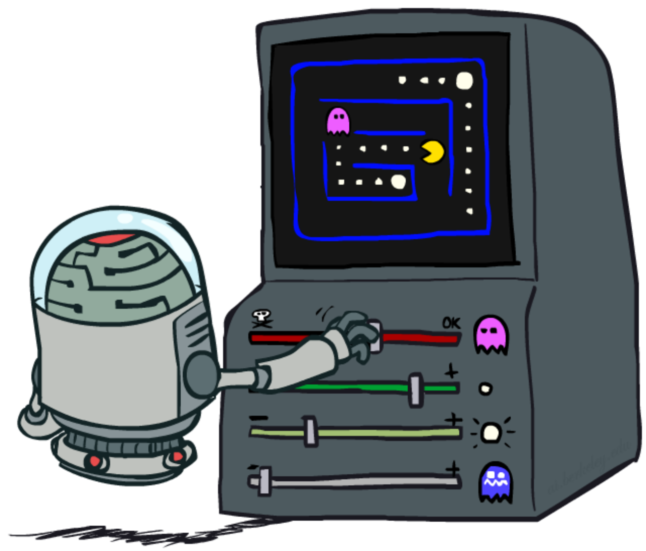
AI: Representation and Problem Solving

Reinforcement Learning II



Instructors: Tuomas Sandholm and Nihar Shah

Slide credits: CMU AI and http://ai.berkeley.edu

Logistics

- Midterm 2 on Wednesday
  - Covers from propositional logic up to (and including) basic Q learning
  - Doesn't include the rest today's lecture (after basic Q learning), which will be covered on the Final Exam
- Extra-cool, optional:
  Prof. Bart Selman will give the
  Inaugural Hans Berliner Lecture on
  "Mathematical and Scientific Discovery: A New Frontier for AI"
  - On 4/4/2024 at 4 PM in Rashid Auditorium in GHC

Reinforcement Learning (RL) Review So Far

- We still assume an MDP:
  - A set of states $s \in S$
  - A set of actions (per state) A
  - A model $T(s,a,s')$
  - A reward function $R(s,a,s')$
- Still looking for a policy $\pi(s)$

- The twist: don't know T or R, so must try out actions

- Big idea: Compute all averages over transition probabilities using sample outcomes

Summary so far

- **Passive RL:** agent has to learn from experience
- **Model-based:** Estimate the transition and rewards; run value iteration or policy iteration
- **Model-free:**
  - Direct policy evaluation – empirical average utility
  - Temporal difference learning – sample based policy iteration update via running averages

Temporal Difference learning

- **Main idea:** learn from each experience visiting state s, doing $\pi(s)$
- Update V(s) each time we experience a transition (s, a, s′, R)
  - Not waiting for the whole episode to get utility
- Likely outcomes s′ will contribute updates more often

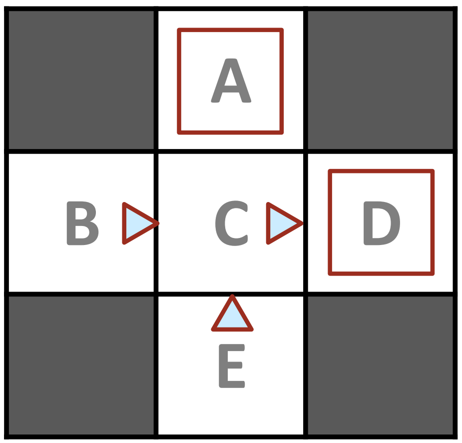
Sample of V^{π}(s): $sample = R + \gamma\, V^{\pi}(s')$

Update to V^{π}(s): $V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + (\alpha)sample$

- Decreasing learning rate (α) towards zero leads to convergence

Example: Model-Based Learning

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

$$\hat{T}(s, a, s')$$

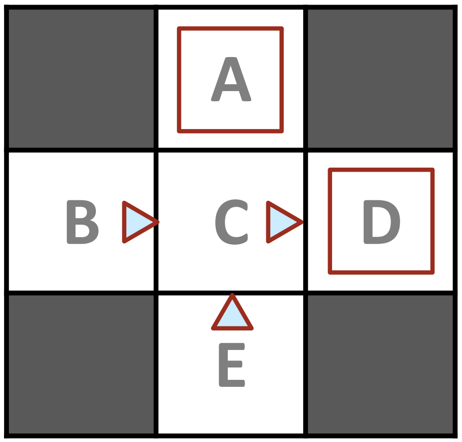
T(B, east, C) =
T(C, east, D) =
T(C, east, A) =
...

$$\hat{R}(s, a, s')$$

R(B, east, C) =
R(C, east, D) =
R(D, exit, x) =
...

