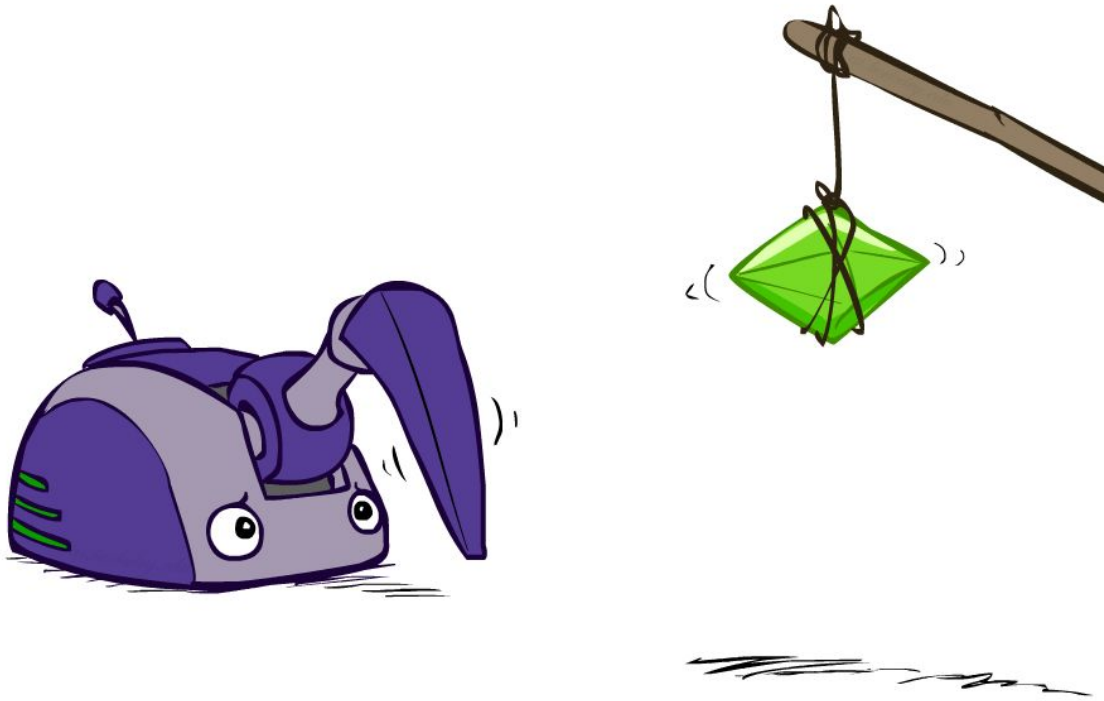
AI: Representation and Problem Solving

Reinforcement Learning I



Instructors: Tuomas Sandholm and Vincent Conitzer

Slide credits: CMU AI and http://ai.berkeley.edu

Reinforcement learning

What if we didn't know $P(s'|s,a)$ and $R(s,a,s')$?

Value iteration: $V_{k+1}(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')], \quad \forall\, s$

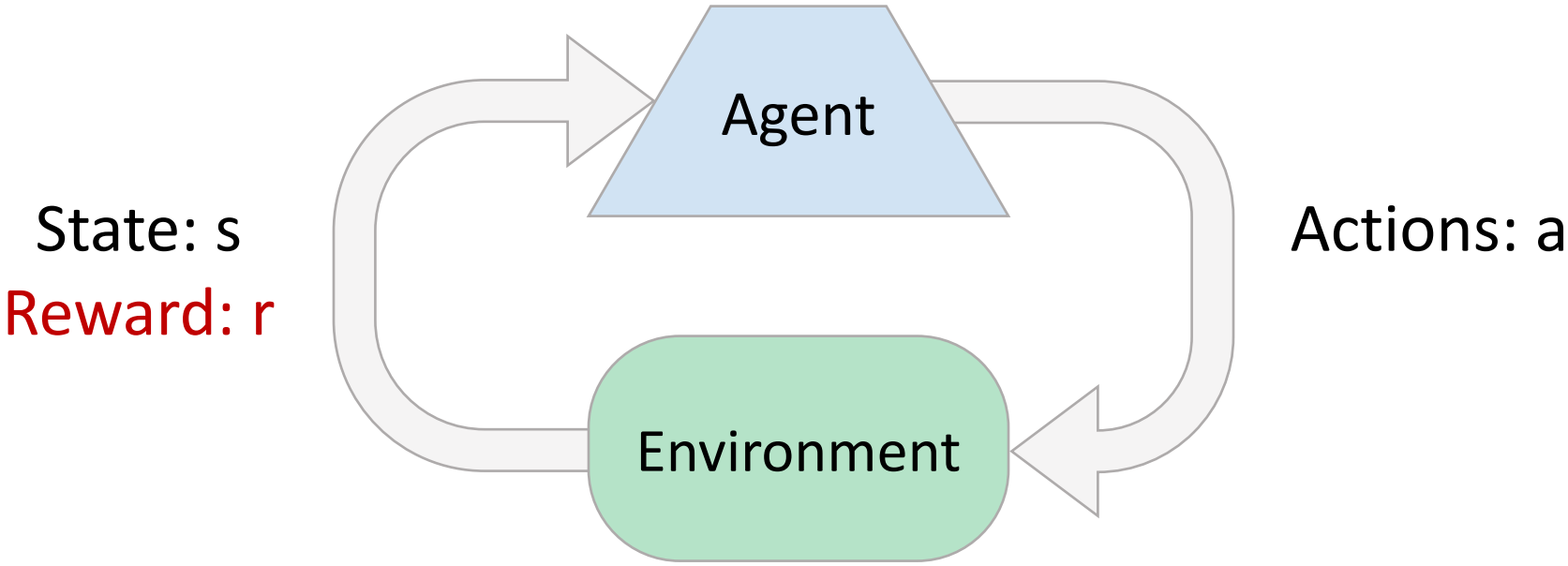
Q-iteration: $Q_{k+1}(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')], \quad \forall\, s,a$

