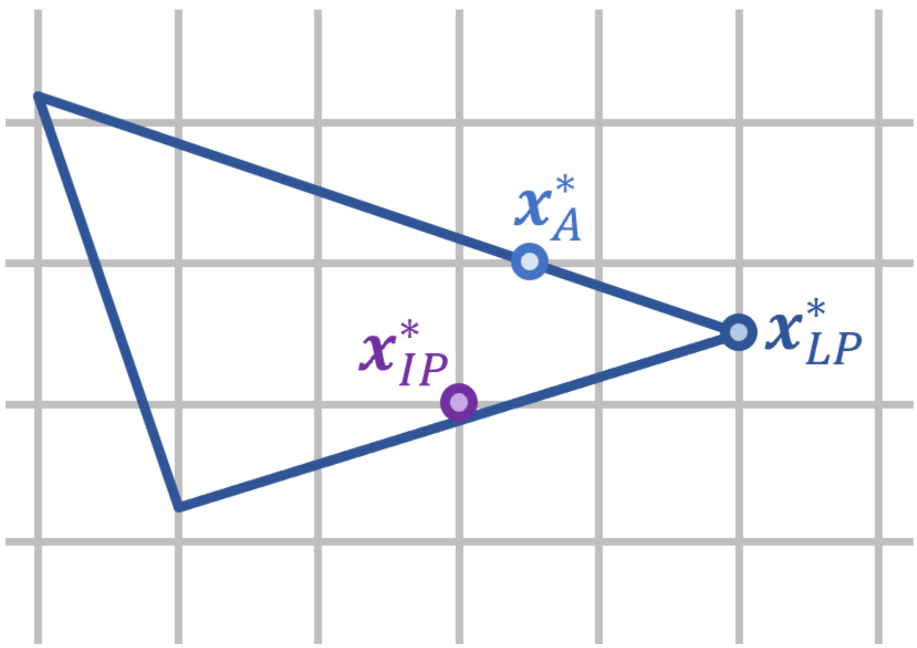
1 Integer and Linear Programming

1. What is the relationship between branch & bound, and pruning?

2. T/F. As the magnitude of of **c** increases, the distance between the contour lines of the objective $c^T x$ increases as well. Explain your answer below.

3. Consider the integer programming problem illustrated by the figure below.



The bold diagonal lines represent the boundaries of the feasible region and the gray vertical and horizontal lines represent the integer values for each axis.

- $\boldsymbol{x}_{IP}^*$ is the point that minimizes the integer program.
- $\boldsymbol{x}_{LP}^*$ is the point that minimizes the relaxed linear program. It happens to lie on a vertical gray line.
- $\boldsymbol{x}_A^*$ is a specific point that lies on both the top constraint boundary as well as a horizontal gray line.

We will use the branch and bound algorithm to find the solution, $\boldsymbol{x}_{IP}^*$, to this IP. When executing branch and bound, we will always explore the x_i less than constraint subtree before the x_i greater than constraint subtree ("less" is to the left and down if that is not already clear).

(a) At what depth of the branch and bound tree will the IP solution be found? Note that we define depth of the root node to be zero.

(b) If the minimum LP objective value at $\boldsymbol{x}_A$ is less than the minimum IP objective value at $\boldsymbol{x}^*_{IP}$, how many times must an LP solver be run to find the solution?

(c) If the minimum LP objective value at $\boldsymbol{x}_A$ is greater than the minimum IP objective value at $\boldsymbol{x}^*_{IP}$, how many times must an LP solver be run to find the solution?

2 All About Logic

1. Propositional Logic

   (a) Vocab check: are you familiar with the following terms?

      i. Symbols
      ii. Operators
      iii. Sentences
      iv. Equivalence
      v. Literals
      vi. Knowledge Base
      vii. Entailment
      viii. Query
      ix. Satisfiable
      x. Valid
      xi. Clause - Definite, Horn clauses
      xii. Model Checking
      xiii. Theorem Proving

   (b) List the 5 operators in propositional logic. Are there any other operators other than these five?