Example: Model-Based Learning

Input Policy π



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D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

$\hat{T}(s, a, s')$

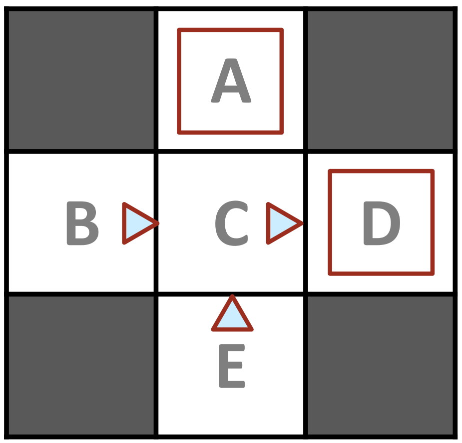
T(B, east, C) = 1
T(C, east, D) = 0.75
T(C, east, A) = 0.25
…

$\hat{R}(s, a, s')$

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = 10
…

Example: Model-Free Direct Evaluation

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Output Values

A:

B:

C:

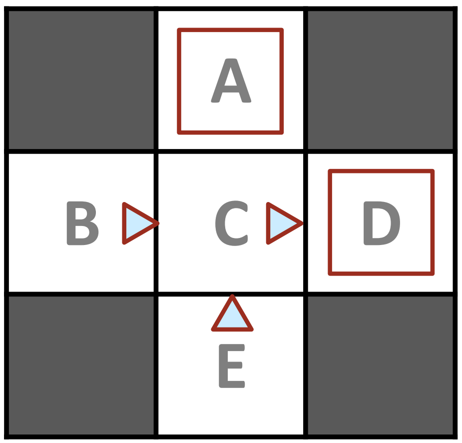
D:

E:

Algorithm: Average all total/future rewards that start at each state

Example: Model-Free Direct Evaluation

Input Policy π



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Observed Episodes (Training)

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E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Algorithm: Average all total/future rewards that start at each state

