Linear algebra review: vector

- vector: ordered m-tuple $\vec{u} = (u_1, u_2 \dots u_m)$;
- vector space $V = \{\vec{u} \in R^m\}$: the set of all ordered m-tuples;
- basis of V: the set of *independent* vectors that span V
- dimension: the number of vectors in a basis of V
- vector p-norms ($p \geq 1$):

$$\|\vec{u}\|_p = (|u_1|^p + |u_2|^p \dots |u_m|^p)^{1/p}$$

$$\|\vec{u}\|_2 = \sqrt{u_1^2 + u_2^2 \dots u_m^2}$$

- Euclidean inner product (dot product):

$$\vec{u} \cdot \vec{v} = \sum_i^m u_i v_i = \|\vec{u}\|\|\vec{v}\| cos(\theta)$$

  where θ is the angle between $\vec{u}$ and $\vec{v}$;

- cosine:

$$cos(\vec{u}, \vec{v}) = \frac{\sum_i^m u_i v_i}{\sqrt{u_1^2 + u_2^2 \dots u_m^2}\sqrt{v_1^2 + v_2^2 \dots v_m^2}} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{\vec{u}}{\|\vec{u}\|} \cdot \frac{\vec{v}}{\|\vec{v}\|};$$

- Euclidean distance:

$$d(\vec{u}, \vec{v}) = \|\vec{u} - \vec{v}\| = \sqrt{\sum_i^m (u_i - v_i)^2};$$