

# Announcements

## Assignments

- HW10
  - Due Wed 4/17
- HW11
  - Plan: Out tomorrow, due Wed 4/24
- P5
  - Plan: Out tonight, due 5/2

# Sampling Wrap-up

# Likelihood Weighting

IN: evidence instantiation

$w = 1.0$

for  $i=1, 2, \dots, n$

▪ if  $X_i$  is an evidence variable

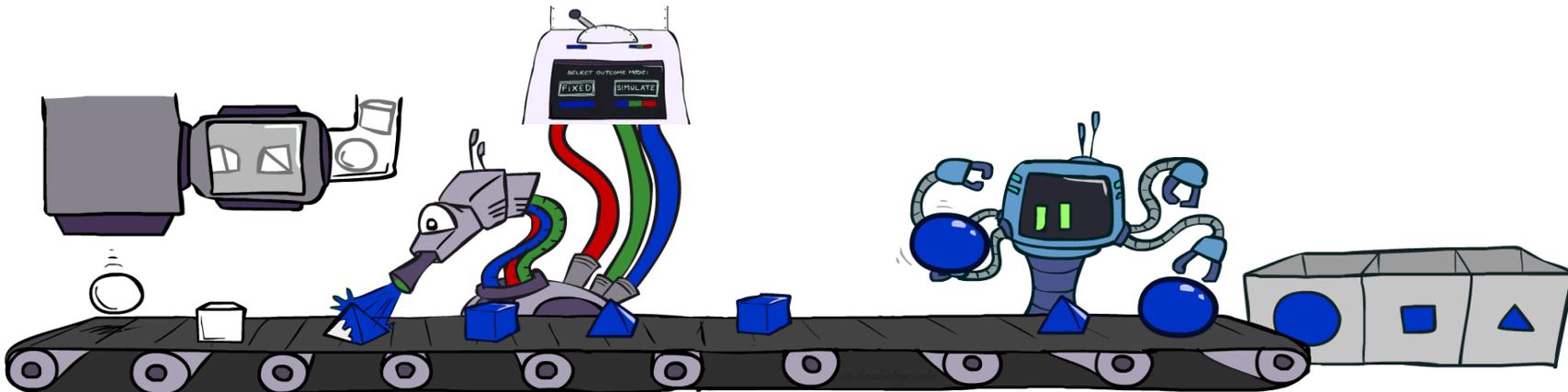
▪  $X_i =$  observation  $x_i$  for  $X_i$

▪ Set  $w = w * P(x_i | \text{Parents}(X_i))$

▪ else

▪ Sample  $x_i$  from  $P(X_i | \text{Parents}(X_i))$

return  $(x_1, x_2, \dots, x_n), w$



# Likelihood Weighting

No evidence:  
Prior Sampling

Input: no evidence

for  $i=1, 2, \dots, n$

- Sample  $x_i$  from  $P(X_i \mid \text{Parents}(X_i))$

return  $(x_1, x_2, \dots, x_n)$

Some evidence:  
Likelihood Weighted Sampling

Input: evidence instantiation

$w = 1.0$

for  $i=1, 2, \dots, n$

if  $X_i$  is an evidence variable

- $X_i = \text{observation } x_i \text{ for } X_i$
- Set  $w = w * P(x_i \mid \text{Parents}(X_i))$

else

- Sample  $x_i$  from  $P(X_i \mid \text{Parents}(X_i))$

return  $(x_1, x_2, \dots, x_n), w$

All evidence:  
Likelihood Weighted

Input: evidence instantiation

$w = 1.0$

for  $i=1, 2, \dots, n$

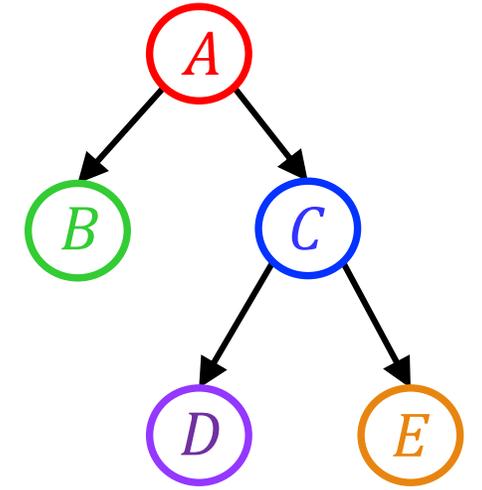
- Set  $w = w * P(x_i \mid \text{Parents}(X_i))$

return  $w$

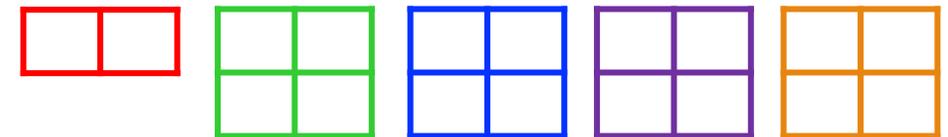
# Likelihood Weighting Distribution

Consistency of likelihood weighted sampling distribution

Joint from Bayes nets



$$\underline{P(A, B, C, D, E)} = P(A) P(B|A) P(C|A) P(D|C) P(E|C)$$

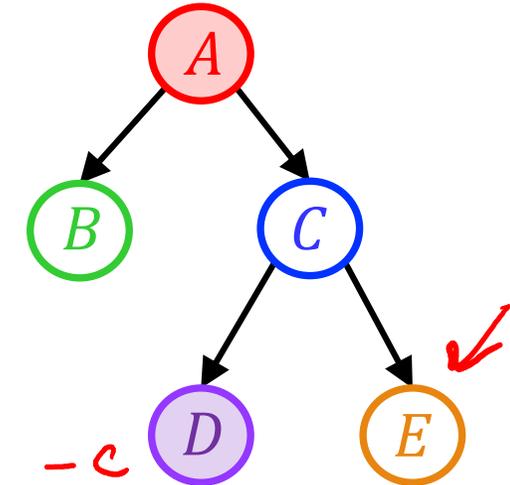


# Likelihood Weighting Distribution

Consistency of likelihood weighted sampling distribution

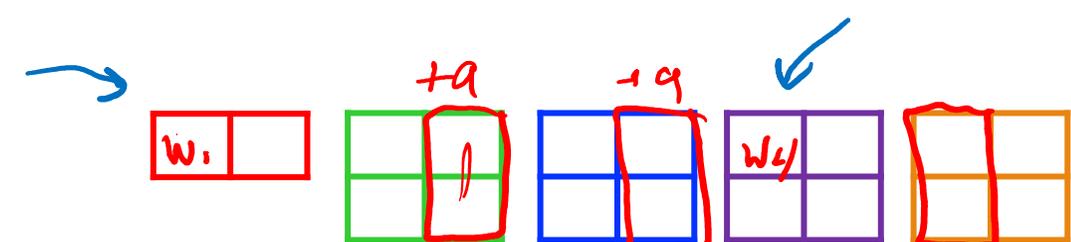
Evidence:  $+a, -d$

Joint from Bayes nets



$$P(A, B, C, D, E) = P(+a) P(\underline{B}|+a) P(C|+a) P(-d|C) P(E|C)$$

$$\begin{array}{cccccc}
 w_1 & +b & -c & w_4 & +e & \\
 \uparrow & & & \uparrow & & \\
 +a & +b & -c & -d & +e & \\
 & & & & & w = w_1 \cdot w_4
 \end{array}$$

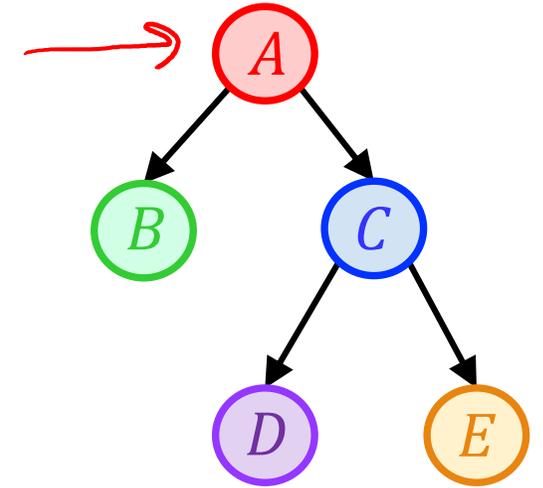


# Likelihood Weighting Distribution

Consistency of likelihood weighted sampling distribution

Evidence:  $+a, +b, -c, -d, +e$

Joint from Bayes nets

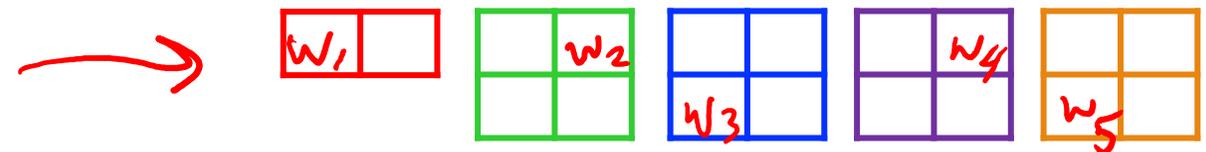


$$P(A, B, C, D, E) = P(+a) P(+b|+a) P(-c|+a) P(-d|-c) P(+e|-c)$$

$w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5$

→  $P(+a, +b, -c, -d, +e)$

$$w = w_1 \cdot w_2 \cdot w_3 \cdot w_4 \cdot w_5$$



# Likelihood Weighting Distribution

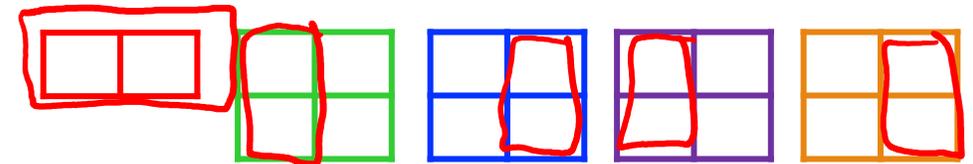
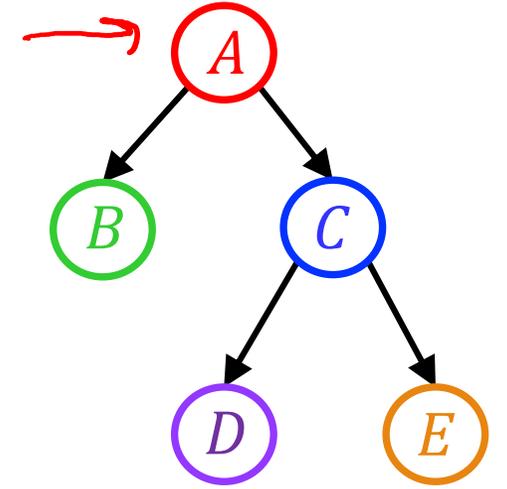
Consistency of likelihood weighted sampling distribution

Evidence: None

Joint from Bayes nets

$$P(A, B, C, D, E) = P(A) P(B|A) P(C|A) P(D|C) P(E|C)$$

$\mathcal{P}(-a, +b, -c, -d, +e)$



# Piazza Poll 1

Two identical samples from likelihood weighted sampling will have the same exact weights.

- A. True
- B. False
- C. It depends
- D. I don't know

# Piazza Poll 1

Two identical samples from likelihood weighted sampling will have the same exact weights.

A. True

B. False

C. It depends

D. I don't know

	<u>+a</u>	B	C	<u>-d</u>	E	
→	+a	+b	-c	-d	+e	$w = w_1 w_4$
→	+a	+b	-c	-d	+e	$w = w_1 w_4$

## Piazza Poll 2

What does the following likelihood weighted value approximate?

$$\text{weight}_{(+a, -b, +c)} \cdot \frac{N(+a, -b, +c)}{N}$$

- A.  $P(+a, -b, +c)$
- B.  $P(+a, -b \mid +c)$
- C. I'm not sure

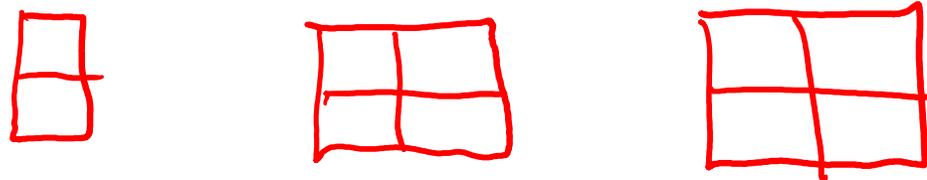
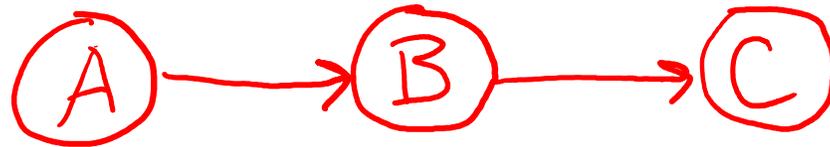
# Piazza Poll 2

What does the following likelihood weighted value approximate?

$$\text{weight}_{(+a, -b, +c)} \cdot \frac{N(+a, -b, +c)}{N}$$

$P(\text{evid} \mid \text{parents}(\text{evid})) \cdot P(\text{not evid} \mid \text{parents}(\text{not evid}))$   
 $= P(\text{evid}, \text{not evid})$

- A.  $P(+a, -b, +c)$
- B.  $P(+a, -b \mid +c)$
- C. I'm not sure



# Likelihood Weighting

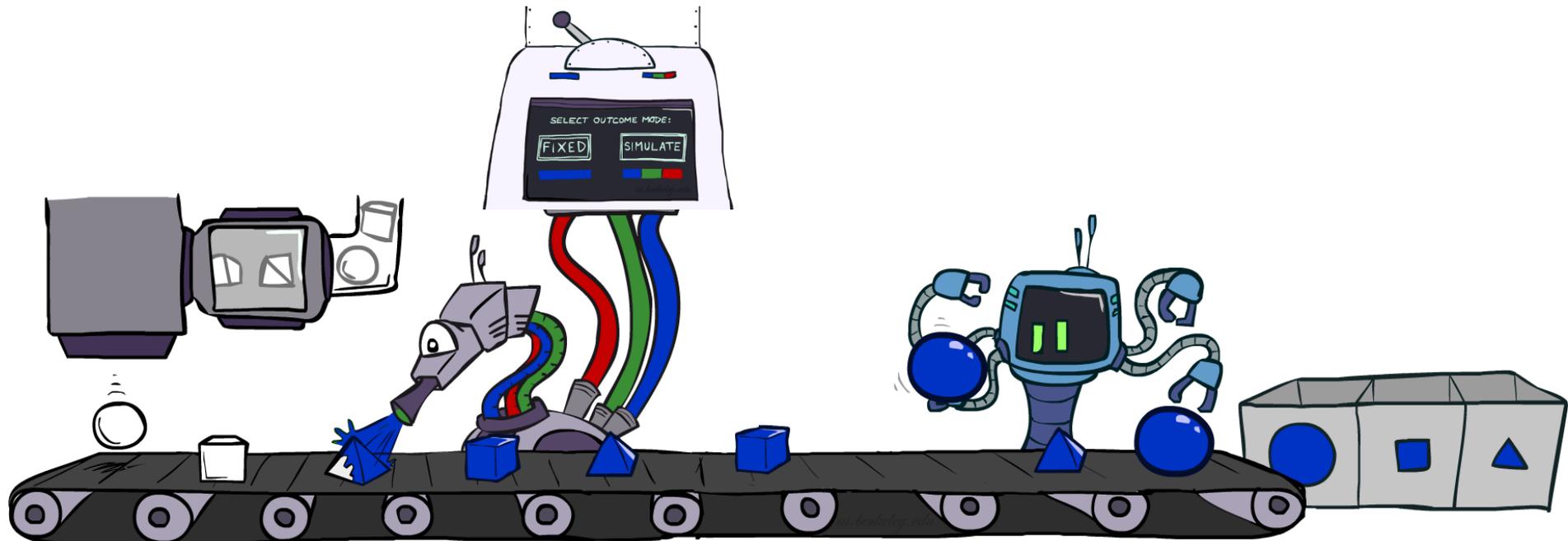
## Likelihood weighting is good

- We have taken evidence into account as we generate the sample
- E.g. here,  $W'$ 's value will get picked based on the evidence values of  $S, R$
- More of our samples will reflect the state of the world suggested by the evidence