   (c) Which of the following have true/false values?

      i. Model
      ii. Sentence
      iii. Knowledge Base
      iv. Query
      v. Literal

   (d) Determine whether the sentences below are satisfiable, valid, or unsatisfiable.

      i. $(\neg(y \vee \neg y) \vee x) \wedge (x \vee (z \iff \neg z))$

      ii. $\neg(x \vee \neg(x \wedge (z \vee T))) \implies \neg(y \wedge (\neg y \vee (\top \implies \bot)))$

      iii. $((\top \iff \neg(x \vee \neg x)) \vee z) \vee z) \wedge \neg(z \wedge ((z \wedge \neg z) \implies x))$

2. First Order Logic

   (a) Vocab check: are you familiar with the following terms?

      i. Objects
      ii. Relations
      iii. Functions
      iv. Constants
      v. Predicates
      vi. Variables

vii. Connectives
viii. Equality
ix. Quantifiers
x. Atomic Sentence
xi. Unification

(b) Consider the following two expressions in first order logic: Fatherof(a, b), Fatherof(a). Which one is a relation and which one is a function?

(c) Which of the following FOL sentences correctly expressed the idea of corresponding English sentences?

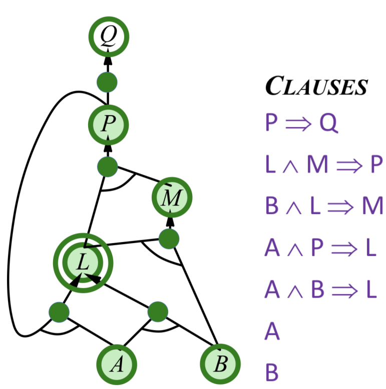
i. There was a student at CMU that never did 381 homework but passed the class.
$\exists x, \text{IsStudent}(x, \text{CMU}) \wedge \neg\text{DoesHW}(x, 381) \implies \text{Pass}(x, 381)$

ii. If a student likes Pat, he'll pass the class.
$\forall x, \text{Student}(x) \wedge \text{Likes}(x, \text{Pat}) \implies \text{Pass}(x, 381)$

3. Forward Chaining

(a) What are the requirements for Knowledge Base in Forward Chaining?

(b) A toy example from class for Forward Chaining:



(c) Is Forward chaining a sound algorithm? Is it a complete algorithm?

(d) What does FOL-FC return?

4. Planning

   (a) Vocab check: are you familiar with the following terms?

      i. Predicates
      ii. Operators
      iii. Linear Planning
      iv. Non-linear Planning
      v. Inconsistency
      vi. Interference
      vii. Competing Needs
      viii. Complete
      height A
      ix. Sound
      height A
      x. Optimal
      height A

   (b) What are in the knowledge base of logic agents and classical planning problem, respectively?

   (c) What are the 3 components when defining an operator?

   (d) Is linear planning complete? Optimal? What about non-linear planning?

5. **Facebook or Chipotle?**

Ethan gets easily bored in class, and often starts to get distracted. Two things usually pop up in his mind: whether or not he should go to Chipotle for lunch after class, and if he should check Facebook. Once he starts thinking about these options, he'll definitely choose to do at least one, because they're so tempting.

Ethan knows that going on Facebook is a mistake. He's so addicted that if he logs on, he'll lose track of time scrolling through his NewsFeed and probably fail the class. If he goes to Chipotle after class, he'll feel happy and satisfied. And, if he manages not to feel bored at all, he'll pass the class no matter what.

Let B be the symbol representing if Ethan gets bored, C representing going to Chipotle, F representing going on Facebook, and P representing passing the class. Below is the truth table for the four symbols.

B	C	F	P
$\bot$	$\top$	$\bot$	(1)
$\bot$	$\top$	$\top$	(2)
(3)	$\bot$	$\top$	$\top$
$\top$	$\top$	(4)	$\bot$
$\top$	(5)	$\bot$	(6)
$\top$	$\bot$	$\top$	(7)

(a) Fill in the holes in the chart based on the given information.

(b) Sean is another student in class. In his knowledge base, he admits that if he doesnt get bored, he will pass the class no matter what he does later on. However, if he gets bored, he will go directly to Chipotle and believes he can always pass the class after getting food. Represent Sean's knowledge base in propositional logic. (Select all that can represent his Knowledge Base)

i. $\neg B \Rightarrow P; B \wedge C \Rightarrow P$

ii. $\neg B \Rightarrow P; \neg P \Rightarrow \neg C \vee F$

iii. $B \vee P; (\neg B \wedge \neg C \wedge F) \vee P$

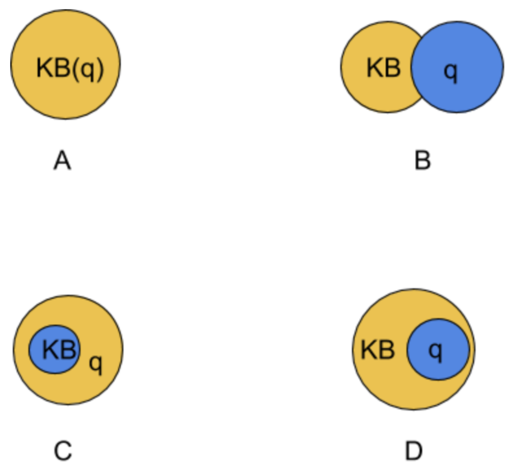
iv. $B \vee P; (\neg B \vee \neg C) \vee P$

