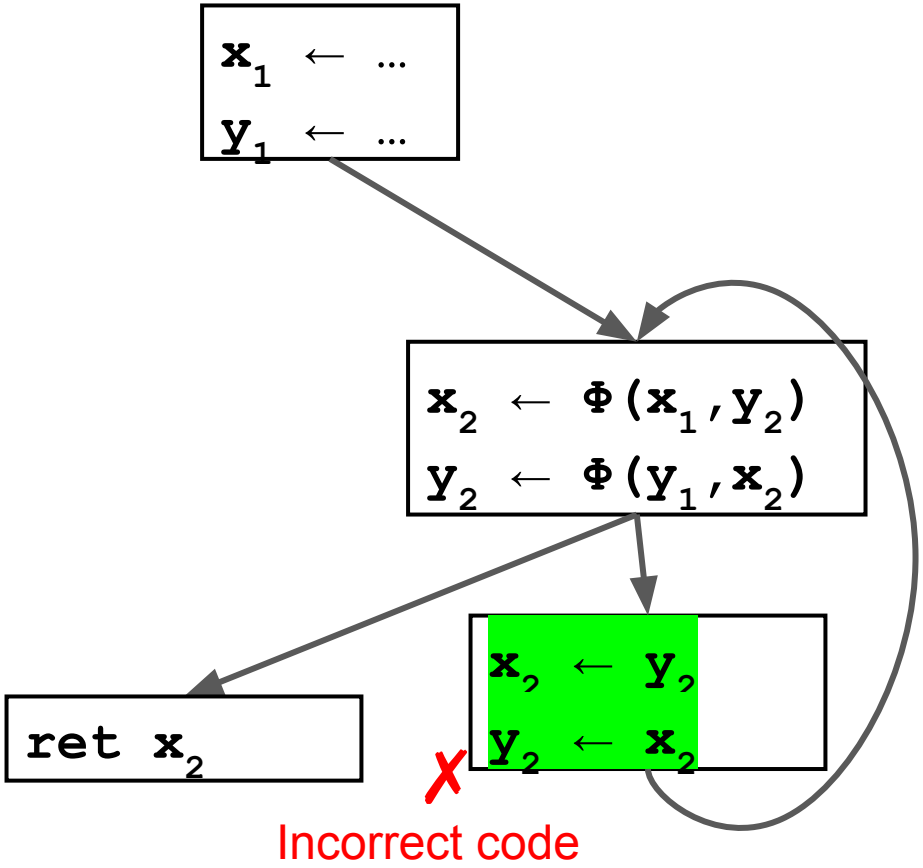

SSA deconstruction using the naïve move insertion will always generate incorrect code, regardless of the order.

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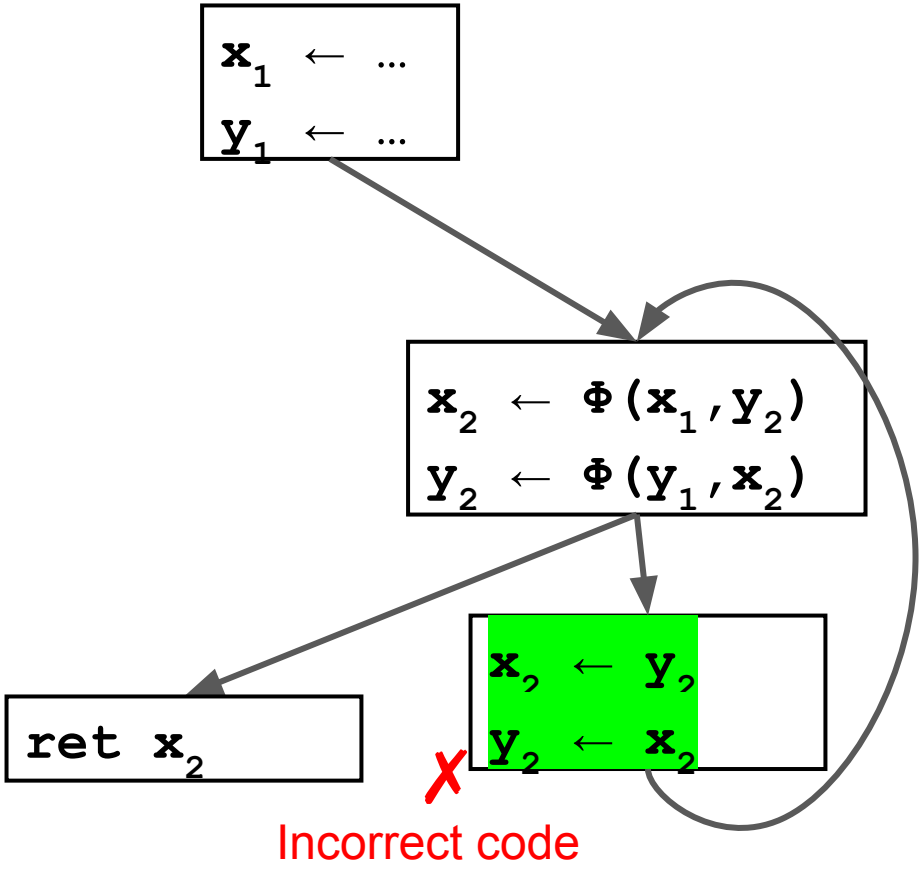

