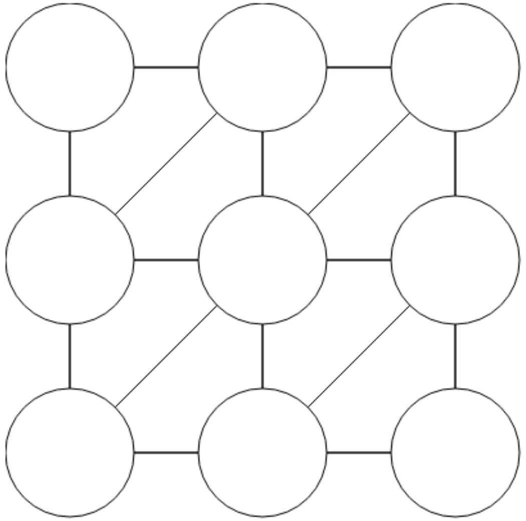
CSP Warm-up

Assign Red, Green, or Blue
Neighbors must be different



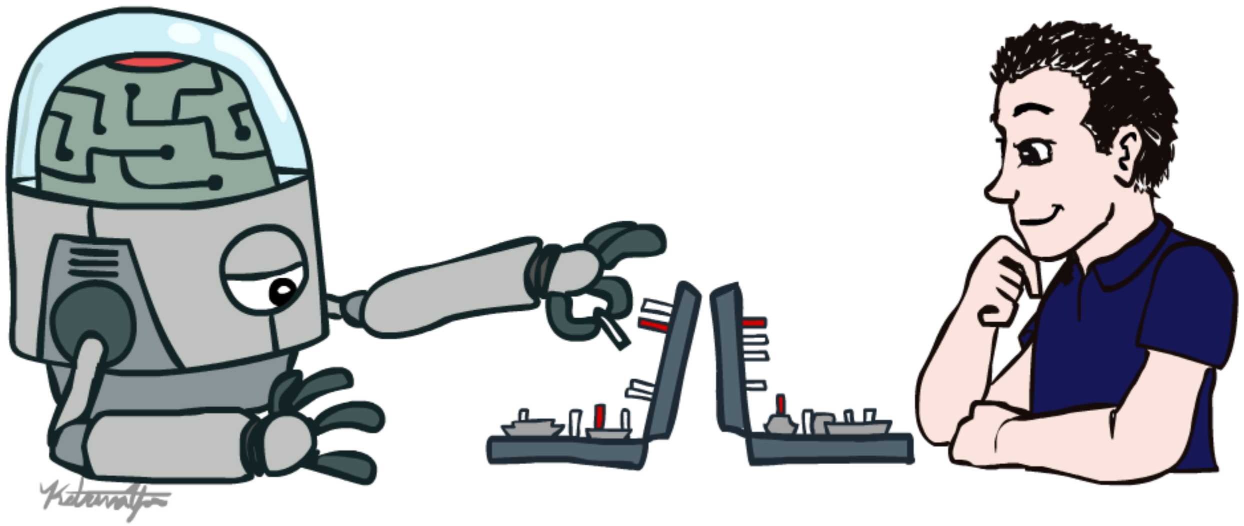
Sudoku

1			
	2	1	
		3	
			4

1) What is your brain doing to solve these?
2) How would you solve these with search (BFS, DFS, etc.)?

AI: Representation and Problem Solving

Constraint Satisfaction Problems (CSPs)



Instructors: Pat Virtue & Stephanie Rosenthal

Slide credits: Pat Virtue, http://ai.berkeley.edu

Announcements

- HW3 due Wednesday!
- P1 due Thursday, you can work in pairs!

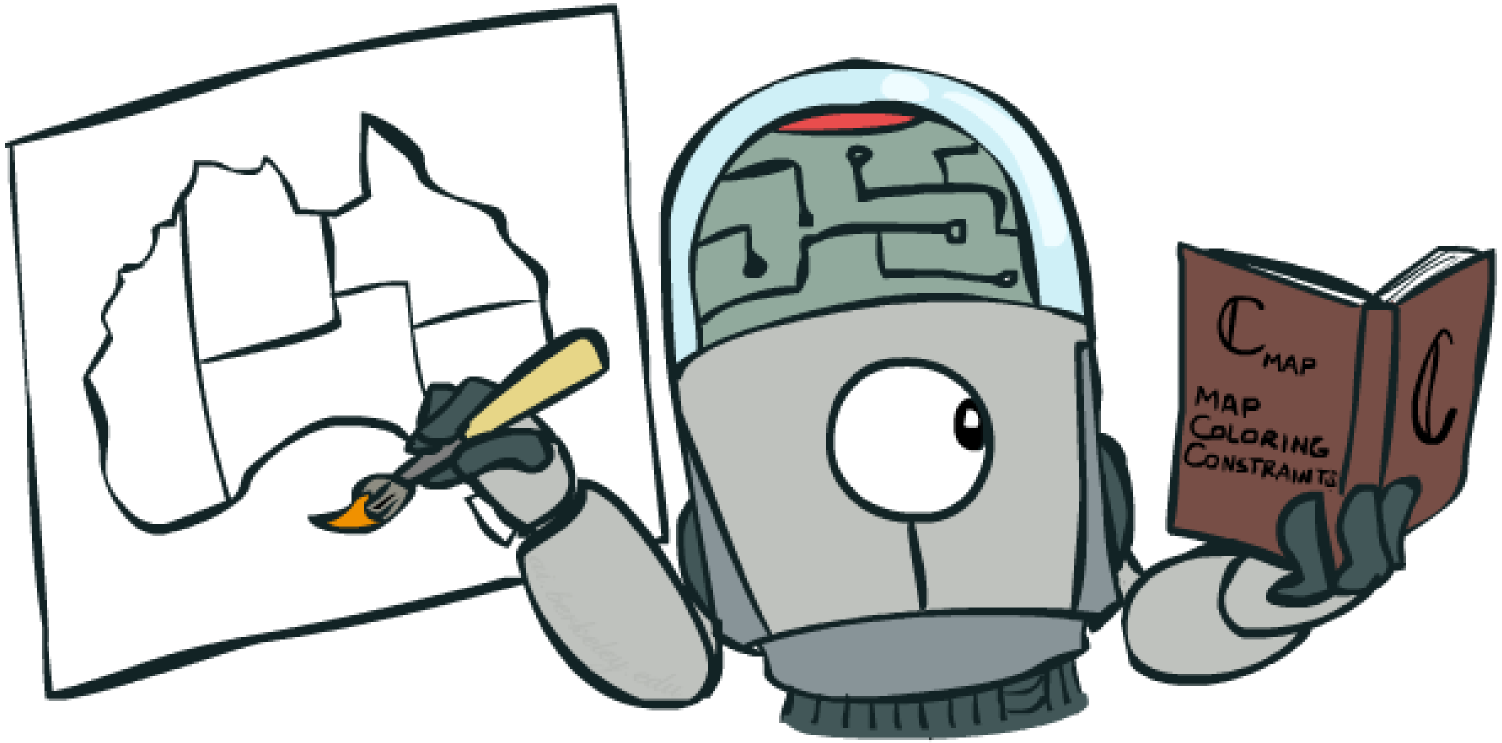
- Watch your time management!

What is Search For?

- Planning: sequences of actions
  - The path to the goal is the important thing
  - Paths have various costs, depths
  - Heuristics give problem-specific guidance

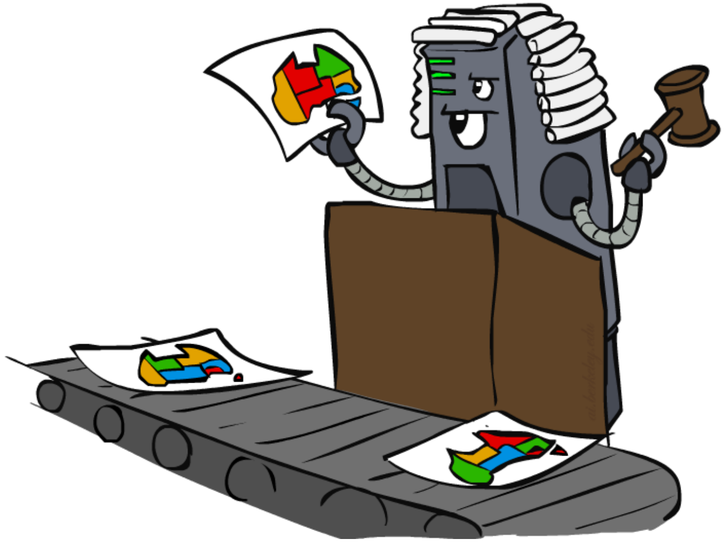
- Identification: assignments to variables
  - The goal itself is important, not the path
  - All paths at the same depth (for some formulations)
  - CSPs are specialized for identification problems

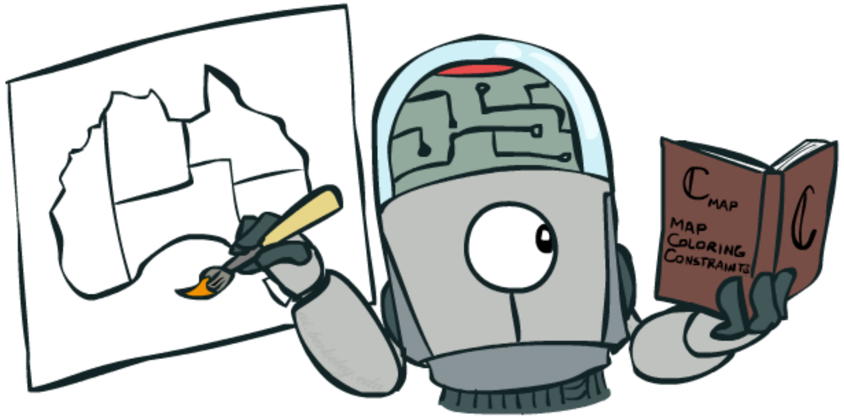
Constraint Satisfaction Problems



Constraint Satisfaction Problems

- Standard search problems:
  - State is a “black box”: arbitrary data structure
  - Goal test can be any function over states
  - Successor function can also be anything

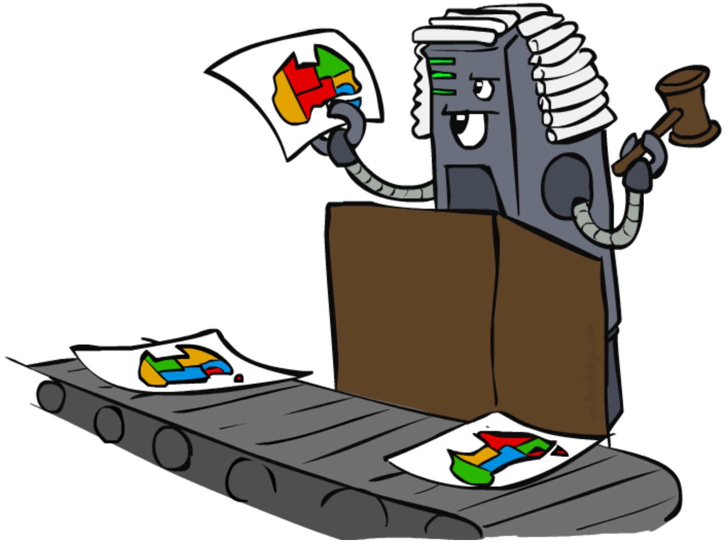


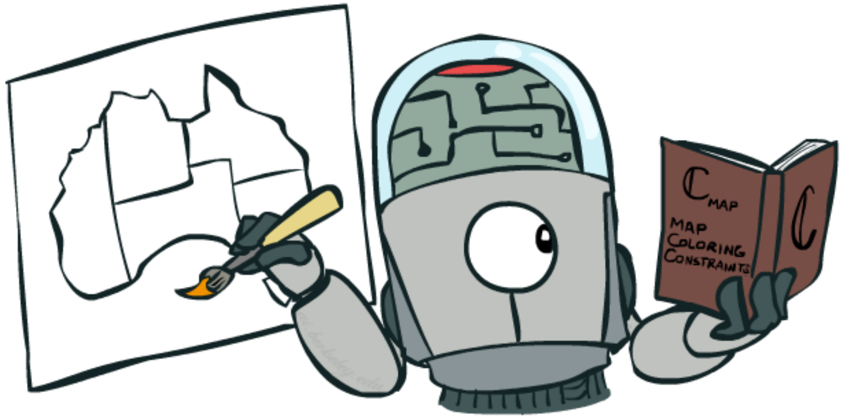


Constraint Satisfaction Problems

- Standard search problems:
  - State is a "black box": arbitrary data structure
  - Goal test can be any function over states
  - Successor function can also be anything

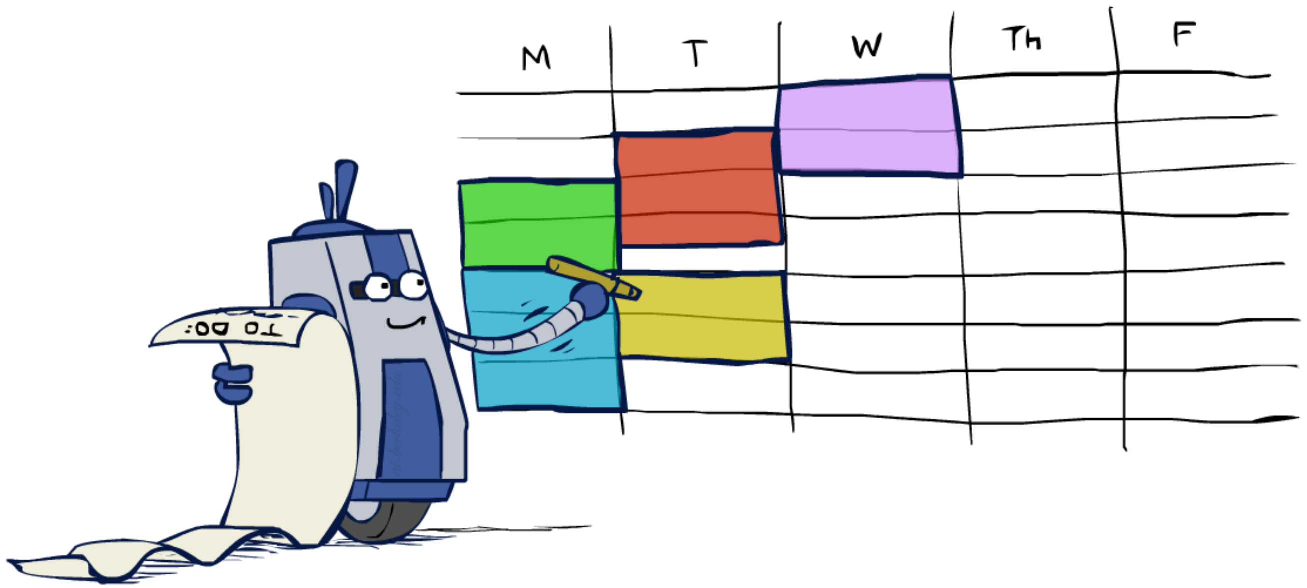
- Constraint satisfaction problems (CSPs):
  - A special subset of search problems
  - State is defined by variables X_i with values from a domain D (sometimes D depends on i)
  - Goal test is a set of constraints specifying allowable combinations of values for subsets of variables





Real-World CSPs

- Assignment problems: e.g., who teaches what class
- Timetabling problems: e.g., which class is offered when and where?
- Hardware configuration
- Transportation scheduling
- Factory scheduling
- Circuit layout
- Fault diagnosis
- … lots more!

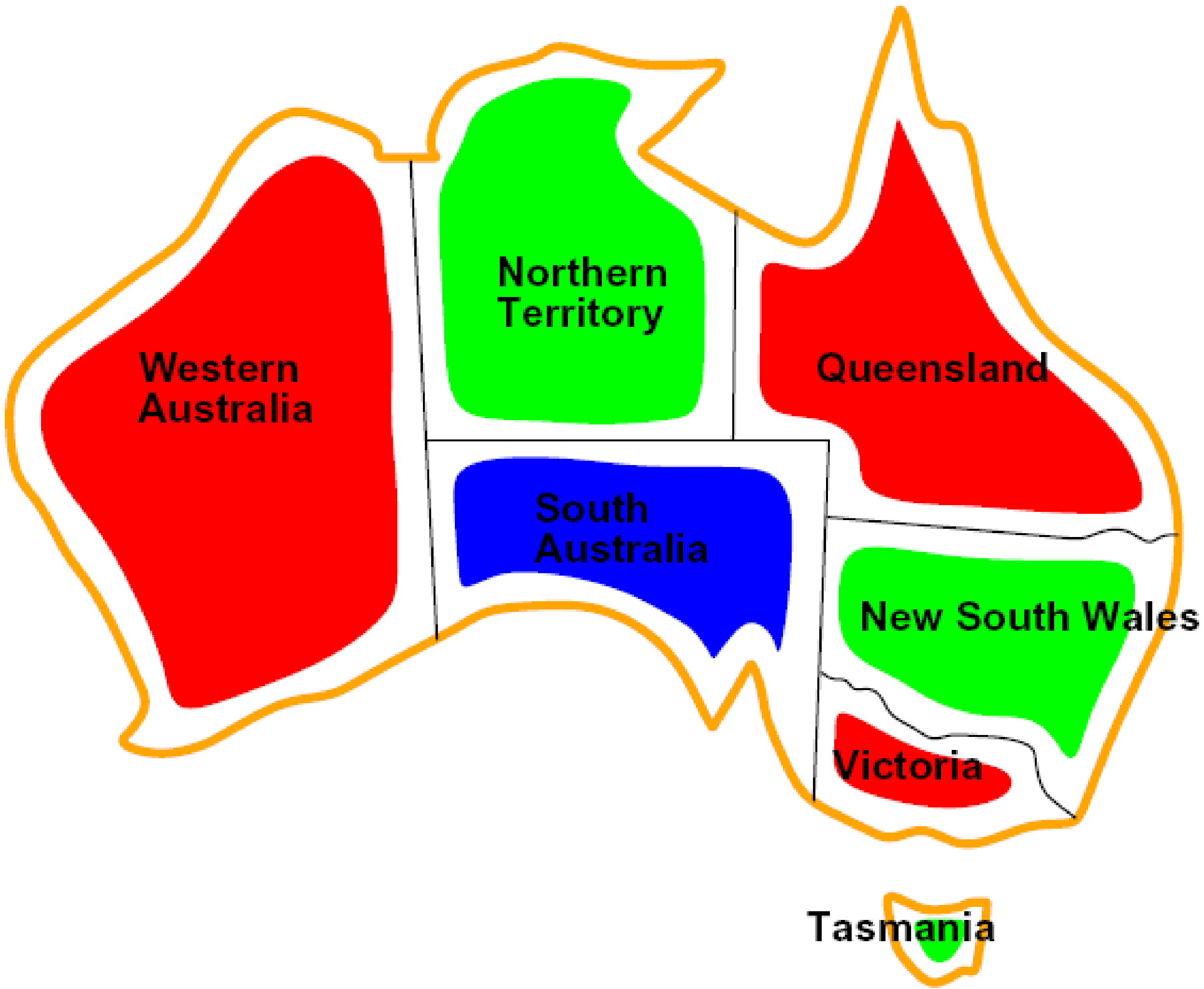


- Many real-world problems involve real-valued variables…

Shelf Organization

The shelves that store products that will be shipped to you (e.g., Amazon) are optimized so that items that ship together are stored on the same shelf.