(c) For each row in the chart in (a), Fill in the corresponding value in the chart below for the "KB" column, where "KB" represents the propositional value where both sentences in Sean's KB are assigned to True.

B	C	F	P	KB	Query
$\bot$	$\top$	$\bot$		(1)	(7)
$\bot$	$\top$	$\top$		(2)	(8)
	$\bot$	$\top$	$\top$	(3)	(9)
$\top$	$\top$		$\bot$	(4)	(10)
$\top$		$\bot$		(5)	(11)
$\top$	$\bot$	$\top$		(6)	(12)

(d) A query q for this problem is the sentence $B \implies \neg P$. Fill in the truth assignment for the query in the chart above according to value assignments on each row for the "Query" column in chart (a).

(e) Which of the following graph would correctly represent relationship between truth values of the knowledge base KB and query q?



(f) In general, if statement A implying statement B is always True for all truth assignments, what can we infer about the entailment relationship between A and B? (multiple choice)

i. B entails A
ii. A entails B
iii. Neither of A or B entails the other
iv. Need more information of values assigned to A and B

3 MDPs and RL

1. In a certain country there are N cities, all connected by roads in a circular fashion. A wandering poet is travelling around the country and staging shows in its different cities. He can choose to move from a city to each of the neighboring ones or he can stay in his current city i and perform, getting a reward r_i. If he chooses to travel, he will have a success probability of p_i. There is a $1 - p_i$ chance he will encounter a dragon along the way, which means he will have to turn back and wait the next day. If he is successful in travelling, he gains a reward of 0 for the day. And if he is unsuccessful at travelling, he can still perform a little bit when he gets back, giving him a reward of $r_i/2$.

   a) Let $r_i = 1$ and $p_i = 0.5$ for all i and let $\gamma = 0.5$. For $1 \leq i \leq N$, answer the following questions with real numbers:
   Hint: Recall that $\forall u \in (1, 0),\ \sum_{j=0}^{\infty} u^j = \frac{1}{1-u}$

      i) What is the value $V^{\text{stay}}(i)$ under the policy the wandering poet always chooses to stay?

      ii) What is the value $V^{\text{next}}(i)$ under the policy the wandering poet always chooses to go to the next city?

   b) Let N be even, and let $p_i = 1$ for all i, and for all i, let the reward for cities be given as:

   $$r_i = \begin{cases} a & i \text{ is even} \\ b & i \text{ is odd} \end{cases}$$

   where a and b are constants and $a > b > 0$.

      i) Suppose we start at an even-numbered city. What is the range of values of the discount factor γ for such that the optimal policy is to stay at the current city forever? Your answer may depend on a and b.

      ii) Suppose we start at an odd-numbered city. What is the range of values of the discount factor γ such that the optimal policy is to stay at the current city forever? Your answer may depend on a and b.

iii) Suppose we start at an odd-numbered city and γ does not lie in the range you computed. Describe the optimal policy.

c) Let N be even, $r_i \geq 0$, and the optimal value of being in the city 1 be positive, i.e., $V^*(1) > 0$. Define $V_k(i)$ to be the value of city i after the k^{th} time-step. Letting $V_0(i) = 0$ for all i, what is the largest k for which $V_k(1)$ could still be 0? Be wary of off-by-one errors.

d) Let $N = 3$, and $[r_1, r_2, r_3] = [0, 2, 3]$ and $p_1 = p_2 = p_3 = 0.5$ and $\gamma = 0.5$. Compute:

i) $V^*(3)$

ii) $V^*(1)$

iii) $Q^*(1, \text{stay})$

2. Vocab check: are you familiar with the following terms?

(a) What are the Bellman Equations, and when are they used?

(b) What does ϵ greedy mean, and in what context is it used?

(c) α, γ, ϵ, living rewards, ... how do these affect an agent's policy search?

(d) Exploration, exploitation, and the difference between them? Why are they both useful?

(e) What is off-policy learning? Why, and how, is this used?