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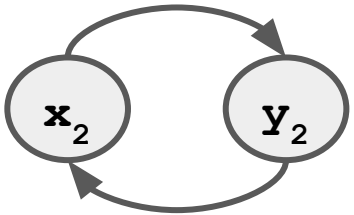

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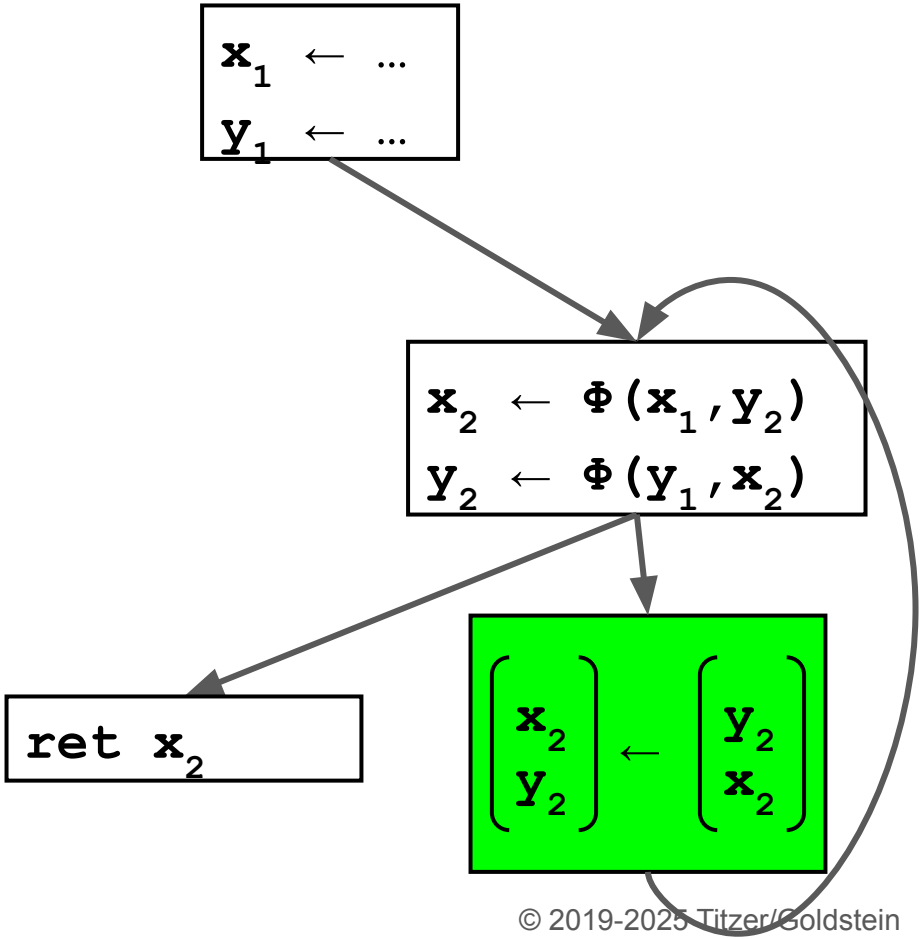
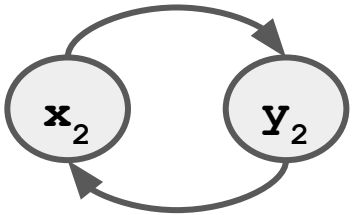


The reason is that phi resolution moves have ***parallel move*** semantics.



Implementing Parallel Moves

- Φ resolution moves must be done in parallel, without overwriting old versions.
- One simple solution: introduce new temps again.

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \leftarrow \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

generates

$$\begin{aligned} t_0 &\leftarrow y_0 \\ t_1 &\leftarrow y_1 \\ t_2 &\leftarrow y_2 \\ t_3 &\leftarrow y_3 \\ x_0 &\leftarrow t_0 \\ x_1 &\leftarrow t_1 \\ x_2 &\leftarrow t_2 \\ x_3 &\leftarrow t_3 \end{aligned}$$

Works every time.

Generates ***a lot*** of temporaries, but maybe the register allocator / copy propagation can clean them up?

Implementing Parallel Moves

- Φ resolution moves must be done in parallel, without overwriting old versions.
- Better solution: order moves more intelligently.

$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} \leftarrow \begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

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Notice that because parallel moves originate from SSA deconstruction, variables on the LHS appear only once on the LHS.

$$x_0 \neq x_1 \neq x_2 \neq x_3$$

Implementing Parallel Moves

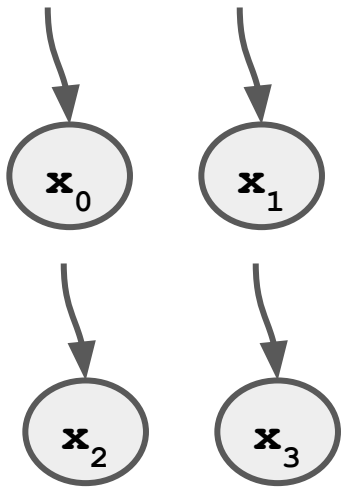
- Φ resolution moves must be done in parallel, without overwriting old versions.
- Better solution: order moves more intelligently ***using LTG***.

