15-381 Artificial Intelligence HW4

INSTRUCTIONS

- **Due: Tuesday, 12 February 2019 at 10:00 PM EDT.** Remember that you have NO slip days for Written Homework, but you may turn it in up to 24 hours late with 50% Penalty.
- **Format:** Submit the answer sheet pdf containing your answers. You should solve the questions on this handout (either through a pdf annotator, or by printing, then scanning). Make sure that your answers (typed or handwritten) are within the dedicated regions for each question/part. If you do not follow this format, we may deduct points.
- **How to submit:** Submit a pdf with your answers on Gradescope. Log in and click on our class 15-381 and click on the submission titled HW4 and upload your pdf containing your answers.
- **Policy:** See the course website for homework policies and Academic Integrity.

Last Name	
First Name	
Andrew ID	

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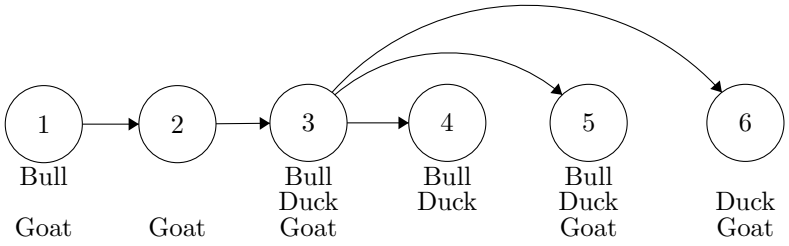
Q1	Q2	Q3	Q4	Total
/20	/30	/20	/30	/100

Q1. [20 pts] Animals of Farmland

The animals in Farmland aren't getting along and the farmers have to assign them to different pens. To avoid fighting, animals of the same type cannot be in connected pens. Fortunately the Farmland pens are connected in a tree structure.

(a) [5 pts] Consider the following constraint diagram that shows six pens with lines indicating connected pens. The remaining domains for each pen are listed below each node.



After assigning a bull to pen 5, enforce arc consistency on this CSP considering only the *directed arcs* shown in the figure. What are the remaining values for each pen?

Pen	Values
1	
2	
3	
4	
5	Bull
6	

(b) [5 pts] (True or False and **Explain** Briefly) If root to leaf arcs are consistent on a general tree structured CSP, assigning values to nodes from root to leaves will not backtrack if a solution exists.

True / False:	**Explain:**

(c) [5 pts] For CSPs with n nodes and d values in the domain, what is the computational complexity of solving general fully connected CSPs versus general tree structured CSPs? Give your answer by defining $g_{full}(n, d)$ and $g_{tree}(n, d)$, such that $O(g_{full}(n, d))$ and $O(g_{tree}(n, d))$ are the simplest and tightest bounds on the computational complexity for fully connected and tree structured CSPs, respectively.

$g_{full}(n, d)$**:**	$g_{tree}(n, d)$**:**

(d) [5 pts] Given your answers for g_{full} and g_{tree} above: with 3 animal types, what is the most number of pens a tree structured CSP could have such that $g_{tree}(n, d)$ is no greater than $g_{full}(n, d)$ for a fully connected CSP with 3 animal types and 10 pens?

Answer:

Q2. [30 pts] Two Mines Company

The Two Mines Company own two different mines that produce an ore which, after being crushed, is graded into three classes: high-, medium- and low-grade. The company has contracted to provide a smelting plant with ***at least*** 12 tons of high-grade, ***at least*** 8 tons of medium-grade and ***at least*** 24 tons of low-grade ore per week. The two mines have different operating characteristics as detailed below. Additionally, we cannot work the same mine for more than 6 days a week.

How many days per week should each mine be operated to fulfill the smelting plant contract while minimizing the total cost?

Note: We have (implicitly) assumed that it is permissible to work in fractions of days.

Mine name	**Cost per day**	**High-grade per day**	**Medium-grade per day**	**Low-grade per day**
Heigh Ho	180	4 tons	5 tons	4 tons
Kessel	160	2 tons	4 tons	8 tons

(a) [15 pts] Write this problem as an LP in **inequality form** as defined in lecture. Define any variables you use in your formulation. *Warning:* Be sure to strictly follow the inequality form, including the proper use of less than or equal, or you will lose points.

Inequality Form:

(b) [10 pts] Accurately plot the graphical representation of this linear program. Specifically:

- Plot the boundary of each halfspace as a line (no need to shade or draw normal vectors), and
- Plot the cost vector as an arrow with magnitude one, somewhere within the feasible region.