Policy extraction: $\pi_V(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')], \quad \forall\, s$

Policy evaluation: $V^{\pi}_{k+1}(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma V^{\pi}_k(s')], \quad \forall\, s$

Policy improvement: $\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')], \quad \forall\, s$

Reinforcement Learning



Basic idea:

- All learning is based on observed samples of rewards and next states!
- Receive feedback in the form of rewards
- Must (learn to) act so as to maximize expected rewards

Example: Learning to Walk

Early on in training

[Kohl and Stone, ICRA 2004]

[Video: AIBO WALK – initial]

Example: Learning to Walk

Finished

[Kohl and Stone, ICRA 2004]

[Video: AIBO WALK – finished]

Example: Sidewinding

[Andrew Ng]

[Video: SNAKE – climbStep+sidewinding]

Example: Toddler Robot

[Tedrake, Zhang and Seung, 2005]

[Video: TODDLER – 40s]

Reinforcement Learning

Still assume a Markov decision process (MDP):

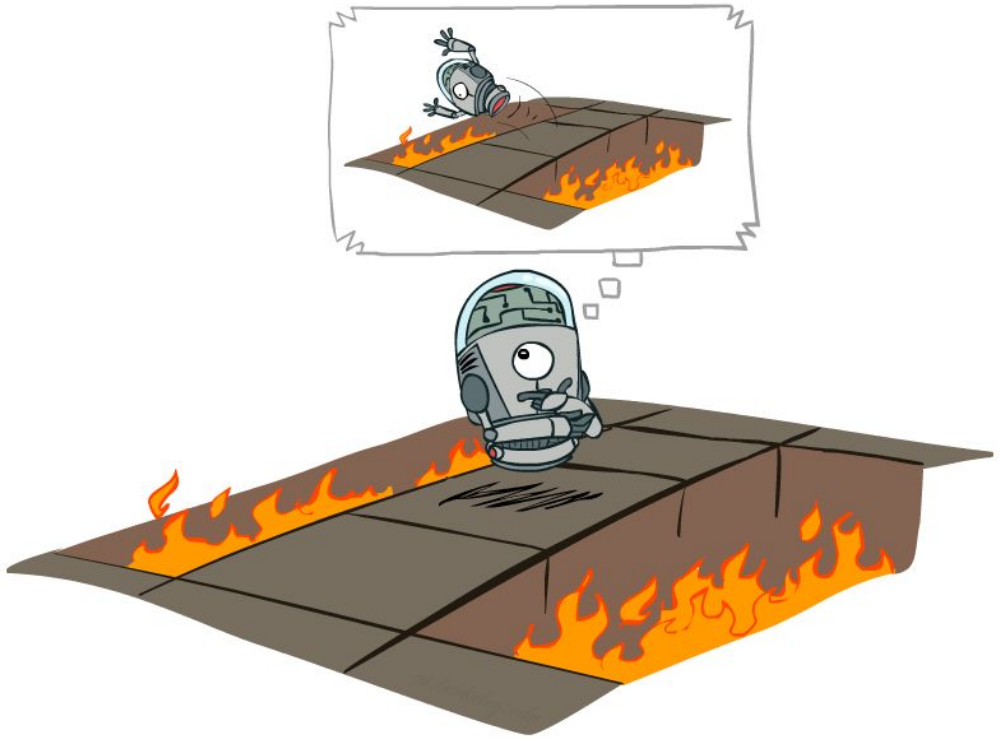
- A set of states $s \in S$
- A set of actions (per state) A
- A model T(s,a,s')
- A reward function R(s,a,s')

Still looking for a policy $\pi(s)$

New twist: don't know T or R

- I.e., we don't know which states are good or what the actions do
- Must actually try actions and states out to learn

Offline (MDPs) vs. Online (RL)



Offline Solution
(Known MDP)

Online Learning
(Unknown MDP)