CSP Examples



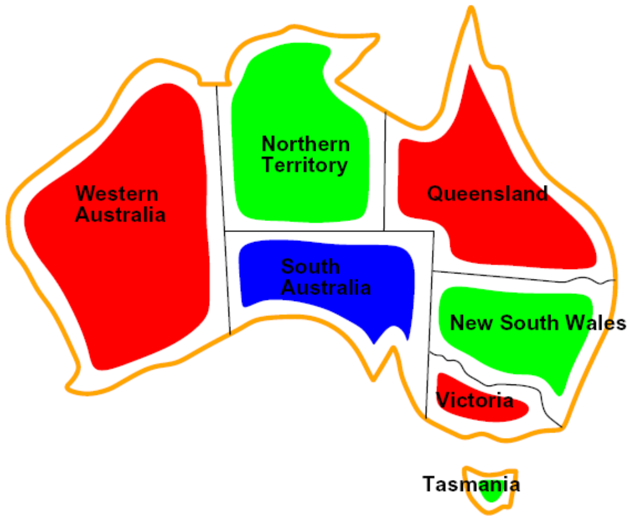
Example: Map Coloring

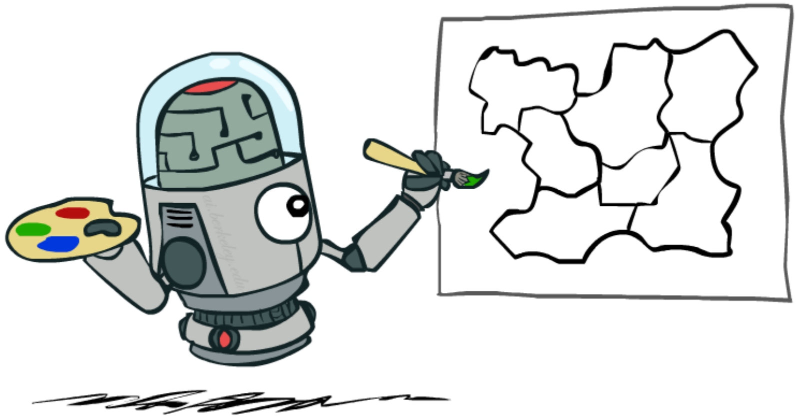
- Variables: $WA,\ NT,\ Q,\ NSW,\ V,\ SA,\ T$
- Domains: $D = \{\text{red}, \text{green}, \text{blue}\}$
- Constraints: adjacent regions must have different colors

  Implicit: $WA \neq NT$

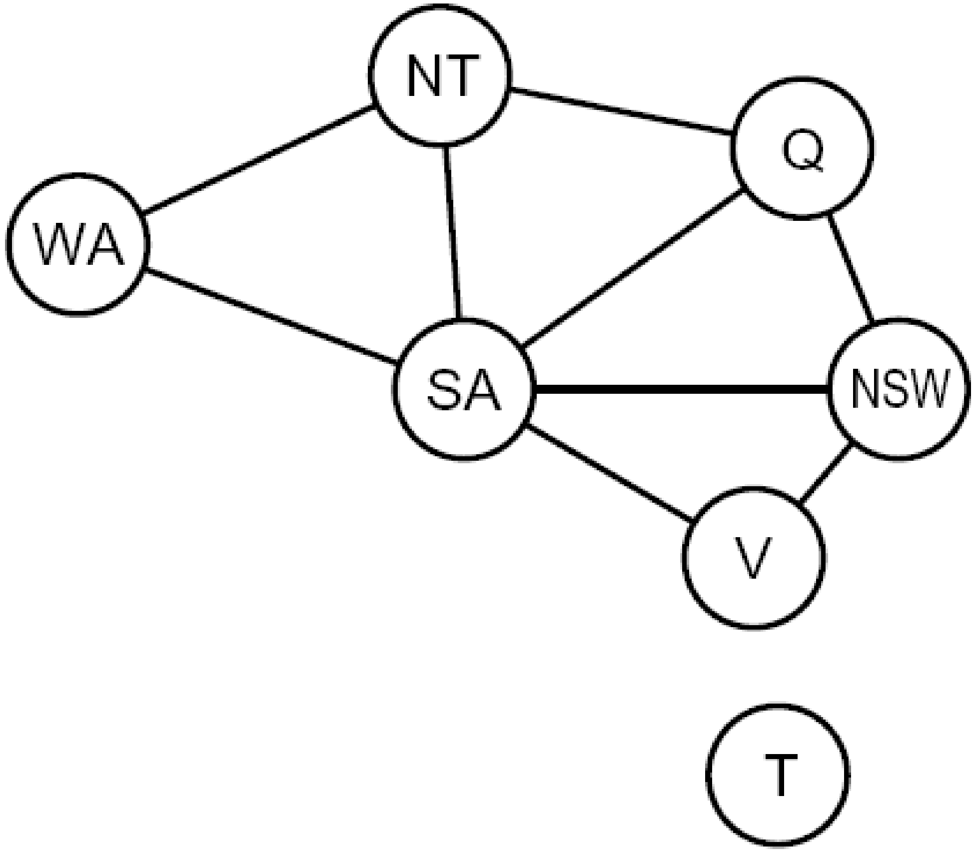
  Explicit: $(WA, NT) \in \{(\text{red}, \text{green}), (\text{red}, \text{blue}), \ldots\}$
- Solutions are assignments satisfying all constraints, e.g.:

  $\{WA{=}\text{red},\ NT{=}\text{green},\ Q{=}\text{red},\ NSW{=}\text{green},\ V{=}\text{red},\ SA{=}\text{blue},\ T{=}\text{green}\}$



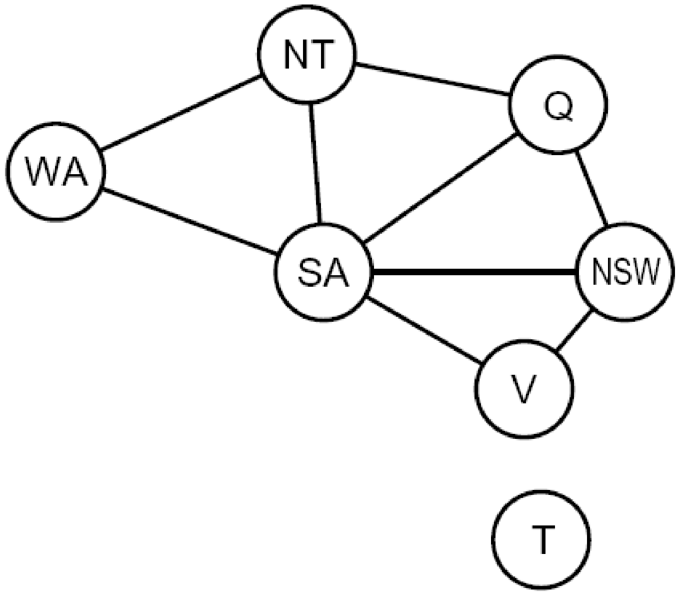


Constraint Graphs



Constraint Graphs

- Binary CSP: each constraint relates (at most) two variables
- Binary constraint graph: nodes are variables, arcs show constraints
- General-purpose CSP algorithms use the graph structure to speed up search. E.g., Tasmania is an independent subproblem!

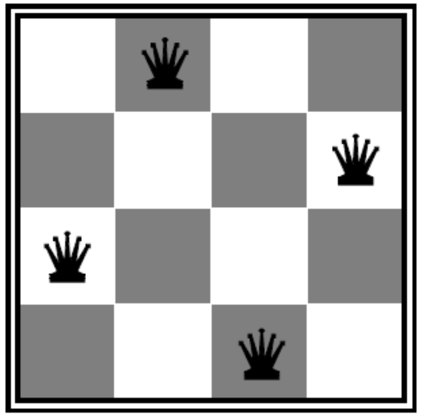


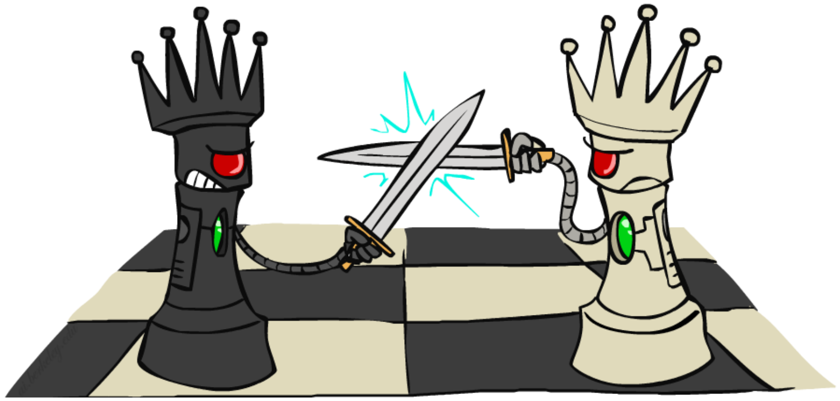
[Demo: CSP applet (made available by aispace.org) -- n-queens]

Varieties of CSPs and Constraints

Example: N-Queens

- Formulation 1:
  - Variables: X_{ij}
  - Domains: $\{0, 1\}$
  - Constraints



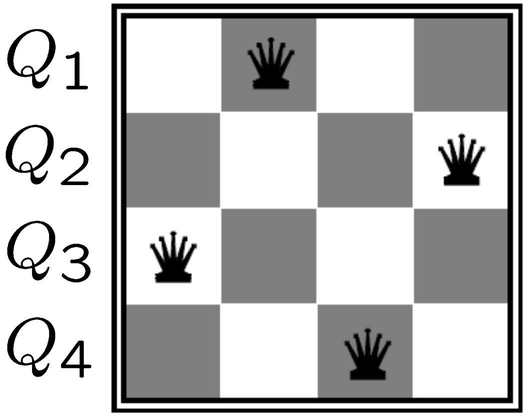


$$\forall i, j, k \;\; (X_{ij}, X_{ik}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k \;\; (X_{ij}, X_{kj}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k \;\; (X_{ij}, X_{i+k,j+k}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k \;\; (X_{ij}, X_{i+k,j-k}) \in \{(0,0), (0,1), (1,0)\}$$

$$\sum_{i,j} X_{ij} = N$$

Example: N-Queens

- Formulation 2:
  - Variables: Q_k
  - Domains: $\{1, 2, 3, \ldots N\}$
  - Constraints:



Implicit: $\forall i, j \;\; \text{non-threatening}(Q_i, Q_j)$

Explicit: $(Q_1, Q_2) \in \{(1, 3), (1, 4), \ldots\}$

$\ldots$

Example: Cryptarithmetic

- Variables:

  $F \ T \ U \ W \ R \ O \ X_1 \ X_2 \ X_3$

- Domains:

  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

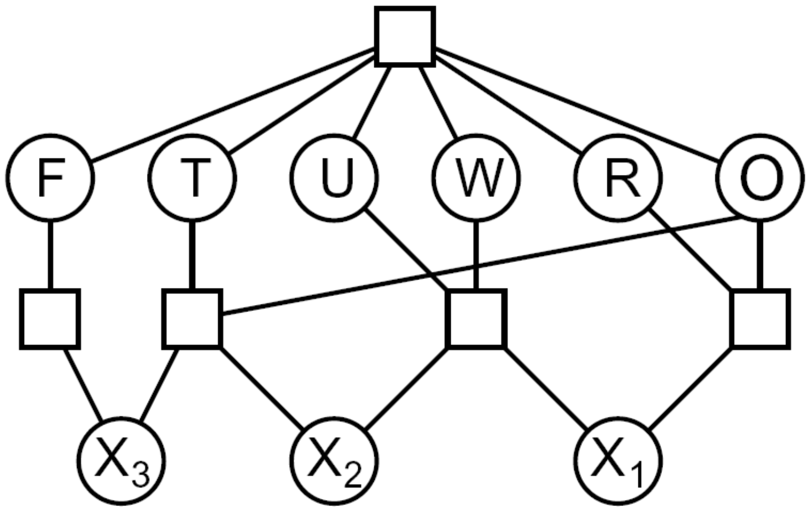
- Constraints:

  $\text{alldiff}(F, T, U, W, R, O)$

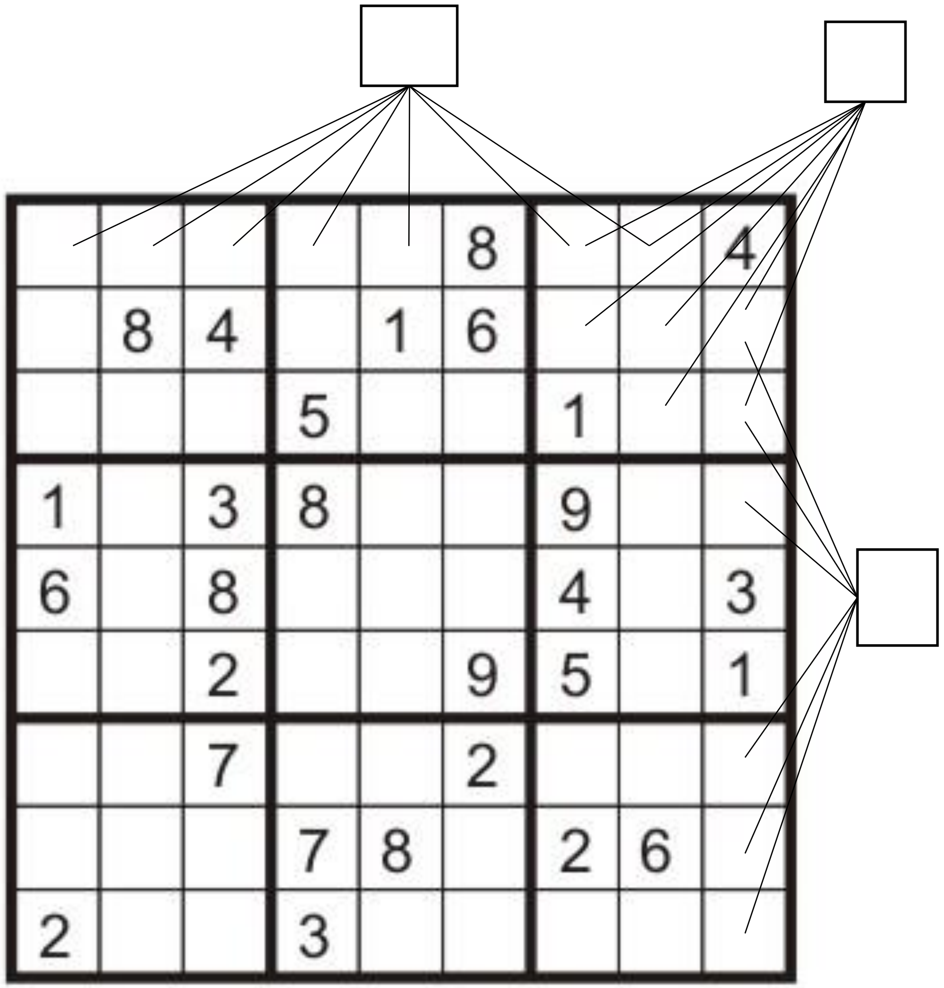
  $O + O = R + 10 \cdot X_1$

  $\cdots$

```
    T W O
+   T W O
---------
  F O U R
```



Example: Sudoku



- Variables:
  - Each (open) square
- Domains:
  - {1,2,…,9}
- Constraints:

9-way alldiff for each column

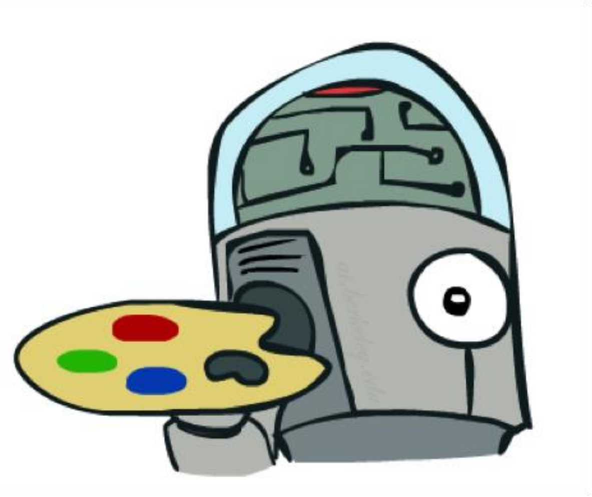
9-way alldiff for each row

9-way alldiff for each region

(or can have a bunch of pairwise inequality constraints)

Varieties of CSPs

- Discrete Variables
  - Finite domains
    - Size d means $O(d^n)$ complete assignments
    - E.g., Boolean CSPs, including Boolean satisfiability (NP-complete)
  - Infinite domains (integers, strings, etc.)
    - E.g., job scheduling, variables are start/end times for each job
    - Linear constraints solvable, nonlinear undecidable



- Continuous variables
  - E.g., start/end times for Hubble Telescope observations
  - Linear constraints solvable in polynomial time by LP methods

Varieties of Constraints

- Varieties of Constraints
  - Unary constraints involve a single variable (equivalent to reducing domains), e.g.:

$$SA \neq green$$

  - Binary constraints involve pairs of variables, e.g.:

$$SA \neq WA$$

  - Higher-order constraints involve 3 or more variables:
    e.g., cryptarithmetic column constraints

- Preferences (soft constraints):
  - E.g., red is better than green
  - Often representable by a cost for each variable assignment
  - Gives constrained optimization problems
  - (We'll ignore these until we get to Bayes' nets)

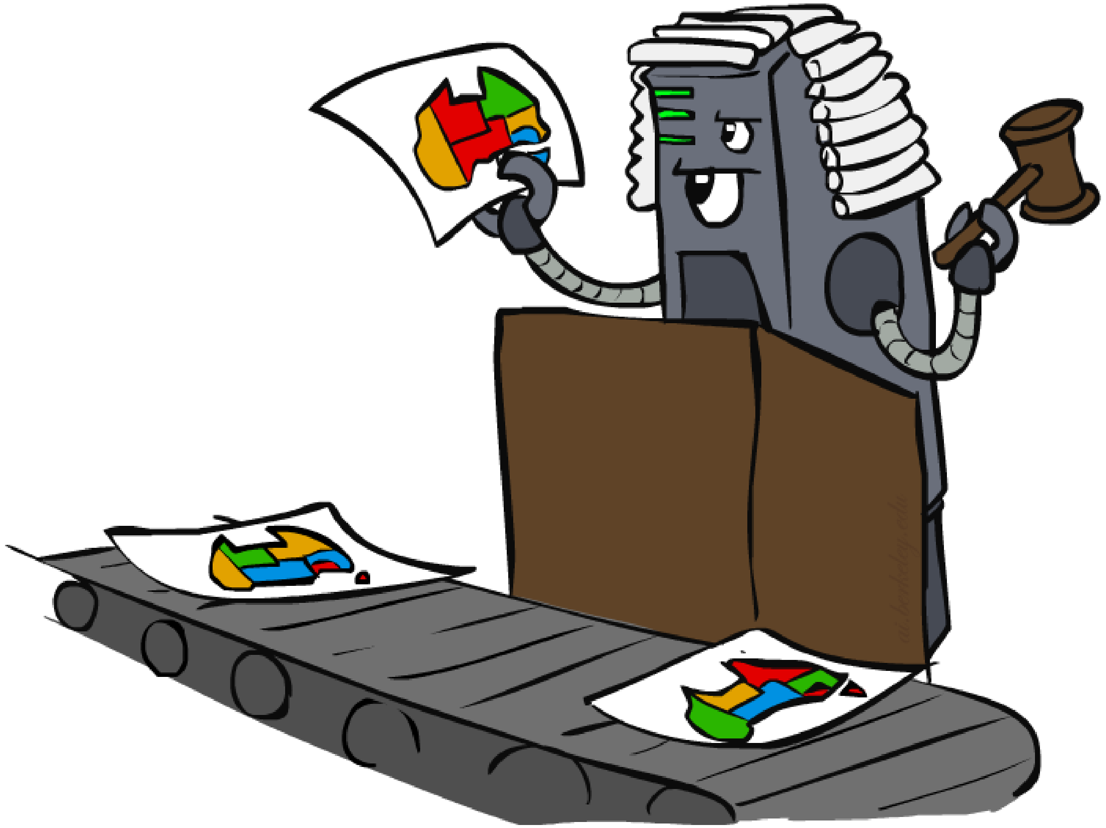
Solving CSPs

Standard Search Formulation

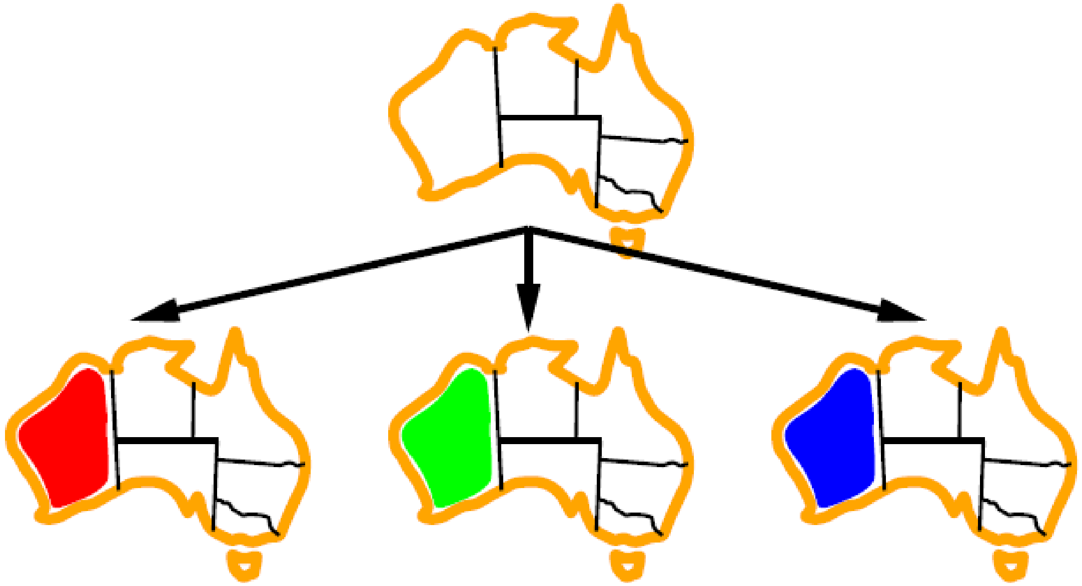
- Standard search formulation of CSPs

- States defined by the values assigned so far (partial assignments)
  - Initial state: the empty assignment, {}
  - Successor function: assign a value to an unassigned variable
  - Goal test: the current assignment is complete and satisfies all constraints

