15-381 Problem Set 2: Logic

Handed out: 10 February, 2000
Due back: 24 February, 2000 (by beginning of class)

This assignment, unlike the first, will contain no programming portion. Please follow the directions for each problem. Handin will be accomplished via hardcopy; typed or neatly written, please. Remember, if we can't read it, we're going to assume that it's incorrect. Direct all questions regarding the format/semantics/correctness of the questions to babailey@andrew.cmu.edu; for help on the assignment, feel free to ask any TA or one of the professors. All updates to the assignment will be posted on the web page, although we will post to the bboard when these updates appear. As always, note any help that you receive from any source, and the nature of that help. As a final warning, please note that the homework is not meant to be comprehensive. There will be questions on the midterm that have not been covered on the homework; use these as your only study guides at your own risk. And now, our main attraction:

1. First Order Logic

A. Clause Form

Reduce the FOL statements below to clause form. In the case that two or more clauses would result from this reduction, label them a, b, etc. (Ex. If statement 0 reduced to 3 clauses, they would be labeled 0a, 0b, 0c). Note that here, "and" is marked with "^", "or" is marked by "v", and "not" is marked with "~".

1. Bob, a treefrog, has launched a nuclear missile at Joe.
   Treefrog(Bob) ^ Nuked(Bob, Joe)
2. Treefrogs, who cannot die, all dislike Elvis.
   ∀x [Treefrog(x) ⇒ ~Likes(x, Elvis) ^ ~Dead(x)]
3. Those who drive Fords do not drive Hondas
   ∀x [Drives(x, Ford) ⇒ ~Drives(x, Honda)]
4. People cannot be treefrogs.
   ∀x [Person(x) ⇒ ~Treefrog(x)]
5. Every person either likes Elvis, or they are dead.
   ∀x [Person(x) ⇒ Likes(x, Elvis) v Dead(x)]
6. Everyone is a person or a treefrog.
   ∀x [Person(x) v Treefrog(x)]
7. If a person is the King of Rock, then they are not dead.
   ∀x [Person(x) ^ KingOfRock(x) ⇒ ~Dead(x)]
8. Elvis is the King of Rock.
   KingOfRock(Elvis)
9. Bob drives a Honda.
   Drives(Bob, Honda)
10. Elvis is not a treefrog.
    ~Treefrog(Elvis)
11. Treefrogs who do not think Elvis is dead drive Hondas.
    ∀x [Treefrog(x) ^ ~ThinksDead(x, Elvis) ⇒ Drives(x, Honda)]
12. Every treefrog drives some car.
    ∀x∃y [Treefrog(x) ^ Car(y) ⇒ Drives(x, y)]
13. Treefrogs only launch nuke those who do not like Elvis and drive Fords.
    ∀x∀y [Treefrog(x) ^ Nuke(x, y) ⇒ ~Likes(y, Elvis) ^ Drives(y, Ford)]
14. People think Elvis is not dead.
    ∀x [Person(x) ⇒ ~ThinksDead(x, Elvis)]
15. Every treefrog thinks that someone is dead.
    ∀x∃y [Treefrog(x) ⇒ ThinksDead(x, y)]

B. Resolution

Using the knowledge base obtained in part A, resolve the following statements, or show that they cannot be resolved. Use the format of p. 19-20 of the 2/8/00 lecture notes (labeled p. 127-128 of Rich/Knight, Ch. 5) where the numbers assigned the clauses are those generated in part A. (Also use the letter generated, such as 0b, if necessary.) Try to find the shortest path to resolution. Be sure to mark substitution of variables (as in the example in the lecture notes).

1. ~Dead(Elvis)	Elvis is not dead.
2. ~ThinksDead(Bob, Elvis)	Bob does not think that Elvis is dead.
3. ~Treefrog(Joe)	Joe is not a Treefrog.

2. Natural Language and First Order Logic

Translate the following natural language sentences to FOL.
Use uncomplicated predicates; for example, “Every TA works hard sometime,” should become $\forall x \exists t$ [TA(x) ^ Time(t) $\Rightarrow$ WorksHard(x,t)] rather than $\forall x$ [TA(x) $\Rightarrow$ WorksHardSometime(x)]; in this case WorksHardSometime can and should be broken down into a separate WorksHard predicate and an existential quantifier on a variable in the Time predicate.

1. No one who writes on whiteboards is a TA.
2. No TA wants to fail all of the students.
3. The Cincinnati Reds will win a World Series.
4. Anyone who wears gray socks owns a briefcase.
5. There is a place where no one loves all computer games.
6. Any TA can fool some of the students some of the time.
7. No TA can fool all of the students all of the time.

3. Beyond First Order Logic

A. 4-Value Logic

1. In 4-value logic (explained on p. 10 of the 2/3/00 lecture notes), what value do you assign to a statement if there is both evidence that it is true, and evidence that it is false?
2. You become convinced that, despite the evidence, the statement is actually false. What is the best course of action?

B. Extending 3-Value Logic to a 4th Value (other than contradiction)

Assume that 3-value logic (p. 9 of the 2/3/00 lecture notes) has been extended to include a value @ which is used when the trueness of the statement fluctuates with respect to time. Fill in the following truth table, drawing as much inference as possible:

x	y	x ^ y	x v y	x $\Rightarrow$ y	x xor y
T	T				
T	F				
T	?				
T	@				
F	T				
F	F				
F	?				
F	@				
?	T				
?	F				
?	?				
?	@				
@	T				
@	F				
@	?				
@	@				