Overview: MDPs and Reinforcement Learning

Known MDP: Offline Solution

Value Iteration / Policy Iteration

Unknown MDP: Online Learning

Model-Based

Estimate MDP T(s,a,s') and R(s,a,s') from samples of environment

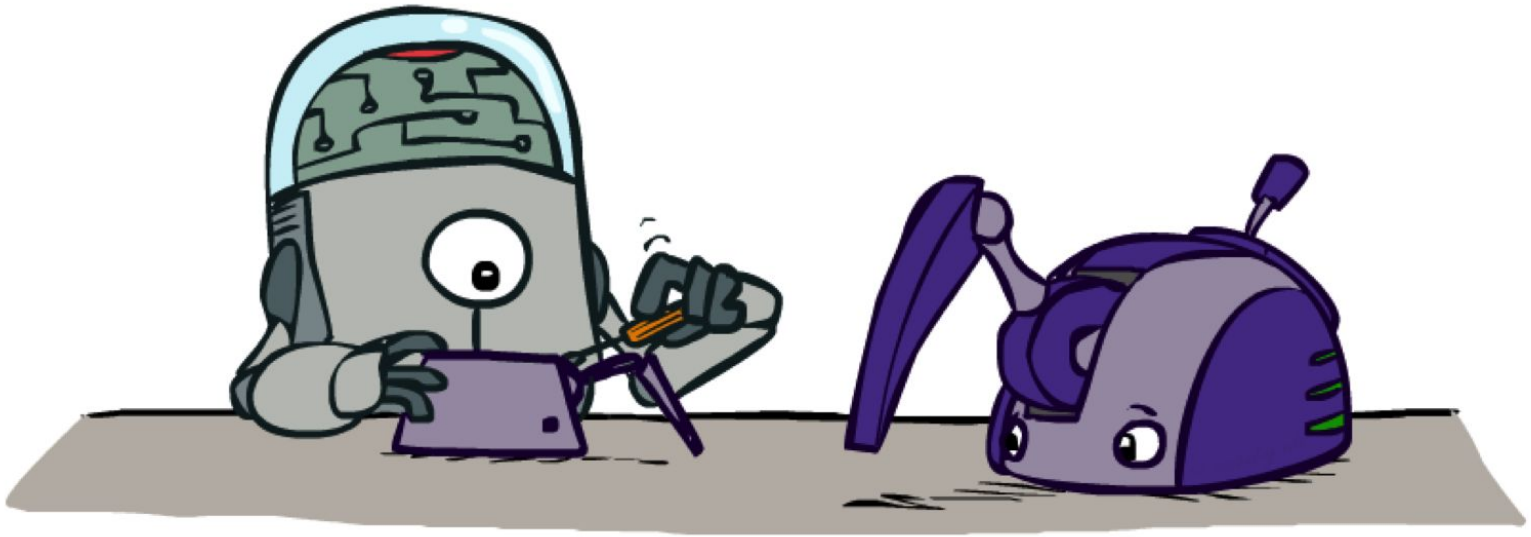
Model-Free

Passive Reinforcement Learning

- Direct Evaluation (simple)
- TD Learning

Active Reinforcement Learning

- Q-Learning



Online Learning

Model-based Learning

Model-Based Learning

Model-Based Idea:

- Learn an approximate model based on experiences
- Solve for values as if the learned model were correct

Step 1: Learn empirical MDP model

- Count outcomes s’ for each s, a
- Normalize to give an estimate of $\hat{T}(s, a, s')$
- Discover each $\hat{R}(s, a, s')$ when we experience (s, a, s’)

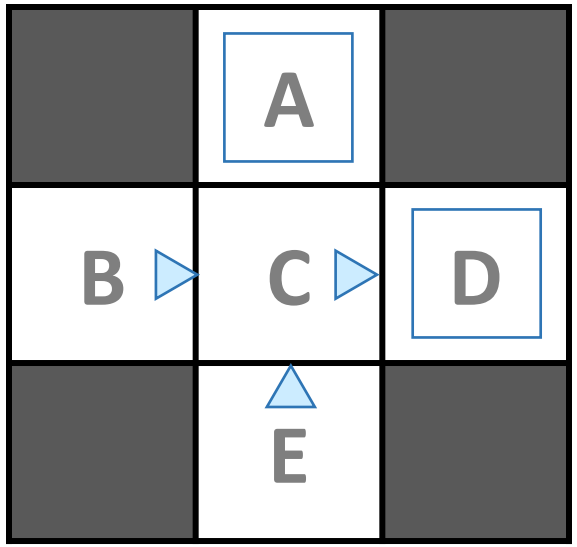
Step 2: Solve the learned MDP

- For example, use value iteration, as before

Example: Model-Based Learning

Episode: a sequence of states actions and rewards sampled from the environment

Input Policy π



Assume: γ = 1

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

$\hat{T}(s, a, s')$

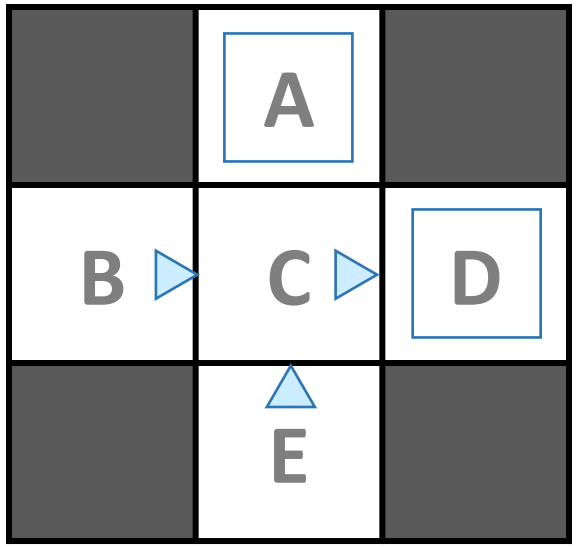
T(B, east, C) =
T(C, east, D) =
T(C, east, A) =
…

$\hat{R}(s, a, s')$

R(B, east, C) =
R(C, east, D) =
R(D, exit, x) =
…

Example: Model-Based Learning

Input Policy π



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Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

$\hat{T}(s, a, s')$

T(B, east, C) = 1.00
T(C, east, D) = 0.75
T(C, east, A) = 0.25
…

$\hat{R}(s, a, s')$

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
…

Example: Expected Age

Goal: Compute expected age of 15-281 students

Known P(A)

$$E[A] = \sum_a P(a) \cdot a \qquad = 0.35 \times 20 + \ldots$$

Without P(A), instead collect samples [a_1, a_2, … a_N]

Unknown P(A): "Model Based"

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

Unknown P(A): "Model Free"

$$E[A] \approx \frac{1}{N} \sum_i a_i$$

Why does this work? Because samples appear with the right frequencies.