- We'll start with the straightforward, naïve approach, then improve it

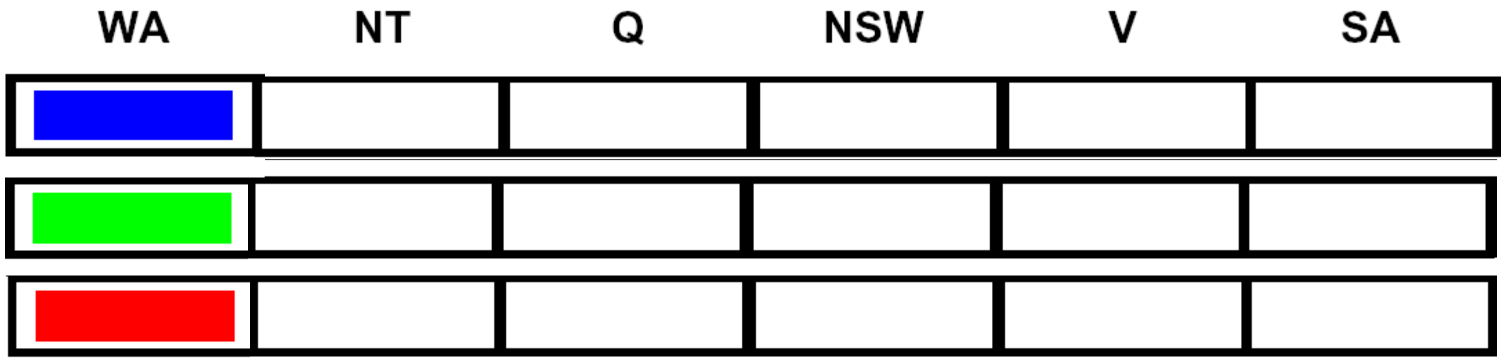


Breadth First Search



... All possible first variables
Check: Is there a solution?

Breadth First Search



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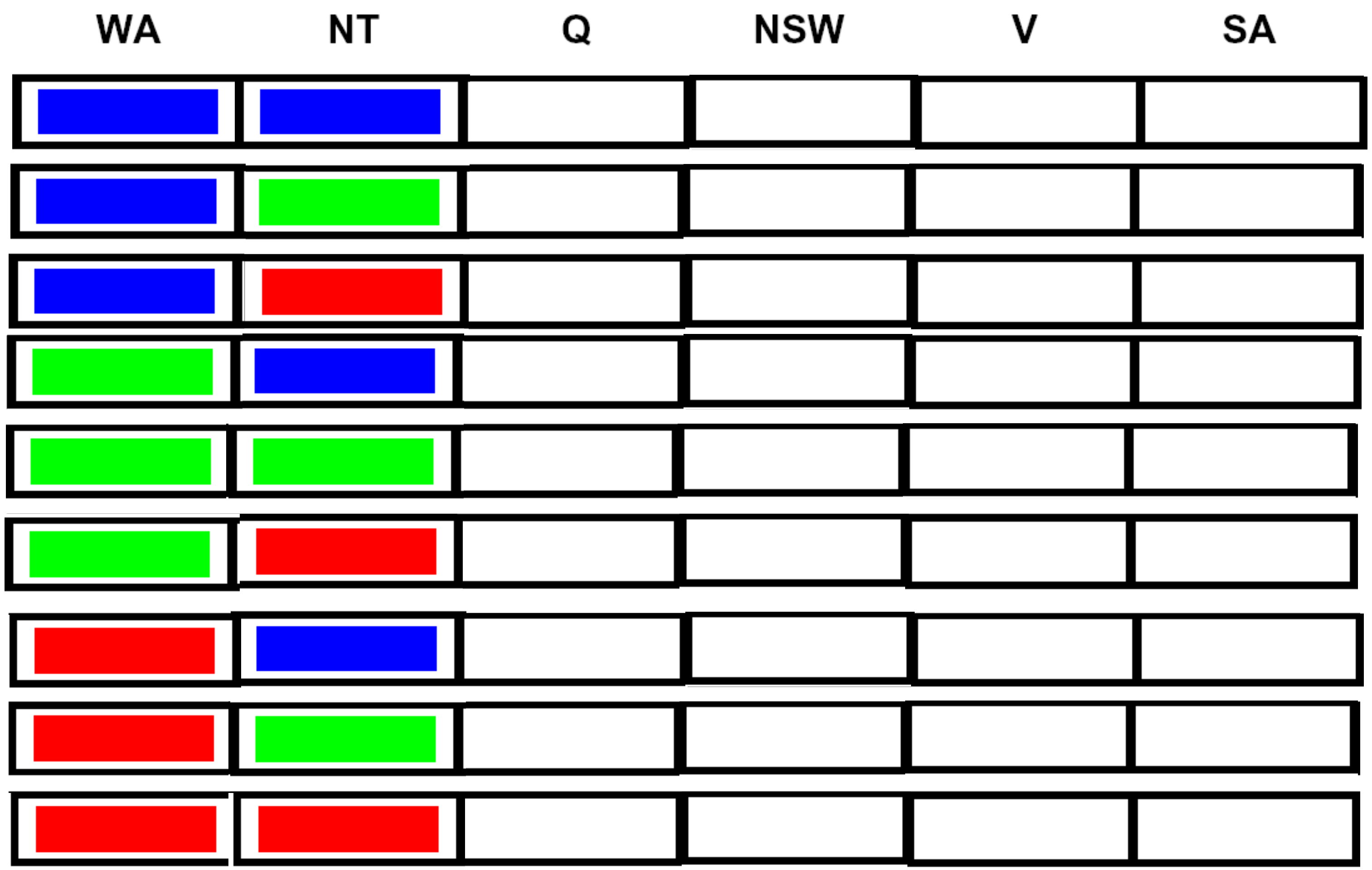

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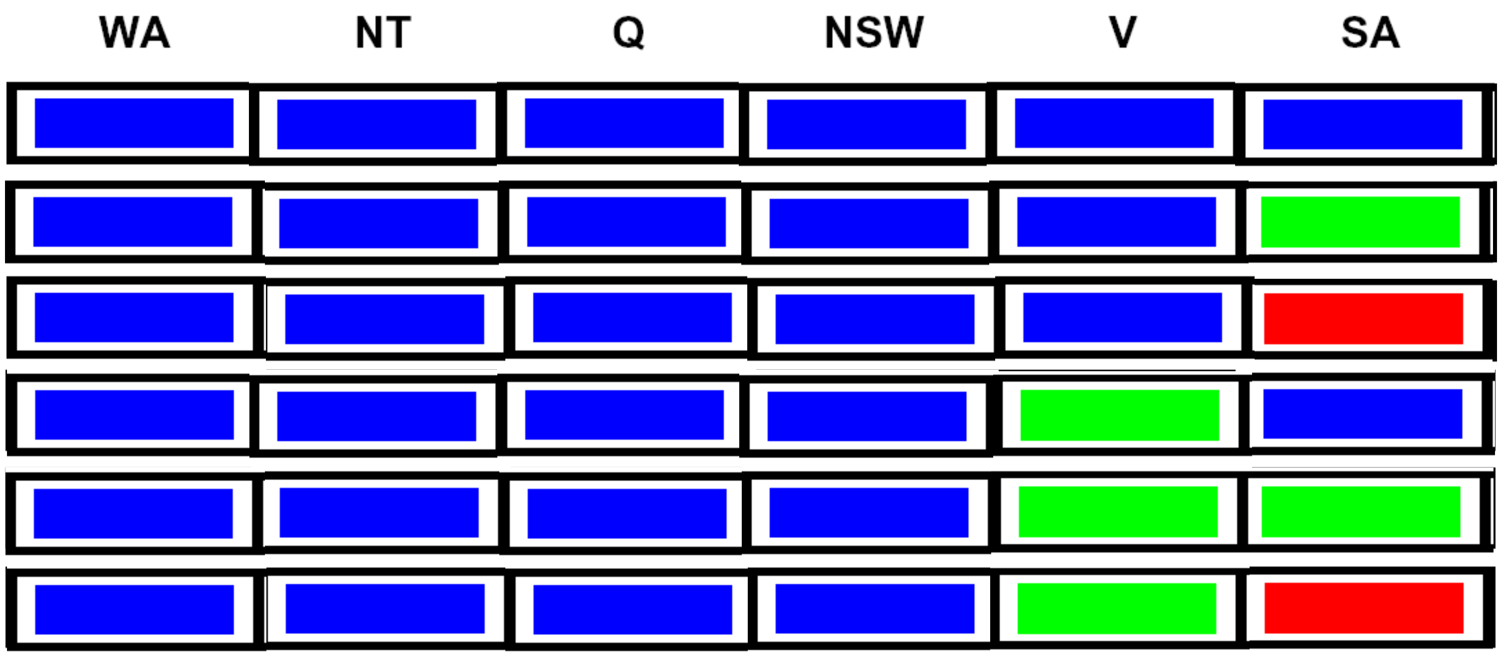

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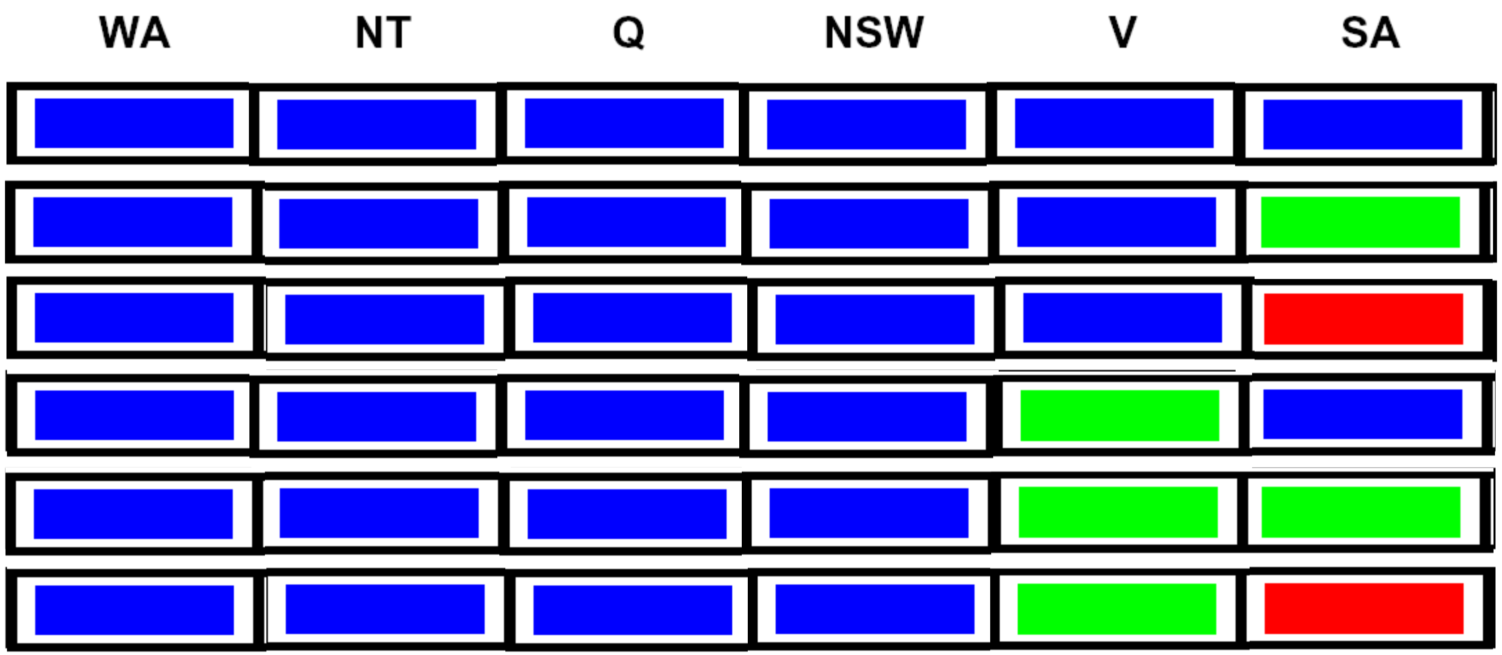
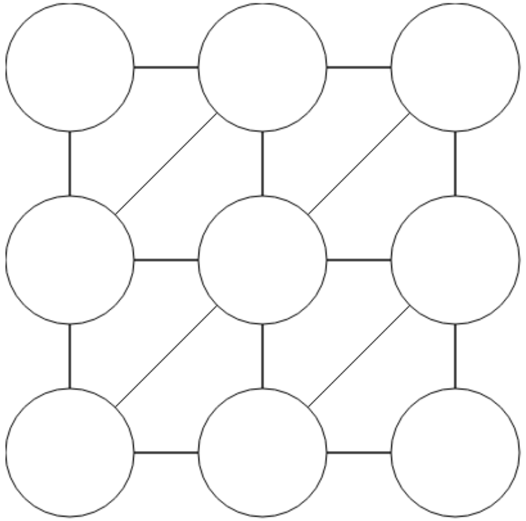
Depth First Search



...

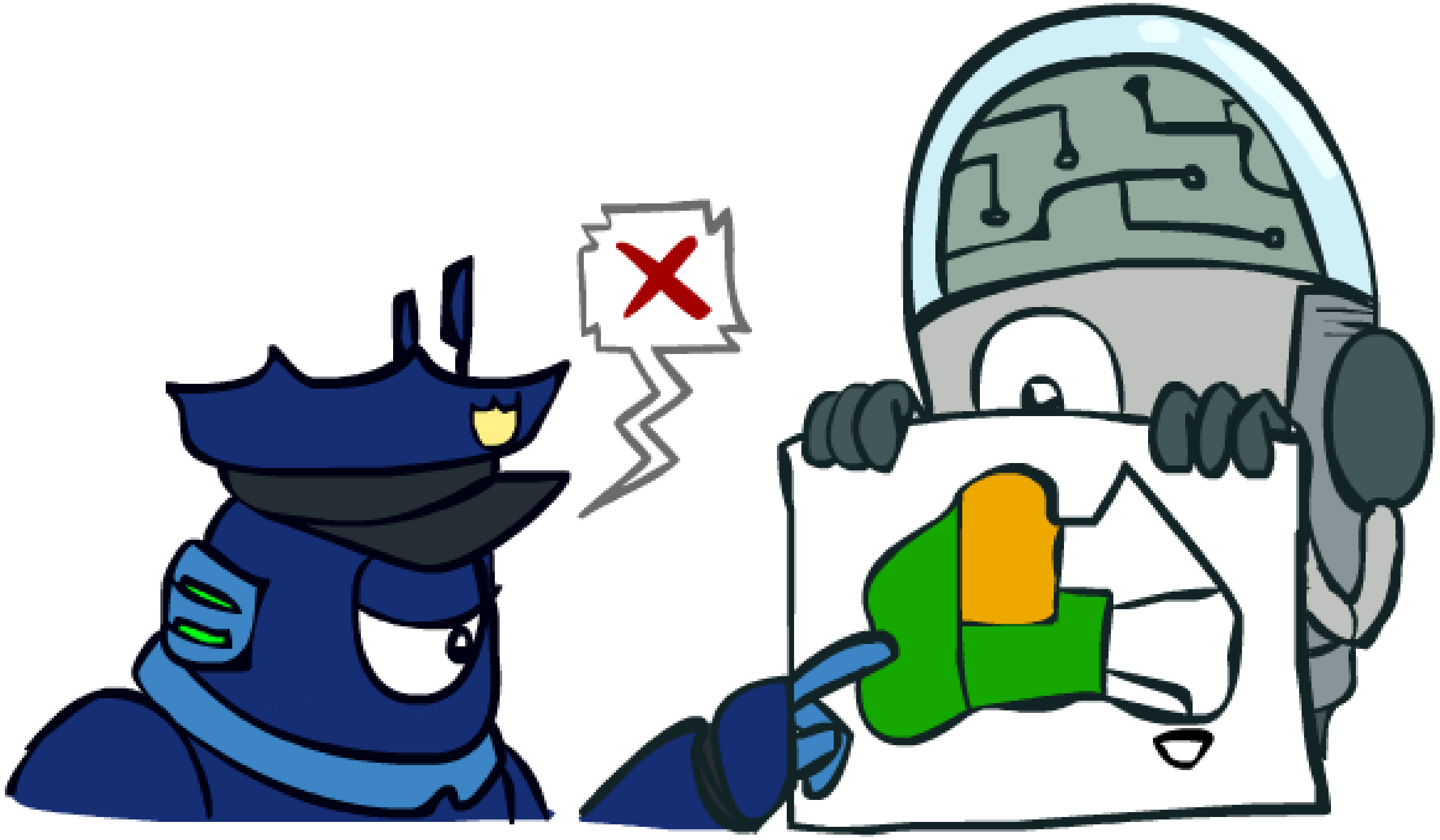
Demo



What is wrong with general search?

- When do you fail?