Do *not* draw; use a plotting tool such as Python matplotlib and include the resulting image here. Be sure to label the axes of your plot, including tick-marks. Display your plot with a *square* aspect ratio, e.g. in matplotlib: `plt.axis("equal")`. Additionally, zoom your plot to make the entire feasible region visible.

Tip to a plot vector $[v_1, v_2]^T$ in matplotlib starting at some point (x_1, x_2):

```
plt.quiver(x1, x2, v1, v2, angles="xy", scale_units="xy", scale=1)
```

Plot:

(c) [5 pts] Find the optimal solution to the LP problem. Give the solution as days per week per mine as well as the corresponding cost.

Heigh Ho:	**Kessel:**	**Cost:**

Q3. [20 pts] Graphing LPs

For the inequality form of a linear program, and a given A matrix and $\boldsymbol{b}$ vector,

$$\begin{aligned} \min_{\boldsymbol{x}} \ & \boldsymbol{c}^T \boldsymbol{x} \\ \text{s.t.} \ & A\boldsymbol{x} \preceq \boldsymbol{b} \end{aligned}$$

For each row i of A and $\boldsymbol{b}$, accurately plot 1) the line $a_{i,1}x_1 + a_{i,2}x_2 = b_i$ and 2) the vector $[a_{i,1}, a_{i,2}]^T$ as an arrow beginning at any point on its respective line.

Tip to a plot vector $[v_1, v_2]^T$ in matplotlib starting at some point (x_1, x_2):

```
plt.quiver(x1, x2, v1, v2, angles="xy", scale_units="xy", scale=1)
```

Do *not* draw; use a plotting tool such as Python matplotlib and include the resulting image here. Be sure to label the axes of your plot, including tick-marks. Display your plot with a *square* aspect ratio, e.g. in matplotlib: `plt.axis("equal")`. Additionally, zoom your plot such that all of the vectors are visible.

(a) [10 pts]

$$A = \begin{bmatrix} 3 & 5 \\ 7 & 6 \\ 12 & 6 \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} 10 \\ 17 \\ 27 \end{bmatrix}$$

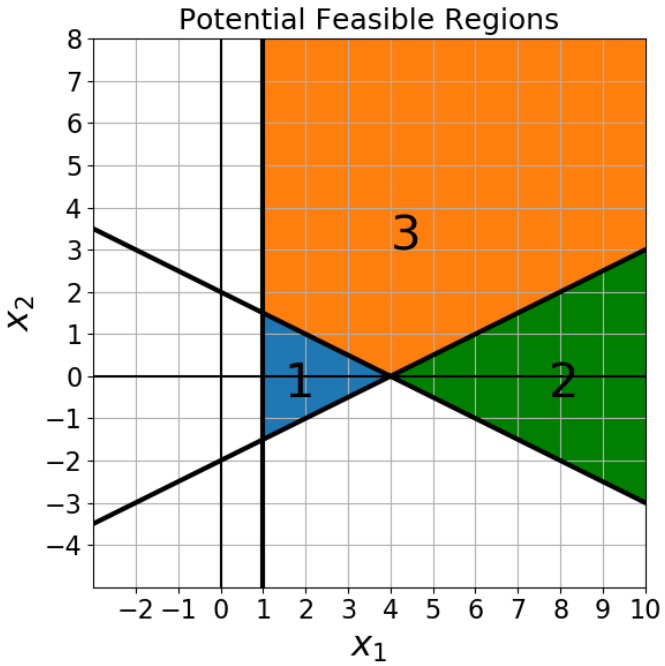
Plot:

(b) [10 pts]

$$A = \begin{bmatrix} -2 & -1 \\ 2 & 5 \\ 7 & 2 \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} -4 \\ 10 \\ 11 \end{bmatrix}$$

Plot:

Q4. [30 pts] Feasible Regions



In this problem, you are given a graph with constraint boundary lines (**bolded**) and potential feasible regions. You may assume shaded regions at the edge of the plot continue to infinity. Provide the corresponding A and $\boldsymbol{b}$ based on the inequality form below for each feasible region in the boxes below:

$$\min_{\boldsymbol{x}} \ \boldsymbol{c}^T\boldsymbol{x}$$
$$\text{s.t. } A\boldsymbol{x} \preceq \boldsymbol{b}$$

(a) [10 pts] Feasible Region 1

A:

$\boldsymbol{b}$:

(b) [10 pts] Feasible Region 2

A:

$\boldsymbol{b}$:

(c) [10 pts] Feasible Region 3

A:

$\boldsymbol{b}$: