1 MDPs: Warm-Up

1. What is the Markov Property?

2. What are the Bellman equations, and when are they used?

3. What is a policy? What is an optimal policy?

4. How does the discount factor γ affect how the agent finds the optimal policy? Why do we restrict gamma $\gamma < 1$?

5. Fill in the following table explaining the effects of having different gamma values:

γ	**Effect on policy search:**
$\gamma = 0$	
$\gamma = 1$	

6. What are the two steps to Policy Iteration?

7. What is the relationship between $V^*(s)$ and $Q^*(s,a)$?

8. (MDP Notation Review) Draw a line connecting each term in the left column with its corresponding equation in the right column.

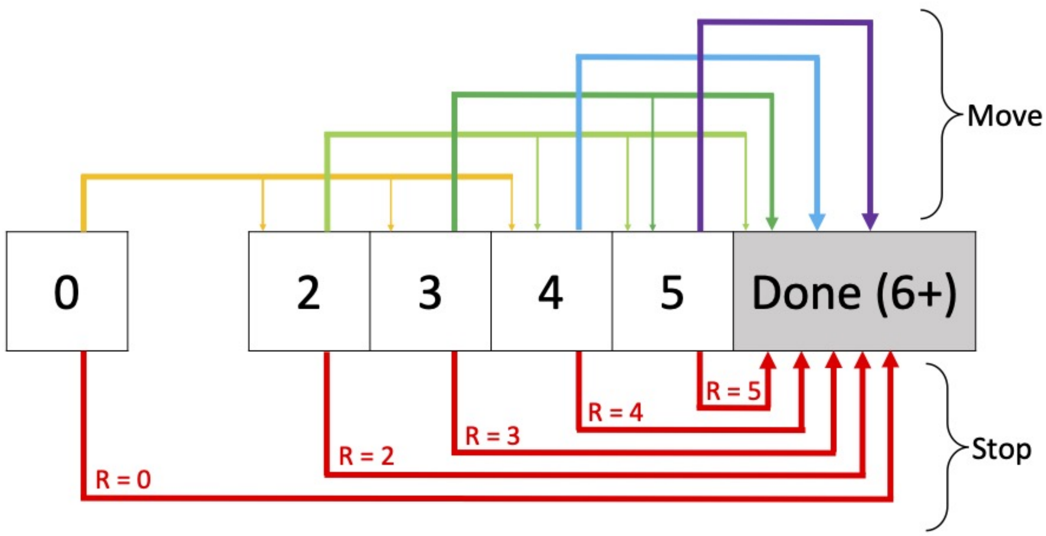
Term	*Equation*
Standard Expectimax	$\pi_V(s) = \text{argmax}_a \sum_{s'} P(s'\|s,a)[R(s,a,s')] + \gamma V(s')], \forall s$
Bellman Equation	$V_{k+1}(s) = \max_a \sum_{s'} P(s'\|s,a)[R(s,a,s')] + \gamma V_k(s')], \forall s$
Value Iteration	$V^*(s) = \max_a \sum_{s'} P(s'\|s,a)[R(s,a,s')] + \gamma V^*(s')]$
Q-Iteration	$Q_{k+1}(s,a) = \sum_{s'} P(s'\|s,a)[R(s,a,s')] + \gamma \max_{a'} Q_k(s',a')], \forall s,a$
Policy Extraction	$V_{k+1}^{\pi}(s) = \sum_{s'} P(s'\|s,\pi(s))[R(s,\pi(s),s')] + \gamma V_k^{\pi}(s')], \forall s$
Policy Evaluation	$\pi_{new}(s) = \text{argmax}_a \sum_{s'} P(s'\|s,a)[R(s,a,s')] + \gamma V^{\pi_{old}}(s')], \forall s$
Policy Improvement	$V(s) = \max_a \sum_{s'} P(s'\|s,a)V(s')$

2 MDPs: Racing

Consider a modification of the racing robot car example seen in lecture. In this game, the car repeatedly moves a random number of spaces that is equally likely to be 2, 3, or 4. The car can either Move or Stop if the total number of spaces moved is less than 6.