Backtracking Search



Backtracking Search

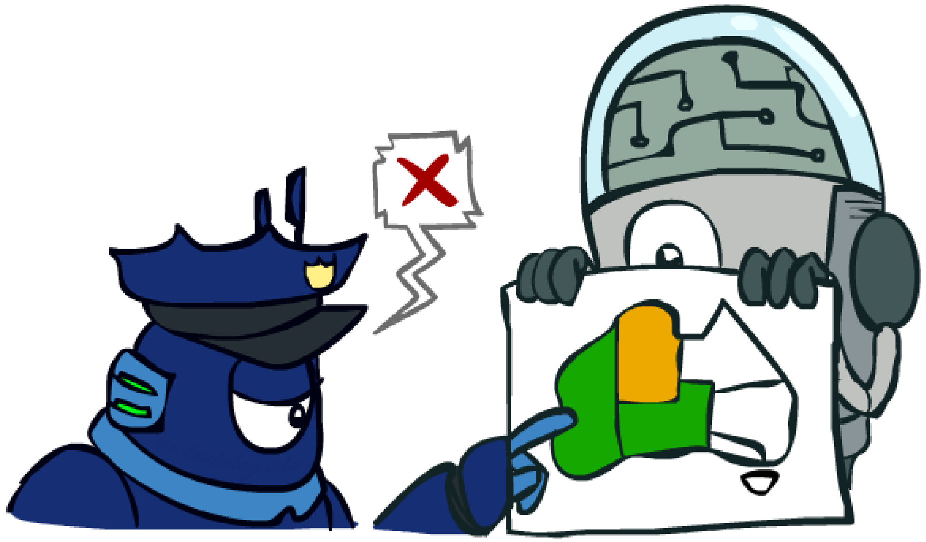
- Backtracking search is the basic uninformed algorithm for solving CSPs

- Idea 1: One variable at a time
  - Variable assignments are commutative, so fix ordering
  - I.e., [WA = red then NT = green] same as [NT = green then WA = red]
  - Only need to consider assignments to a single variable at each step

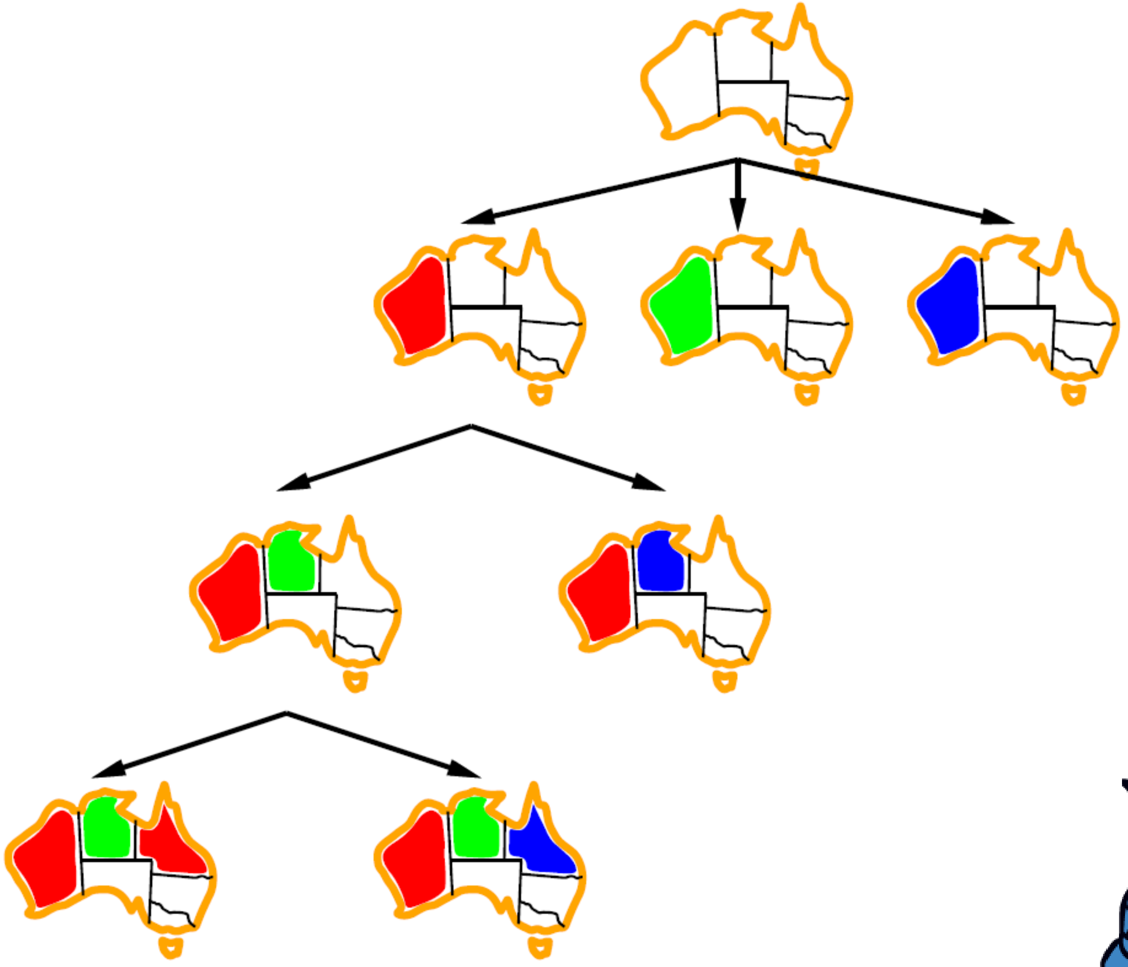
- Idea 2: Check constraints as you go
  - I.e. consider only values which do not conflict previous assignments
  - Might have to do some computation to check the constraints
  - “Incremental goal test”

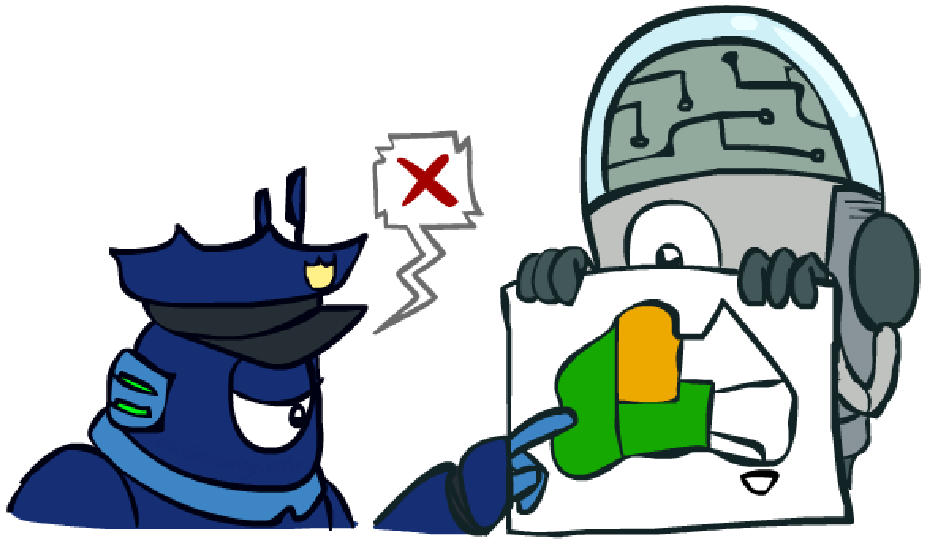
- Depth-first search with these two improvements is called *backtracking search* (not the best name)

- Can solve n-queens for n $\approx$ 25



Backtracking Example





Backtracking Search

```
function BACKTRACKING-SEARCH(csp) returns solution/failure
   return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
   if assignment is complete then return assignment
   var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
   for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
      if value is consistent with assignment given CONSTRAINTS[csp] then
         add {var = value} to assignment
         result ← RECURSIVE-BACKTRACKING(assignment, csp)
         if result ≠ failure then return result
         remove {var = value} from assignment
   return failure
```