Overview: MDPs and Reinforcement Learning

Known MDP: Offline Solution

Value Iteration / Policy Iteration

Unknown MDP: Online Learning

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Estimate MDP T(s,a,s') and R(s,a,s') from samples of environment

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Passive Reinforcement Learning

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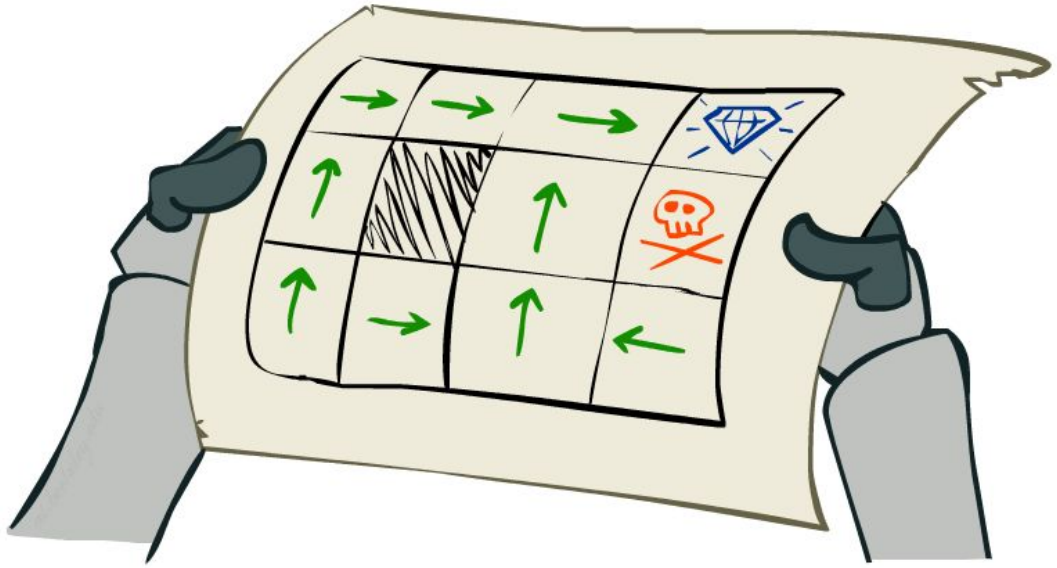
Online Learning



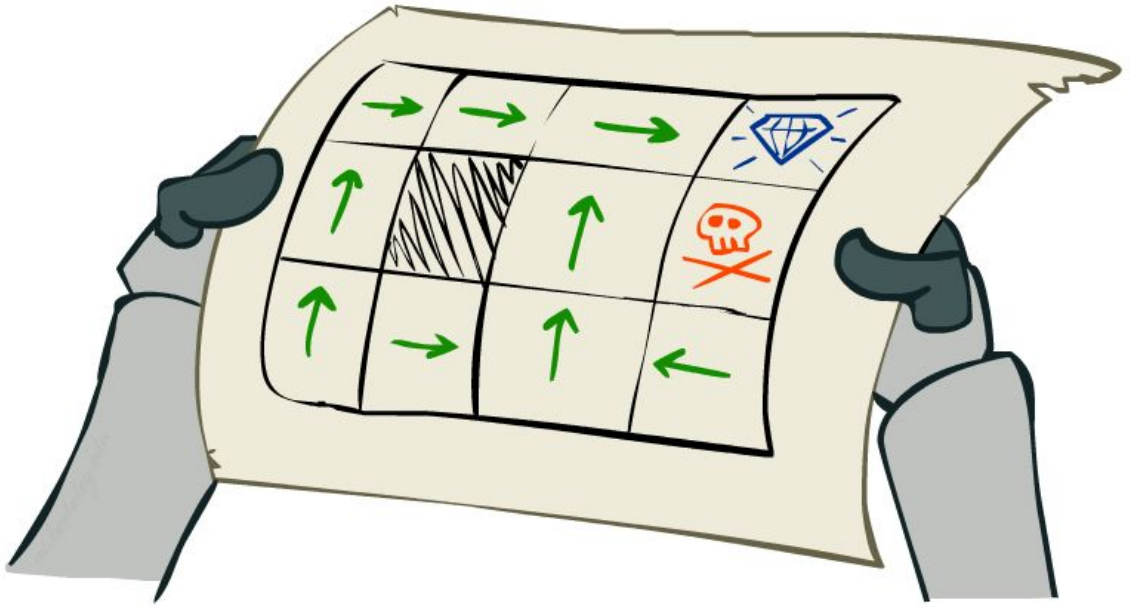
Model-free Learning

Passive Reinforcement Learning

Passive Reinforcement Learning

Simplified task: policy evaluation

- Input: a fixed policy π(s)
- You don't know the transitions T(s,a,s')
- You don't know the rewards R(s,a,s')
- Goal: learn the state values



In this case:

- Learner is "along for the ride"
- No choice about what actions to take
- Just execute the policy and learn from experience
- This is NOT offline planning! You actually take actions in the world.