If the total spaces moved is 6 or higher, the game automatically ends, and the car receives a reward of 0. When the car Stops, the reward is equal to the total spaces moved (up to 5), and the game ends. There is no reward for the Move action.

Let's formulate this problem as an MDP with the states $\{0, 2, 3, 4, 5, Done\}$.



1. What is the transition function for this MDP? (You should specify discrete values for specific state/action inputs.)
2. What is the reward function for this MDP?

3. Recall the value iteration update equation:

$$V_{k+1}(s) = \max_a \sum_{s'} T(s, a, s')[R(s, a, s') + \gamma V_k(s')]$$

Perform value iteration for 4 iterations with $\gamma = 1$.

States	**0**	**2**	**3**	**4**	**5**	**Done**
V_0						0
V_1						0
V_2						0
V_3						0
V_4						0

4. You should have noticed that value iteration converged above. What is the optimal policy?

States	**0**	**2**	**3**	**4**	**5**
π^*					

5. How would our results change with $\gamma = 0.1$?

6. Now imagine we changed the rules so that moving to spaces after 5 does not immediately end the game, but rather wraps back around to the beginning states (this is the case where the states exist side by side in a loop).

   For example if you start in state 5, select the action Move, and move 2 spaces, you would end up in state 1 (this is a new state we would need introduce).

   To be clear, the states would now be states {0,1,2,3,4,5, *Done*}.

   How would this modification change our results? (Assume we again use γ=1)

3 MDPs: Policy Iteration

Recall the racing MDP from the prior problem. In this game, the car repeatedly moves a random number of spaces that is equally likely to be 2, 3, or 4. The car can either Move or Stop if the total number of spaces moved is less than 6.

If the total spaces moved is 6 or higher, the game automatically ends, and the car receives a reward of 0. When the car Stops, the reward is equal to the total spaces moved (up to 5), and the game ends. There is no reward for the Move action.

States: $\{0, 2, 3, 4, 5, Done\}$

Transition	Reward
$T(s, Stop, Done) = 1, \quad \forall s \neq Done$	$R(s, Stop, Done) = s, \quad \forall s \leq 5$
$T(0, Move, s') = \frac{1}{3}, \quad \forall s' \in \{2, 3, 4\}$	$R(s, a, s') = 0$ otherwise
$T(2, Move, s') = \frac{1}{3}, \quad \forall s' \in \{4, 5, Done\}$	
$T(3, Move, 5) = \frac{1}{3}$	
$T(3, Move, Done) = \frac{2}{3}$	
$T(4, Move, Done) = 1$	
$T(5, Move, Done) = 1$	
$T(s, a, s') = 0$ otherwise	

Now recall the policy evaluation and policy improvement equations, which together make up policy iteration.

$$\text{Policy Evaluation: } V_{k+1}^{\pi}(s) = \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$

$$\text{Policy Improvement: } \pi_{new}(s) \leftarrow \text{argmax}_a \sum_{s'} T(s, a, s')[R(s, a, s') + \gamma V^{\pi_{old}}(s')]$$

Perform two iterations of policy iteration for one step of this MDP, starting from the fixed policy below. Use the initial $\gamma = 1$.

States	0	2	3	4	5
π_0	Move	Stop	Move	Stop	Move
$V_0^{\pi_0}$					
$V_1^{\pi_0}$					
$V_2^{\pi_0}$					
$V_3^{\pi_0}$					
π_1					
$V_0^{\pi_1}$					
$V_1^{\pi_1}$					
$V_2^{\pi_1}$					
$V_3^{\pi_1}$					
π_2					

4 MDPs: Conceptual Questions

1. Which MDP methods are optimal (assuming sufficient convergence when iterating)? [Value Iteration, Policy Iteration, Both, Neither]

2. What are some limitations of value iteration? What are some limitations of policy iteration?

3. When does policy iteration end? Immediately after policy iteration ends (without performing additional computation), do we have the values of the optimal policy?

4. What changes if during policy iteration, you only run one iteration of Bellman update instead of running it until convergence? Do you still get an optimal policy?