Backtracking Search

General Search checks consistency on full assignment

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```

Backtracking Search

Backtracking Search checks consistency at each assignment

```
function BACKTRACKING-SEARCH(csp) returns solution/failure
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- Backtracking = DFS + variable-ordering + fail-on-violation
- What are the choice points?

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- Backtracking = DFS + variable-ordering + fail-on-violation
- What are the choice points?

Demo Coloring – Backtracking

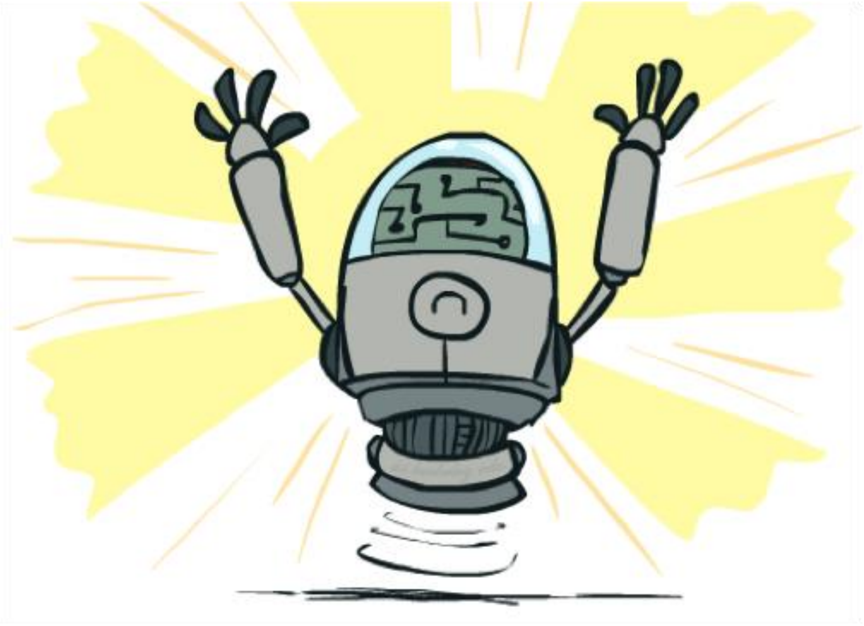
Improving Backtracking

- General-purpose ideas give huge gains in speed

- Filtering: Can we detect inevitable failure early?

- Ordering:
  - Which variable should be assigned next?
  - In what order should its values be tried?

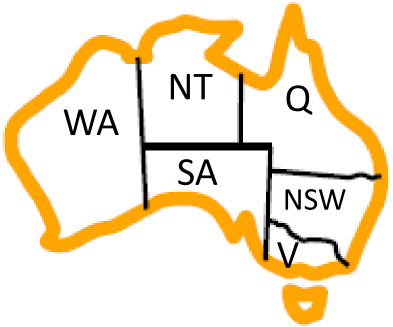
- Structure: Can we exploit the problem structure?

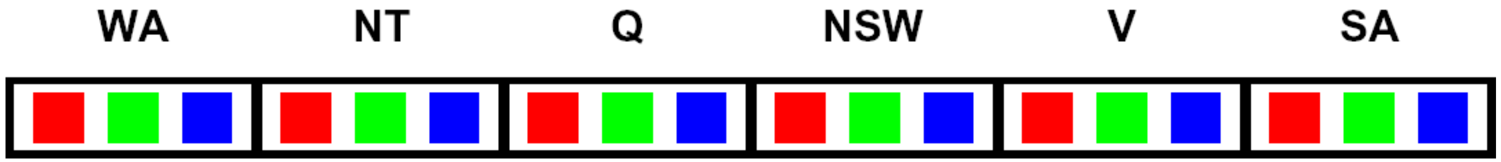


Filtering

Filtering: Forward Checking

- Filtering: Keep track of domains for unassigned variables and cross off bad options
- Forward checking: Cross off values that violate a constraint when added to the existing assignment





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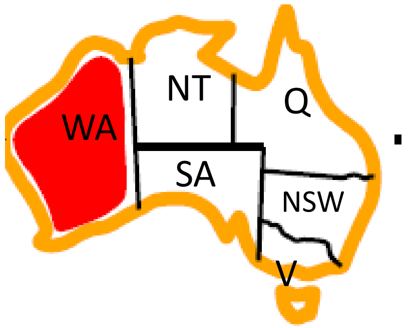

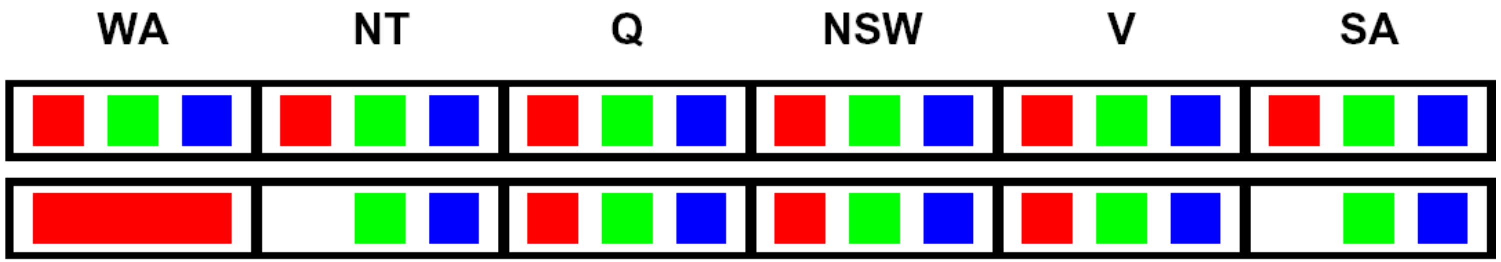

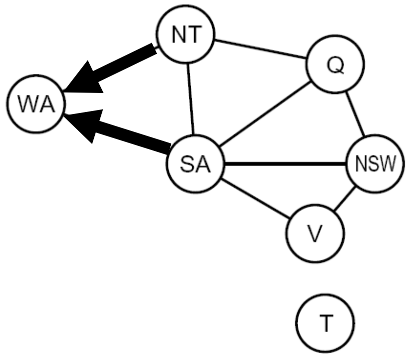

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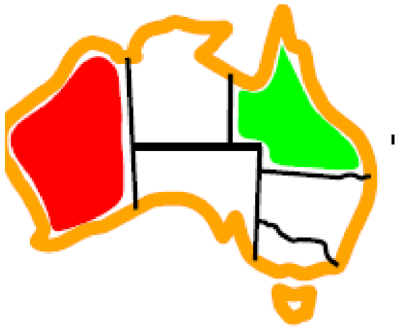

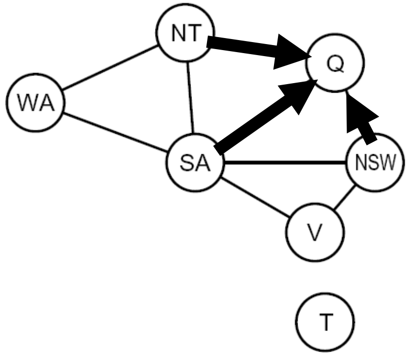

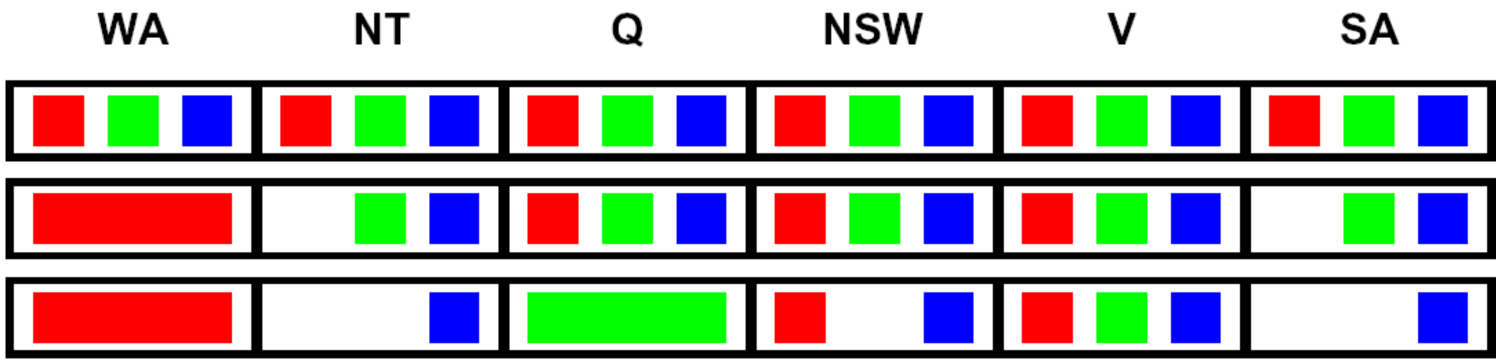

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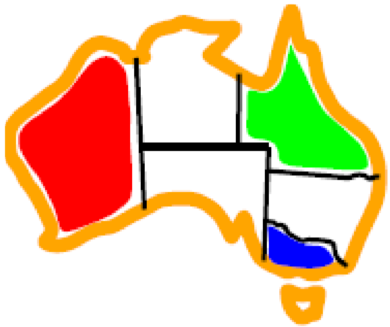

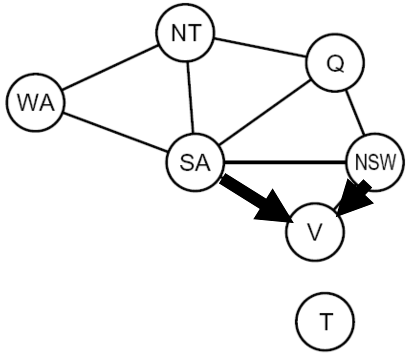

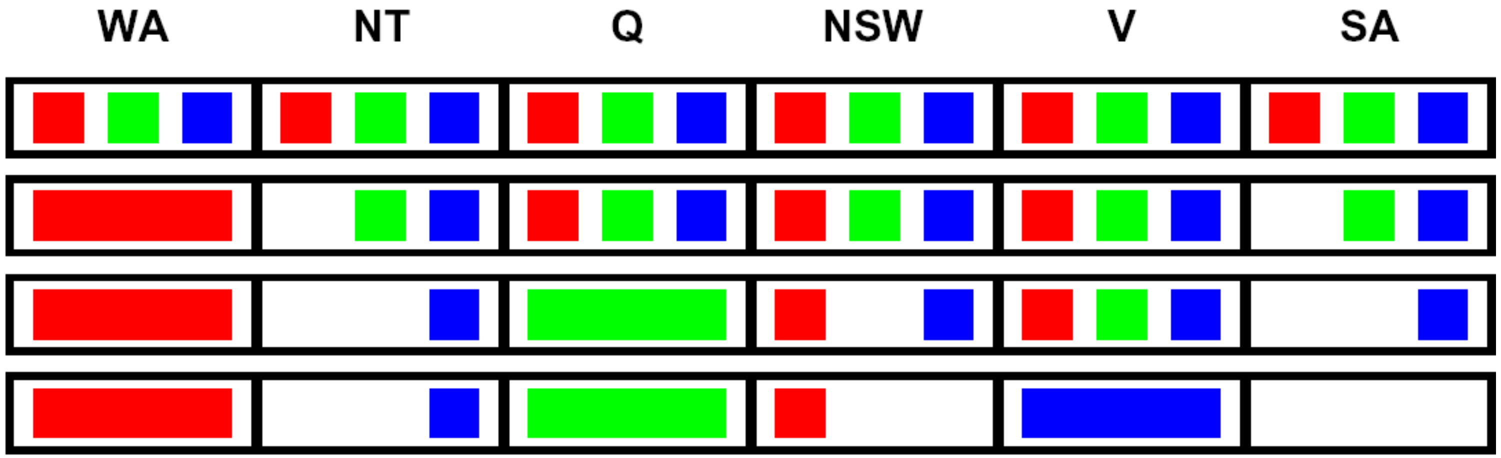

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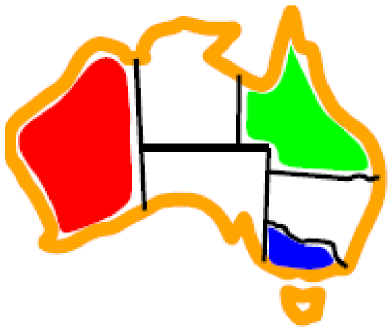
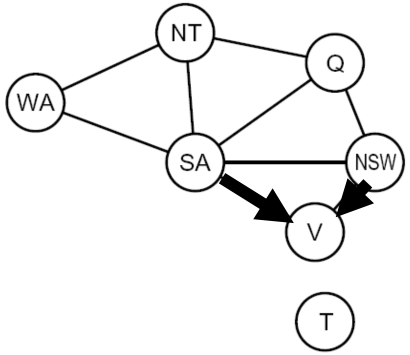
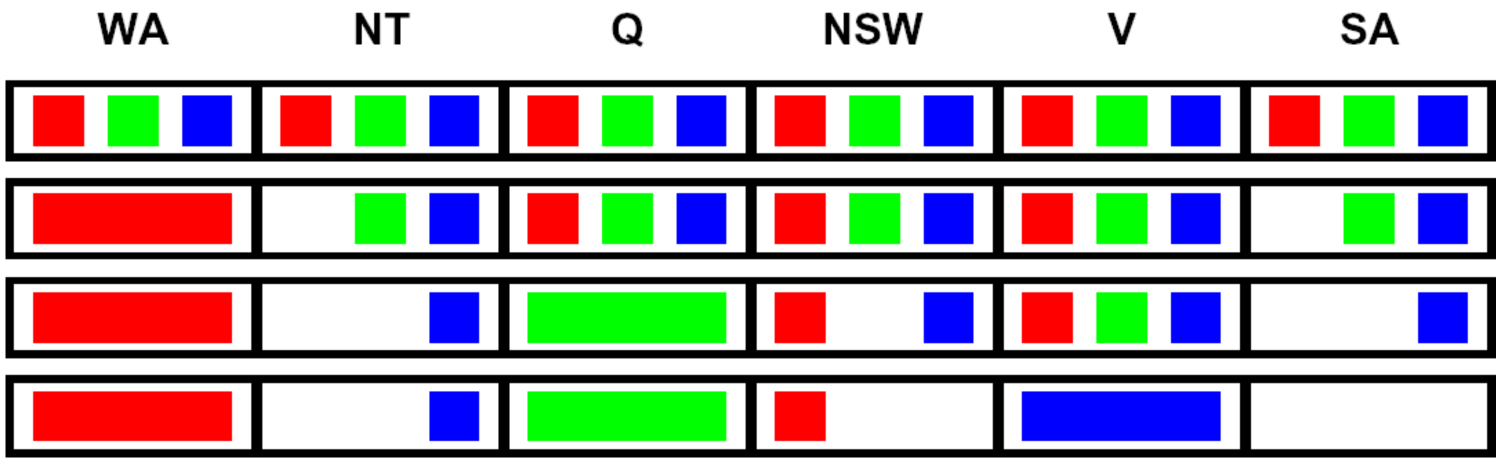




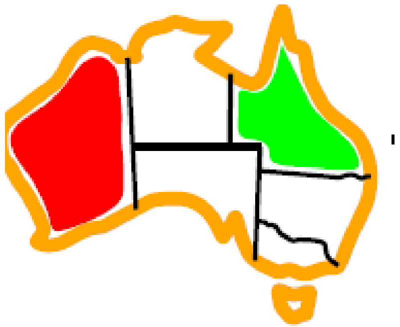


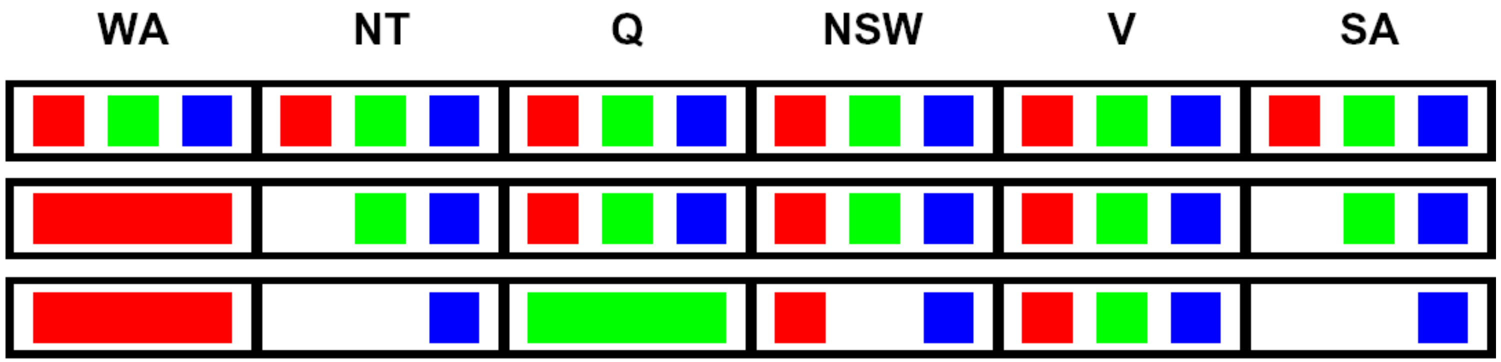
FAIL – variable with no possible values

Demo Coloring – Backtracking with Forward Checking

Filtering: Constraint Propagation

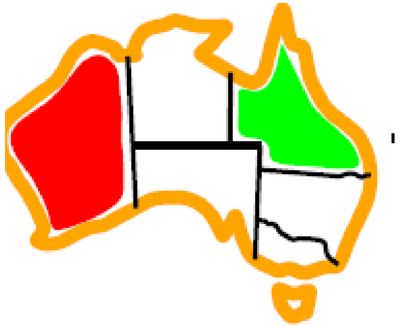
- Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures

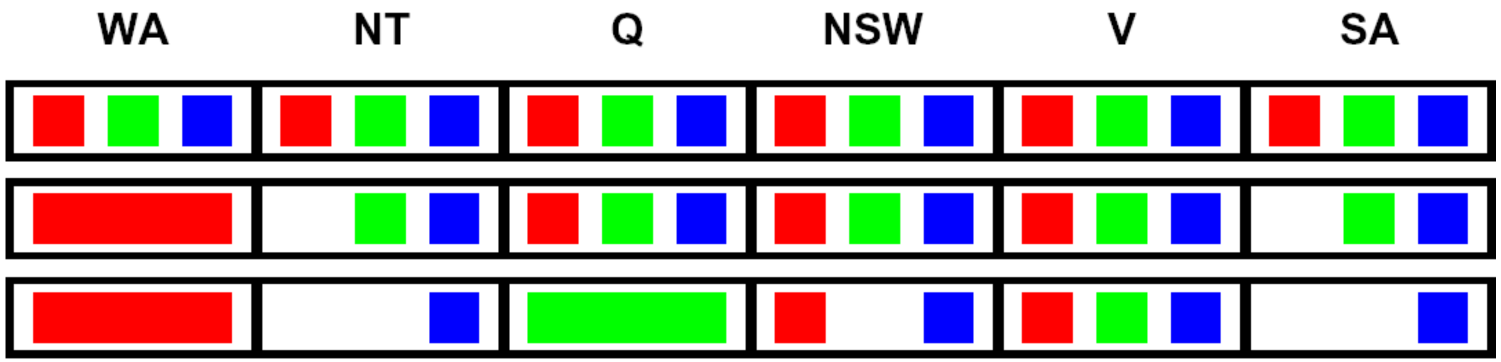




Filtering: Constraint Propagation

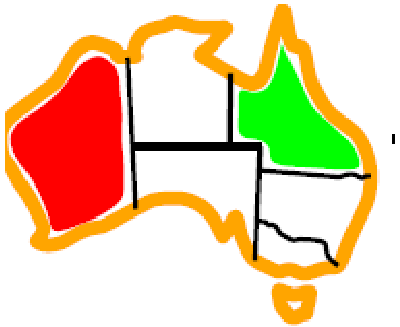
- Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures
  - NT and SA cannot both be blue! Why didn’t we detect this yet?

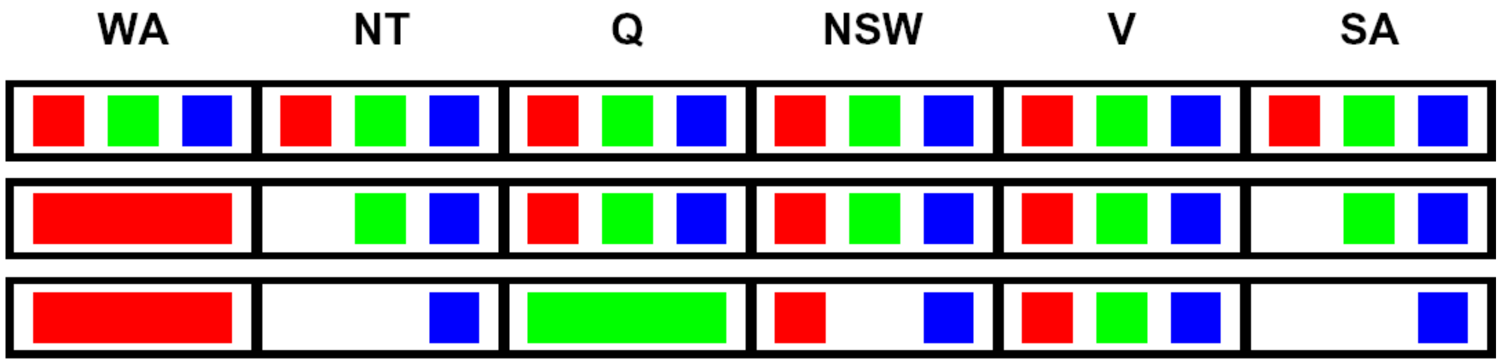




Filtering: Constraint Propagation

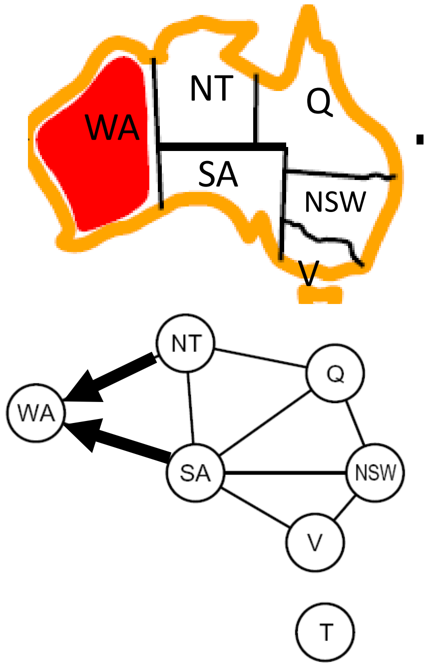
- Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures
  - NT and SA cannot both be blue! Why didn’t we detect this yet?
- *Constraint propagation:* reason from constraint to constraint

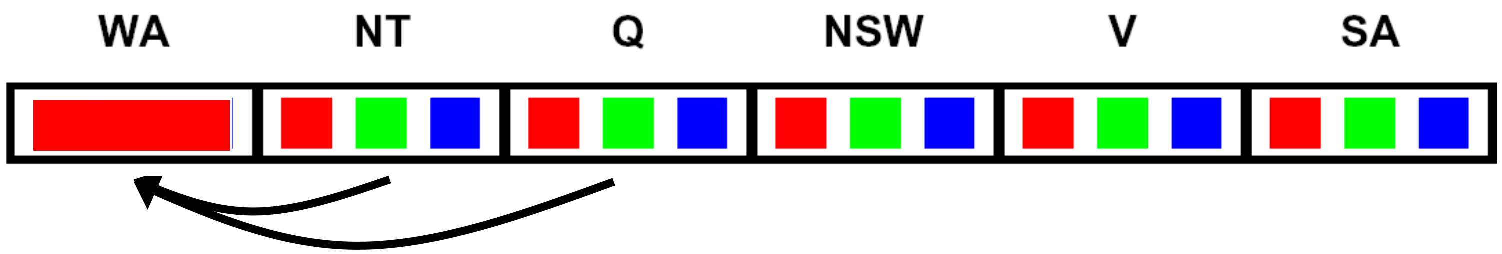




Consistency of A Single Arc