## Likelihood weighting doesn't solve all our problems

- Evidence influences the choice of downstream variables, but not upstream ones ( $C$  isn't more likely to get a value matching the evidence)

We would like to consider evidence when we sample every variable



# Likelihood Weighting

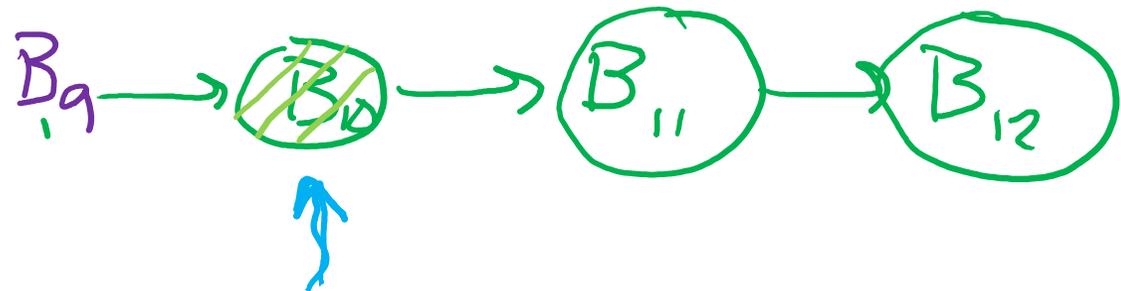
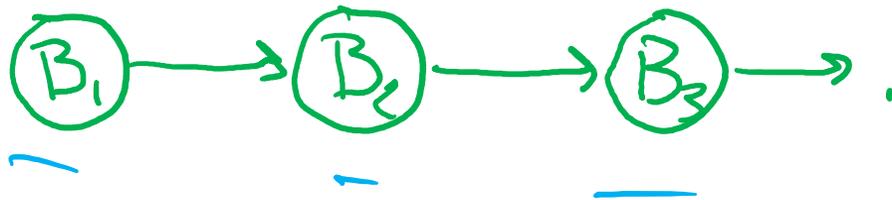


Likelihood weighting doesn't solve all our problems

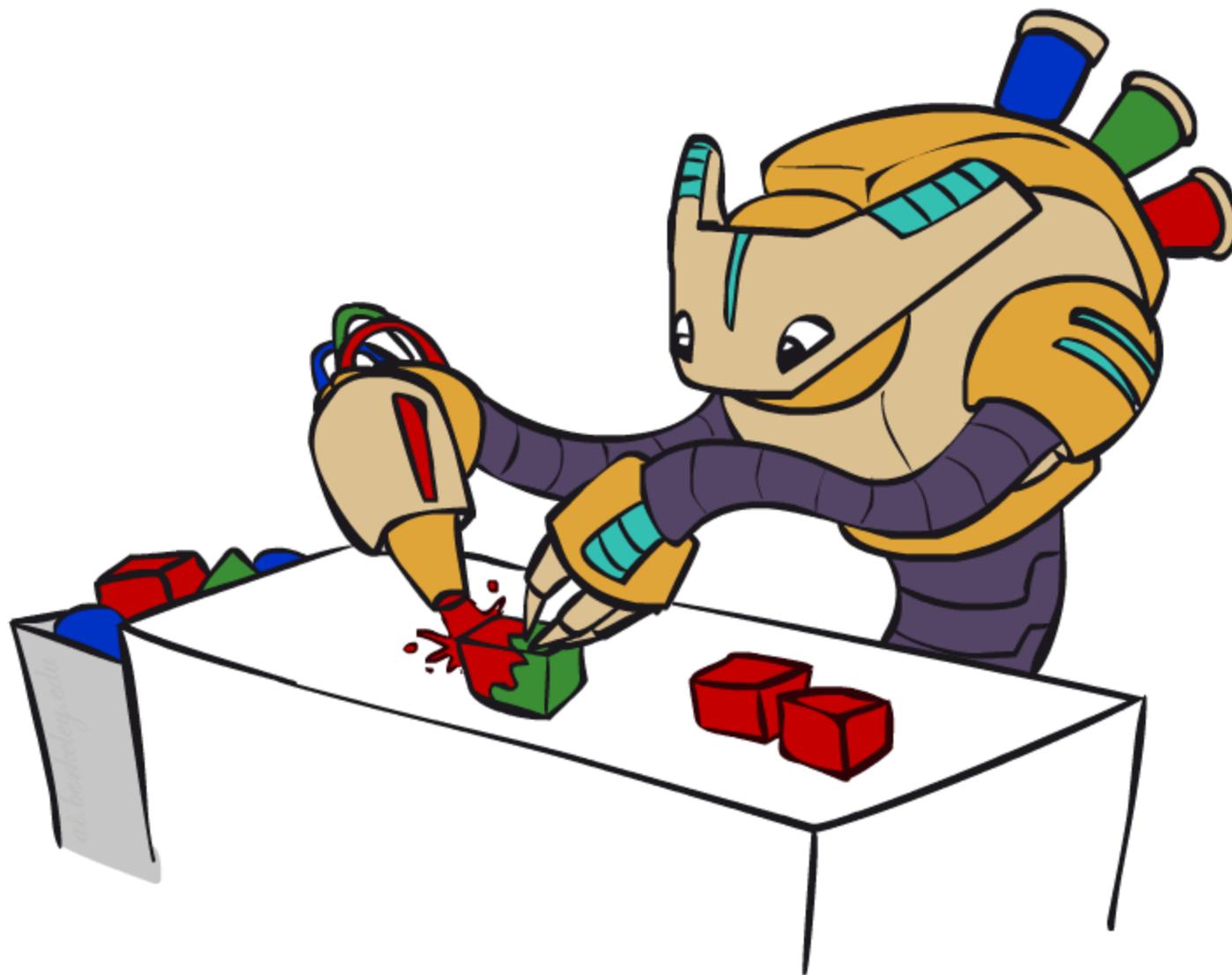
- Evidence influences the choice of downstream variables, but not upstream ones (C isn't more likely to get a value matching the evidence)

We would like to consider evidence when we sample every variable

→ Gibbs sampling



# Gibbs Sampling



# Gibbs Sampling

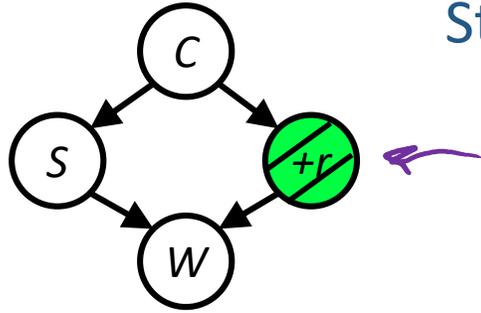
*Procedure:* keep track of a full instantiation  $x_1, x_2, \dots, x_n$ .

1. Start with an arbitrary instantiation consistent with the evidence.
2. Sample one variable at a time, conditioned on all the rest, but keep evidence fixed.
3. Keep repeating this for a long time.

# Gibbs Sampling Example: $P(\underline{S} \mid +r)$

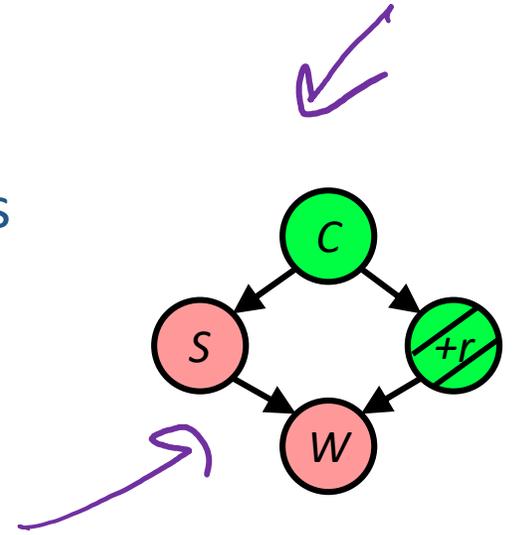
## Step 1: Fix evidence

- $R = +r$



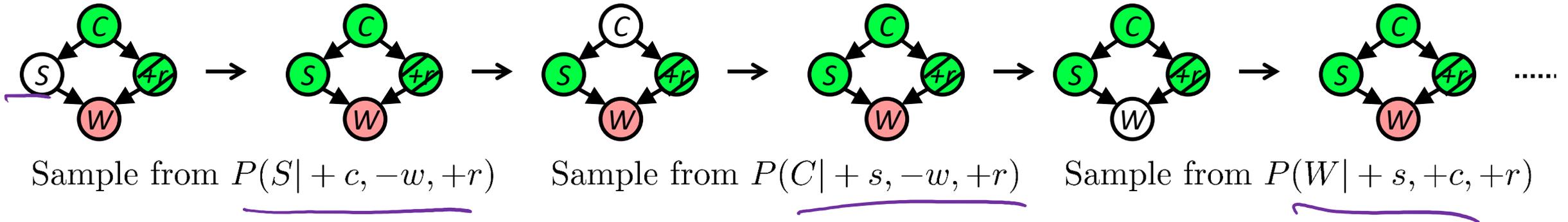
## Step 2: Initialize other variables

- Randomly



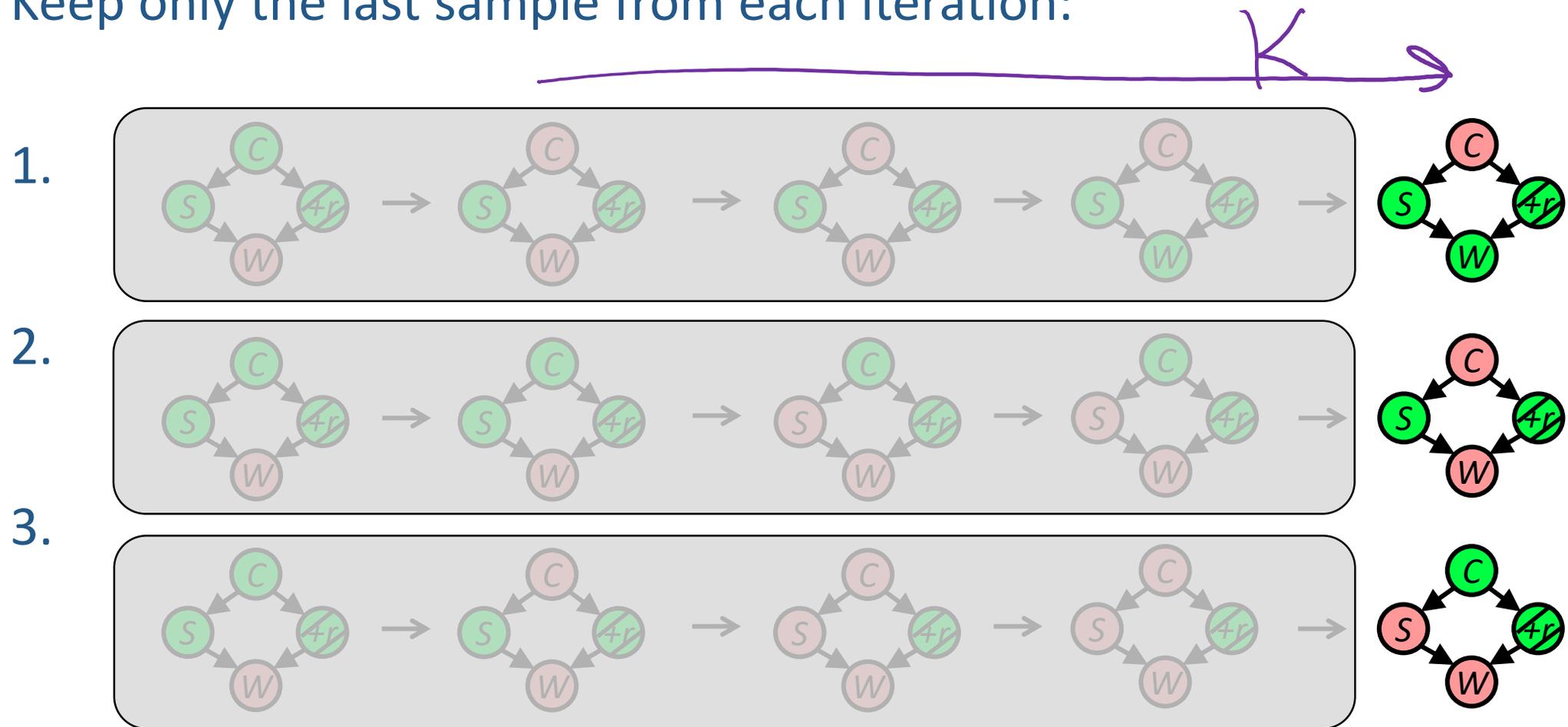
## Steps 3: Repeat

- Choose a non-evidence variable  $X$
- Resample  $X$  from  $P(X \mid \text{all other variables})$



# Gibbs Sampling Example: $P(S \mid +r)$

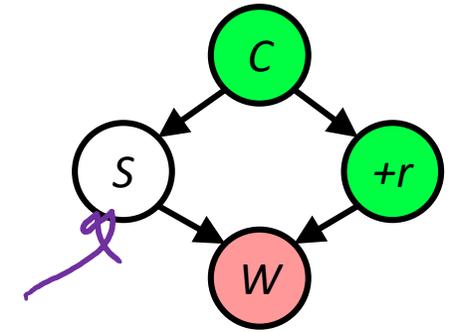
Keep only the last sample from each iteration:



# Efficient Resampling of One Variable

Sample from  $P(S \mid +c, +r, -w)$

$$\begin{aligned}
 \underline{P(S \mid +c, +r, -w)} &= \frac{P(S, +c, +r, -w)}{P(+c, +r, -w)} \\
 &= \frac{P(S, +c, +r, -w)}{\sum_s P(s, +c, +r, -w)} \\
 &= \frac{P(+c)P(S \mid +c)P(+r \mid +c)P(-w \mid S, +r)}{\sum_s P(+c)P(s \mid +c)P(+r \mid +c)P(-w \mid s, +r)} \\
 &= \frac{\cancel{P(+c)}P(S \mid +c)\cancel{P(+r \mid +c)}P(-w \mid S, +r)}{\cancel{P(+c)}\cancel{P(+r \mid +c)}\sum_s P(s \mid +c)P(-w \mid s, +r)} \\
 &= \frac{P(S \mid +c)P(-w \mid S, +r)}{\sum_s P(s \mid +c)P(-w \mid s, +r)}
 \end{aligned}$$



Many things cancel out – only CPTs with S remain!

More generally: only CPTs that have resampled variable need to be considered, and joined together

# Gibbs Sampling

*Procedure:* keep track of a full instantiation  $x_1, x_2, \dots, x_n$ .

1. Start with an arbitrary instantiation consistent with the evidence.
2. Sample one variable at a time, conditioned on all the rest, but keep evidence fixed.
3. Keep repeating this for a long time.

*Property:* in the limit of repeating this infinitely many times the resulting sample is coming from the correct distribution

*Rationale:* both upstream and downstream variables condition on evidence.

In contrast: likelihood weighting only conditions on upstream evidence, and hence weights obtained in likelihood weighting can sometimes be very small. Sum of weights over all samples is indicative of how many “effective” samples were obtained, so want high weight.