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \leftarrow \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

We can build a graph where each node in the parallel moves gets a node, and directed edges represent moves.

$$x_0 \neq x_1 \neq x_2 \neq x_3$$

Location Transfer Graph



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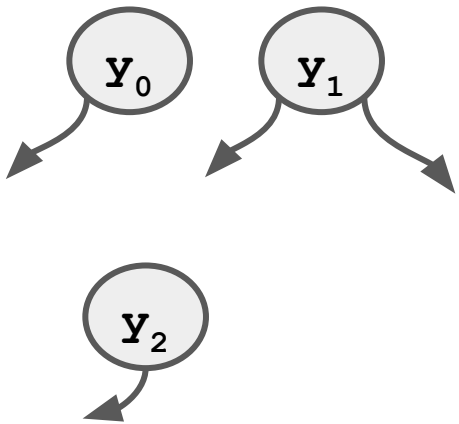
Variables may appear ***multiple times*** on the RHS, and may appear on both LHS and RHS.

$$x_0 \neq x_1 \neq x_2 \neq x_3$$

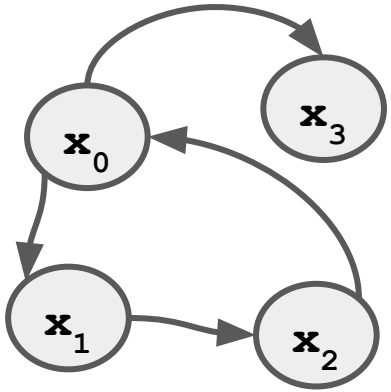
Location Transfer Graph



Location Transfer Graphs

- A location transfer graph represents a set of parallel moves.
- It can be traversed to generate a legal move ordering.
- It's constrained:
  - Every node in the graph has at most one incoming edge.
  - That implies the graph can only have simple cycles.

$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} \leftarrow \begin{pmatrix} x_2 \\ x_0 \\ x_1 \\ x_0 \end{pmatrix}$$



Location Transfer Graphs

- A location transfer graph represents a set of parallel moves.
- It can be traversed to generate a legal move ordering.
- It's constrained:
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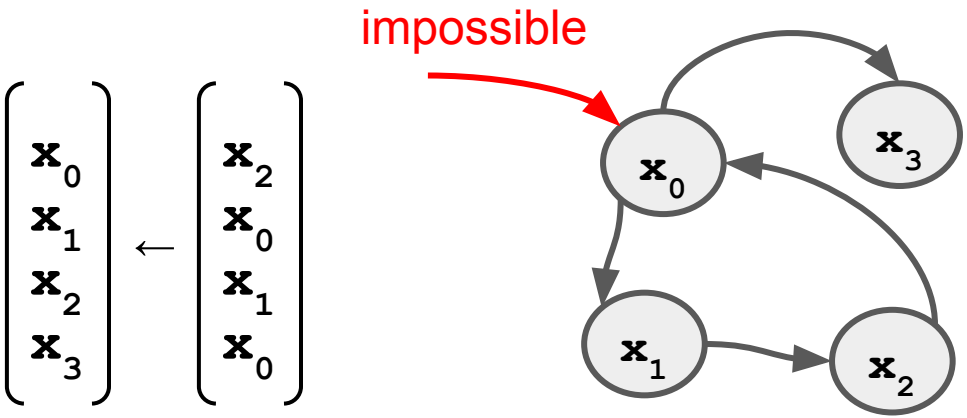

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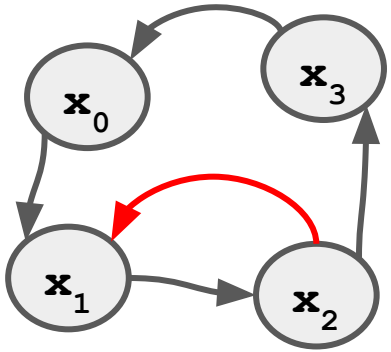

impossible to have nested cycles

Location Transfer Graphs

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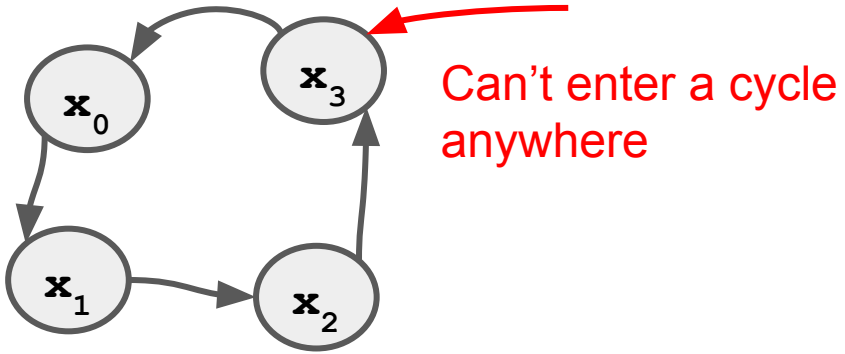

Location Transfer Graphs

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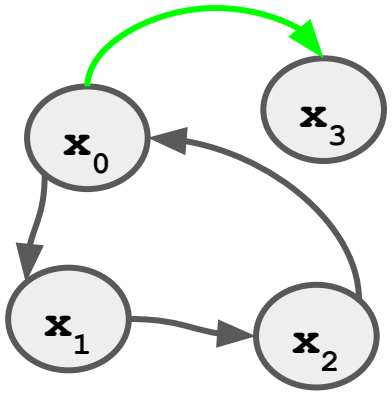

Can leave a cycle though!