Simple Passive Learning: Direct Evaluation

Goal: Compute values for each state under π

Idea: Average together observed sample values

- Act according to π
- Every time you visit a state, write down what the sum of discounted rewards turned out to be
- Average those samples

This is called direct evaluation

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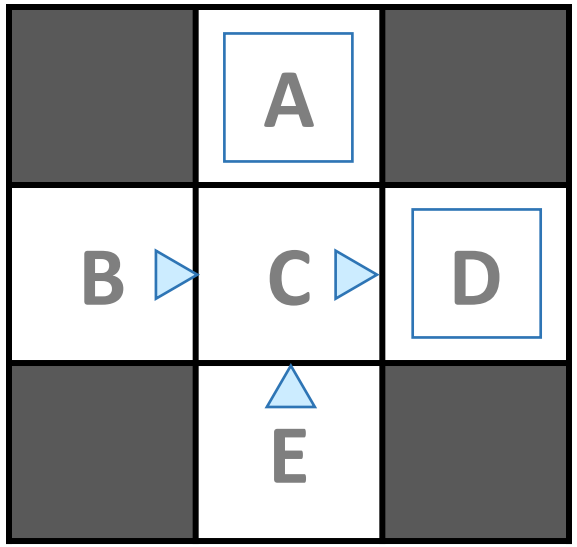
This is called direct evaluation

Pieces Available	Take 1	Take 2
2	0%	**100%**
3	2%	0%
4	**75%**	2%
5	4%	**68%**
6	5%	6%
7	**60%**	5%

Example: Direct Evaluation

Input Policy π



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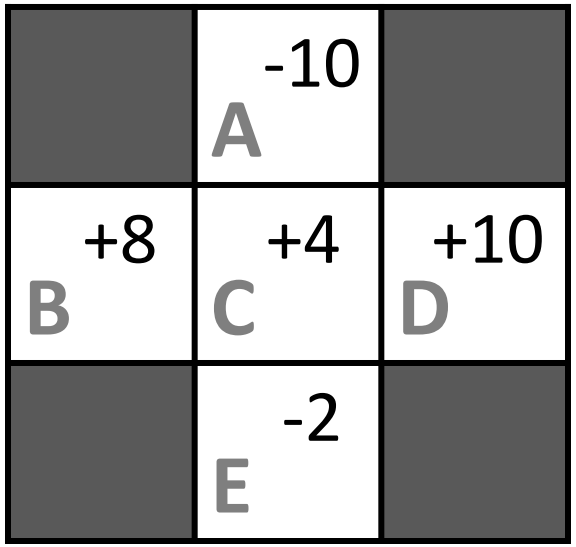
C, east, A, -1
A, exit, x, -10

Output Values



Problems with Direct Evaluation

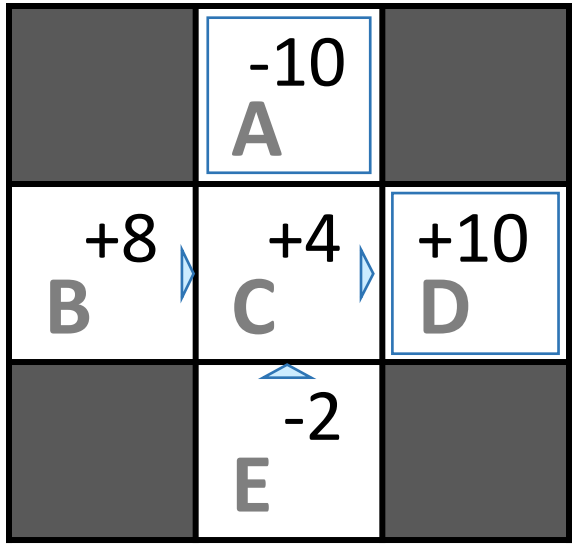
What's good about direct evaluation?

- It's easy to understand
- It doesn't require any knowledge of T, R
- It eventually computes the correct average values, using just sample transitions

What bad about it?

- It wastes information about state connections
- Each state must be learned separately
- So, it takes a long time to learn

Output Values



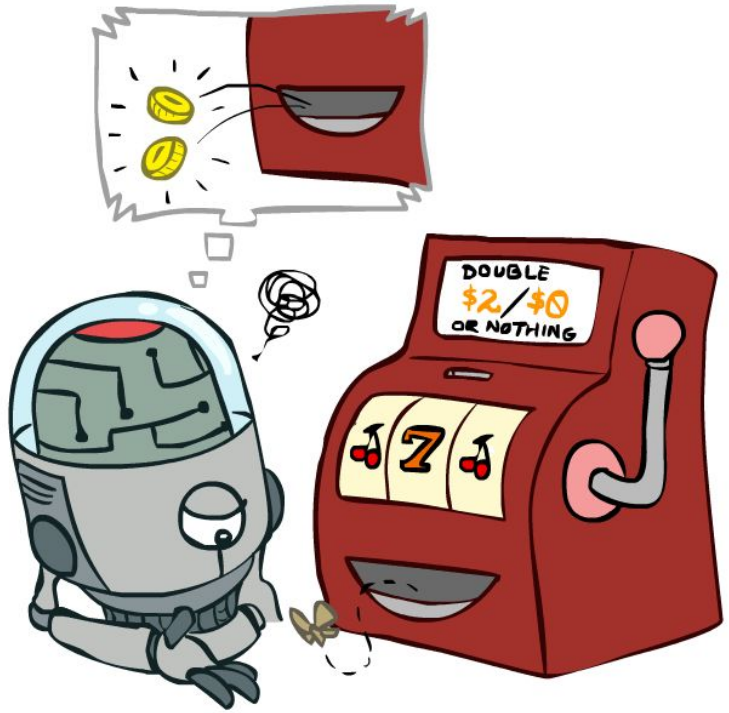
If B and E both go to C under this policy, how can their values be different?

