Artificial Intelligence -- 15-381
Propositional and First-Order Logic

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OUTLINE:

- Knolwedge Representation in AI
- Proposional Logic
- Predicate Logic (first-order)
- Multi-valued logics

Knowledge Representation in AI

Motivation: KR Manifesto

Want to represent knowledge ***explicitely***, ***expressively***, ***unambiguously*** and enabling ***tractable*** computation

Examples to represent:

- If john is not lazy then he will study for AI.
- All men are mortals; Socrates is a man; ...
- Anyone can fool some of the people some of the time, but no one can fool all of the people all of the time.
- Fred is a penguin; penguins are birds; birds fly; Fred does not fly; Henry is a canary; ...
- Success of chemotherapy depends on dosage and intrinsic health, but not on the phase of the moon.

Natural Language => KR is one-many mapping

- I see the man with the telescope in Virginia
- Time flies like an arrow

Types of Knowledge Representations in AI

Logic-Based Representations

1. Propositional Logic (Predicate Calculus)
   grounded instances only.
2. First-Order Logic (FOL, aka predicate logic)
   variables, quantifiers, ...
3. Multi-value logics (beyond just T and F)
4. Modal Logics
5. Higher-order logics

Ontologies, Inheritance and Frames

1. Semantic Networks with Inheritance Operations
2. Frames with Procedural Attachments

Unification and Resolution

1. Clause Form for FOL
2. Unification (a.k.a matching with variables)
3. Resolution (a proof precedure)

Propositional Logic

Ingredients

- Constants: "Blue", "Clyde", "Fido", "Y2K"
- Truth Values: True, False
- Connectives: not, and, or, implies, xor, <=>...
- Functions: color(Fido), significance(Y2K),
- Predicates: Nice(Mary), Has-Color(Fido,Blue),
- Sentences: ((P & Q) implies R)
  where P & Q are truth-values, predicates or sentences
- Tautologies: (P => Q) <=> (~P or Q)

Truth tables

P	Q	(P & Q)	(P or Q)
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

P	Q	(P => Q)	(~P or Q)	~(P xor Q)
T	T			
T	F			
F	T			
F	F			

Inference in Prop. Logic
[See Textbook, p172]

Modus Ponens

```
P => Q,  P
__________
    Q
```

Transitive Implicature

```
P => Q,  Q => R,  P
___________________
              R
```

And Elimination

```
A & B & C & D & E & ...
_______________________
A,  B,  C,  D,  E,  ...
```

Unit Resolution

```
P or Q,  ~P
___________
    Q
```

First Order Logic

Additional Ingredients

- Variables: x, y, a_1, ...
  bound to constants, terms, sentences, lower case
- Functions: color(x), sum(x, y), ...
  n-ary, any value, lower case
- Predicates: HAS-COLOR(x,y), PERSON(x),
  EATS(dog(x), Purina), ...
  n-ary, boolean value, capitalized
- Terms: constant | variable | function
- Quantifier: Universal | Existential
- Sentence: predicate |
  Quantifier(variable) Sentence |
  sentence connective sentence

Examples of Sentences

All humans are mortal
Forall(x) Human(x) => Mortal(x)

There is at least one honest politician
Exists(y) Politician(y) & Honest(y)

Everybody loves somebody sometime
Forall(x) Exists(y) Exists(t)
(Human(x) & Human(y) & Time(t) => Loves(x,y,t)

More on FOL

Equivalence Examples

Double negation: ~(~x)) <=> x

Clause reduction: (x => y) <=> (~x or y)

Equivalence: ~(x xor y) <=> (x <=> y)

DeMorgan’s law: (x & y) <=> ~(~x or ~y)

"Reducing the universe:")
All(x) P(x) <=> ~Exists(x) ~P(x)

Quantifier Extensions

- Unique Existence: Exist!(x) P(x)
  Forall(c) Exists!(p) Country(x) & President(p,x)
- Bounded Existence: At-most(N,x) P(x)
- "That man is worse than Fuzzy Universals: Most(x) P(x)
- λ-operator: λ(x) f(x)
  λ(x) plural(x) (country) = "countries"

Examples of FOL

Not all students are nerds
~All(x)[Student(s) => Nerd(s)]
~All(x)[~Student(s) or Nerd(s)]
~(~Exists(s)~[~(Student(s) or Nerd(s)]
Exists(s) [Student(s) & ~Nerd(s)]

All students who graduated passed at least one exam
All(s)Exists(e) [(Student(s) & Exam(e) & Graduated(s))
=> Passed(s,e)]

There are no cheaters enrolled in AI class

All men are mortal except Elvis

Multi-Valued Logics

The third value: Unknown

- Truth values: T, F. ?
- ? = value of variable is unknown
- Enable inference nonetheless

x	y	(x & y)	(x => y)	(x & y) => y
T	T	T	T	T
T	F	F	F	T
T	?			
F	T			
F	F			
F	?			
?	T			
?	F			
?	?			

Observations

- "Conservation of ?'s"
  => indeterminacy does not increase
- Useful for reasoning under uncertainty
- Useful for pinpointing needed info
- Not useful for probabilistic inference

Multi-Valued Logics II

The Fourth value: Contradiction

- Truth values: T, F, ?, #
- ? means ***either*** T of F (unknown)
- # means ***neither*** T nor F

Why is 4-valued logic useful?

- Know when you ***don't know*** (inconclusive)
- Know when you ***can't know*** (inconsitency)
- *"He knoweth not, and worse, he knoweth not that he knoweth not"*
  --(Some famous dead writer)
- => Metareasoning: reasoning about what you know (part of Epistemology)