# Gibbs Sampling

Gibbs sampling produces sample from the query distribution  $P(Q | e)$  in limit of re-sampling infinitely often

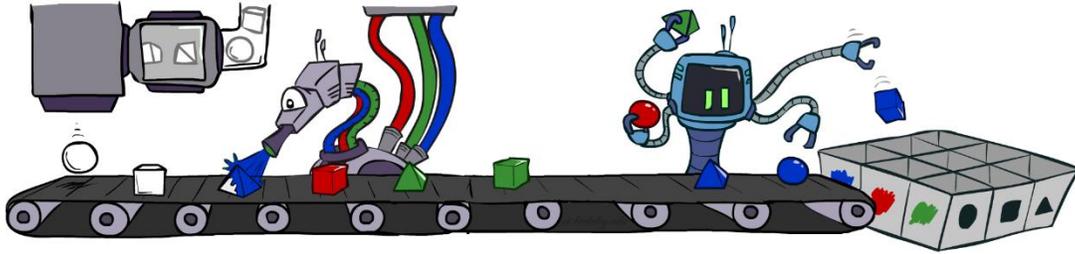
Gibbs sampling is a special case of more general methods called Markov chain Monte Carlo (MCMC) methods

- Metropolis-Hastings is one of the more famous MCMC methods (in fact, Gibbs sampling is a special case of Metropolis-Hastings)

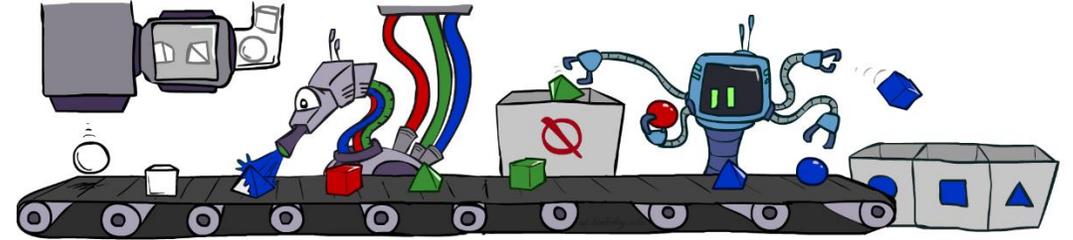
You may read about Monte Carlo methods – they're just sampling

# Bayes' Net Sampling Summary

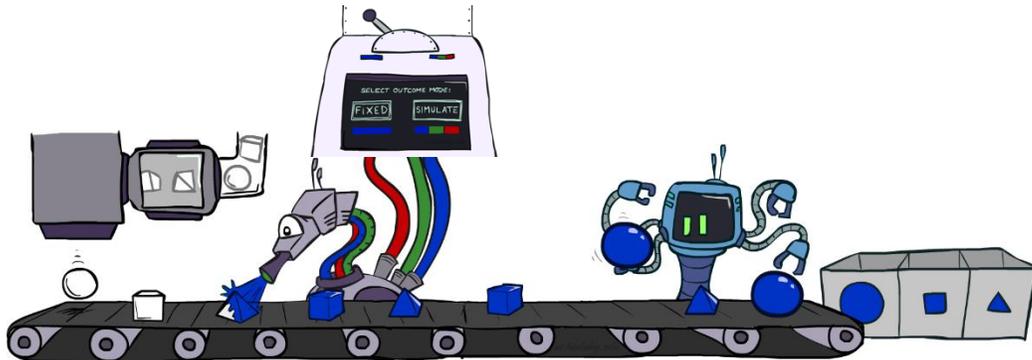
Prior Sampling  $P(Q, E)$



Rejection Sampling  $P(Q | e)$



Likelihood Weighting  $P(Q, e)$



Gibbs Sampling  $P(Q | e)$



# AI: Representation and Problem Solving

## Hidden Markov Models



Instructors: Pat Virtue & Stephanie Rosenthal

Slide credits: CMU AI and <http://ai.berkeley.edu>

# Reasoning over Time or Space

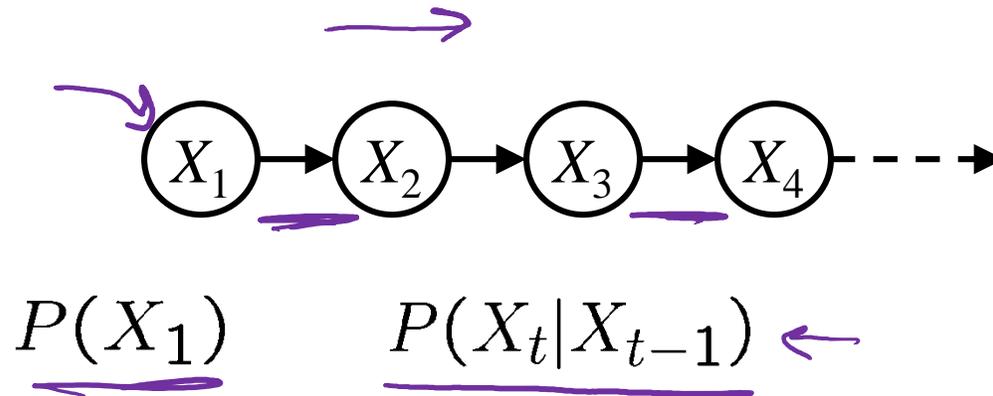
Often, we want to reason about a sequence of observations

- Speech recognition
- Robot localization
- User attention
- Medical monitoring

Need to introduce time (or space) into our models

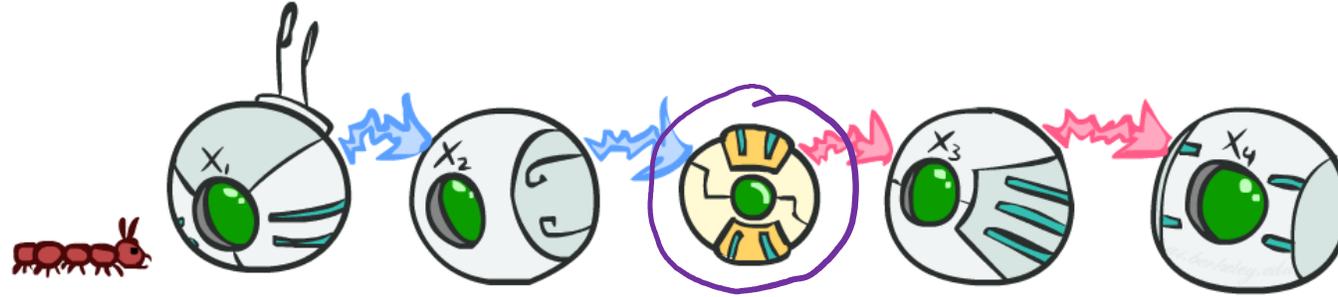
# Markov Models

- Value of  $X$  at a given time is called the **state**



- Parameters: called **transition probabilities** or dynamics, specify how the state evolves over time (also, initial state probabilities)
- Stationarity assumption: transition probabilities the same at all times
- Same as MDP transition model, but no choice of action

# Conditional Independence



## Basic conditional independence:

- Past and future independent given the present
- Each time step only depends on the previous
- This is called the (first order) Markov property

## Note that the chain is just a (growable) BN

- We can always use generic BN reasoning on it if we truncate the chain at a fixed length

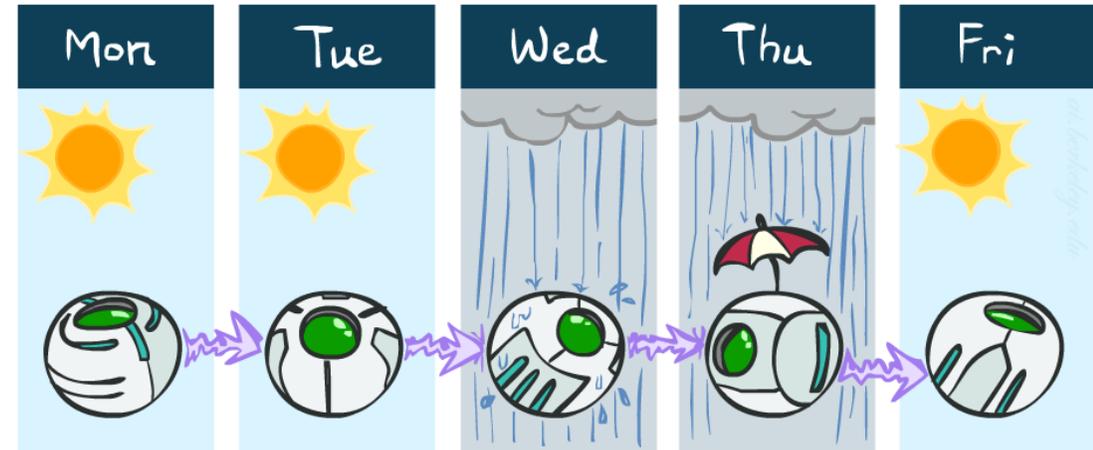
# Example Markov Chain: Weather

States:  $X = \{\text{rain, sun}\}$

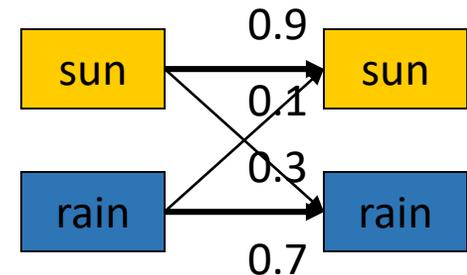
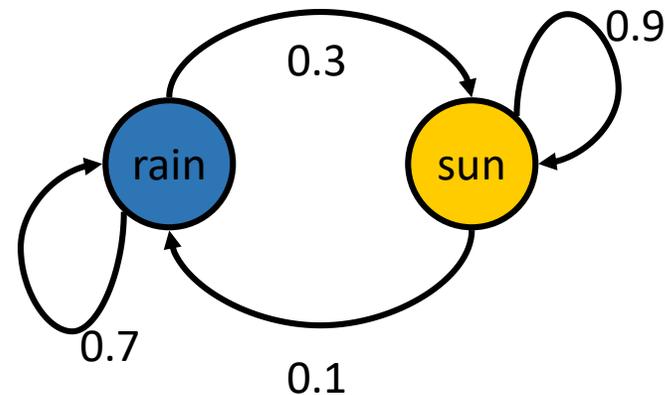
- Initial distribution: 1.0 sun

- CPT  $P(X_t | X_{t-1})$ :

$X_{t-1}$	$X_t$	$P(X_t   X_{t-1})$
sun	sun	0.9
sun	rain	0.1
rain	sun	0.3
rain	rain	0.7



Two new ways of representing the same CPT



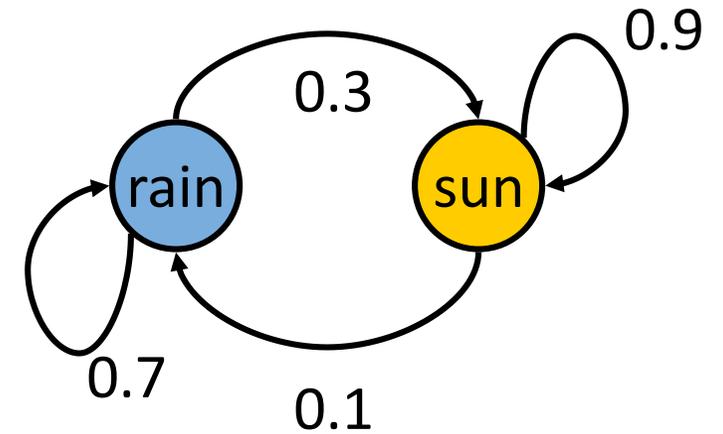
## Piazza Poll 3

Initial distribution:  $P(X_1 = \text{sun}) = 1.0$

What is the probability distribution after one step?

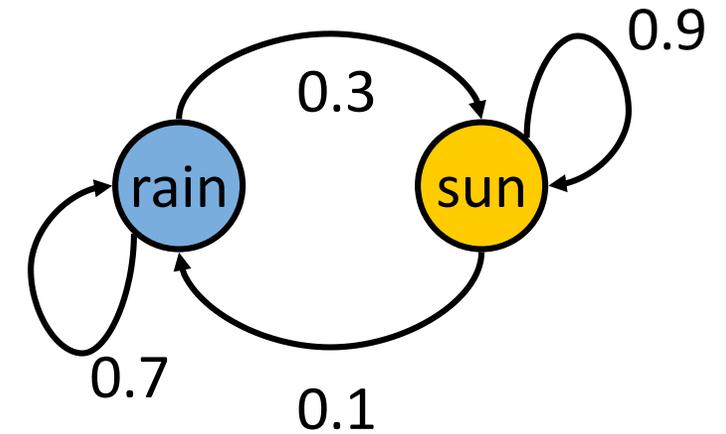
$P(X_2 = \text{sun}) = ?$

- A) 0
- B) 0.3
- C) 0.9
- D) 1.0
- E) 1.2



# Piazza Poll 3

Initial distribution:  $P(X_1 = sun) = 1.0$



What is the probability distribution after one step?

$P(X_2 = sun) = ?$

A) 0

B) 0.3

C) 0.9

D) 1.0

E) 1.2

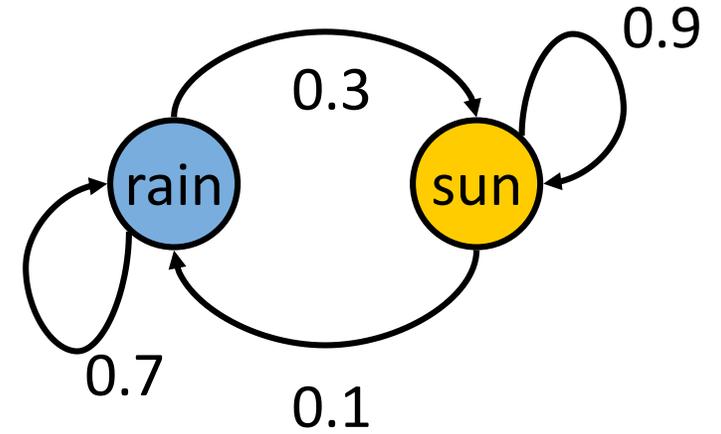
$$\begin{aligned} P(X_2 = sun) &= \sum_{x_1} P(x_1, X_2 = sun) \\ &= P(sun, sun) + P(rain, sun) \\ &= P(x_1 = sun)P(x_2 = sun | x_1 = sun) + \\ &\quad P(x_1 = rain)P(x_2 = sun | x_1 = rain) \end{aligned}$$

# Piazza Poll 3

Initial distribution:  $P(X_1 = \text{sun}) = 1.0$

What is the probability distribution after one step?

$P(X_2 = \text{sun}) = ?$



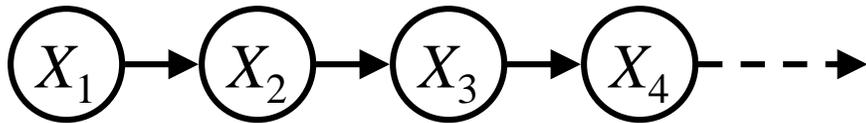
- A) 0
- B) 0.3
- C) 0.9
- D) 1.0
- E) 1.2

$$P(X_2 = \text{sun}) = P(X_2 = \text{sun} | X_1 = \text{sun})P(X_1 = \text{sun}) + P(X_2 = \text{sun} | X_1 = \text{rain})P(X_1 = \text{rain})$$

$$0.9 \cdot 1.0 + 0.3 \cdot 0.0 = 0.9$$

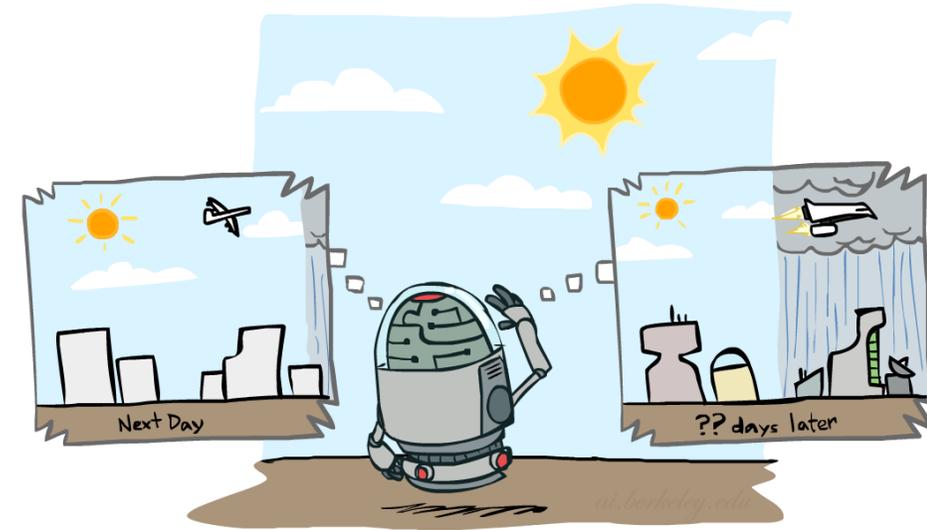
# Mini-Forward Algorithm

Question: What's  $P(X)$  on some day  $t$ ?