- An arc X $\rightarrow$ Y is consistent iff for *every* x in the tail there is *some* y in the head which could be assigned without violating a constraint
- Remove values in the domain of X if there isn't a corresponding legal Y
- Forward checking: Enforcing consistency of arcs pointing to each new assignment





Consistency of A Single Arc

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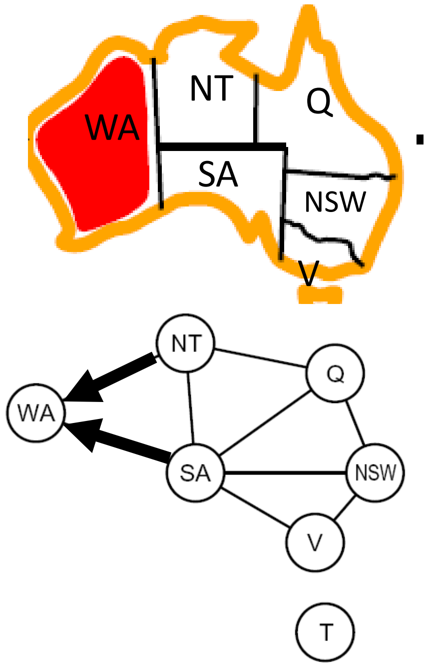
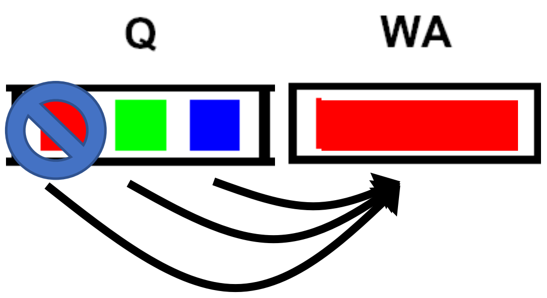
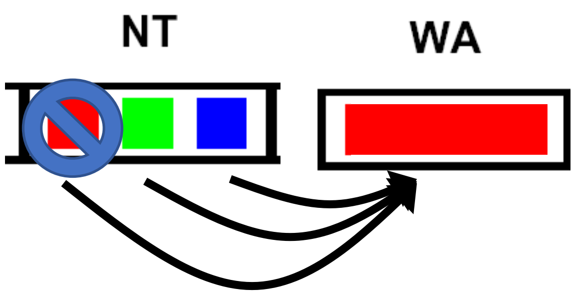
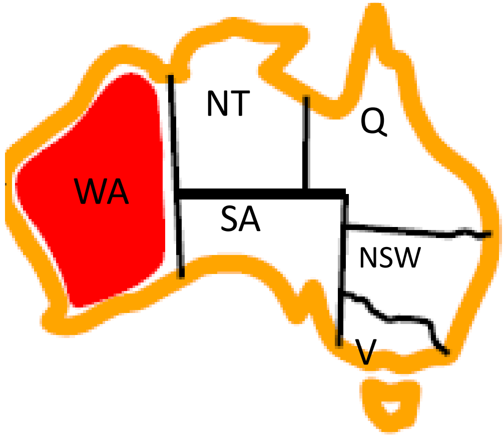


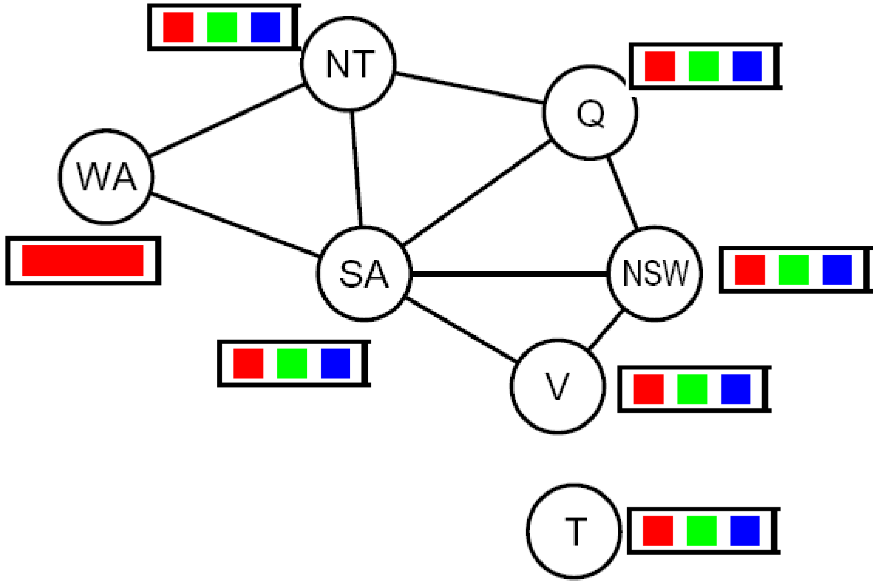




Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are consistent:





Remember: Delete from the tail!

Enforcing Arc Consistency in a CSP

function AC-3(*csp*) **returns** the CSP, possibly with reduced domains
  inputs: *csp*, a binary CSP with variables $\{X_1, X_2, \ldots, X_n\}$
  local variables: *queue*, a queue of arcs, initially all the arcs in *csp*

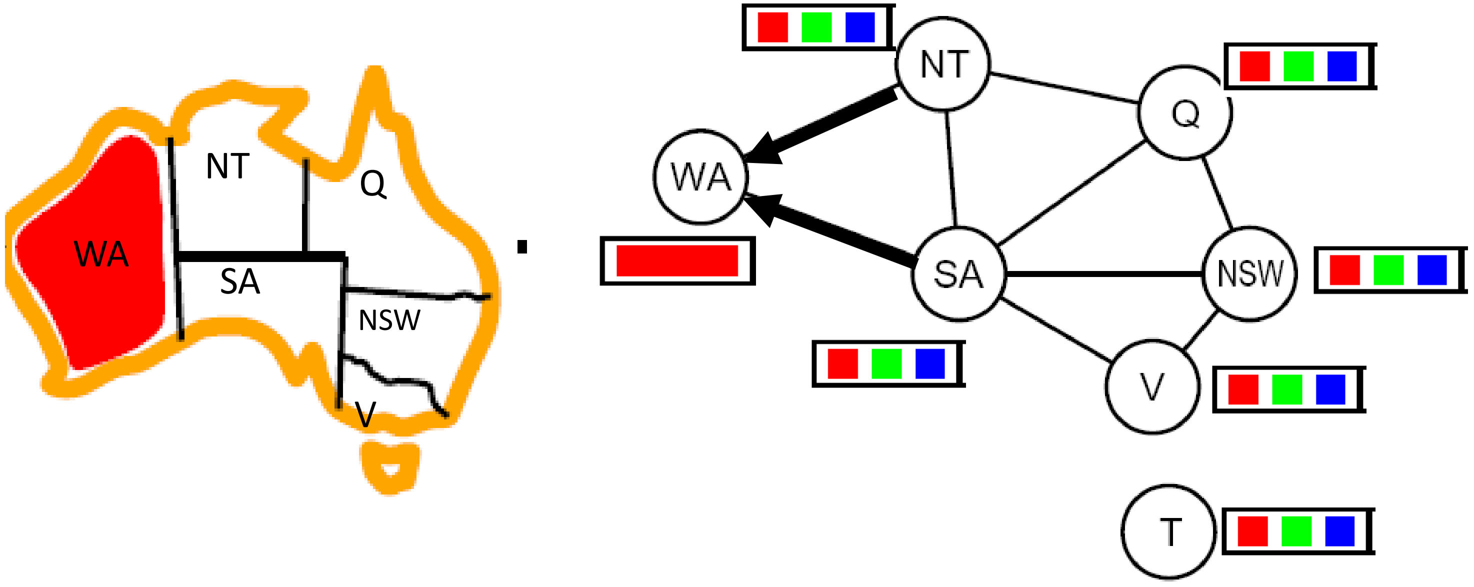
  while *queue* is not empty **do**
    $(X_i, X_j) \leftarrow$ REMOVE-FIRST(*queue*)
    if REMOVE-INCONSISTENT-VALUES(X_i, X_j) **then**
      for each X_k **in** NEIGHBORS[X_i] **do**
        add (X_k, X_i) to *queue*

---

function REMOVE-INCONSISTENT-VALUES(X_i, X_j) **returns** true iff succeeds
  removed $\leftarrow$ *false*
  for each x **in** DOMAIN[X_i] **do**
    if no value y in DOMAIN[X_j] allows (x,y) to satisfy the constraint $X_i \leftrightarrow X_j$
      then delete x from DOMAIN[X_i]; *removed* $\leftarrow$ *true*
  return *removed*

Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are consistent:

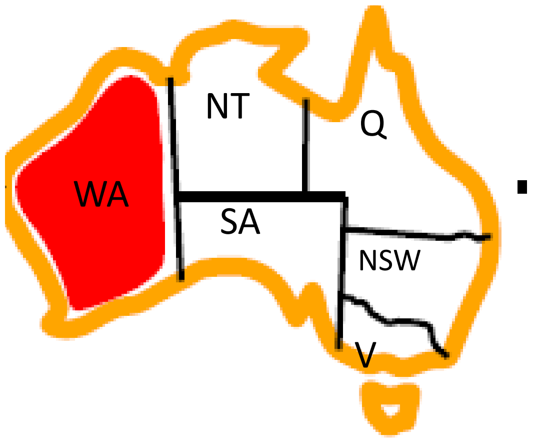


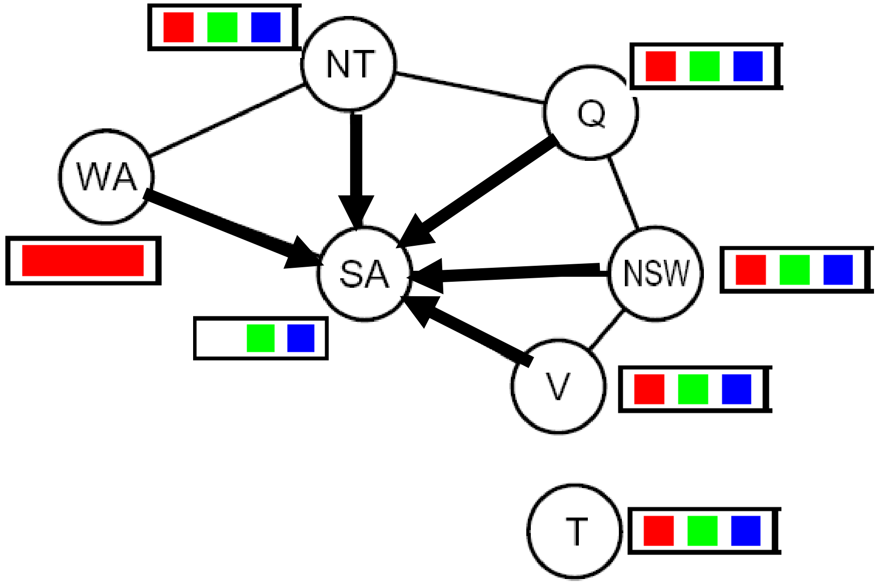
Queue:
SA->WA
NT->WA

Remember: Delete from the tail!

Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are consistent:





Queue:
NT->WA
WA->SA
NT->SA
Q->SA
NSW->SA
V->SA

Remember: Delete from the tail!

Arc Consistency of an Entire CSP

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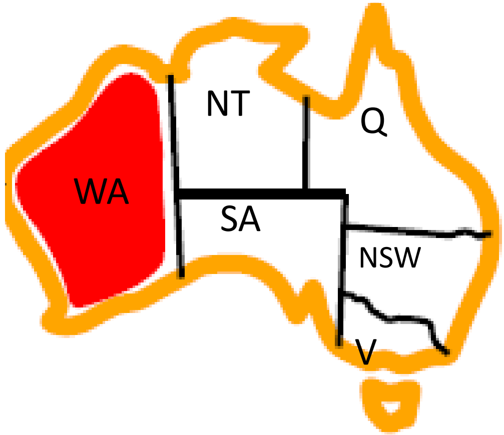

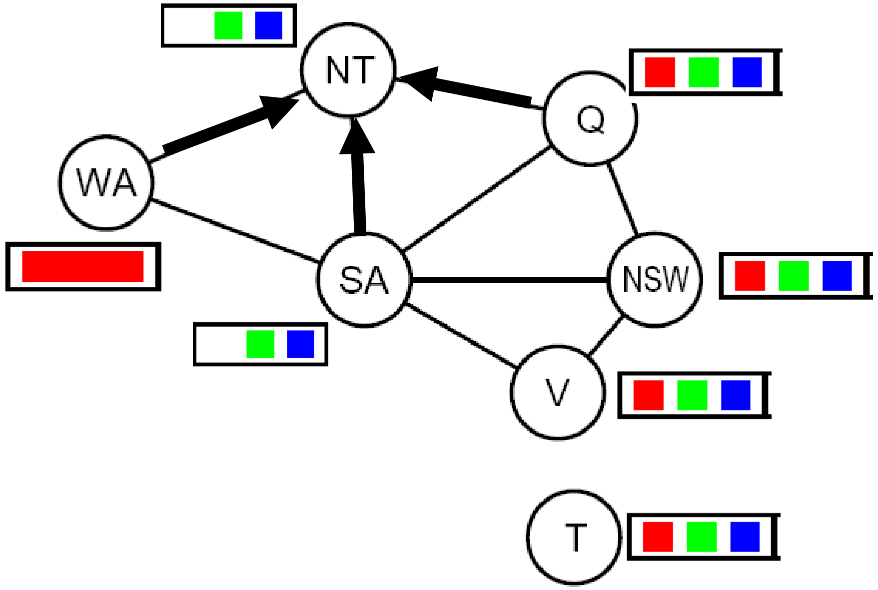

Queue:
WA->SA
NT->SA
Q->SA
NSW->SA
V->SA
WA->NT
SA->NT
Q->NT

Remember: Delete from the tail!

Arc Consistency of an Entire CSP

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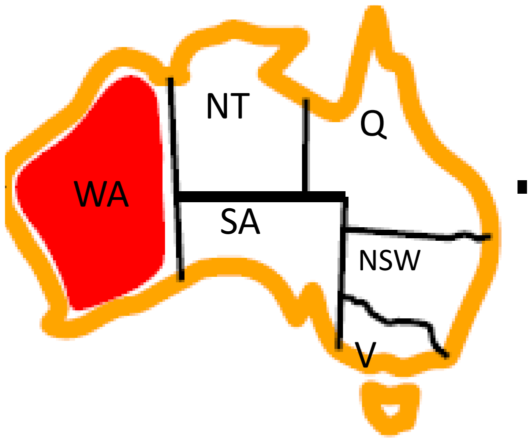
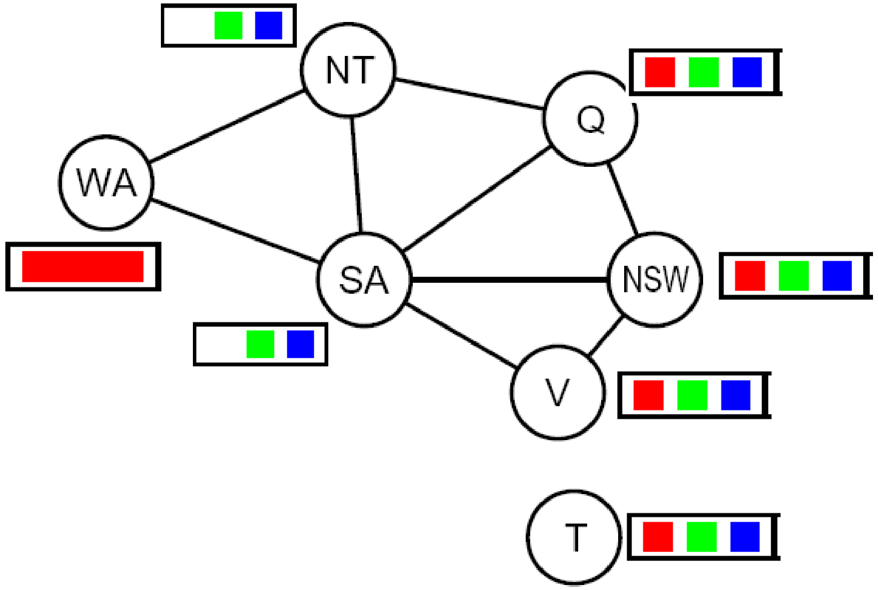


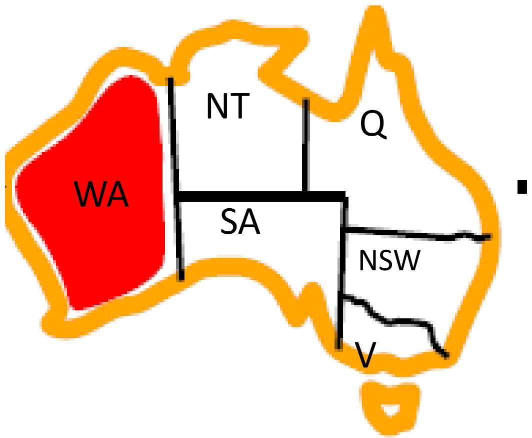


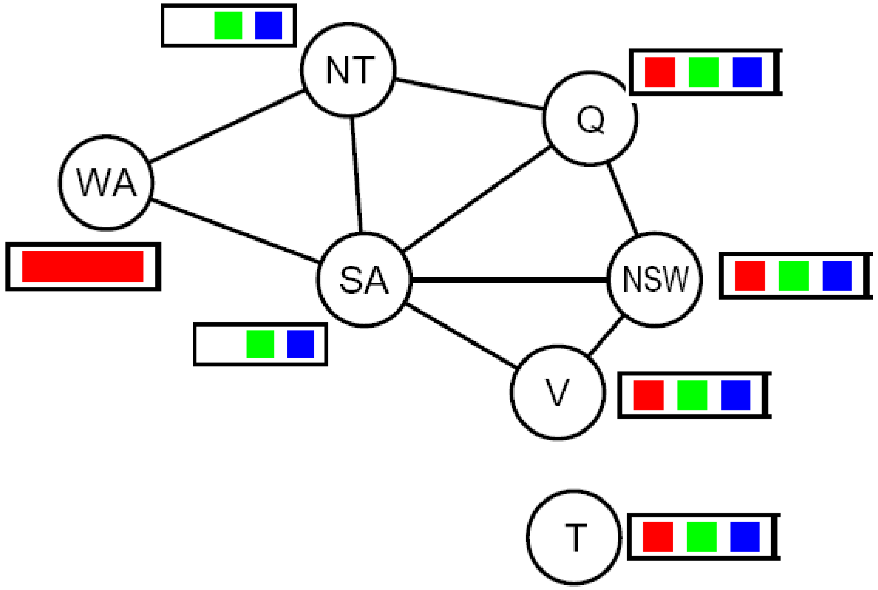
Queue:
WA->SA
NT->SA
Q->SA
NSW->SA
V->SA
WA->NT
SA->NT
Q->NT

Remember: Delete from the tail!

Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are consistent:





Queue:
NT->SA
Q->SA
NSW->SA
V->SA
WA->NT
SA->NT
Q->NT

Remember: Delete from the tail!

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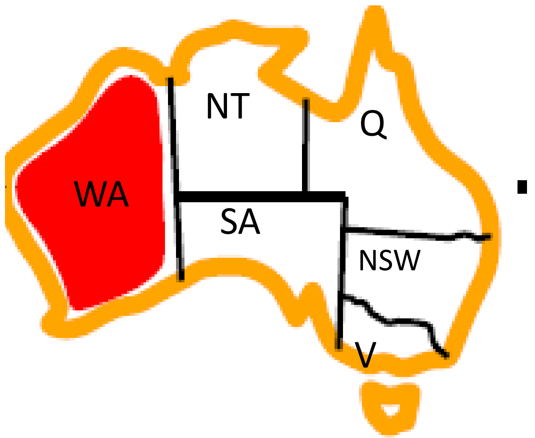

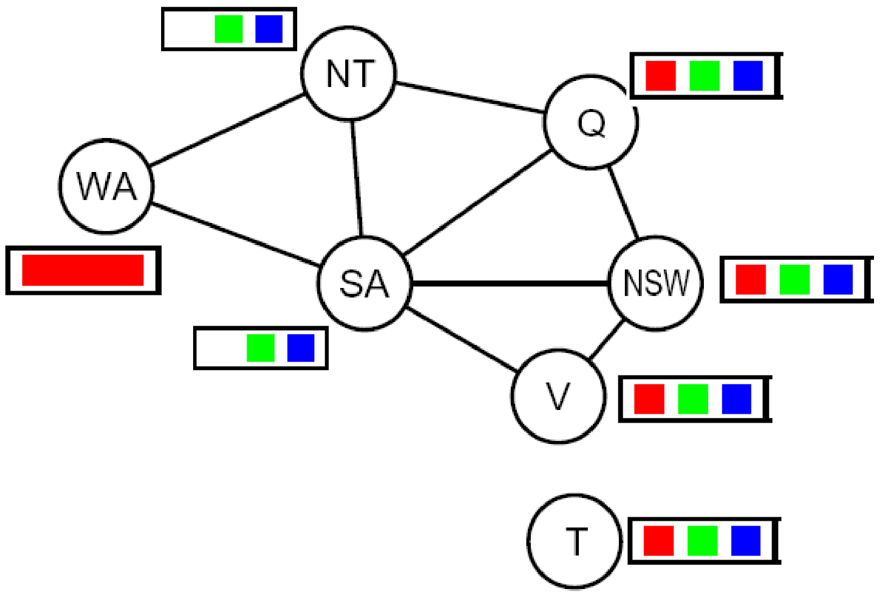

Queue:
Q->SA
NSW->SA
V->SA
WA->NT
SA->NT
Q->NT

Remember: Delete from the tail!

Arc Consistency of an Entire CSP