Location Transfer Graphs

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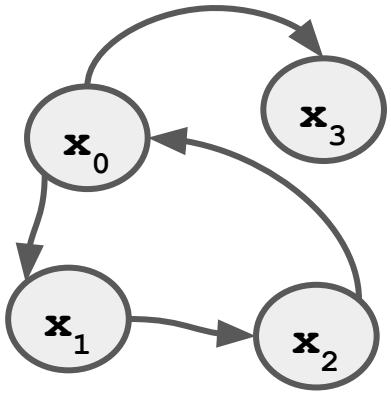


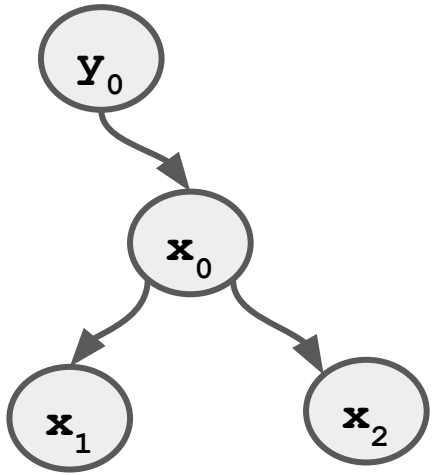
Also known as "windmill graphs" [1]

[1] See "SSA Elimination after Register Allocation" by Pereira and Palsberg, 2009.

Location Transfer Graphs

- Post-order depth-first search (DFS) on this graph yields a legal move ordering.
- Must break cycles with a temporary (or use swaps [1])

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} \leftarrow \begin{bmatrix} y_0 \\ x_0 \\ x_0 \end{bmatrix}$$



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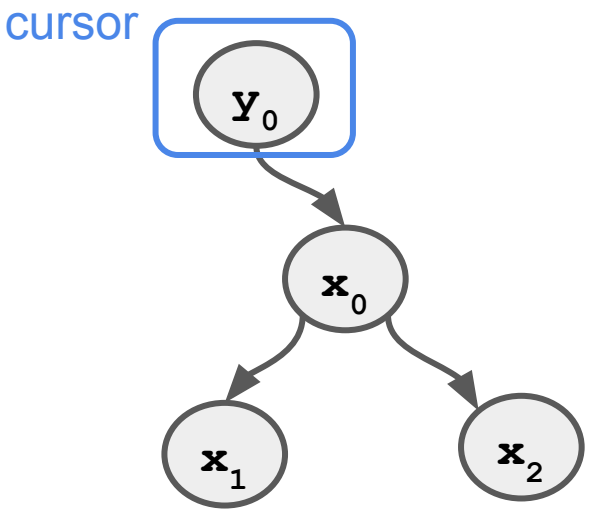

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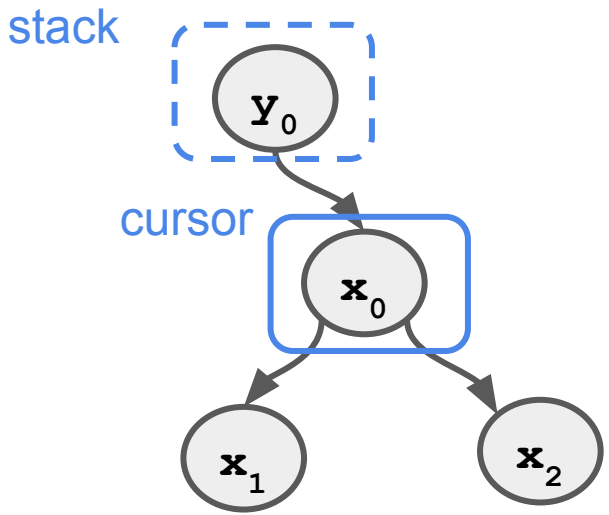

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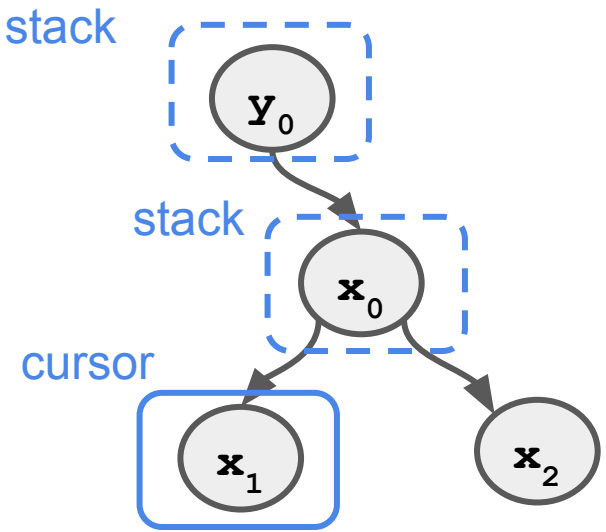

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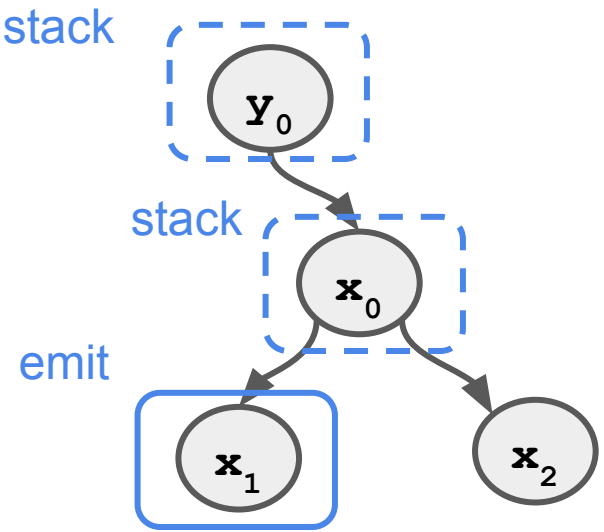

$x_1 \leftarrow x_0$

Location Transfer Graphs

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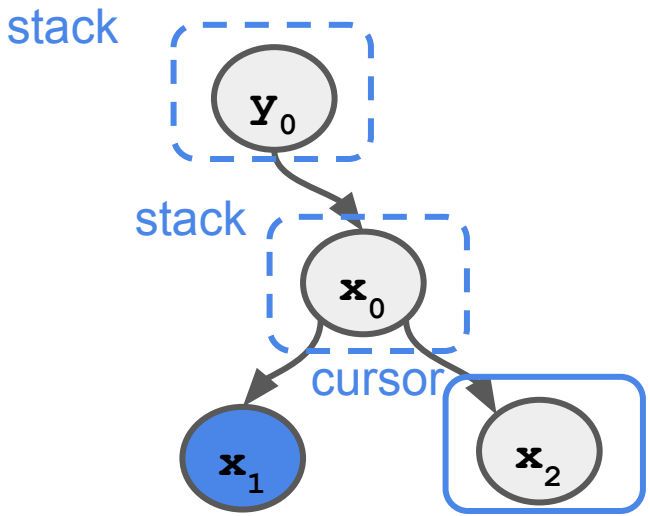
$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} \leftarrow \begin{pmatrix} y_0 \\ x_0 \\ x_0 \end{pmatrix}$$



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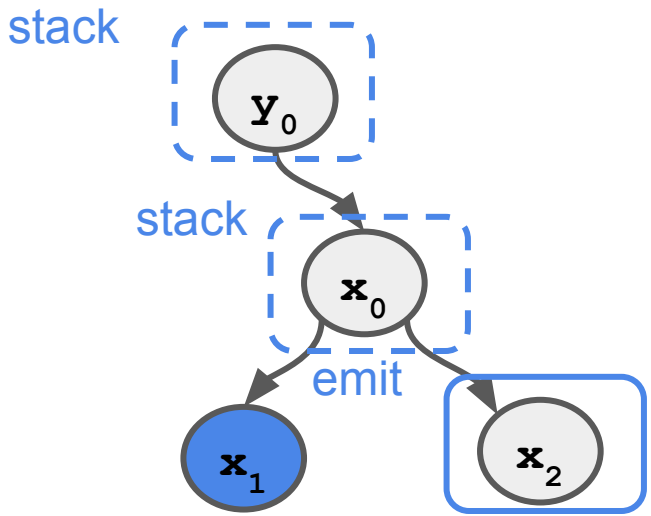

$x_1 \leftarrow x_0$

$x_2 \leftarrow x_0$

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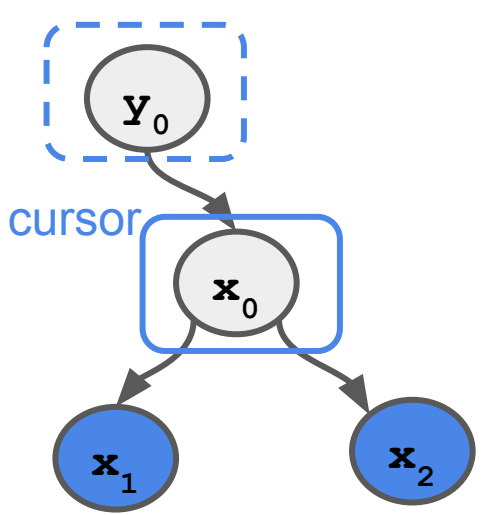

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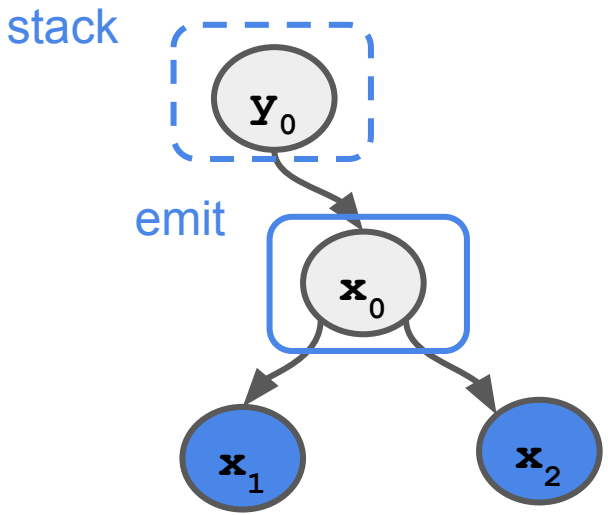

$$x_1 \leftarrow x_0$$
$$x_2 \leftarrow x_0$$
$$x_0 \leftarrow y_0$$

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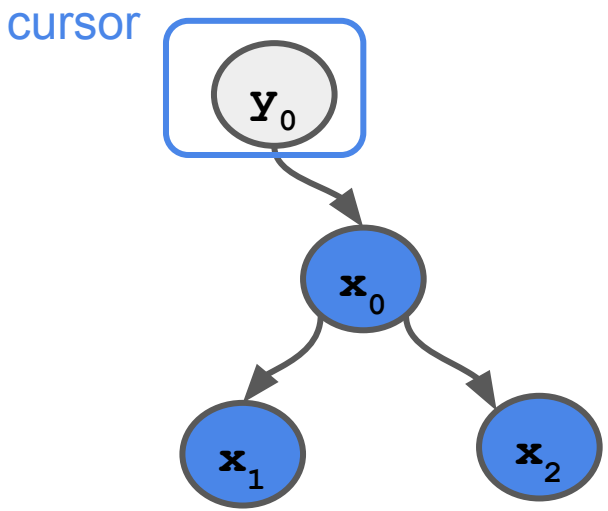

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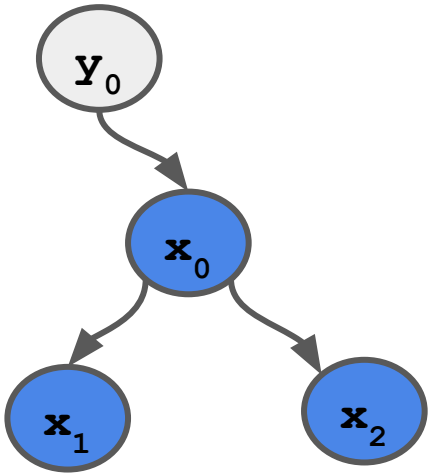

$$x_1 \leftarrow x_0$$
$$x_2 \leftarrow x_0$$
$$x_0 \leftarrow y_0$$

Location Transfer Graphs

- Post-order depth-first search (DFS) on this graph yields a legal move ordering.
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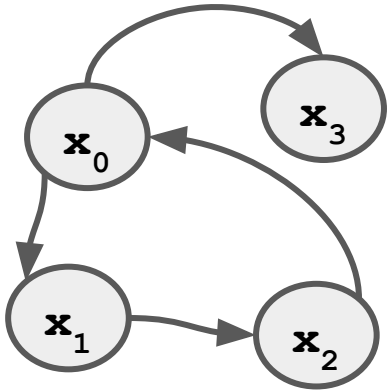

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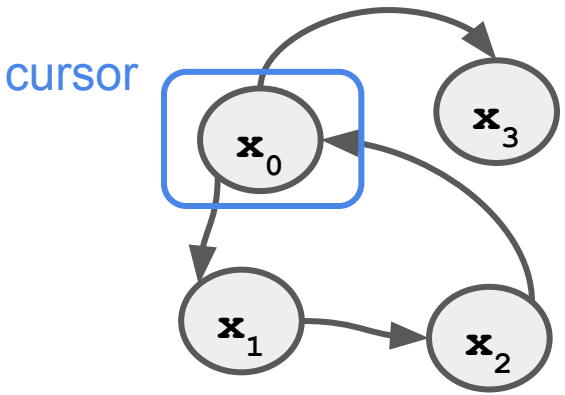

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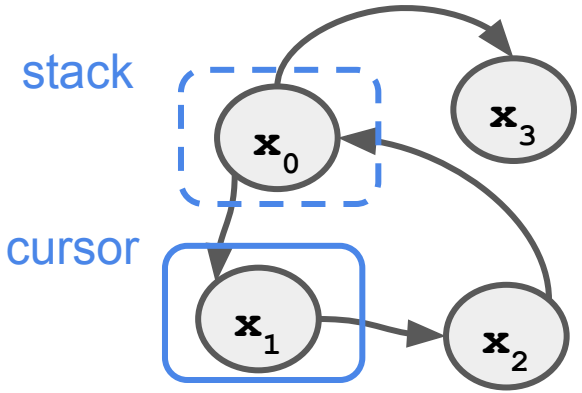
$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} \leftarrow \begin{pmatrix} x_2 \\ x_0 \\ x_1 \\ x_0 \end{pmatrix}$$