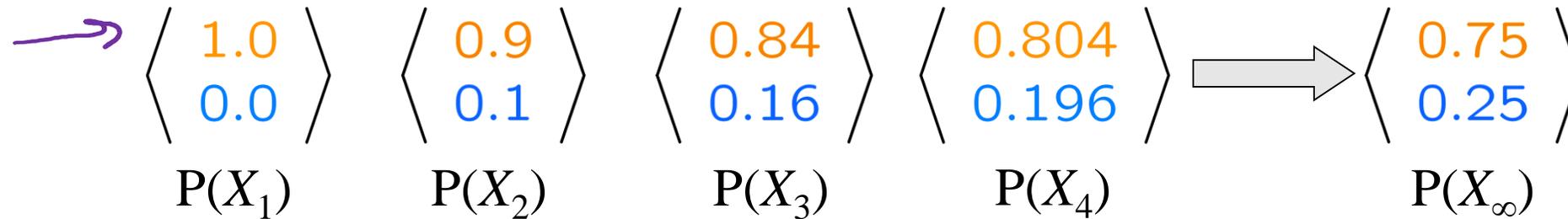
$P(x_1)$  = known

$$\begin{aligned} P(x_t) &= \sum_{x_{t-1}} P(x_{t-1}, x_t) \\ &= \sum_{x_{t-1}} \underbrace{P(x_t | x_{t-1})}_{\text{Forward simulation}} P(x_{t-1}) \end{aligned}$$

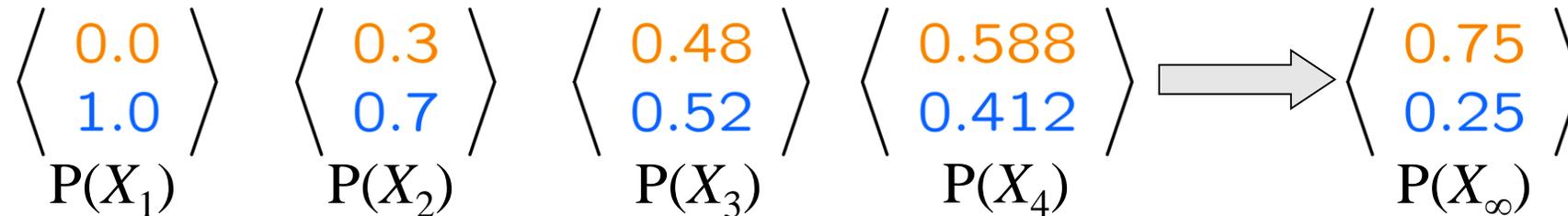


# Example Run of Mini-Forward Algorithm

- From initial observation of sun



- From initial observation of rain



- From yet another initial distribution  $P(X_1)$ :



# Demo Ghostbusters Basic Dynamics

# Demo Ghostbusters Circular Dynamics

# Demo Ghostbusters Whirlpool Dynamics

# Stationary Distributions

## For most chains:

- Influence of the initial distribution gets less and less over time.
- The distribution we end up in is independent of the initial distribution

## ▪ Stationary distribution:

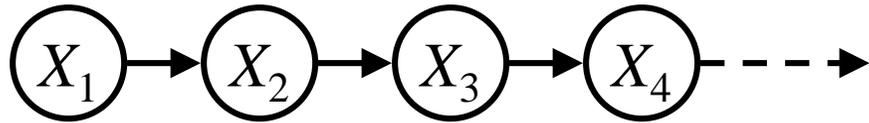
- The distribution we end up with is called the stationary distribution  $P_\infty$  of the chain
- It satisfies

$$P_\infty(X) = P_{\infty+1}(X) = \sum_x P(X|x)P_\infty(x)$$



# Example: Stationary Distributions

Question: What's  $P(X)$  at time  $t = \text{infinity}$ ?



$$P_{\infty}(\text{sun}) = P(\text{sun}|\text{sun})P_{\infty}(\text{sun}) + P(\text{sun}|\text{rain})P_{\infty}(\text{rain})$$

$$P_{\infty}(\text{rain}) = P(\text{rain}|\text{sun})P_{\infty}(\text{sun}) + P(\text{rain}|\text{rain})P_{\infty}(\text{rain})$$

$$P_{\infty}(\text{sun}) = 0.9P_{\infty}(\text{sun}) + 0.3P_{\infty}(\text{rain})$$

$$P_{\infty}(\text{rain}) = 0.1P_{\infty}(\text{sun}) + 0.7P_{\infty}(\text{rain})$$

$$P_{\infty}(\text{sun}) = 3P_{\infty}(\text{rain})$$

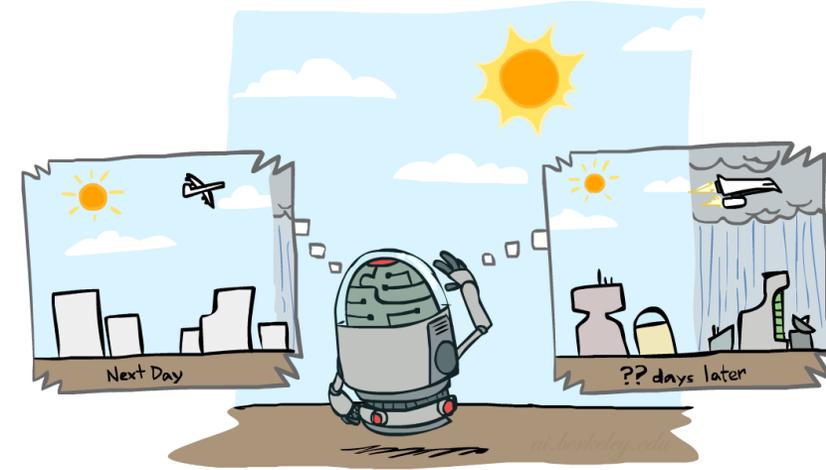
$$P_{\infty}(\text{rain}) = 1/3P_{\infty}(\text{sun})$$

Also:  $P_{\infty}(\text{sun}) + P_{\infty}(\text{rain}) = 1$



$$P_{\infty}(\text{sun}) = 3/4$$

$$P_{\infty}(\text{rain}) = 1/4$$

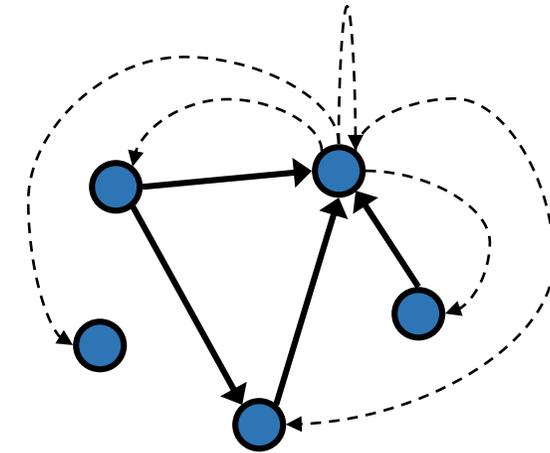


$X_{t-1}$	$X_t$	$P(X_t X_{t-1})$
sun	sun	0.9
sun	rain	0.1
rain	sun	0.3
rain	rain	0.7

# Application of Stationary Distribution: Web Link Analysis

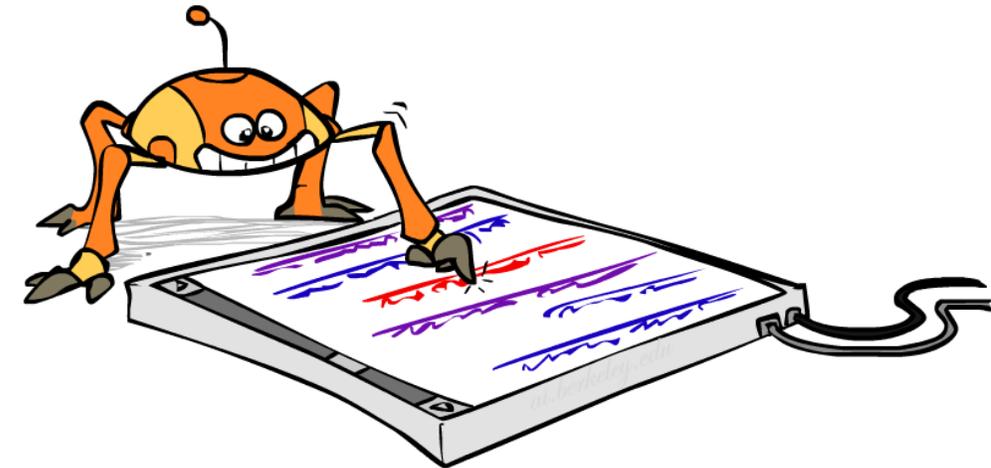
## PageRank over a web graph

- Each web page is a state
- Initial distribution: uniform over pages
- Transitions:
  - With prob.  $c$ , uniform jump to a random page (dotted lines, not all shown)
  - With prob.  $1-c$ , follow a random outlink (solid lines)



## Stationary distribution

- Will spend more time on highly reachable pages
- E.g. many ways to get to the Acrobat Reader download page
- Somewhat robust to link spam
- Google 1.0 returned the set of pages containing all your keywords in decreasing rank, now all search engines use link analysis along with many other factors (rank actually getting less important over time)



# Application of Stationary Distributions: Gibbs Sampling\*

Each joint instantiation over all hidden and query variables is a state:  $\{X_1, \dots, X_n\} = H \cup Q$

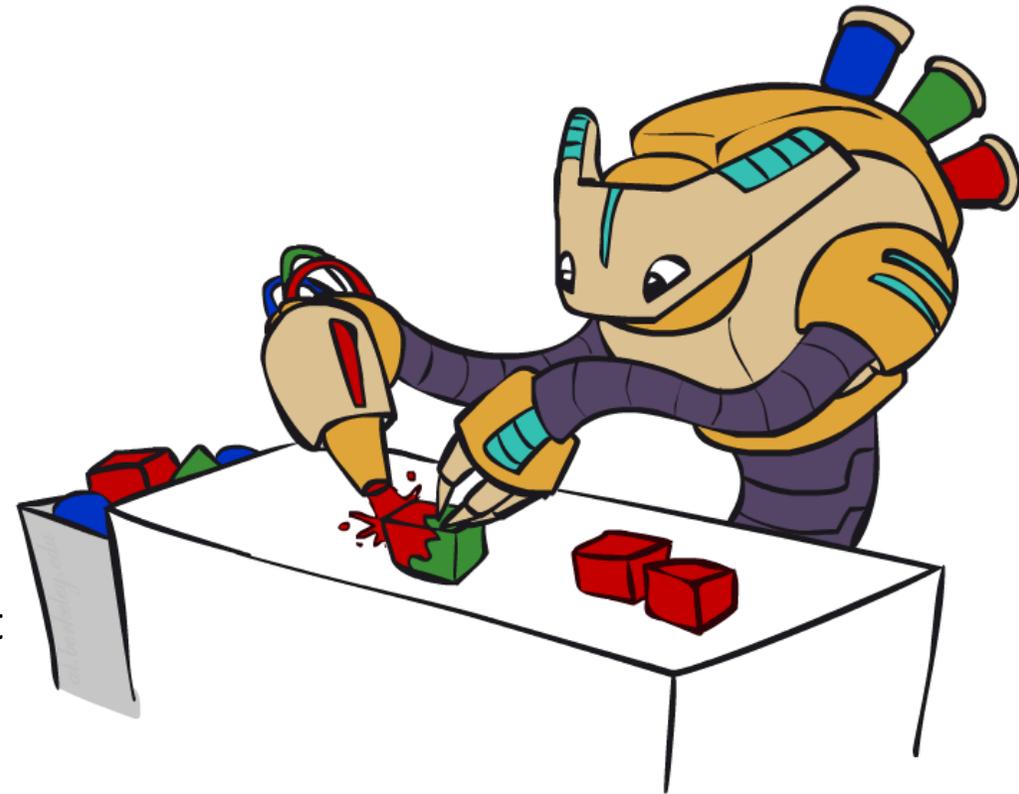
Transitions:

- With probability  $1/n$  resample variable  $X_j$  according to

$$P(X_j \mid x_1, x_2, \dots, x_{j-1}, x_{j+1}, \dots, x_n, e_1, \dots, e_m)$$

Stationary distribution:

- Conditional distribution  $P(X_1, X_2, \dots, X_n \mid e_1, \dots, e_m)$
- Means that when running Gibbs sampling long enough we get a sample from the desired distribution
- Requires some proof to show this is true!



# Quick Break

[How the real Ghostbusters make decisions](#)

# Hidden Markov Models



# Pacman – Sonar (P5)



Demo Pacman – Sonar (no beliefs)

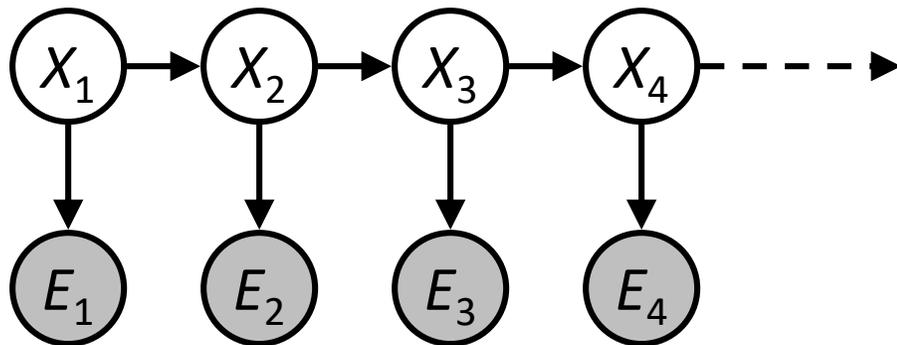
# Hidden Markov Models

Markov chains not so useful for most agents

- Need observations to update your beliefs

Hidden Markov models (HMMs)

- Underlying Markov chain over states  $X$
- You observe outputs (effects) at each time step

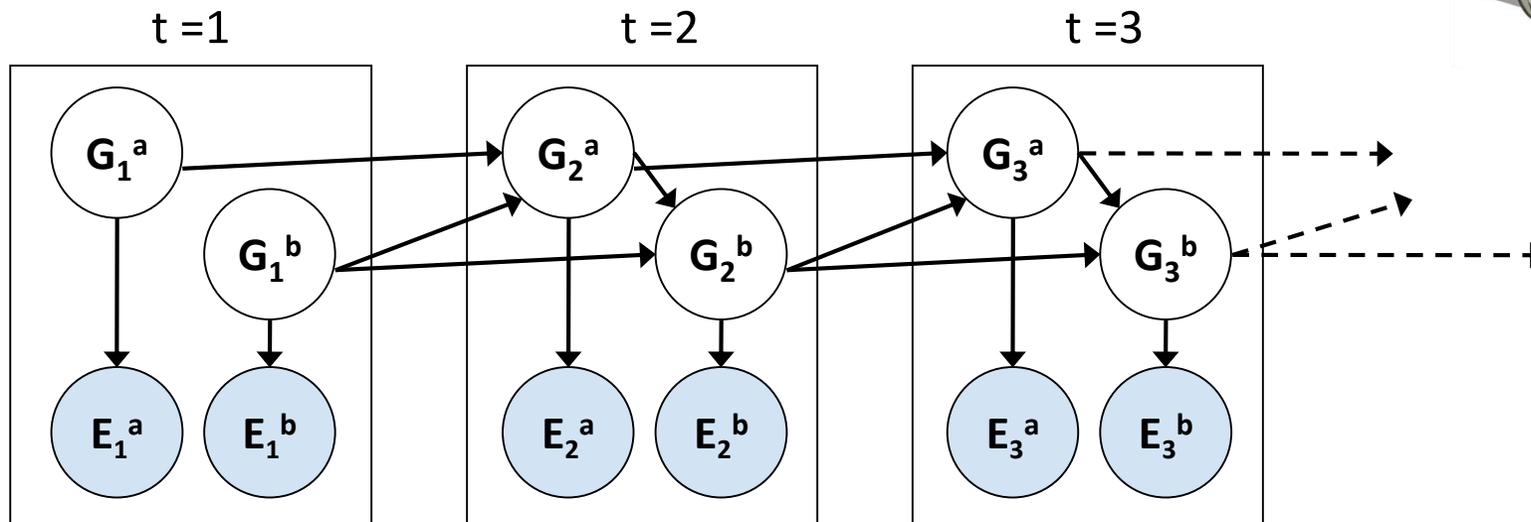
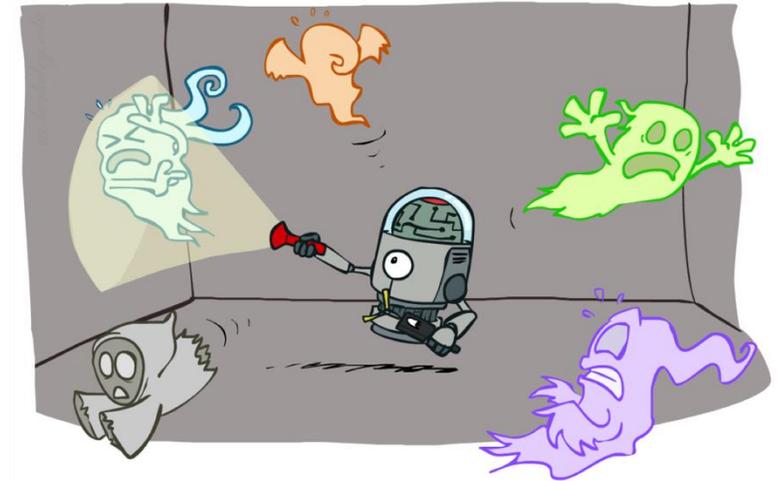