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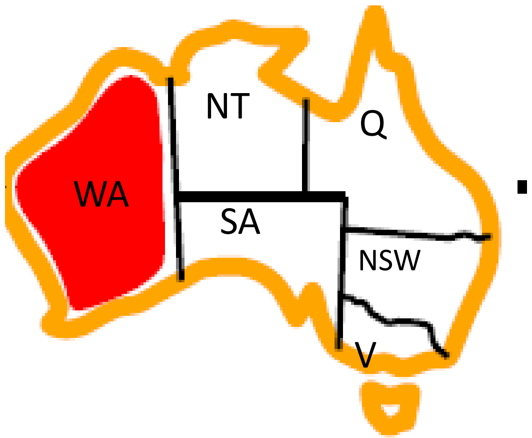

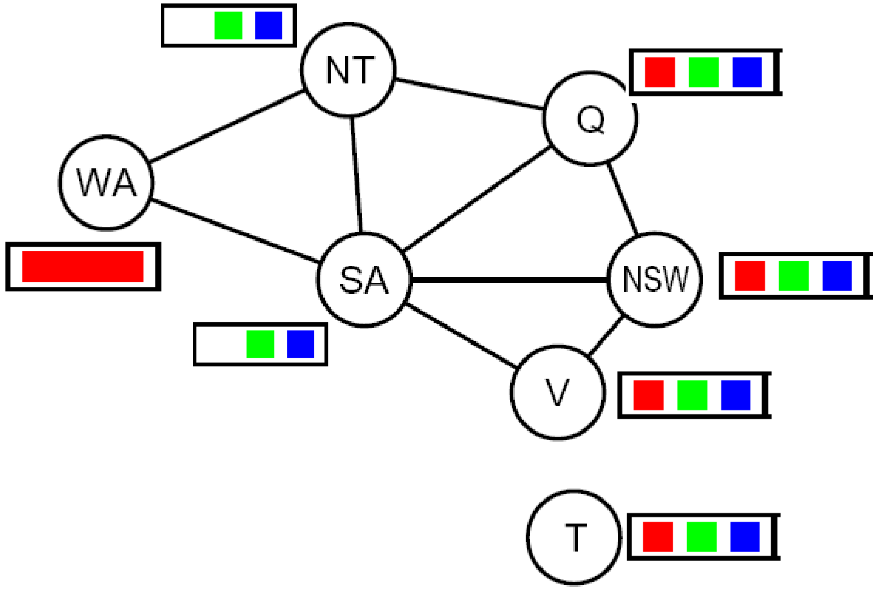

Queue:
NSW->SA
V->SA
WA->NT
SA->NT
Q->NT

Remember: Delete from the tail!

Arc Consistency of an Entire CSP

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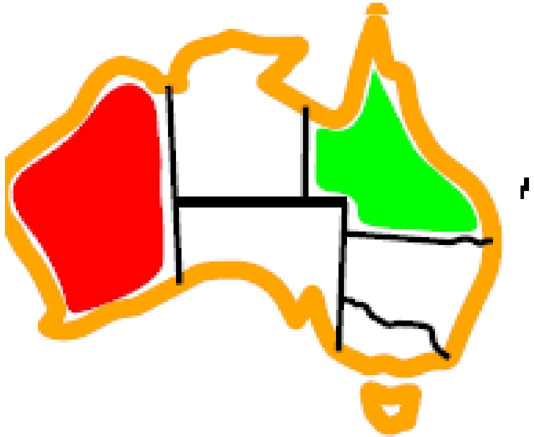
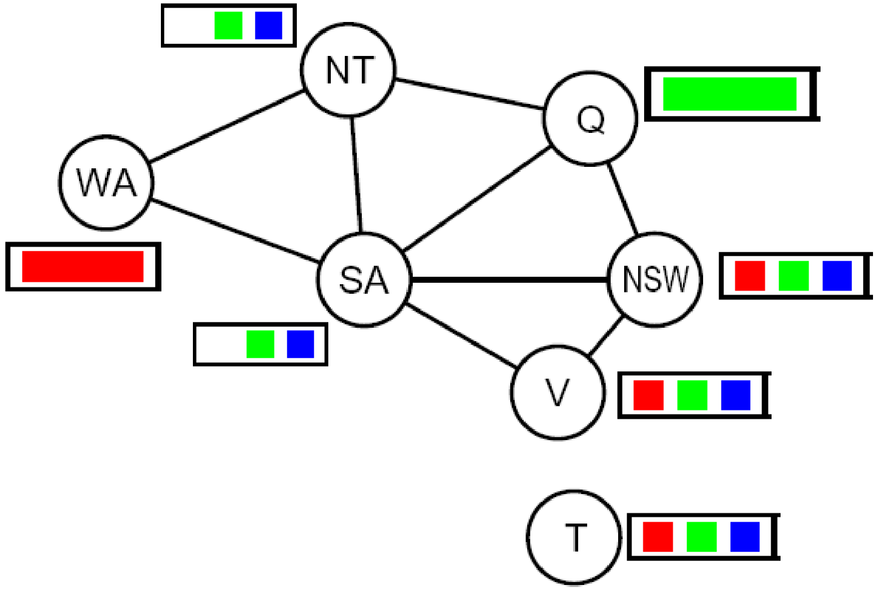


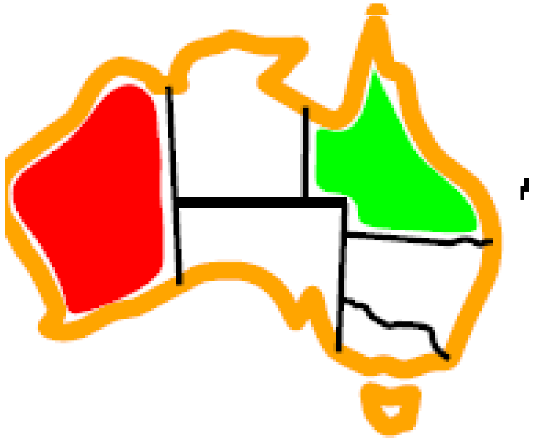


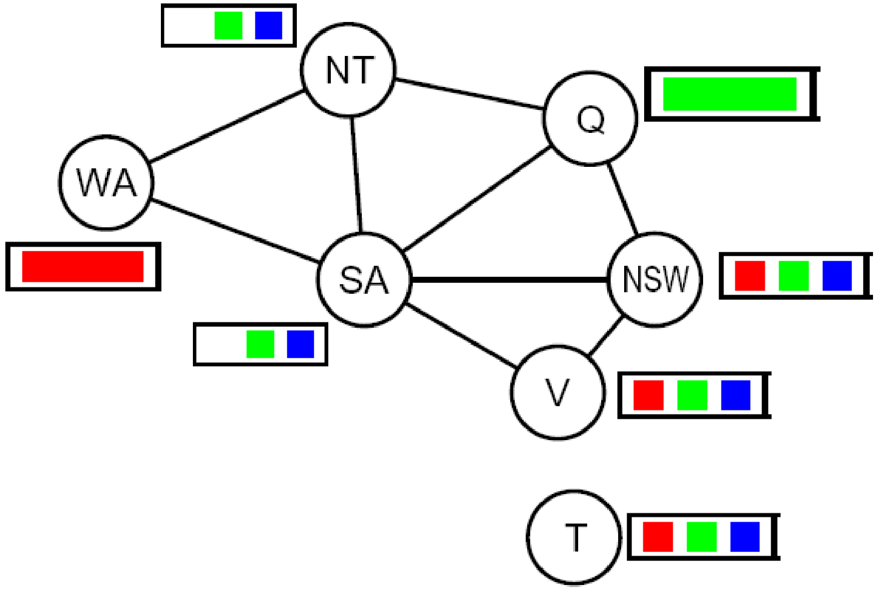
Queue:

Remember: Delete from the tail!

POLL: What gets added to the Queue?

- A simple form of propagation makes sure all arcs are consistent:





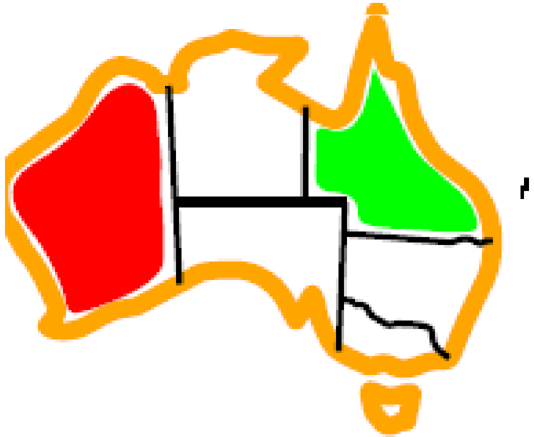
Queue:

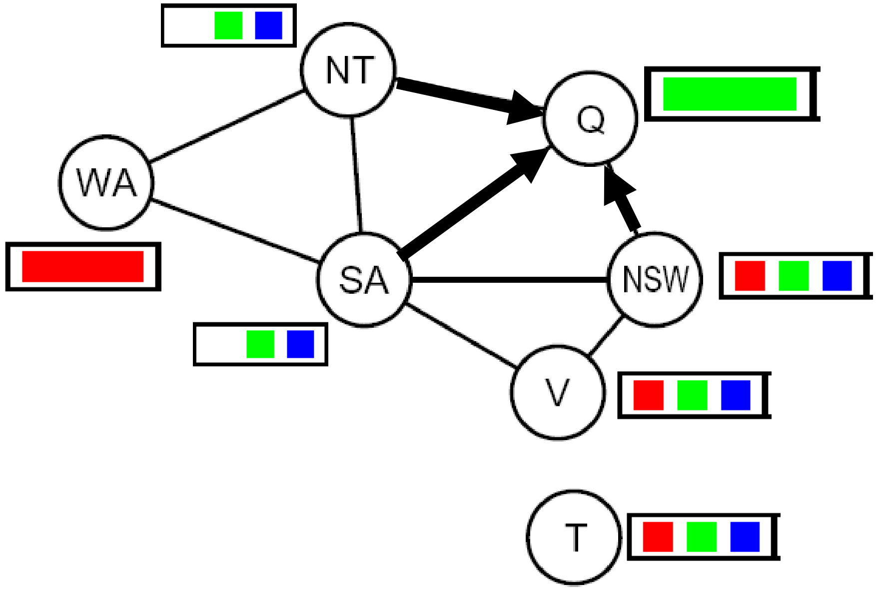
A: NSW->Q, SA->Q, NT->Q

B: Q->NSW, Q->SA, Q->NT

Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are consistent:



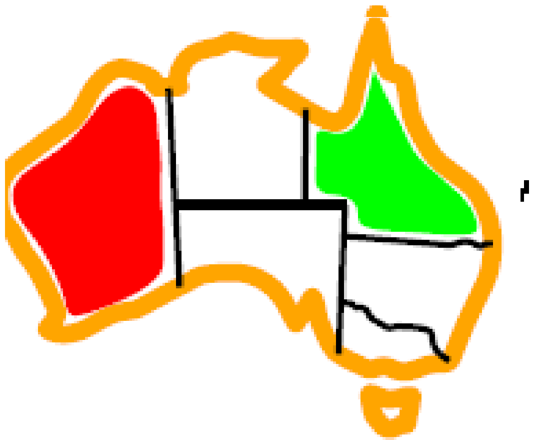


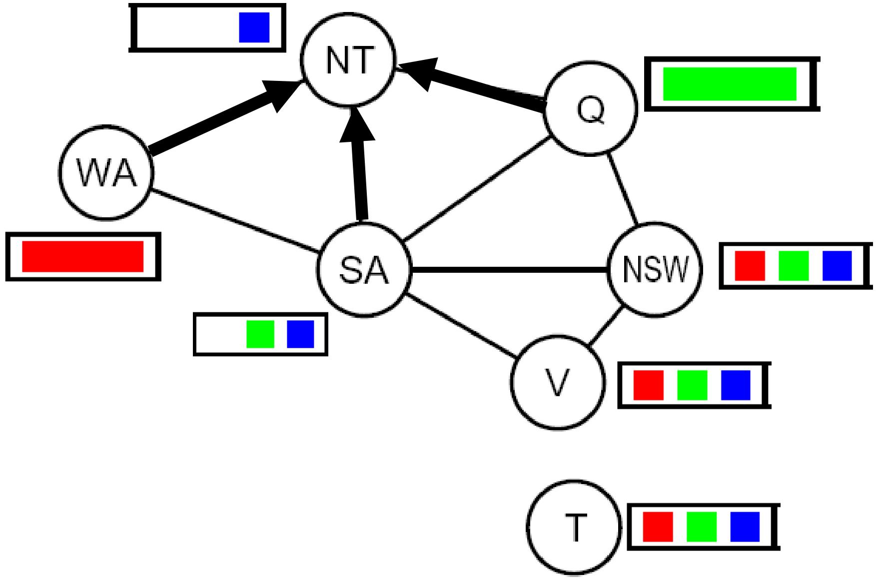
Queue:
NT->Q
SA->Q
NSW->Q

Remember: Delete from the tail!

Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are consistent:





Queue:
SA->Q
NSW->Q
WA->NT
SA->NT
Q->NT

Remember: Delete from the tail!

Arc Consistency of an Entire CSP

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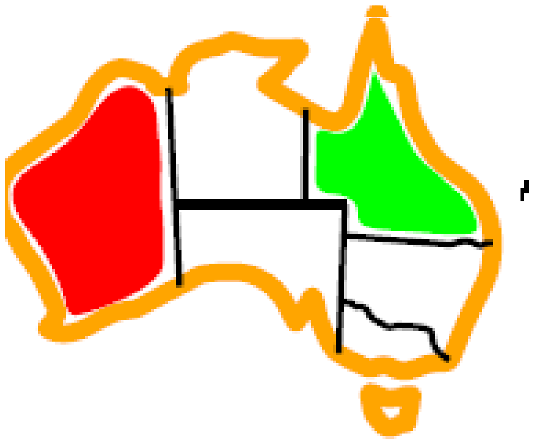
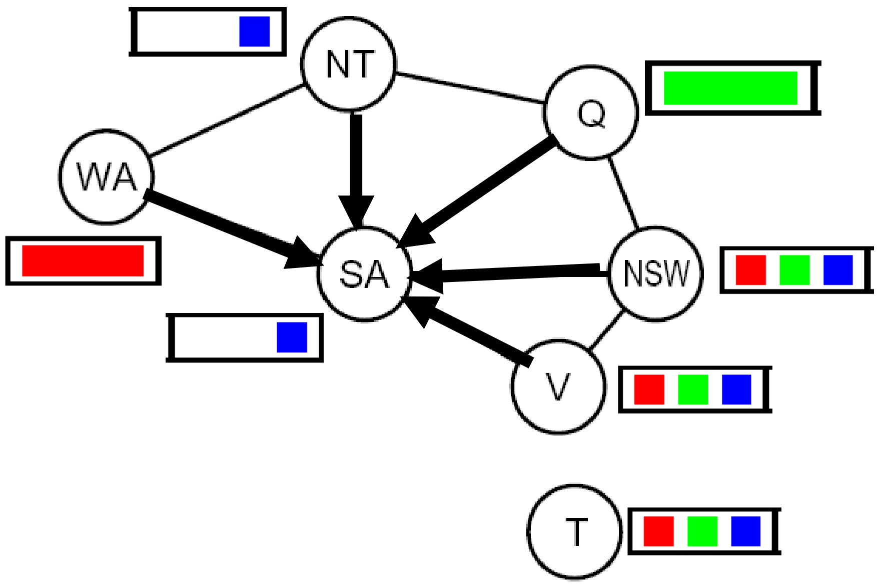


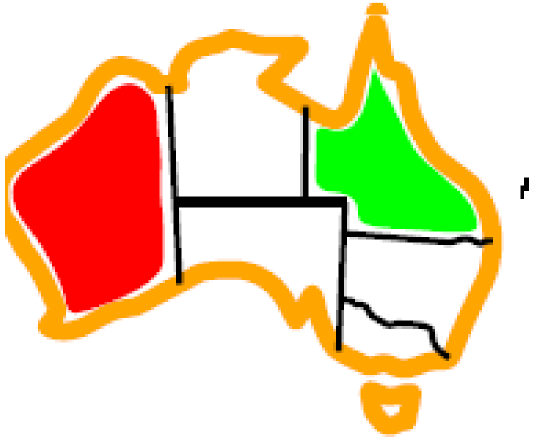


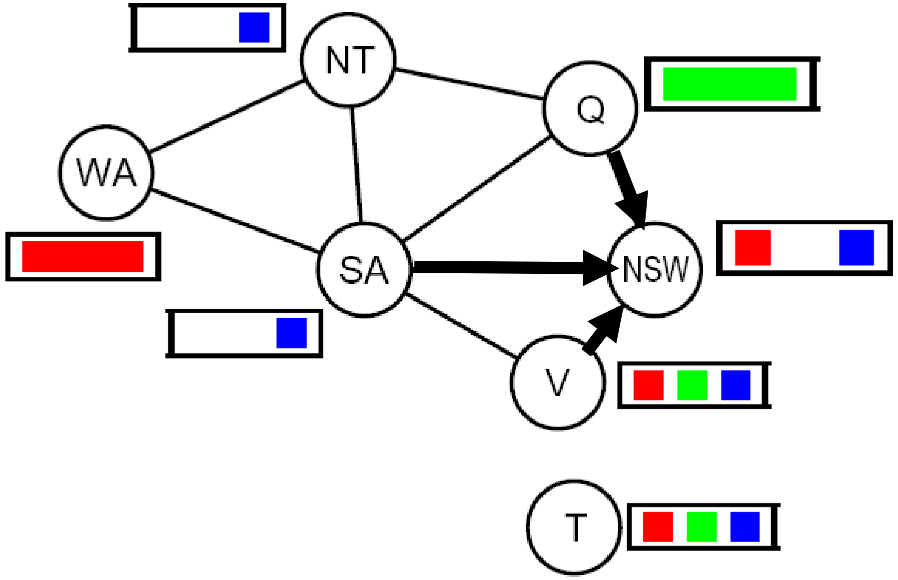
Queue:
NSW->Q
WA->NT
SA->NT
Q->NT
WA->SA
NT->SA
Q->SA
NSW->SA
V->SA

Remember: Delete from the tail!

Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are consistent:





Queue:
WA->NT
SA->NT
Q->NT
WA->SA
NT->SA
Q->SA
NSW->SA
V->SA
V->NSW
Q->NSW
SA->NSW

Remember: Delete from the tail!

Arc Consistency of an Entire CSP

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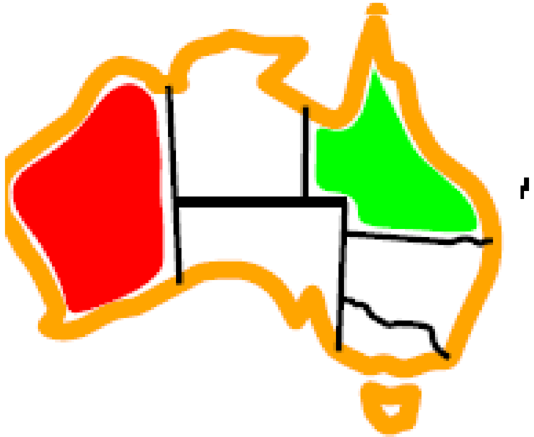

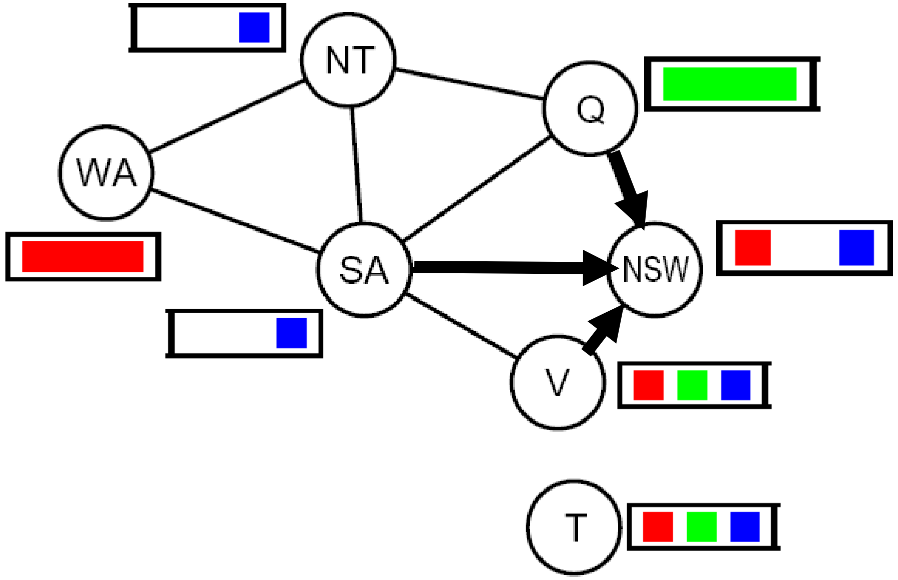

Queue:
WA->NT
SA->NT
Q->NT
WA->SA
NT->SA
Q->SA
NSW->SA
V->SA
V->NSW
Q->NSW
SA->NSW

Remember: Delete from the tail!

Arc Consistency of an Entire CSP

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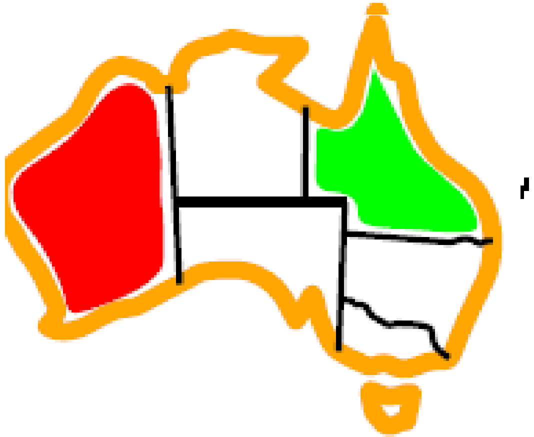

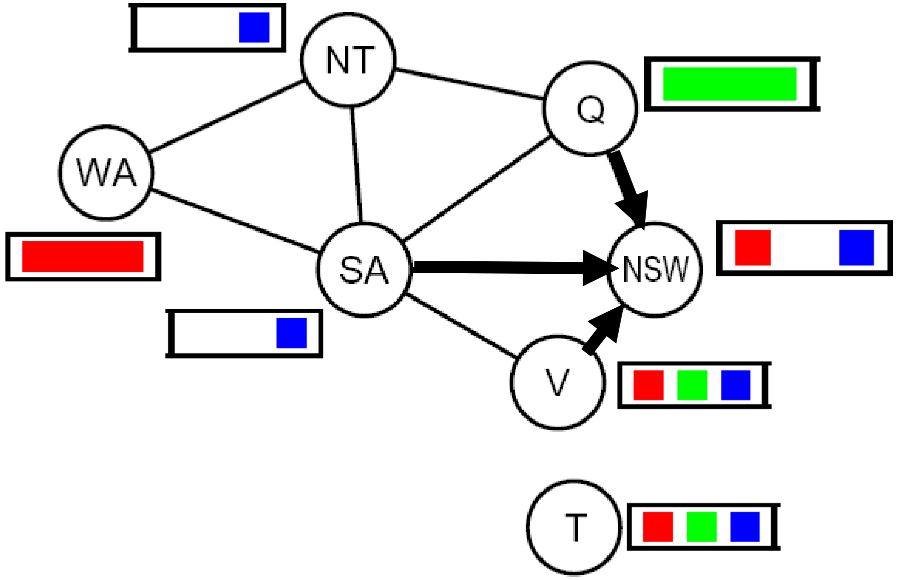

Queue:
SA->NT
Q->NT
WA->SA
NT->SA
Q->SA
NSW->SA
V->SA
V->NSW
Q->NSW
SA->NSW

Remember: Delete from the tail!

Arc Consistency of an Entire CSP

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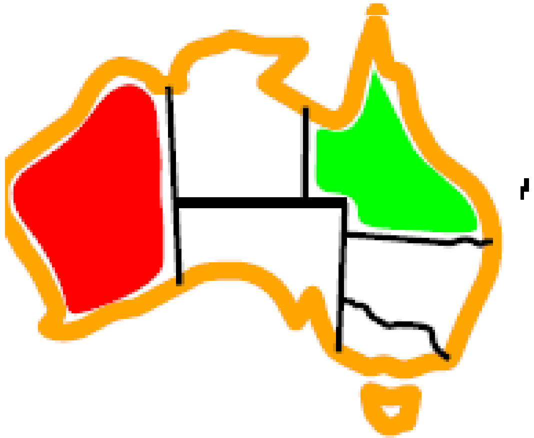

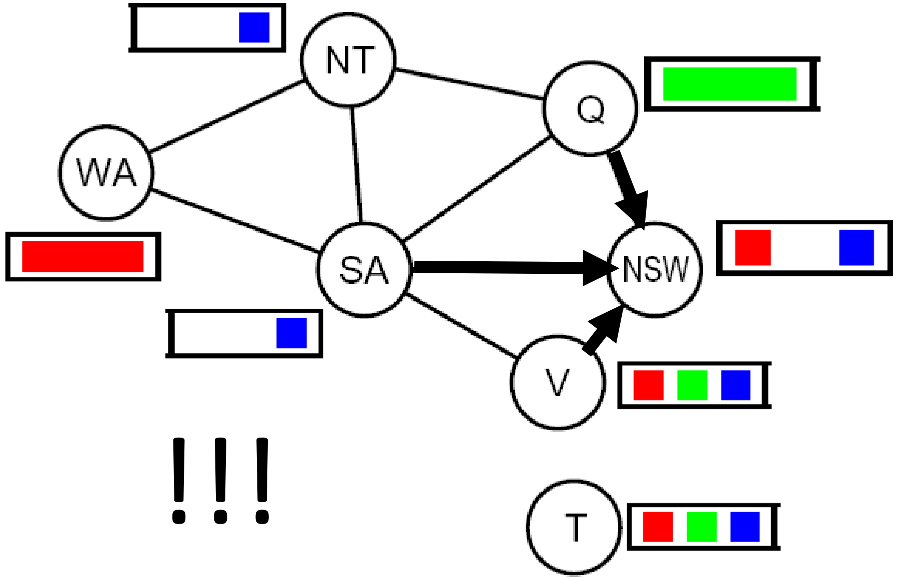

Queue:
SA->NT
Q->NT
WA->SA
NT->SA
Q->SA
NSW->SA
V->SA
V->NSW
Q->NSW
SA->NSW

Remember: Delete from the tail!

Arc Consistency of an Entire CSP

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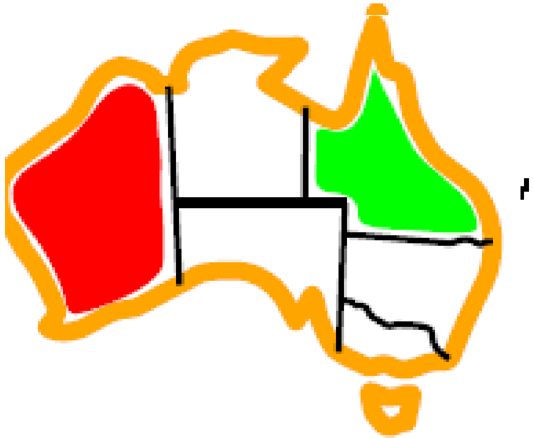
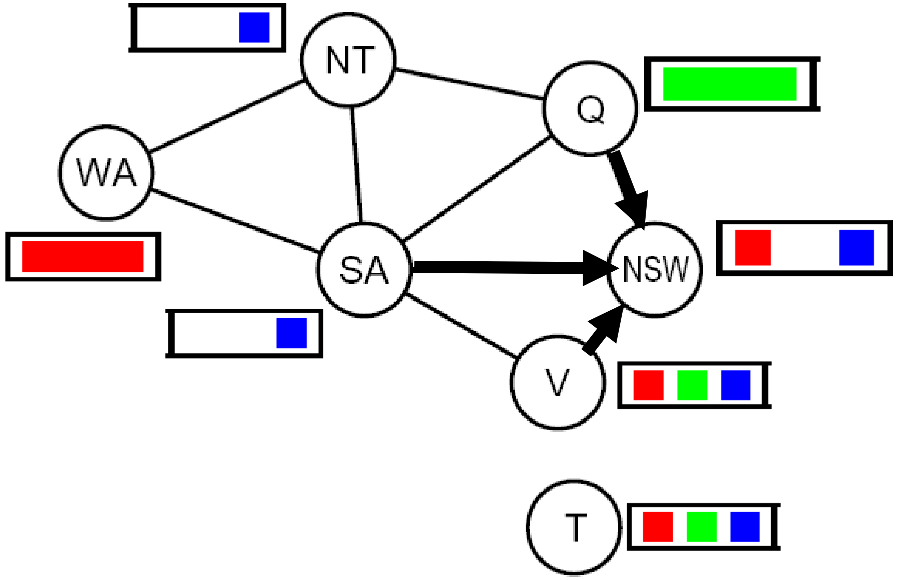




Queue:
SA->NT
Q->NT
WA->SA
NT->SA
Q->SA
NSW->SA
V->SA
V->NSW
Q->NSW
SA->NSW

- Backtrack on the assignment of Q
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment
- What's the downside of enforcing arc consistency?

Remember: Delete from the tail!

Enforcing Arc Consistency in a CSP