Online Learning

Model-free Learning

Passive Reinforcement Learning

Temporal Difference Learning



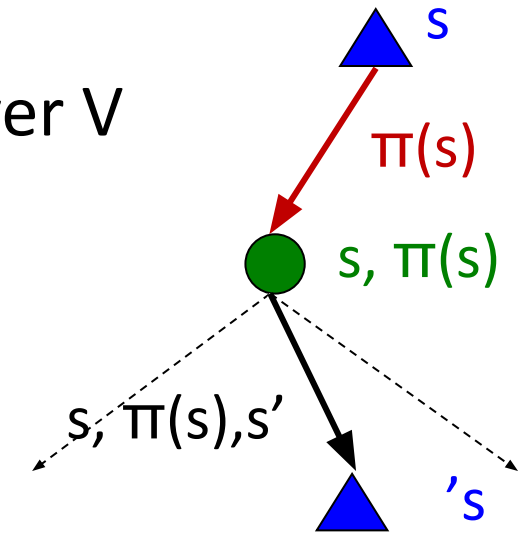
Why Not Use Policy Evaluation?

Simplified Bellman updates calculate V for a fixed policy:

- Each round, replace V with a one-step-look-ahead layer over V

$$V_0^{\pi}(s) = 0$$

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$



- This approach fully exploited the connections between the states
- Unfortunately, we need T and R to do it!

Key question: How can we do this update to V without knowing T and R?

- In other words, how can we take a weighted average without knowing the weights?

Sample-Based Policy Evaluation?

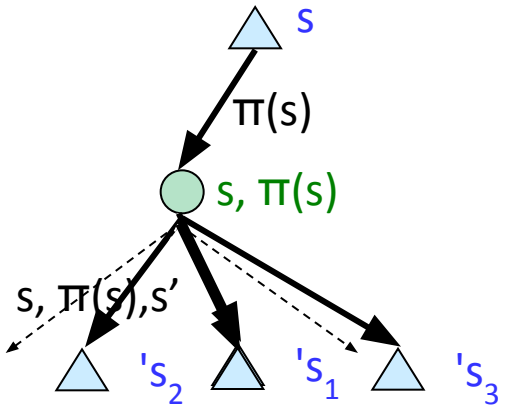
We want to improve our estimate of V by computing these averages:

$$V^{\pi}_{k+1}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V^{\pi}_{k}(s')]$$

Idea: Take samples of outcomes s' (by doing the action!) and average

$$sample_1 = R(s, \pi(s), s'_1) + \gamma V^{\pi}_{k}(s'_1)$$
$$sample_2 = R(s, \pi(s), s'_2) + \gamma V^{\pi}_{k}(s'_2)$$
$$\cdots$$
$$sample_n = R(s, \pi(s), s'_n) + \gamma V^{\pi}_{k}(s'_n)$$

$$V^{\pi}_{k+1}(s) \leftarrow \frac{1}{n} \sum_{i} sample_i$$

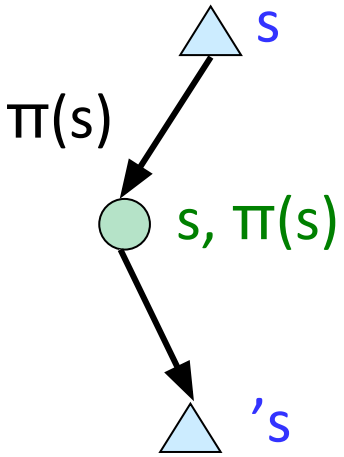


Almost! But we can't rewind time to get sample after sample from state s.

Temporal Difference (TD) Learning

Big idea: learn from every experience!

- Update V(s) each time we experience a transition (s, a, s', r)
- Likely outcomes s' will contribute updates more often



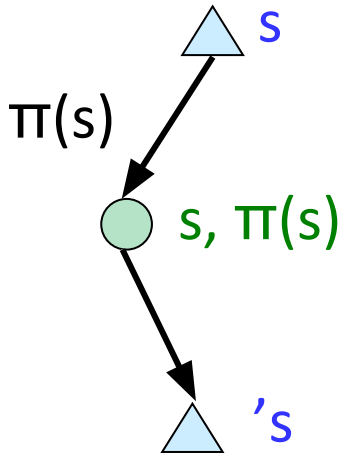
Temporal difference learning of values

- Policy still fixed, still doing evaluation!
- Move values toward value of whatever successor occurs: running average

Sample of V(s): $sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$

Update to V(s):

Temporal Difference Learning



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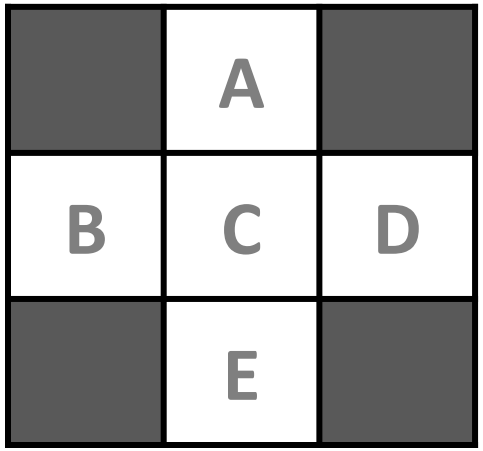
Update to V(s): $V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + (\alpha)sample$

Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$

Example: Temporal Difference Learning

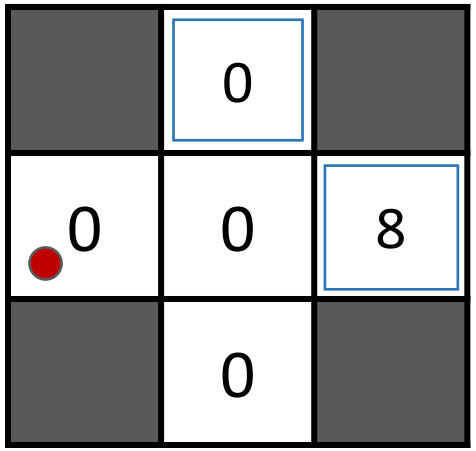
States



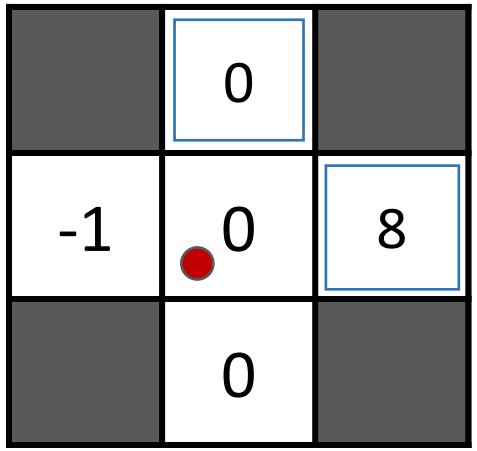
Assume: γ = 1,
α = 1/2

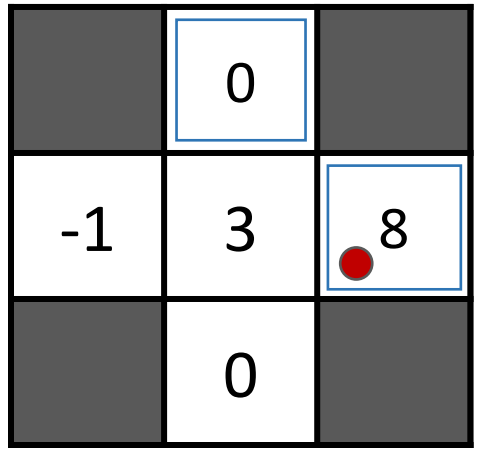
Observed Transitions

B, east, C, -2



C, east, D, -2





Example: Temporal Difference Learning

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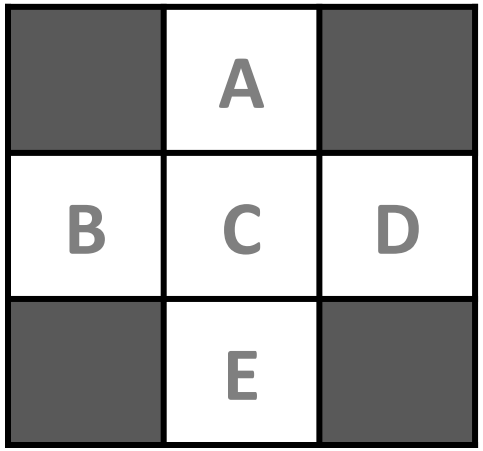

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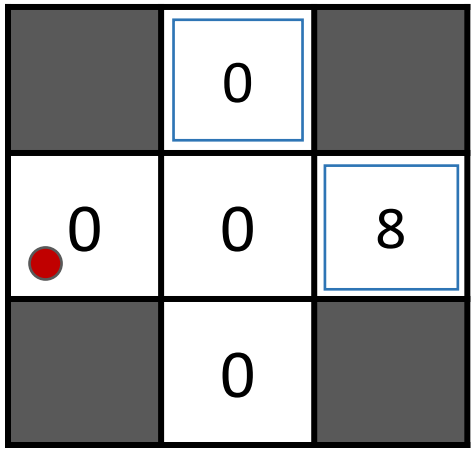
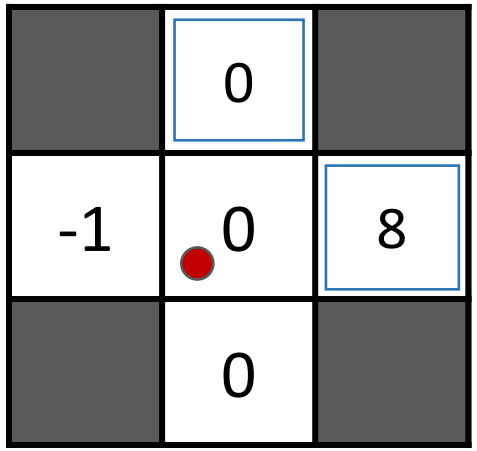
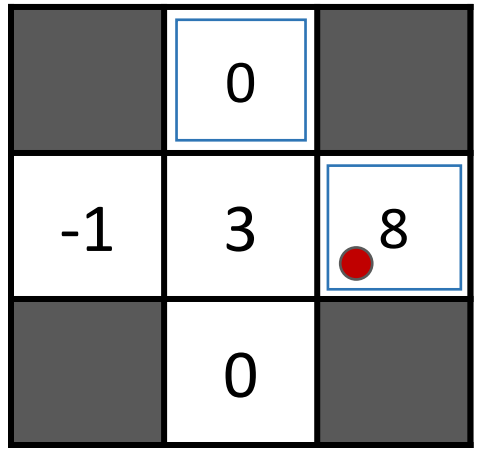


C, east, D, -2





$$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + \alpha\left[R(s,\pi(s),s') + \gamma V^{\pi}(s')\right]$$

