

1 Definitions

1. Conditional Probability: $P(A | B) =$
2. Product Rule: $P(A, B) =$
3. Bayes' Theorem: $P(A | B) =$
4. Normalization: $P(A | B) =$
5. Chain Rule: $P(A, B, C) =$
6. Law of Total Probability: [using only $P(B)$ and $P(A | B)$] $P(A) =$
7. Independence: A, B independent, $P(A, B) =$
8. Conditional Independence: If A and B are conditionally independent given C , then $P(A, B | C) =$

2 Warm Up

- (a) State the two ways to write the chain rule (conditional probability decomposition) for $P(A, B)$
- (b) Rearrange the above equation to find $P(A | B)$
- (c) Find $P(a)$ in terms of the joint $P(a, b)$ for any $a \in A, b \in B$ (Hint: these are specific values, answer should include a sum)
- (d) Find $P(b | a)$ in terms of the joint $P(a, b)$ for any $a \in A, b \in B$
- (e) Find $P(b | a)$ in terms of the distributions $P(b)$, $P(a | b)$, for any $a \in A, b \in B$
- (f) Assume we had some fixed a and wanted to find each element of $P(b | a)$ (i.e. wanted to find $P(B | a)$). Would the numerator of the fraction in the previous question change for each value of b ? What about the denominator? How could you use this to do the calculation with less steps?
- (g) Assume A is a random variable that can take 3 values, B is a random variable that can take 2 values, and C is a random variable that can take 5 values. What do the following probability tables sum to? Also, how many entries are in the following probability tables?
 - (a) $P(A | b)$
 - (b) $P(A | C)$
 - (c) $P(C | B)$
 - (d) $P(B | a)$
 - (e) $P(B | A)$

3 Cake



Consider the above cake with 12 slices. Let s_1 indicate a slice with no sprinkles and s_2 be a slice with sprinkles. Let c_1 indicate a slice with no candles and c_2 be a slice with candles. Let S be a random variable indicating sprinkles and C be a random variable indicating candles. Calculate the following probabilities.

1. $P(C = c_1)$

2. $P(S = s_1, C = c_2)$

3. $P(C = c_2 \mid S = s_1)$

4. $\sum_{s \in \{s_1, s_2\}} \sum_{c \in \{c_1, c_2\}} P(s, c)$

5. $\sum_{c \in \{c_1, c_2\}} \sum_{s \in \{s_1, s_2\}} P(s \mid c)$

6. $\sum_{s \in \{s_1, s_2\}} \sum_{c \in \{c_1, c_2\}} P(c \mid s)$

4 Queries on a Large Joint Distribution

Consider binary (two outcomes) random variables A, B, C, D , and the following joint distribution table of all four variables.

A	B	C	D	$P(A, B, C, D)$
$+a$	$+b$	$+c$	$+d$	$12/64$
$+a$	$+b$	$+c$	$-d$	$4/64$
$+a$	$+b$	$-c$	$+d$	$2/64$
$+a$	$+b$	$-c$	$-d$	$2/64$
$+a$	$-b$	$+c$	$+d$	$8/64$
$+a$	$-b$	$+c$	$-d$	$4/64$
$+a$	$-b$	$-c$	$+d$	$2/64$
$+a$	$-b$	$-c$	$-d$	$4/64$
$-a$	$+b$	$+c$	$+d$	$6/64$
$-a$	$+b$	$+c$	$-d$	$3/64$
$-a$	$+b$	$-c$	$+d$	$4/64$
$-a$	$+b$	$-c$	$-d$	$6/64$
$-a$	$-b$	$+c$	$+d$	$2/64$
$-a$	$-b$	$+c$	$-d$	$1/64$
$-a$	$-b$	$-c$	$+d$	$3/64$
$-a$	$-b$	$-c$	$-d$	$1/64$

1. Calculate the following probabilities:

(a) $P(+c)$

(b) $P(+a, -b)$

(c) $P(-b \mid +a)$

(d) $P(-a, +b, -d)$

(e) $P(+c \mid -a, +b, -d)$

(f) $P(+c, +d \mid +a, +b)$

2. What value do the following probability tables sum to?

(a) $P(B)$

(b) $P(+b \mid C, +d)$

(c) $P(C, D \mid +a, +b)$

5 Dancing With The Stars: Ilona, Stephen, and Chandler

“Obviously you can’t please everyone. I’m sure some people say, ‘Bloody old Len Goodman gets on my nerves.’” –*Len Goodman*

You are given the following probability tables for binary random variables I , S , C :

C	$P(C)$
$+c$	0.33
$-c$	0.67

C	S	$P(S C)$
$+c$	$+s$	0.5
$+c$	$-s$	0.5
$-c$	$+s$	0.14
$-c$	$-s$	0.86

S	C	I	$P(I S, C)$
$+s$	$+c$	$+i$	0.25
$+s$	$+c$	$-i$	0.75
$+s$	$-c$	$+i$	0.42
$+s$	$-c$	$-i$	0.58
$-s$	$+c$	$+i$	0.15
$-s$	$+c$	$-i$	0.85
$-s$	$-c$	$+i$	0.3
$-s$	$-c$	$-i$	0.7

1. Write the probability of I in terms of $P(C)$, $P(S|C)$, and $P(I|S, C)$
2. What should all the joint probabilities $P(I, S, C)$ add up to?

S	C	I	$P(S, C, I)$
$+s$	$+c$	$+i$	
$+s$	$+c$	$-i$	
$+s$	$-c$	$+i$	
$+s$	$-c$	$-i$	
$-s$	$+c$	$+i$	
$-s$	$+c$	$-i$	
$-s$	$-c$	$+i$	
$-s$	$-c$	$-i$	