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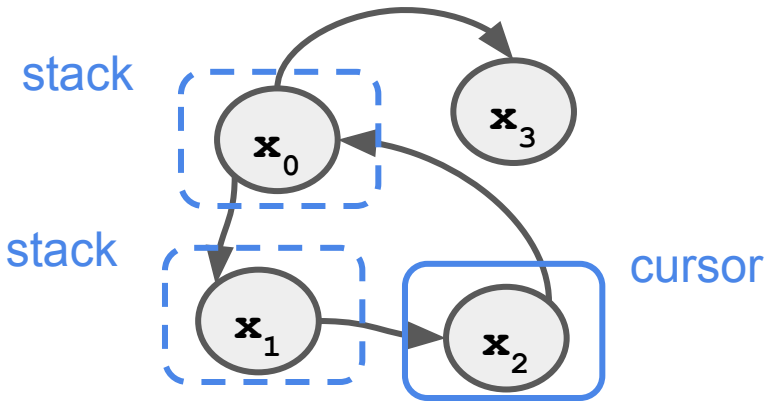
$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} \leftarrow \begin{pmatrix} x_2 \\ x_0 \\ x_1 \\ x_0 \end{pmatrix}$$

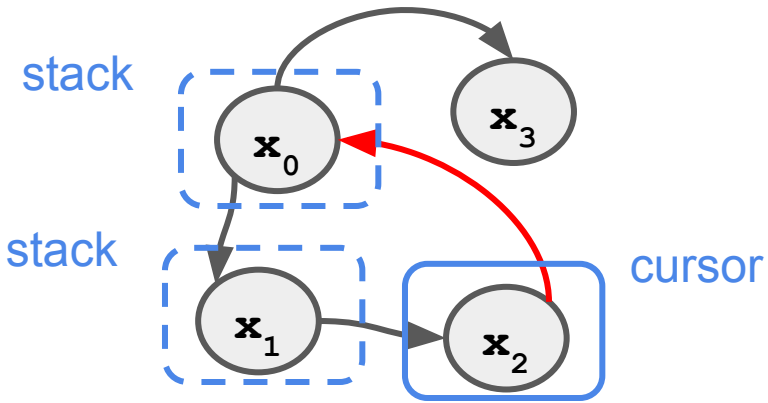


Location Transfer Graphs

- Post-order depth-first search (DFS) on this graph yields a legal move ordering.
- Must break cycles with a temporary (or use swaps [1])

Break cycle by saving in a temp

$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} \leftarrow \begin{pmatrix} x_2 \\ x_0 \\ x_1 \\ x_0 \end{pmatrix}$$

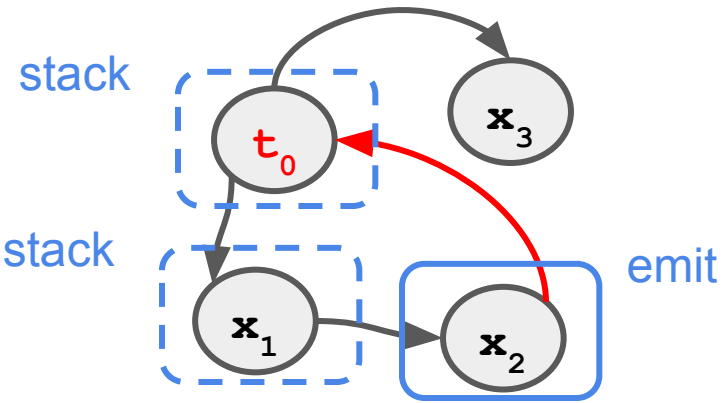


Location Transfer Graphs

- Post-order depth-first search (DFS) on this graph yields a legal move ordering.
- Must break cycles with a temporary (or use swaps [1])

Break cycle by saving in a temp, ***overwriting***, and using the temp instead.

$$\begin{bmatrix} x_2 \\ x_3 \end{bmatrix} \quad \begin{bmatrix} x_1 \\ x_0 \end{bmatrix}$$



$$t_0 \leftarrow x_0$$
$$x_0 \leftarrow x_2$$

Location Transfer Graphs

- Post-order depth-first search (DFS) on this graph yields a legal move ordering.
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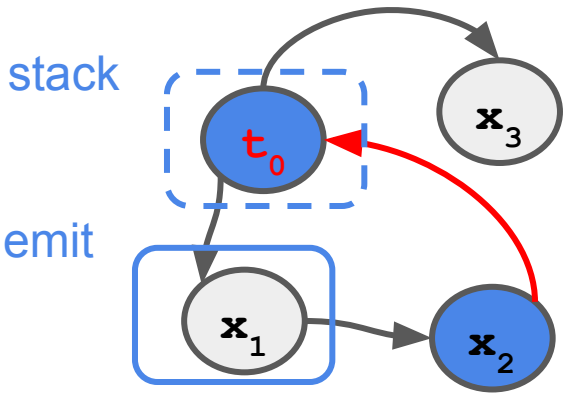
$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} \leftarrow \begin{pmatrix} x_2 \\ x_0 \\ x_1 \\ x_0 \end{pmatrix}$$