# Dynamic Bayes Nets (DBNs)

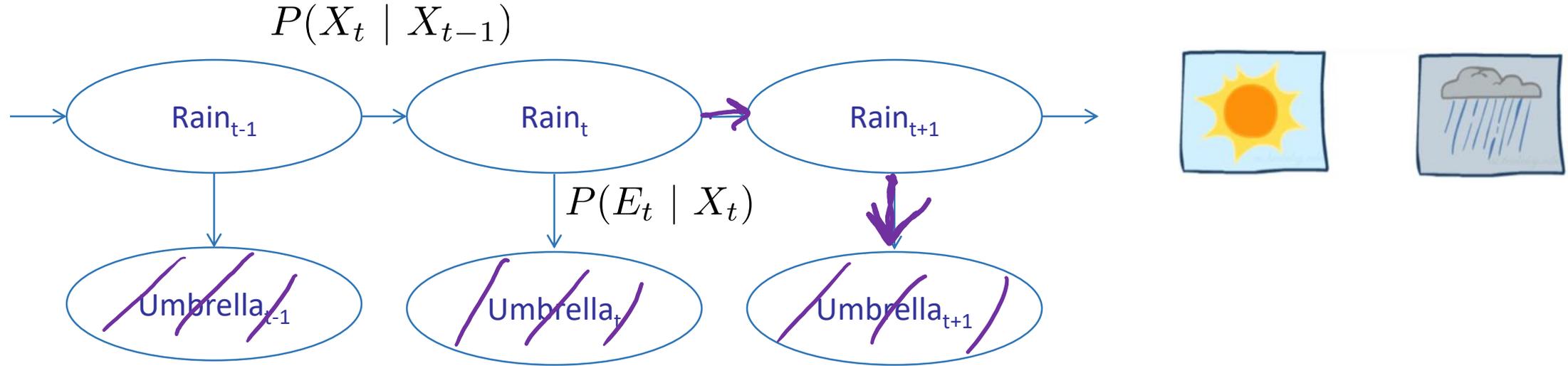
We want to track multiple variables over time, using multiple sources of evidence

Idea: Repeat a fixed Bayes net structure at each time

Variables from time  $t$  can condition on those from  $t-1$



# Example: Weather HMM



An HMM is defined by:

- Initial distribution:
- Transitions:
- Emissions:

$$P(X_1)$$

$$P(X_t | X_{t-1})$$

$$P(E_t | X_t)$$

$R_{t-1}$	$R_t$	$P(R_t   R_{t-1})$
+r	+r	0.7
+r	-r	0.3
-r	+r	0.3
-r	-r	0.7

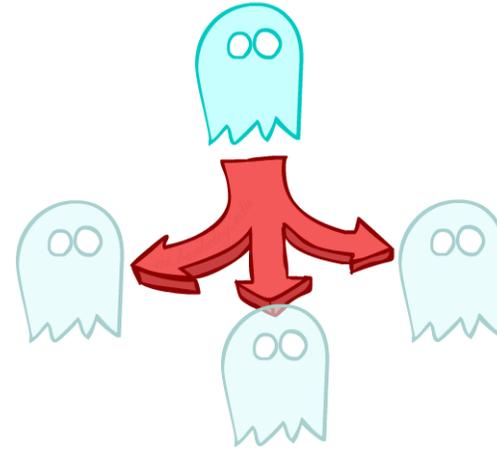
$R_t$	$U_t$	$P(U_t   R_t)$
+r	+u	0.9
+r	-u	0.1
-r	+u	0.2
-r	-u	0.8

# Example: Ghostbusters HMM

$P(X_1) = \text{uniform}$

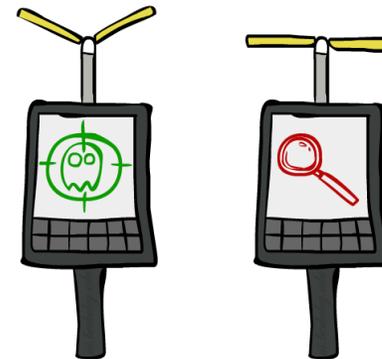
$P(X|X')$  = usually move clockwise, but sometimes move in a random direction or stay in place

$P(R_{ij}|X)$  = same sensor model as before: red means close, green means far away.



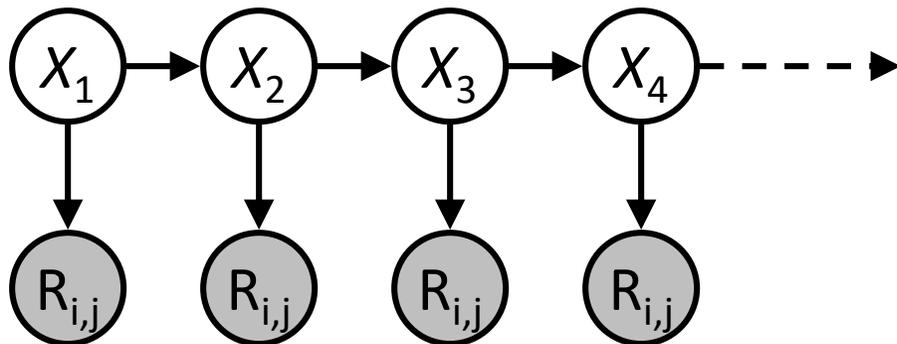
1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

$P(X_1)$



1/6	1/6	1/2
0	1/6	0
0	0	0

$P(X|X'=\langle 1,2 \rangle)$

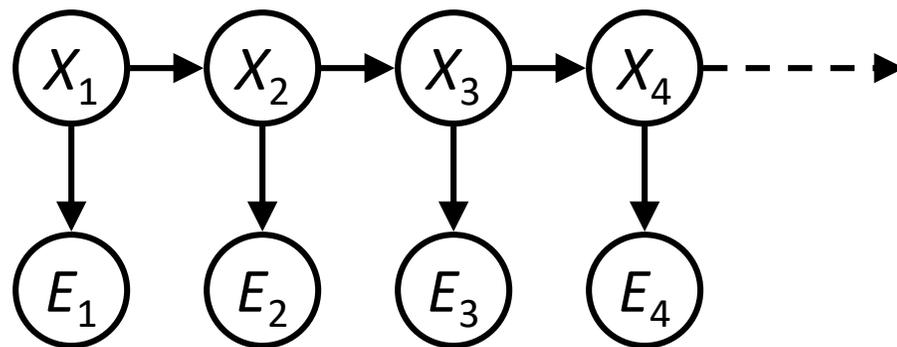


# Demo Ghostbusters – Circular Dynamics -- HMM

# Conditional Independence

HMMs have two important independence properties:

- Markov hidden process: future depends on past via the present
- Current observation independent of all else given current state



Does this mean that evidence variables are guaranteed to be independent?

- [No, they tend to be correlated by the hidden state]

# Real HMM Examples

## Speech recognition HMMs:

- Observations are acoustic signals (continuous valued)
- States are specific positions in specific words (so, tens of thousands)

## Machine translation HMMs:

- Observations are words (tens of thousands)
- States are translation options

## Robot tracking:

- Observations are range readings (continuous)
- States are positions on a map (continuous)

# Filtering / Monitoring

Filtering, or monitoring, is the task of tracking the distribution  $B_t(X) = P_t(X_t | e_1, \dots, e_t)$  (the belief state) over time

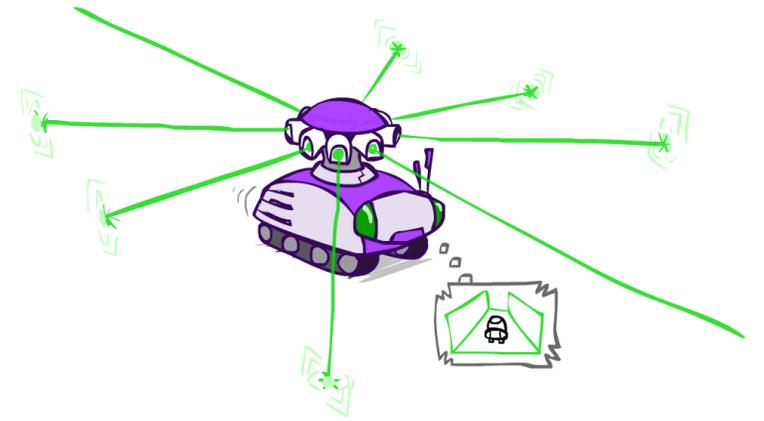
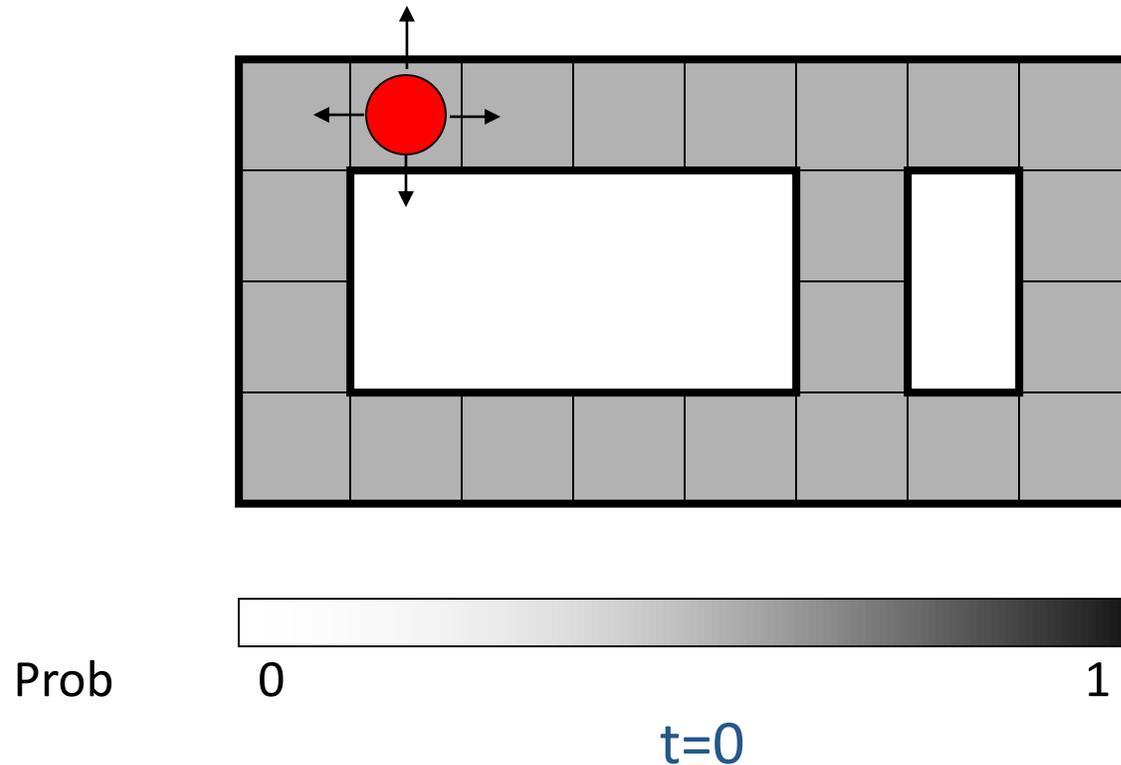
We start with  $B_1(X)$  in an initial setting, usually uniform

As time passes, or we get observations, we update  $B(X)$

The Kalman filter was invented in the 60's and first implemented as a method of trajectory estimation for the Apollo program

# Example: Robot Localization

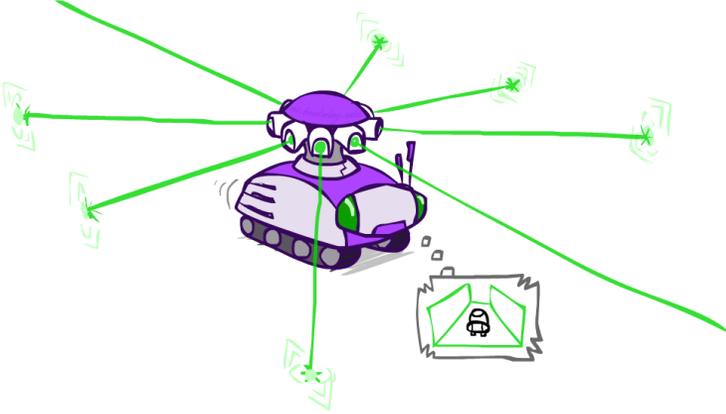
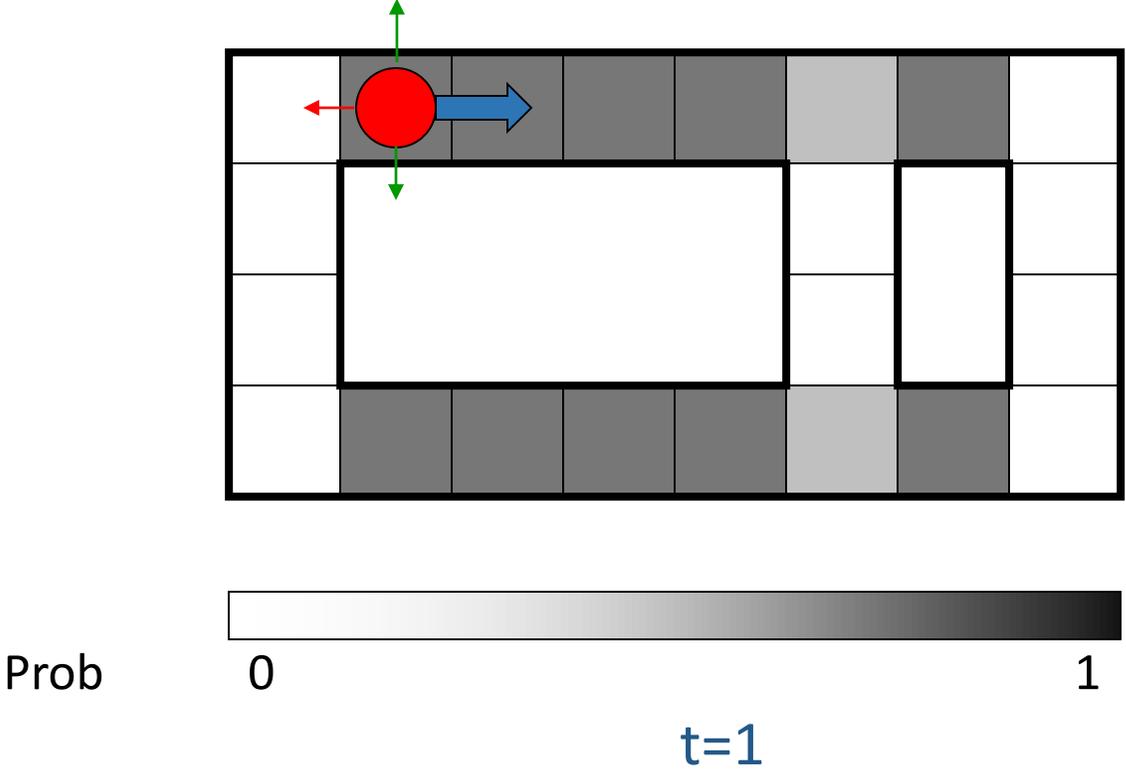
Example from  
Michael Pfeiffer



Sensor model: can read in which directions there is a wall,  
never more than 1 mistake

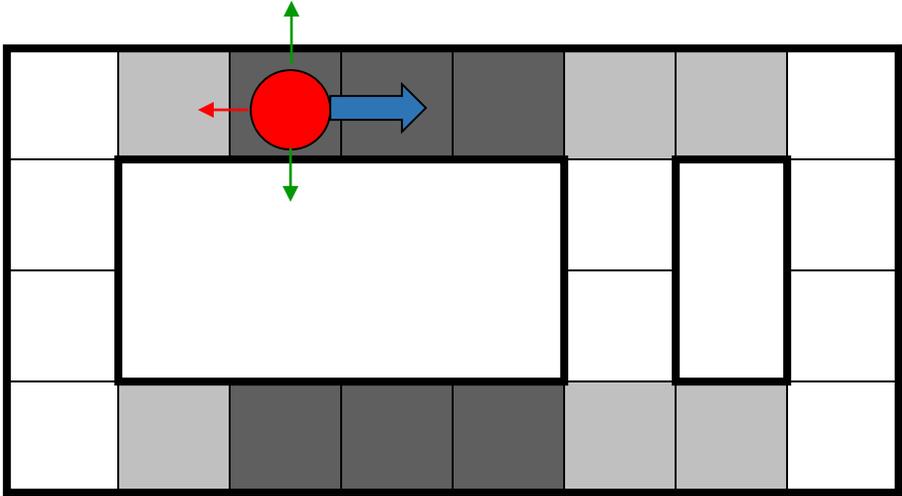
Motion model: may not execute action with small prob.