```
function AC-3( csp) returns the CSP, possibly with reduced domains
   inputs: csp, a binary CSP with variables {X1, X2, ..., Xn}
   local variables: queue, a queue of arcs, initially all the arcs in csp

   while queue is not empty do
      (Xi, Xj) ← REMOVE-FIRST(queue)
      if REMOVE-INCONSISTENT-VALUES(Xi, Xj) then
         for each Xk in NEIGHBORS[Xi] do
            add (Xk, Xi) to queue
```

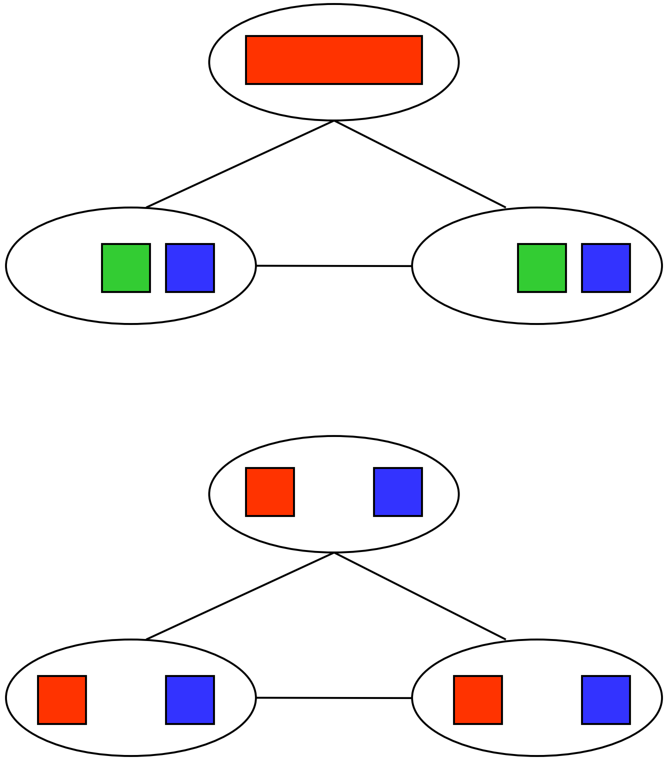
```
function REMOVE-INCONSISTENT-VALUES( Xi, Xj) returns true iff succeeds
   removed ← false
   for each x in DOMAIN[Xi] do
      if no value y in DOMAIN[Xj] allows (x,y) to satisfy the constraint Xi ↔ Xj
         then delete x from DOMAIN[Xi]; removed ← true
   return removed
```

- Runtime: $O(n^2d^3)$, can be reduced to $O(n^2d^2)$
- … but detecting all possible future problems is NP-hard – why?

Limitations of Arc Consistency

- After enforcing arc consistency:
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)

- Arc consistency still runs inside a backtracking search!



What went wrong here?

[Demo: coloring -- forward checking]
[Demo: coloring -- arc consistency]

Demo Coloring – Backtracking with Forward Checking – Complex Graph

Demo Coloring – Backtracking with Arc Consistency – Complex Graph

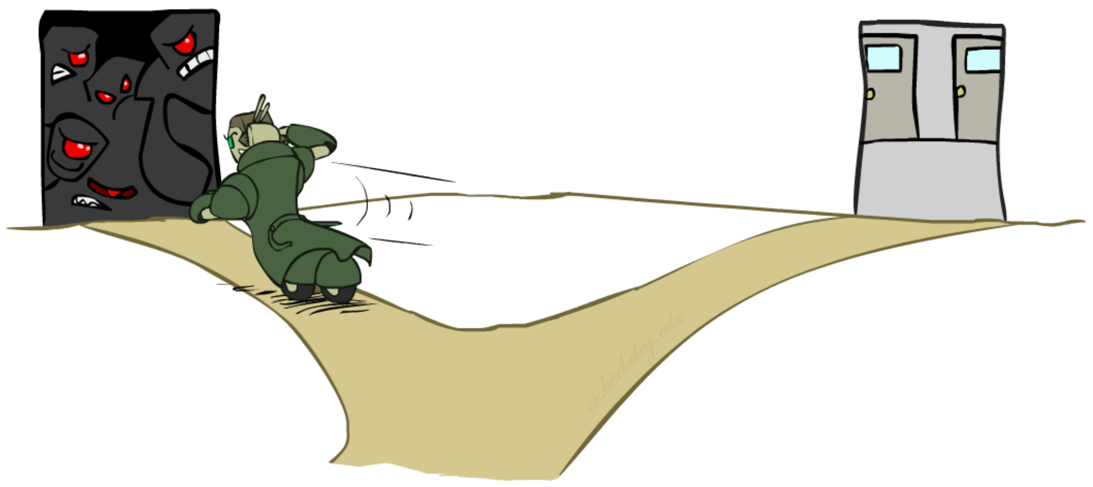
Ordering

Demo: Coloring -- Backtracking + Forward Checking (+ MRV)

Ordering: Minimum Remaining Values

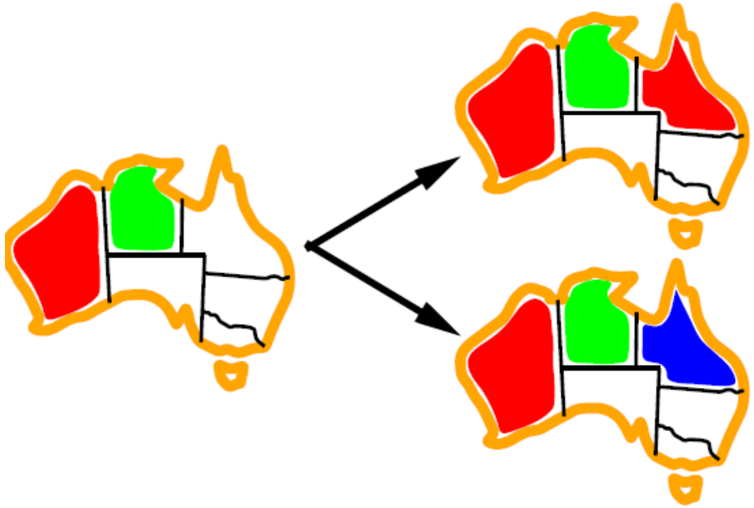
- Variable Ordering: Minimum remaining values (MRV):
  - Choose the variable with the fewest legal left values in its domain

- Why min rather than max?
- Also called “most constrained variable”
- “Fail-fast” ordering



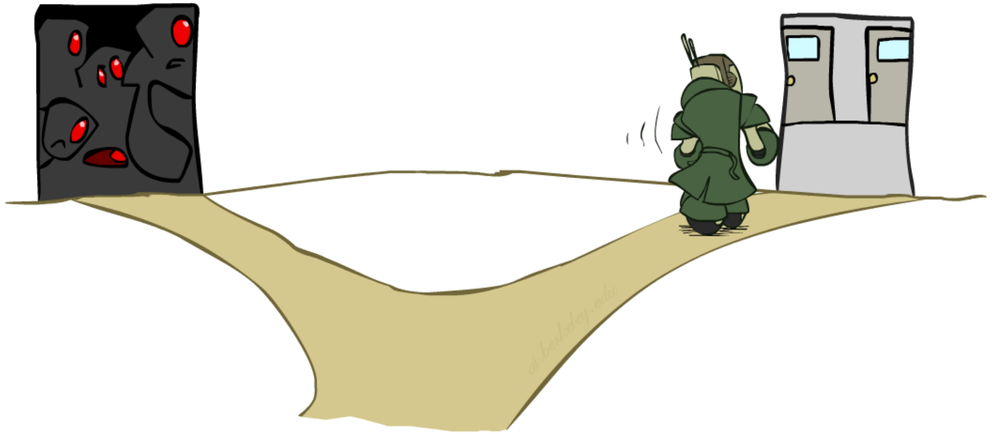
Ordering: Least Constraining Value

- Value Ordering: Least Constraining Value
  - Given a choice of variable, choose the *least constraining value*
  - I.e., the one that rules out the fewest values in the remaining variables
  - Note that it may take some computation to determine this! (E.g., rerunning filtering)



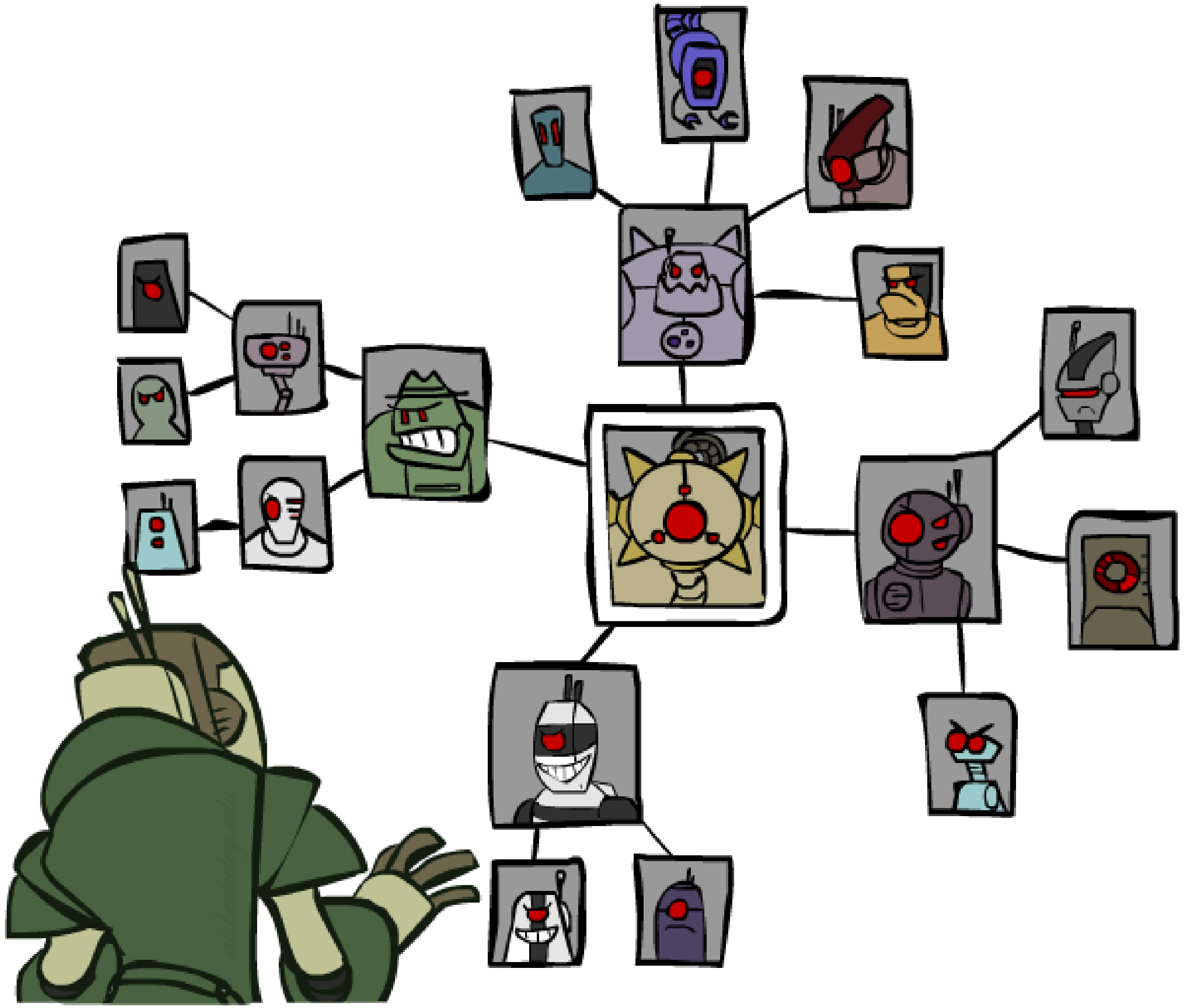
- Why least rather than most?

- Combining these ordering ideas makes 1000 queens feasible



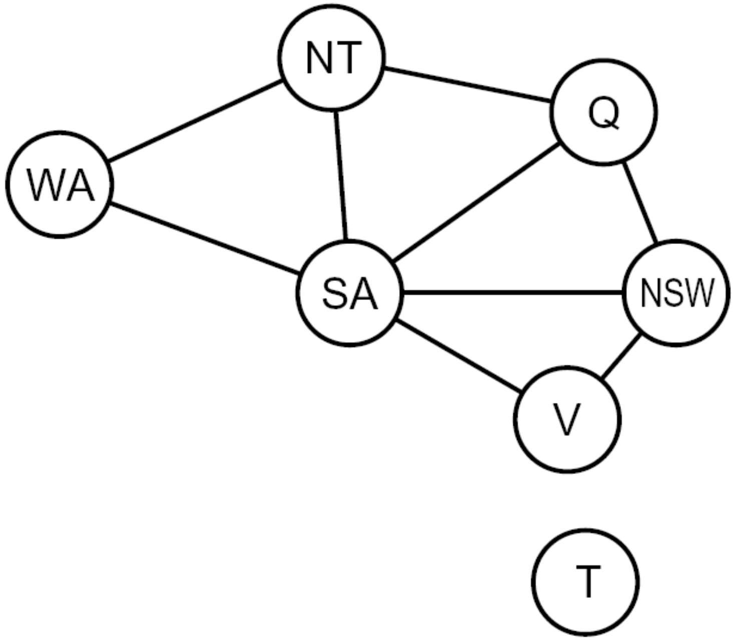
Demo: Coloring -- Backtracking + Arc Consistency + Ordering

Structure

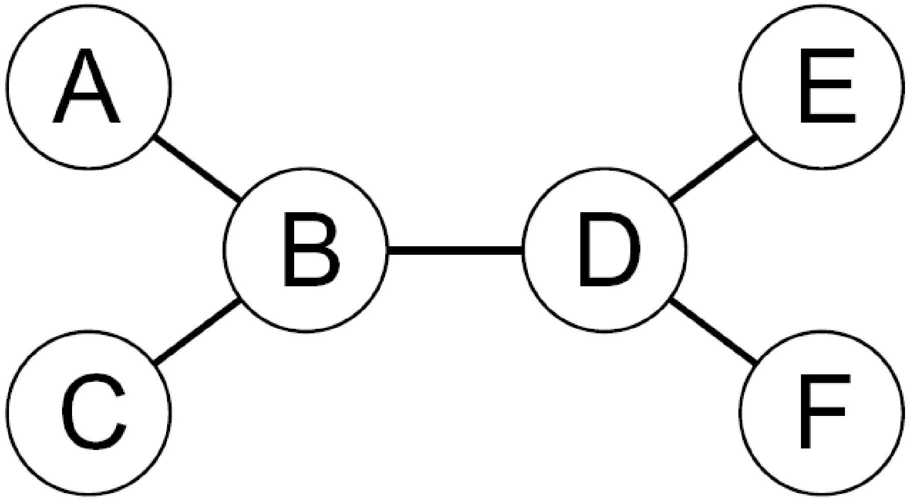


Problem Structure

- Extreme case: independent subproblems
  - Example: Tasmania and mainland do not interact
- Independent subproblems are identifiable as connected components of constraint graph
- Suppose a graph of n variables can be broken into subproblems of only c variables:
  - Worst-case solution cost is $O((n/c)(d^c))$, linear in n
  - E.g., n = 80, d = 2, c =20
  - 2^{80} = 4 billion years at 10 million nodes/sec
  - $(4)(2^{20})$ = 0.4 seconds at 10 million nodes/sec



Tree-Structured CSPs

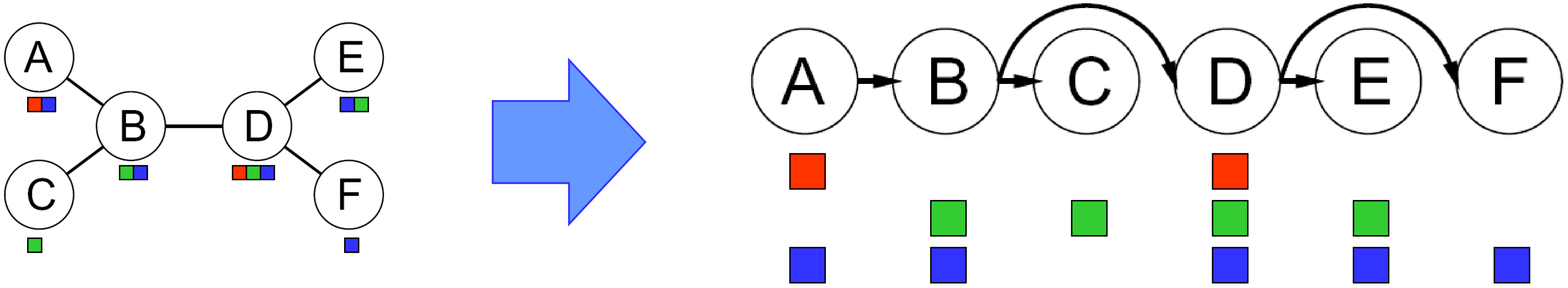


- Theorem: if the constraint graph has no loops, the CSP can be solved in $O(n\ d^2)$ time
  - Compare to general CSPs, where worst-case time is $O(d^n)$

- This property also applies to probabilistic reasoning (later): an example of the relation between syntactic restrictions and the complexity of reasoning

Tree-Structured CSPs

- Algorithm for tree-structured CSPs:
  - Order: Choose a root variable, order variables so that parents precede children

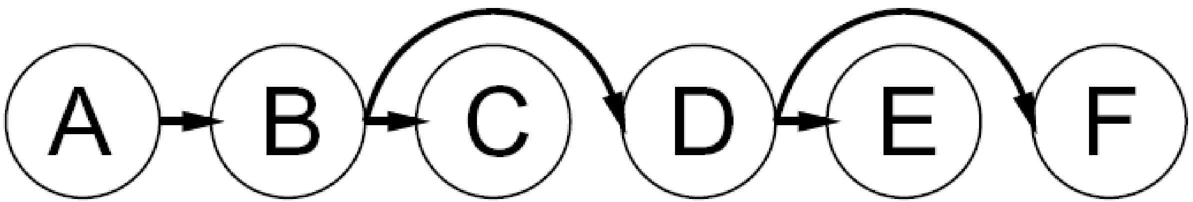


  - Remove backward: For i = n : 2, apply RemoveInconsistent(Parent(X_i),X_i)
  - Assign forward: For i = 1 : n, assign X_i consistently with Parent(X_i)

- Runtime: $O(n\ d^2)$ (why?)

Tree-Structured CSPs

- Claim 1: After backward pass, all root-to-leaf arcs are consistent
- Proof: Each X→Y was made consistent at one point and Y's domain could not have been reduced thereafter (because Y's children were processed before Y)



- Claim 2: If root-to-leaf arcs are consistent, forward assignment will not backtrack
- Proof: Induction on position

- Why doesn't this algorithm work with cycles in the constraint graph?
- Note: we'll see this basic idea again with Bayes' nets

Summary: CSPs

- CSPs are a special kind of search problem:
  - States are partial assignments
  - Goal test defined by constraints
- Basic solution: backtracking search
- Speed-ups:
  - Ordering
  - Filtering
  - Structure