Model-Free Learning

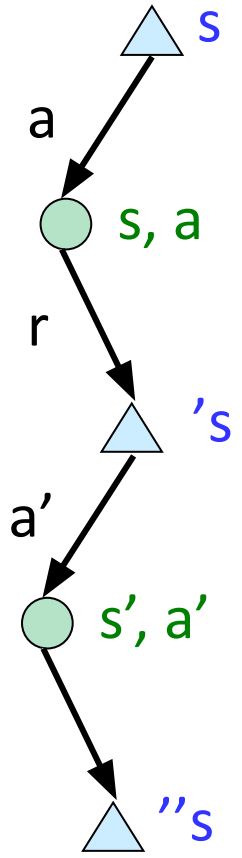
Model-free (temporal difference) learning

- Experience world through episodes

$$(s, a, r, s', a', r', s'', a'', r'', s'''' \ldots)$$

- Update estimates each transition (s, a, r, s')

- Over time, updates will mimic Bellman updates



Exponential Moving Average

Exponential moving average

- The running interpolation update: $\bar{x}_n = (1-\alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n$
- Makes recent samples more important:

$$\bar{x}_n = \frac{x_n + (1-\alpha) \cdot x_{n-1} + (1-\alpha)^2 \cdot x_{n-2} + \dots}{1 + (1-\alpha) + (1-\alpha)^2 + \dots}$$

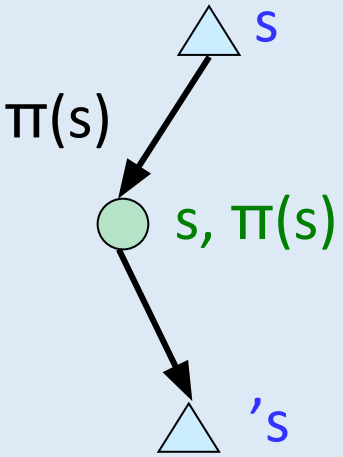
- Forgets about the past (distant past values were wrong anyway)

Decreasing learning rate α can give converging averages

A **Gradient Descent** Perspective on TD learning

Big idea: learn from every experience!

- Update V(s) each time we experience a transition (s, a, s', r)
- Likely outcomes s' will contribute updates more often



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Sample of V(s): $sample = r + \gamma V^{\pi}(s')$

Update to V(s): $V^{\pi}(s) \leftarrow (1-\alpha) V^{\pi}(s) + (\alpha)\, sample$

Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha\,[sample - V^{\pi}(s)]$

Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) - \alpha \nabla Error$ $\qquad Error = \frac{1}{2}\left(sample - V^{\pi}(s)\right)^2$

Quick Calculus Quiz

$$f(x) = \frac{1}{2}(y - x)^2$$

What is $\frac{df}{dx}$?

Gradient Descent

$$f(x) = \frac{1}{2}(y - x)^2$$

Goal: find x that minimizes $f(x)$

$$\frac{df}{dx} = -(y - x)$$

1. Start with initial guess, x_0
2. Update x by taking a step in the direction that $f(x)$ is changing fastest (in the negative direction) with respect to x:

$$Error(V) = \frac{1}{2}\,(sample - V)^2$$

Temporal Difference Learning

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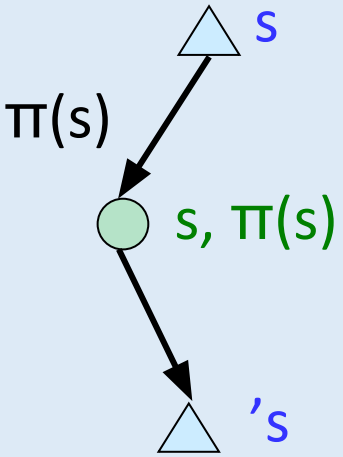

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Poll

TD update: $V^{\pi}(s) = V^{\pi}(s) + \alpha\,[r + \gamma\, V^{\pi}(s') - V^{\pi}(s)]$

Which converts TD values into a policy?

A) Value iteration: $V_{k+1}(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')]\,, \quad \forall\, s$

B) Q-iteration: $Q_{k+1}(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')]\,, \quad \forall\, s,a$

C) Policy extraction: $\pi_V(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')]\,, \quad \forall\, s$

D) Policy evaluation: $V^{\pi}_{k+1}(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma V^{\pi}_k(s')]\,, \quad \forall\, s$

E) Policy improvement: $\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')]\,, \quad \forall\, s$

F) None of the above

Poll

TD update: $V^{\pi}(s) = V^{\pi}(s) + \alpha\,[r + \gamma\, V^{\pi}(s') - V^{\pi}(s)]$

Which converts TD values into a policy?

A) Value iteration: $V_{k+1}(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')]\,, \quad \forall\, s$

B) Q-iteration: $Q_{k+1}(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')]\,, \quad \forall\, s,a$

C) Policy extraction: $\pi_V(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')]\,, \quad \forall\, s$

D) Policy evaluation: $V^{\pi}_{k+1}(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma V^{\pi}_k(s')]\,, \quad \forall\, s$

E) Policy improvement: $\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')]\,, \quad \forall\, s$

F) None of the above

Problems with TD Value Learning

TD value leaning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages

However, if we want to turn values into a (new) policy, we're sunk:

$$\pi(s) = \arg\max_a Q(s,a)$$

$$Q(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V(s') \right]$$

Idea: learn Q-values, not values

Makes action selection model-free too!