# Example: Robot Localization



Lighter grey: was possible to get the reading, but less likely b/c required 1 mistake

# Example: Robot Localization

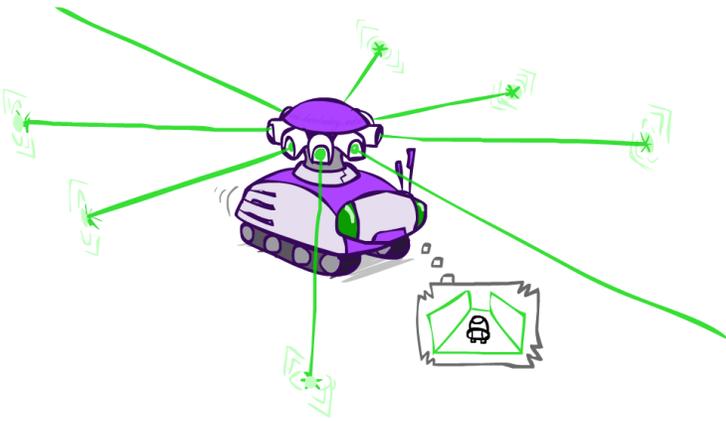


Prob

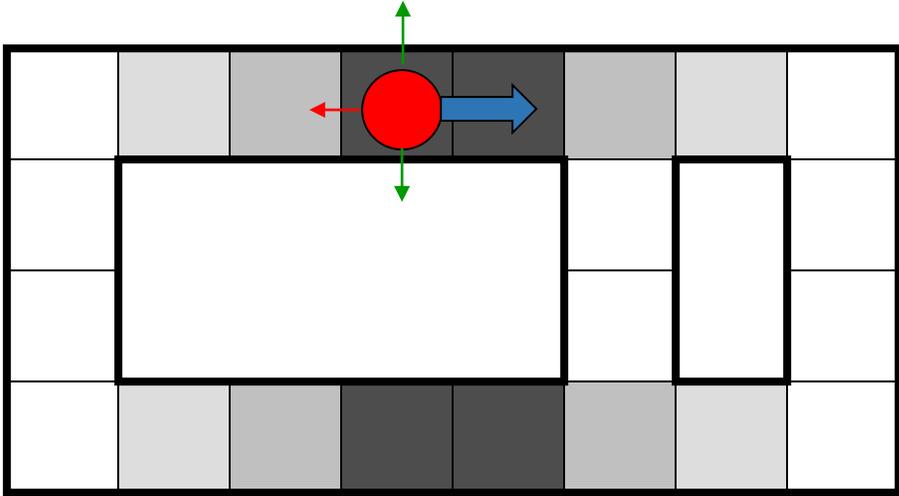
0

1

t=2



# Example: Robot Localization

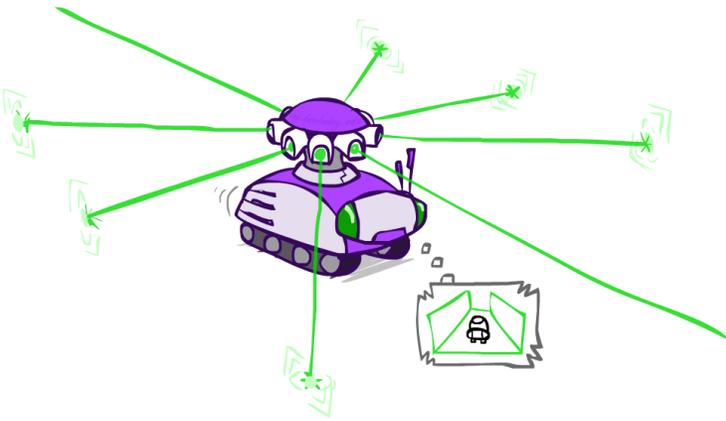


Prob

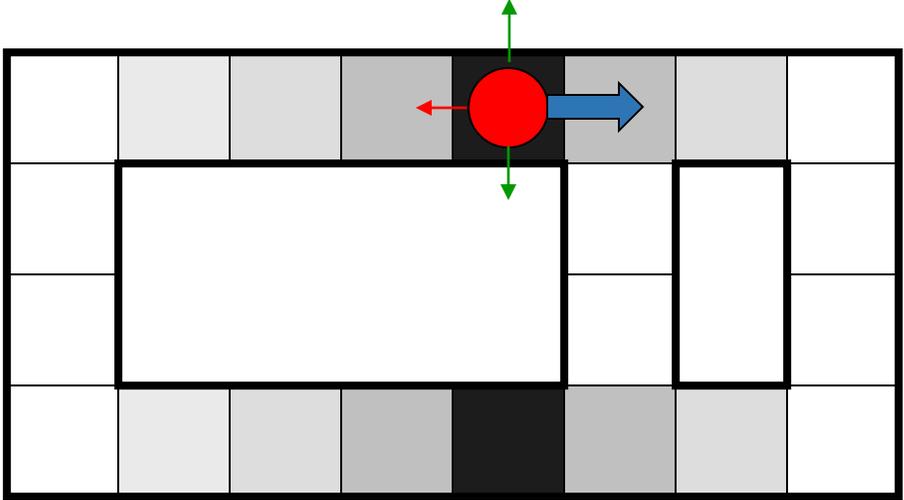
0

1

t=3



# Example: Robot Localization

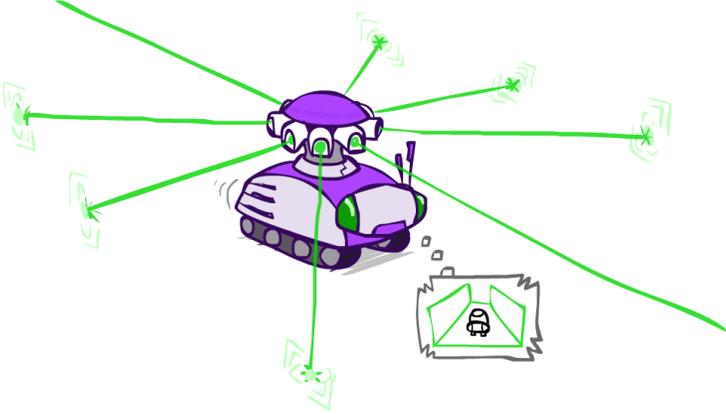


Prob

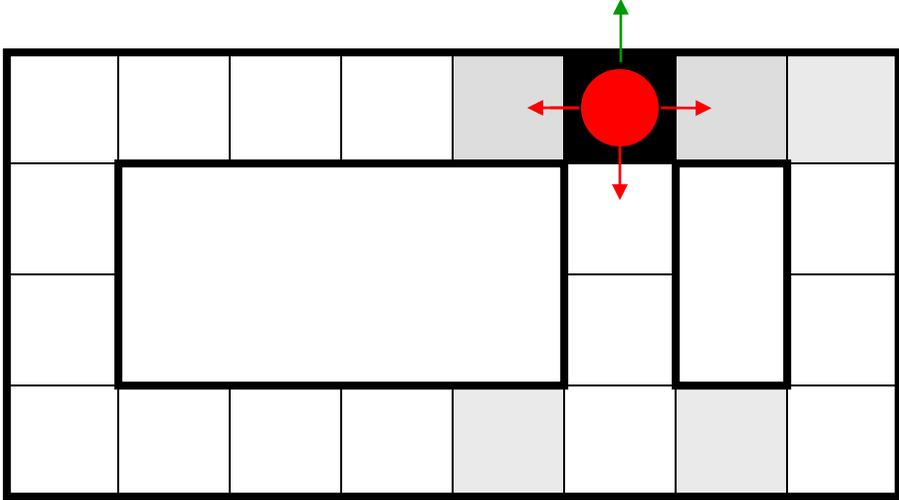
0

1

t=4



# Example: Robot Localization

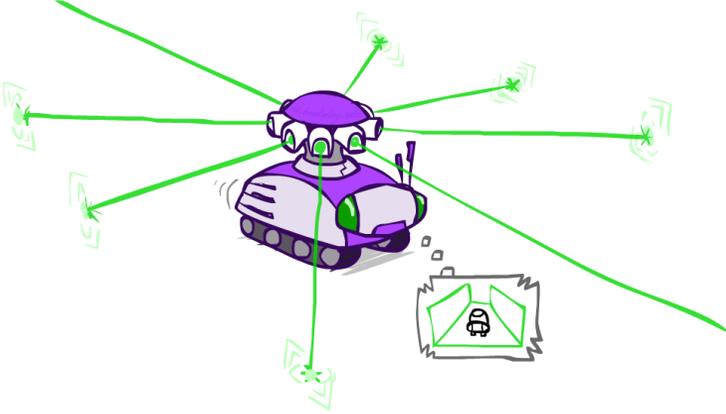


Prob

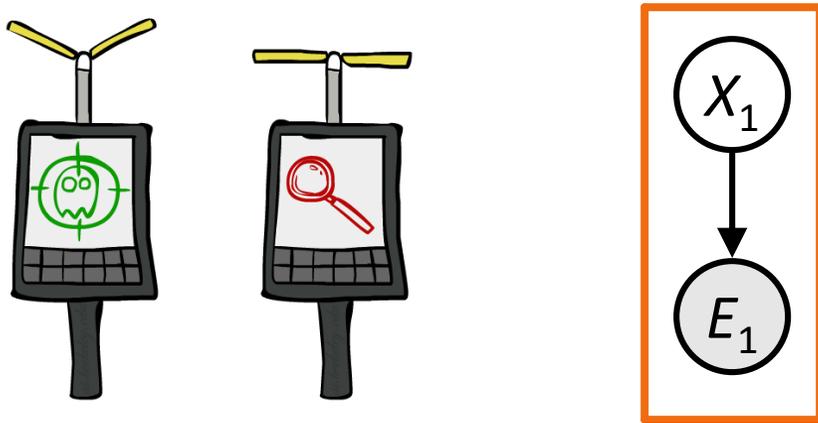
0

1

t=5

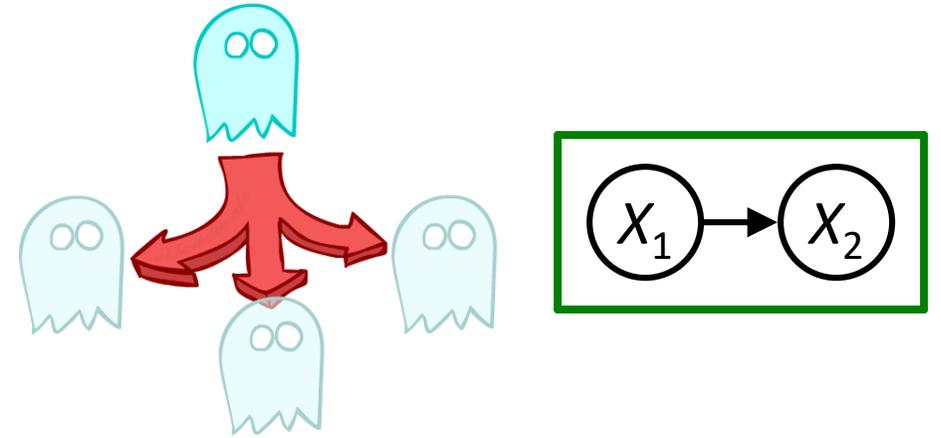


# Inference: Base Cases



$$P(\underline{X_1} | \underline{e_1})$$

$$\begin{aligned} P(x_1 | e_1) &= P(x_1, e_1) / P(e_1) \\ &\propto_{X_1} P(x_1, e_1) \\ &= P(x_1) P(\underline{e_1} | x_1) \end{aligned}$$



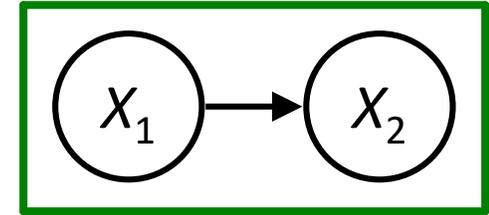
$$P(X_2)$$

$$\begin{aligned} P(x_2) &= \sum_{x_1} P(x_1, x_2) \\ &= \sum_{x_1} P(x_1) P(x_2 | x_1) \end{aligned}$$

# Passage of Time

Assume we have current belief  $P(X \mid \text{evidence to date})$

$$B(X_t) = P(X_t | e_{1:t})$$



Then, after one time step passes:

$$\begin{aligned} P(X_{t+1} | e_{1:t}) &= \sum_{x_t} P(X_{t+1}, x_t | e_{1:t}) \\ &= \sum_{x_t} P(X_{t+1} | x_t, e_{1:t}) P(x_t | e_{1:t}) \\ &= \sum_{x_t} P(X_{t+1} | x_t) P(x_t | e_{1:t}) \end{aligned}$$

■ Or compactly:

$$B'(X_{t+1}) = \sum_{x_t} P(X' | x_t) B(x_t)$$

Basic idea: beliefs get “pushed” through the transitions

- With the “B” notation, we have to be careful about what time step  $t$  the belief is about, and what evidence it includes

# Example: Passage of Time

As time passes, uncertainty “accumulates”

(Transition model: ghosts usually go clockwise)

<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
<0.01	<0.01	1.00	<0.01	<0.01	<0.01
<0.01	<0.01	<0.01	<0.01	<0.01	<0.01

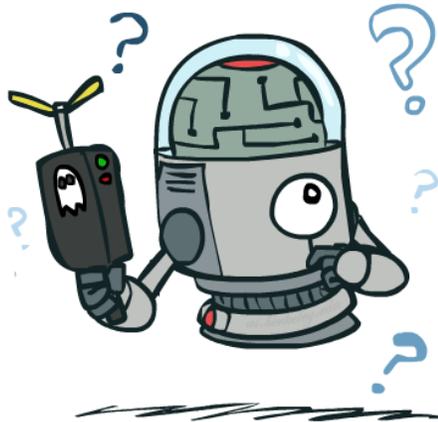
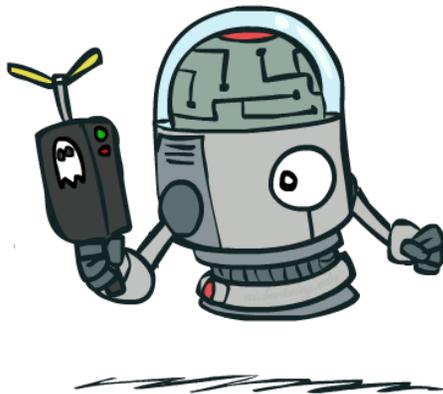
T = 1

<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
<0.01	<0.01	0.06	<0.01	<0.01	<0.01
<0.01	0.76	0.06	0.06	<0.01	<0.01
<0.01	<0.01	0.06	<0.01	<0.01	<0.01