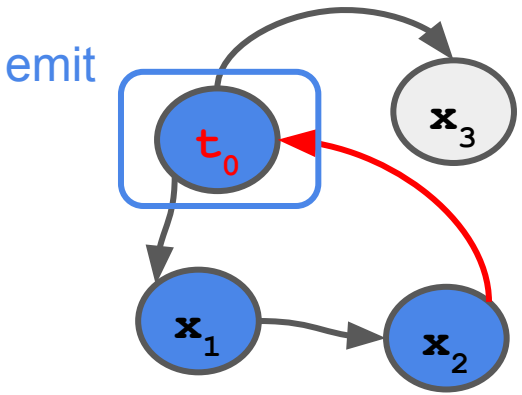


$$t_0 \leftarrow x_0$$
$$x_0 \leftarrow x_2$$
$$x_2 \leftarrow x_1$$

Location Transfer Graphs

- Post-order depth-first search (DFS) on this graph yields a legal move ordering.
- Must break cycles with a temporary (or use swaps [1])

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \leftarrow \begin{bmatrix} x_2 \\ x_0 \\ x_1 \\ x_0 \end{bmatrix}$$



$$t_0 \leftarrow x_0$$
$$x_0 \leftarrow x_2$$
$$x_2 \leftarrow x_1$$
$$x_1 \leftarrow t_0$$

Location Transfer Graphs

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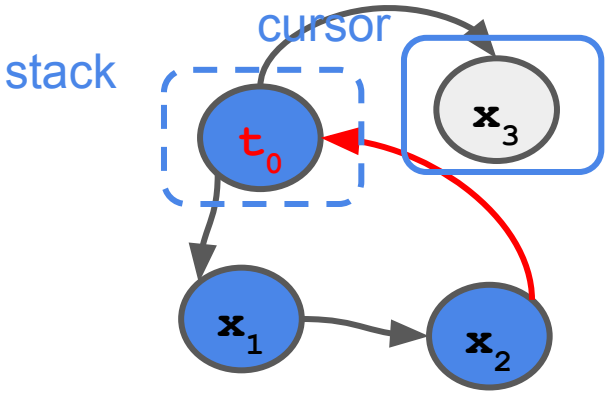

$$t_0 \leftarrow x_0$$
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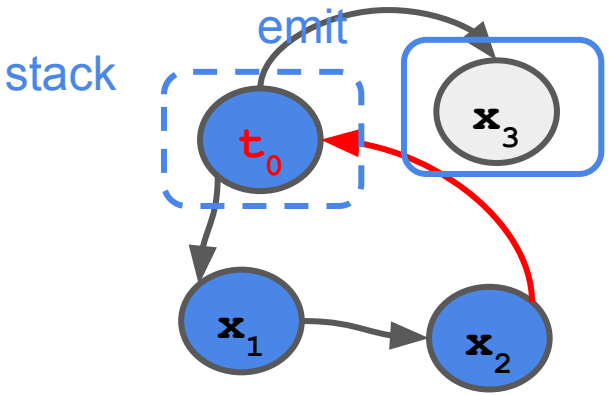

$$\begin{aligned} t_0 &\leftarrow x_0 \\ x_0 &\leftarrow x_2 \\ x_2 &\leftarrow x_1 \\ x_1 &\leftarrow t_0 \\ x_3 &\leftarrow t_0 \end{aligned}$$

Location Transfer Graphs

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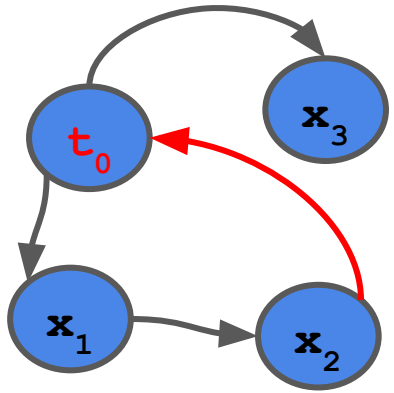

$$\begin{aligned} t_0 &\leftarrow x_0 \\ x_0 &\leftarrow x_2 \\ x_2 &\leftarrow x_1 \\ x_1 &\leftarrow t_0 \\ x_3 &\leftarrow t_0 \end{aligned}$$

SSA Elimination After Register Allocation

- Breaking cycles in LTG is easy with a temporary
- Before regalloc, temporaries are plentiful
- If deconstructing after register allocation, all regs may be used
  - How do we know? May need to save something from regalloc
- It's possible to implement LTG with swaps and/or sequences of xor's
- Another option is to use a temporary slot
- See paper for details:
  - "SSA Elimination after Register Allocation" by Pereira and Palsberg, 2009.

Syntax-directed SSA Construction

- Is it possible to skip dominators/dominance frontier?
- “Single-Pass Generation of Static Single-Assignment Form for Structured Languages” by Brandis and Mössenbock (1994).
  - https://dl.acm.org/doi/10.1145/197320.197331
- Also done in my Virgil compiler
  - https://github.com/titzer/virgil