T = 2

0.05	0.01	0.05	<0.01	<0.01	<0.01
0.02	0.14	0.11	0.35	<0.01	<0.01
0.07	0.03	0.05	<0.01	0.03	<0.01
0.03	0.03	<0.01	<0.01	<0.01	<0.01

T = 5



# Observation

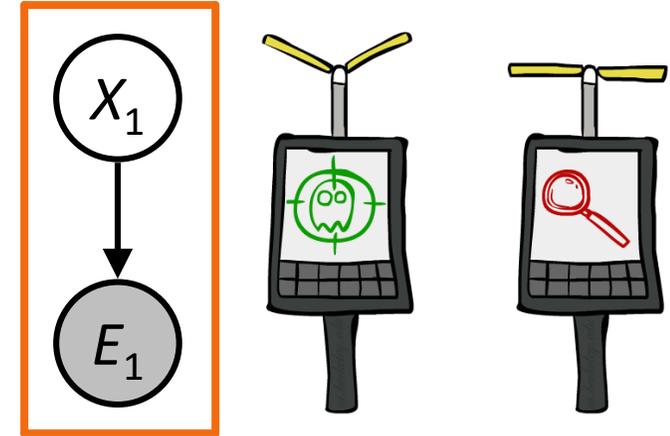
Assume we have current belief  $P(X \mid \text{previous evidence})$ :

$$B'(X_{t+1}) = P(X_{t+1} | e_{1:t})$$

Then, after evidence comes in:

$$\begin{aligned} P(X_{t+1} | e_{1:t+1}) &= P(X_{t+1}, e_{t+1} | e_{1:t}) / P(e_{t+1} | e_{1:t}) \\ &\propto_{X_{t+1}} P(X_{t+1}, e_{t+1} | e_{1:t}) \\ &= P(e_{t+1} | e_{1:t}, X_{t+1}) P(X_{t+1} | e_{1:t}) \\ &= P(e_{t+1} | X_{t+1}) P(X_{t+1} | e_{1:t}) \end{aligned}$$

Or, compactly:  $B(X_{t+1}) \propto_{X_{t+1}} P(e_{t+1} | X_{t+1}) B'(X_{t+1})$



- Basic idea: beliefs “reweighted” by likelihood of evidence
- Unlike passage of time, we have to renormalize

# Example: Observation

As we get observations, beliefs get reweighted, uncertainty “decreases”

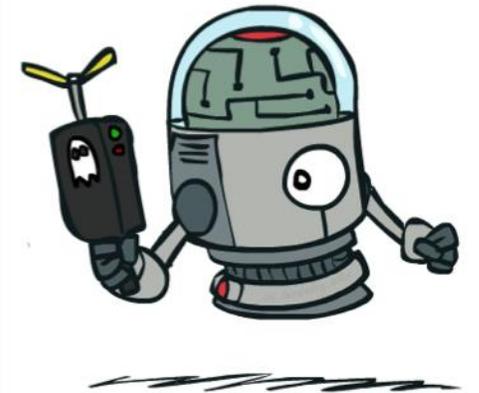
0.05	0.01	0.05	<0.01	<0.01	<0.01
0.02	0.14	0.11	0.35	<0.01	<0.01
0.07	0.03	0.05	<0.01	0.03	<0.01
0.03	0.03	<0.01	<0.01	<0.01	<0.01

Before observation

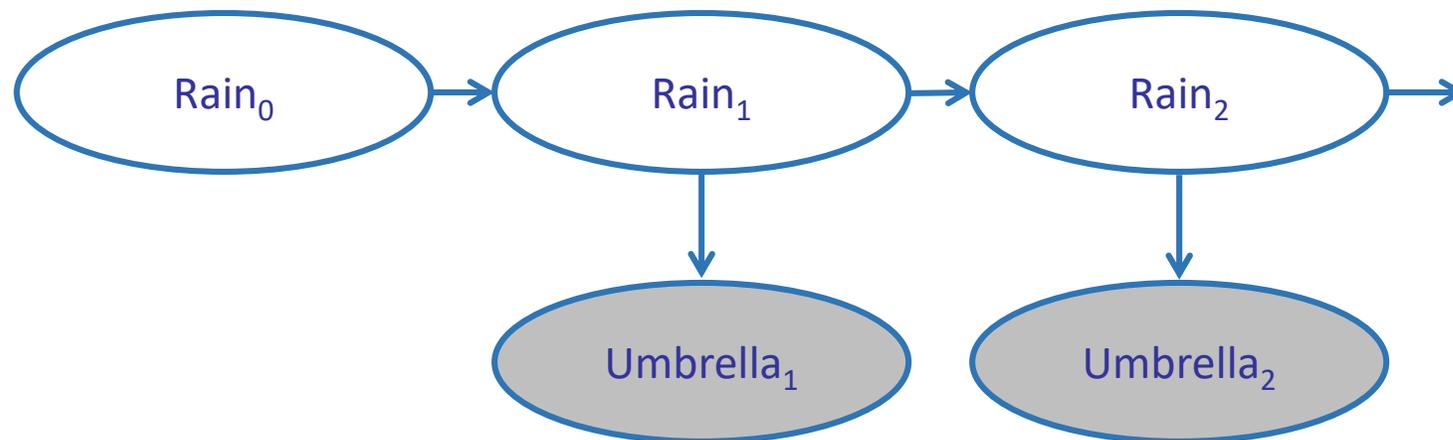
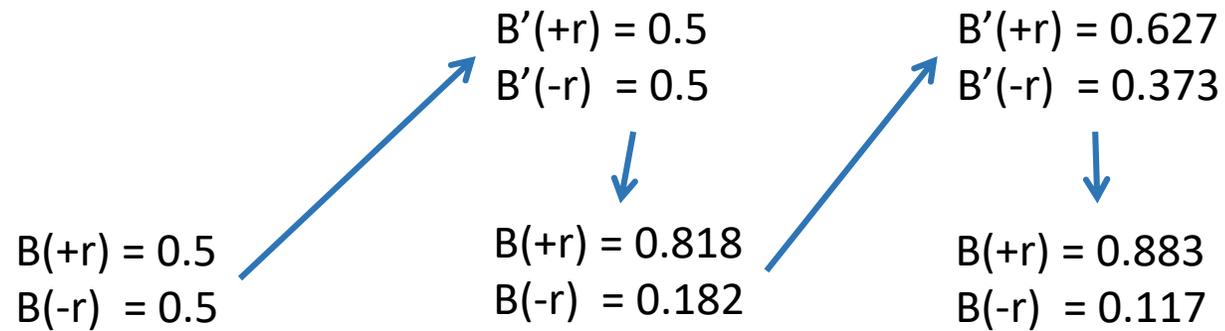
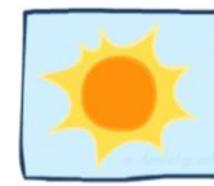
<0.01	<0.01	<0.01	<0.01	0.02	<0.01
<0.01	<0.01	<0.01	0.83	0.02	<0.01
<0.01	<0.01	0.11	<0.01	<0.01	<0.01
<0.01	<0.01	<0.01	<0.01	<0.01	<0.01

After observation

$$B(X) \propto P(e|X)B'(X)$$



# Example: Weather HMM

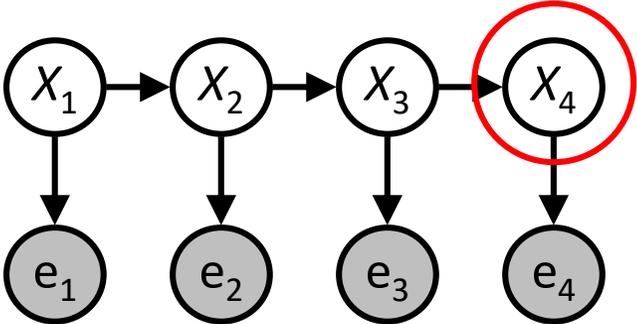


$R_t$	$R_{t+1}$	$P(R_{t+1}   R_t)$
+r	+r	0.7
+r	-r	0.3
-r	+r	0.3
-r	-r	0.7

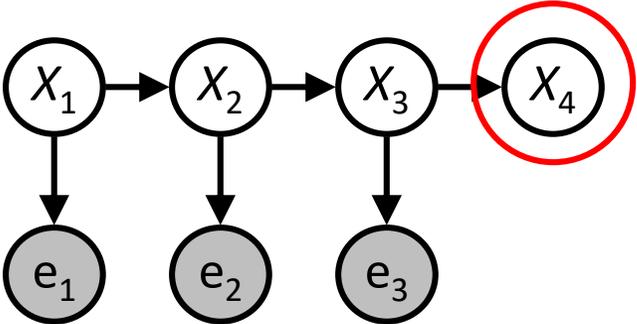
$R_t$	$U_t$	$P(U_t   R_t)$
+r	+u	0.9
+r	-u	0.1
-r	+u	0.2
-r	-u	0.8

# Other HMM Queries

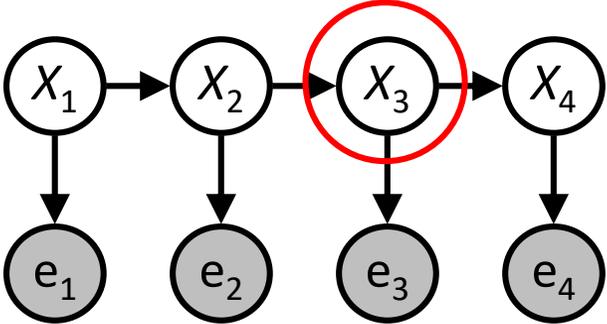
Filtering:  $P(X_t | e_{1:t})$



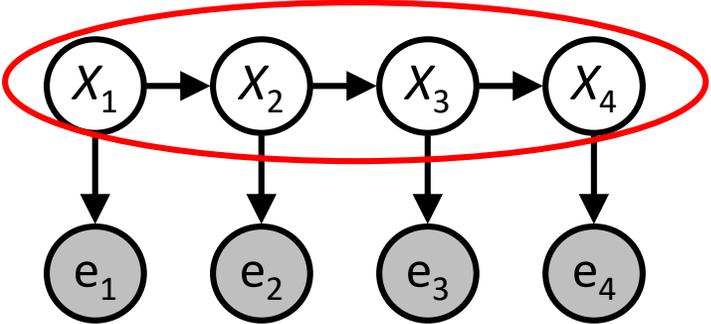
Prediction:  $P(X_t | e_{1:t-1})$



Smoothing:  $P(X_t | e_{1:N}), t < N$



Explanation:  $P(X_{1:N} | e_{1:N})$

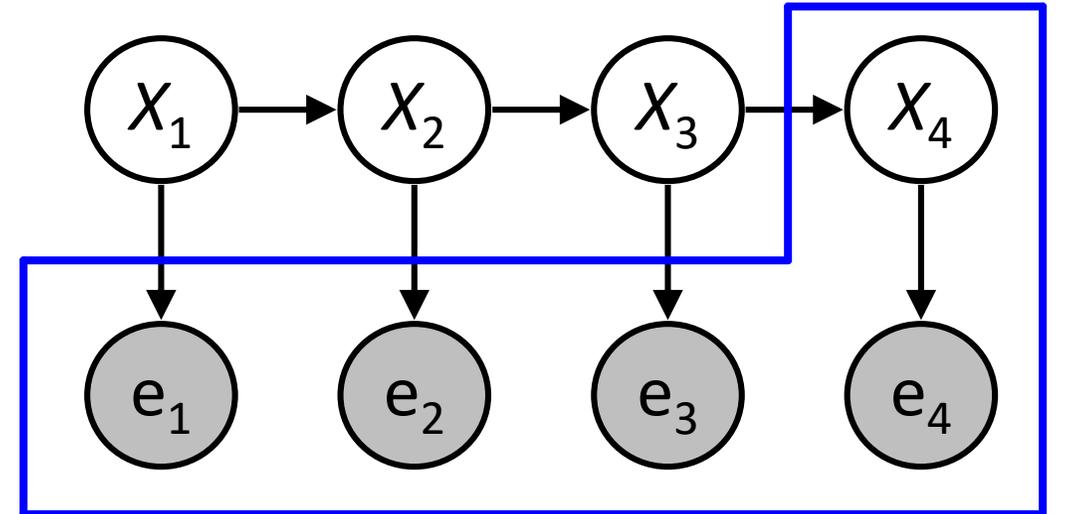


# The Forward Algorithm

Query: What is the current state, given all of the current and past evidence?

Matching math with Bayes net

$$P(x_t | e_{1:t}) \propto_{X_t} P(x_t, e_{1:t})$$

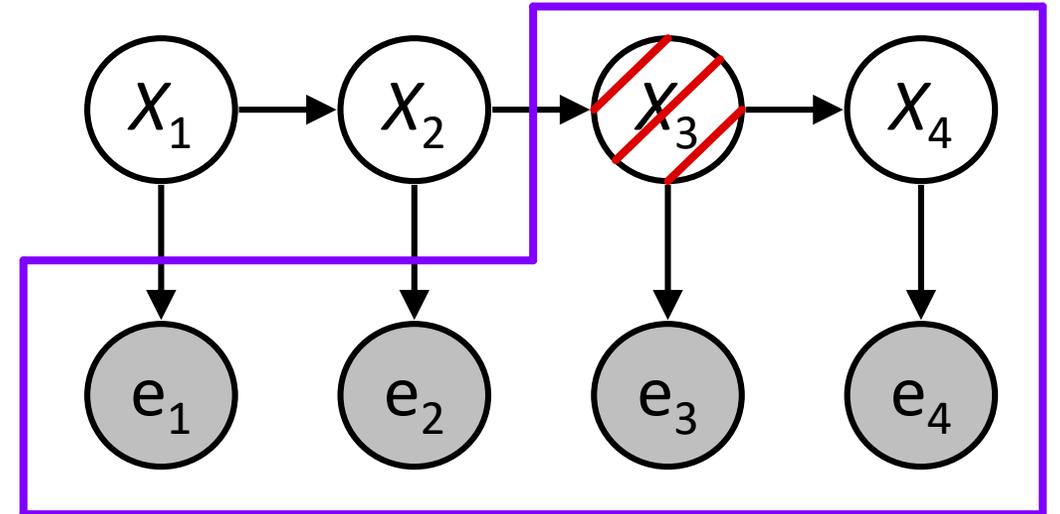


# The Forward Algorithm

Query: What is the current state, given all of the current and past evidence?

Matching math with Bayes net

$$\begin{aligned} P(x_t | e_{1:t}) &\propto_{X_t} P(x_t, e_{1:t}) \\ &= \sum_{x_{t-1}} P(x_{t-1}, x_t, e_{1:t}) \end{aligned}$$

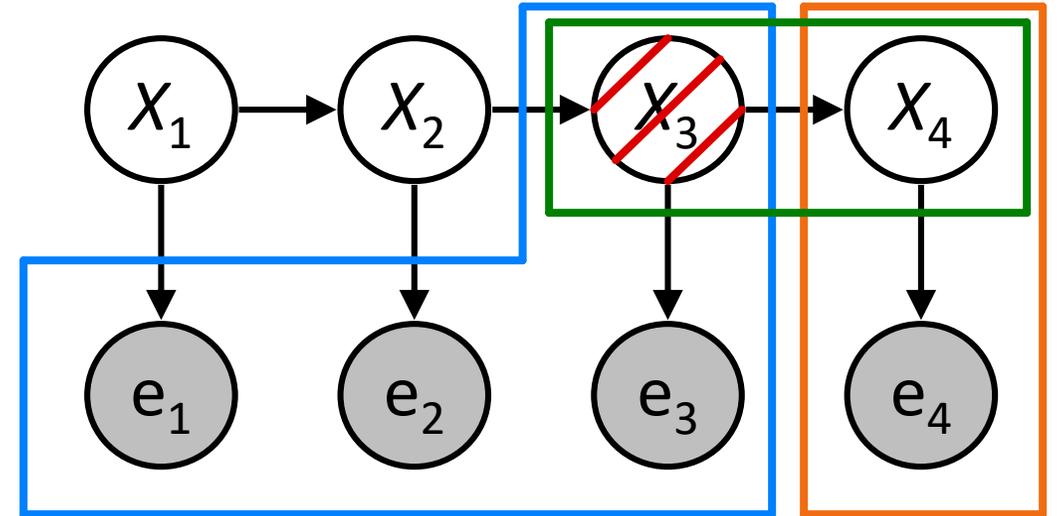


# The Forward Algorithm

Query: What is the current state, given all of the current and past evidence?

Matching math with Bayes net

$$\begin{aligned} P(x_t | e_{1:t}) &\propto_{X_t} P(x_t, e_{1:t}) \\ &= \sum_{x_{t-1}} P(x_{t-1}, x_t, e_{1:t}) \\ &= \sum_{x_{t-1}} P(x_{t-1} | e_{1:t-1}) P(x_t | x_{t-1}) P(e_t | x_t) \\ &= P(e_t | x_t) \sum_{x_{t-1}} P(x_t | x_{t-1}) P(x_{t-1} | e_{1:t-1}) \end{aligned}$$



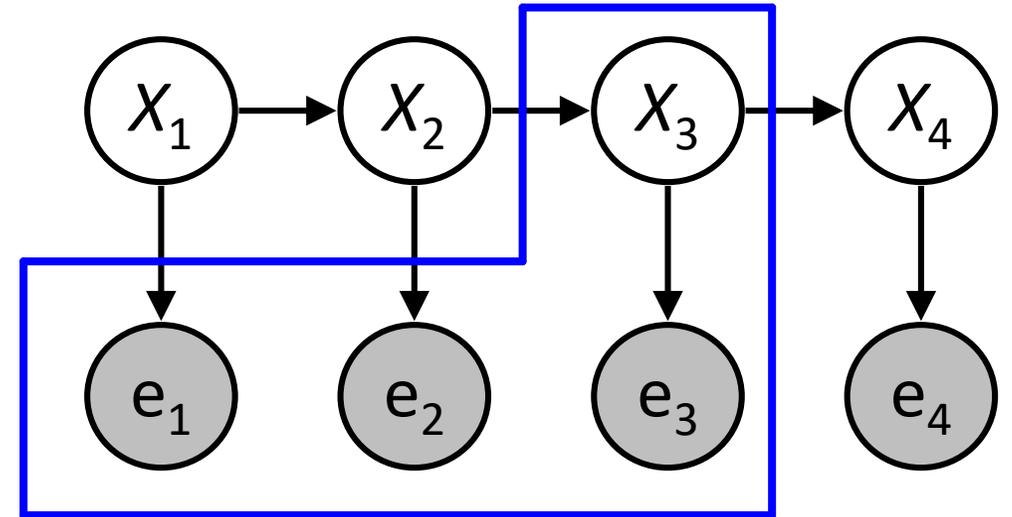
# The Forward Algorithm

Query: What is the current state, given all of the current and past evidence?

Matching math with Bayes net

Recursive

$$\begin{aligned} P(x_t | e_{1:t}) &\propto_{X_t} P(x_t, e_{1:t}) \\ &= \sum_{x_{t-1}} P(x_{t-1}, x_t, e_{1:t}) \\ &= \sum_{x_{t-1}} P(x_{t-1} | e_{1:t-1}) P(x_t | x_{t-1}) P(e_t | x_t) \\ &= P(e_t | x_t) \sum_{x_{t-1}} P(x_t | x_{t-1}) P(x_{t-1} | e_{1:t-1}) \end{aligned}$$

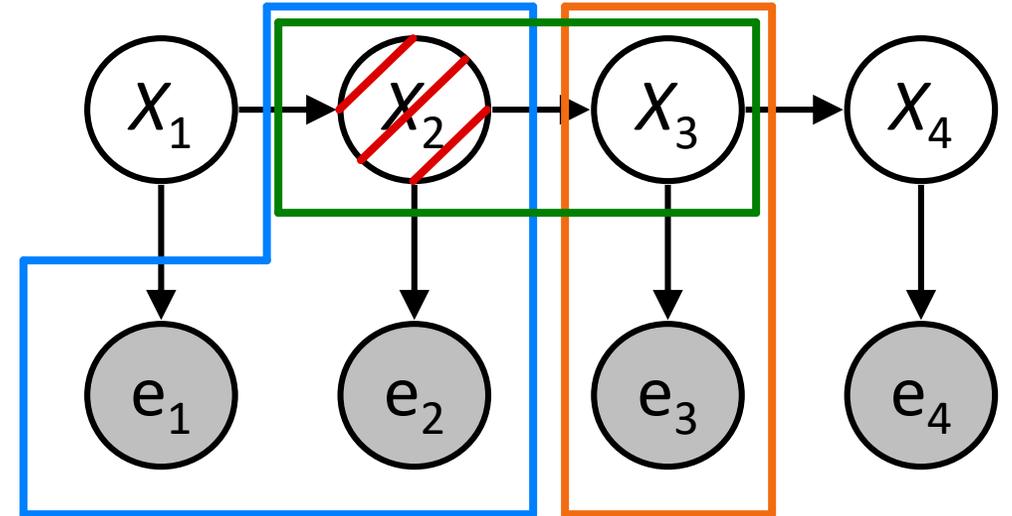


# The Forward Algorithm

Query: What is the current state, given all of the current and past evidence?

Matching math with Bayes net

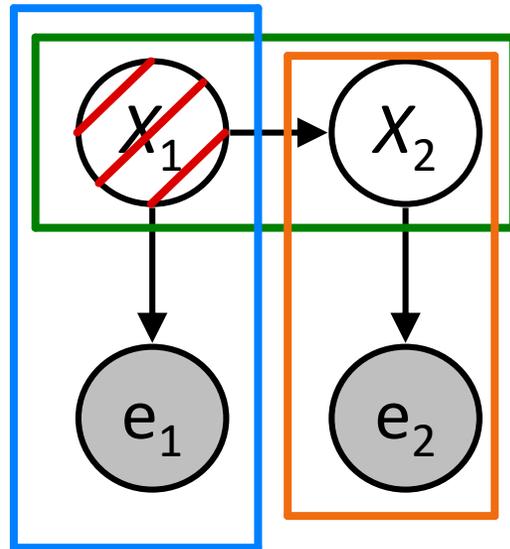
$$\begin{aligned} P(x_t | e_{1:t}) &\propto_{X_t} P(x_t, e_{1:t}) \\ &= \sum_{x_{t-1}} P(x_{t-1}, x_t, e_{1:t}) \\ &= \sum_{x_{t-1}} P(x_{t-1} | e_{1:t-1}) P(x_t | x_{t-1}) P(e_t | x_t) \\ &= P(e_t | x_t) \sum_{x_{t-1}} P(x_t | x_{t-1}) P(x_{t-1} | e_{1:t-1}) \end{aligned}$$



# The Forward Algorithm

Query: What is the current state, given all of the current and past evidence?

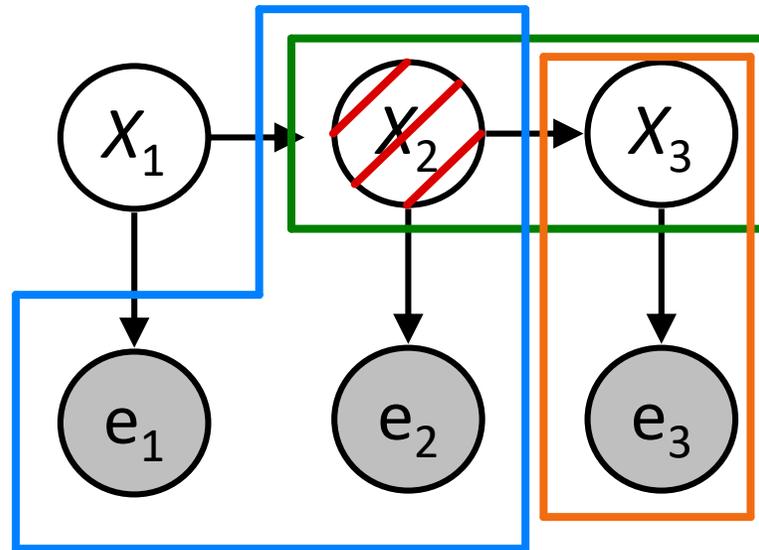
Marching **forward** through the HMM network



# The Forward Algorithm

Query: What is the current state, given all of the current and past evidence?

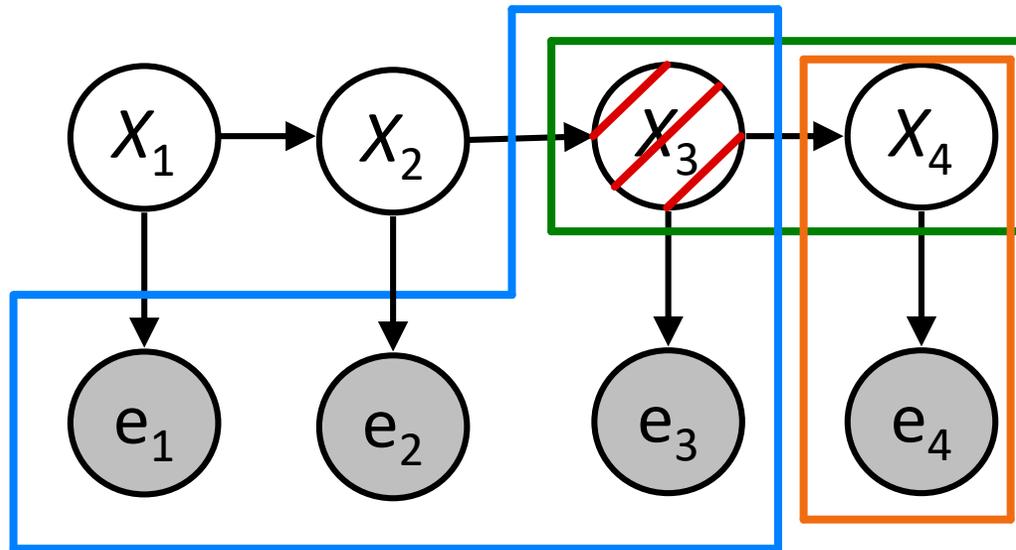
Marching **forward** through the HMM network



# The Forward Algorithm

Query: What is the current state, given all of the current and past evidence?

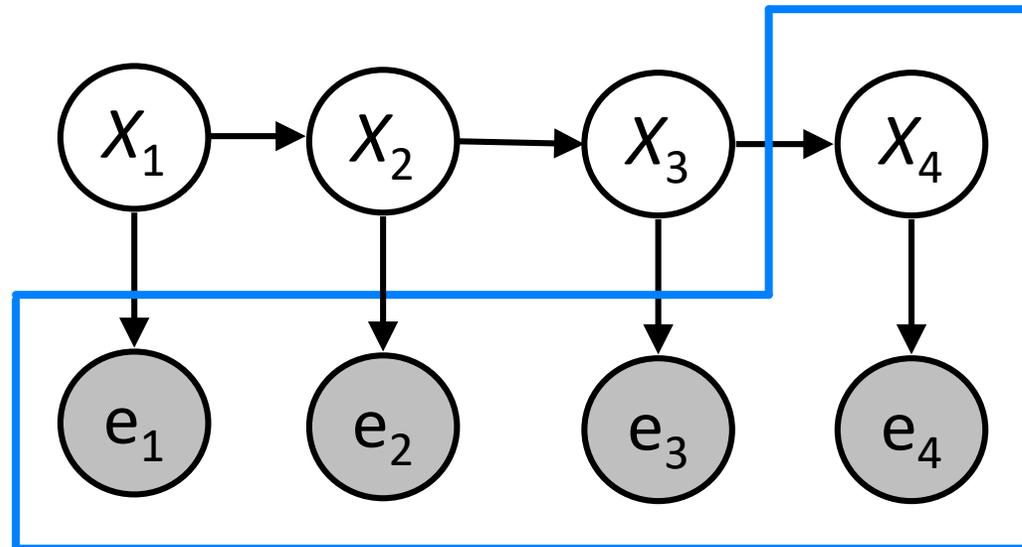
Marching **forward** through the HMM network



# The Forward Algorithm

Query: What is the current state, given all of the current and past evidence?

Marching **forward** through the HMM network



# Online Belief Updates

Every time step, we start with current  $P(X \mid \text{evidence})$

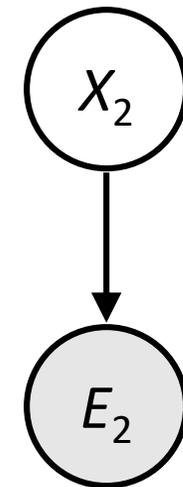
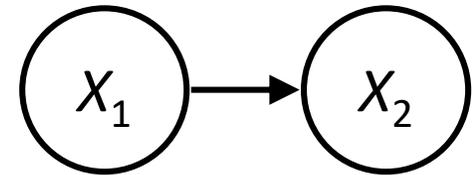
We update for time:

$$P(x_t | e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1} | e_{1:t-1}) \cdot P(x_t | x_{t-1})$$

We update for evidence:

$$P(x_t | e_{1:t}) \propto_X P(x_t | e_{1:t-1}) \cdot P(e_t | x_t)$$

The forward algorithm does both at once (and doesn't normalize)



# Pacman – Sonar (P5)

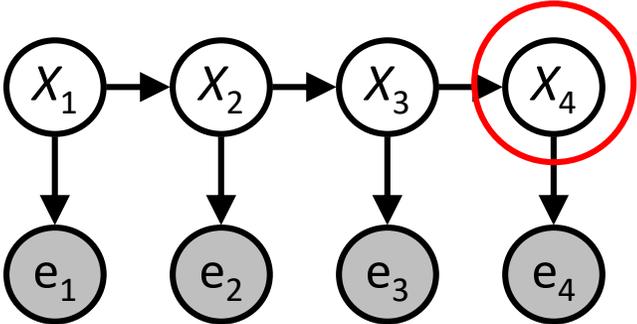


[Demo: Pacman – Sonar – No Beliefs(L14D1)]

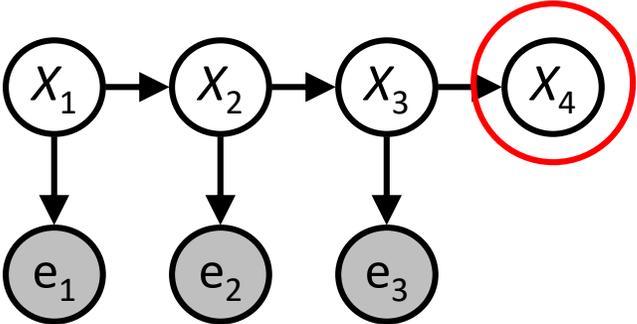
Video of Demo Pacman – Sonar (with beliefs)

# Other HMM Queries

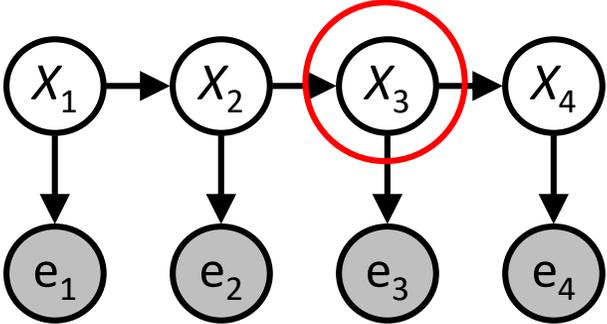
Filtering:  $P(X_t | e_{1:t})$



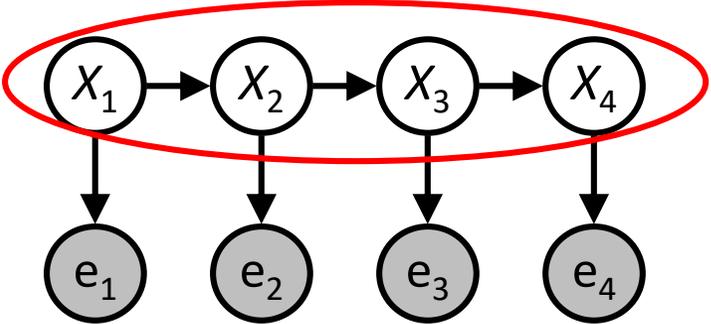
Prediction:  $P(X_t | e_{1:t-1})$



Smoothing:  $P(X_t | e_{1:N}), t < N$



Explanation:  $P(X_{1:N} | e_{1:N})$



Next Time: Particle Filtering and Applications of HMMs