Output Values

A: -10 **[-10]**

B: 8, 8 **[8]**

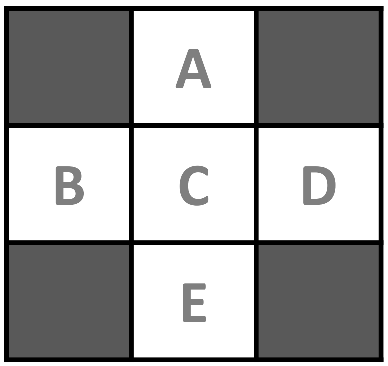
C: 9, 9, 9, -11 **[4]**

D: 10, 10, 10 **[10]**

E: 8, -12 **[-2]**

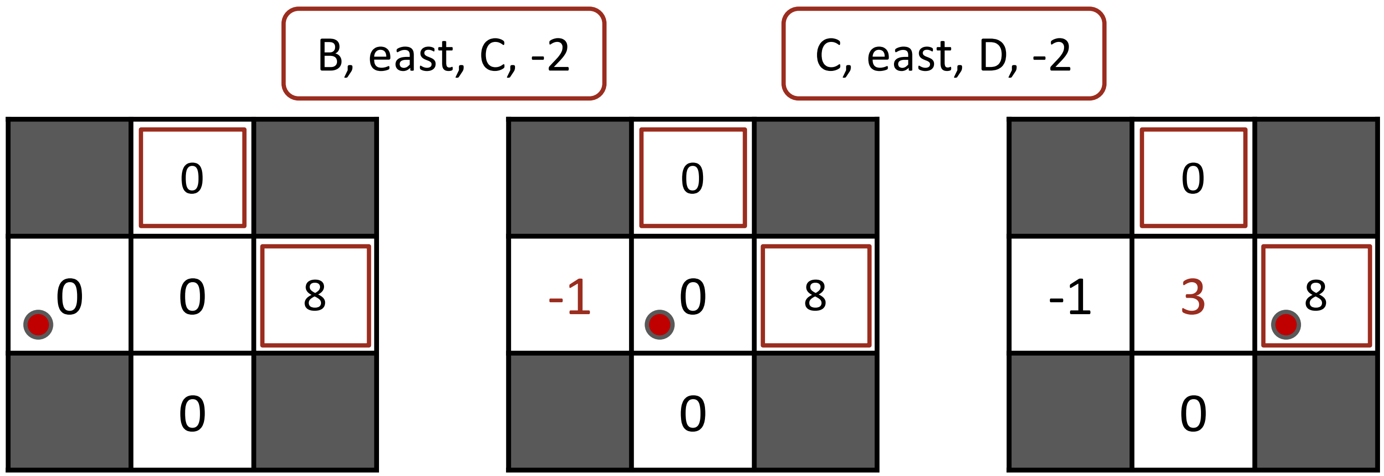
Example: Temporal Difference Learning

States



Assume: $\gamma = 1$, $\alpha = 0.5$

Observed Transitions



$$sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$$

$$V^{\pi}(s) \leftarrow (1 - \alpha) V^{\pi}(s) + (\alpha) sample$$

Poll

Which of the following allows you to estimate the **optimal policy**?

- (A) Model-based RL
- (B) Model-free RL : direct policy evaluation
- (C) Model-free RL : temporal difference value learning

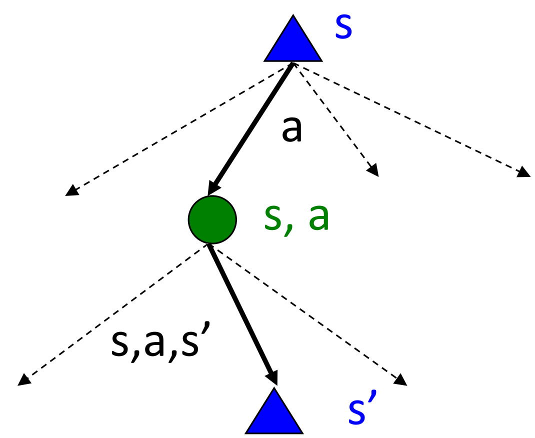
From TD Value Learning to Q-learning

- TD value learning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages
- If we want to turn values into a (new) policy, we can learn Q-values, not state values

$$\pi(s) = \arg\max_a Q(s,a)$$

$$Q(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V(s')\right]$$

Conceptual; actual details in next slides



Bootstrapped prediction of Q-values

<u>Estimating $V^{\pi}(s)$ from (s, a, s′, r)</u>

Sample of V^{π}(s): $sample = r + \gamma\, V^{\pi}(s')$

Update to V^{π}(s): $V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + (\alpha)sample$

<u>Estimating $Q^{\pi}(s)$ from (s, a, s′, r, a′)</u>

Sample of Q^{π}(s): $sample = r + \gamma\, Q^{\pi}(s', a')$

Update to Q^{π}(s): $Q^{\pi}(s) \leftarrow (1 - \alpha)\, Q^{\pi}(s) + \alpha\, sample$

Q-learning

- Expectimax update for **optimal** Q-values

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

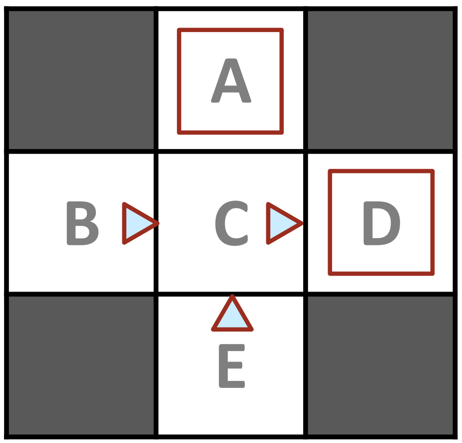
- Q-learning: sample-based Q-value iteration
- Given data (s, a, s′, R):
  - sample $= \mathrm{R} + \gamma \; \max_{a'} \; Q(s, a')$ *(consider new sample estimate)*
  - $Q(s,a) = (1 - \alpha) Q(s,a) + \alpha \; sample$ *(incorporate into running avg)*

Q-learning properties

- Important property: Q-learning converges to values of the optimal policy even if you are acting suboptimally
- This is called off-policy learning
  - Learning about the **optimal policy** while the experience is obtained via a **different (suboptimal) policy**
- Caveats:
  - Data-collecting policy has to explore enough
  - Have to lower the learning rate α eventually
    - But not too quickly
- Basically, in the limit, doesn't matter how you select actions!

In-class activity

Input S,A



Assume: $\gamma = 1$

$\alpha = 0.5$

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Episode 4

E, north, C, -1
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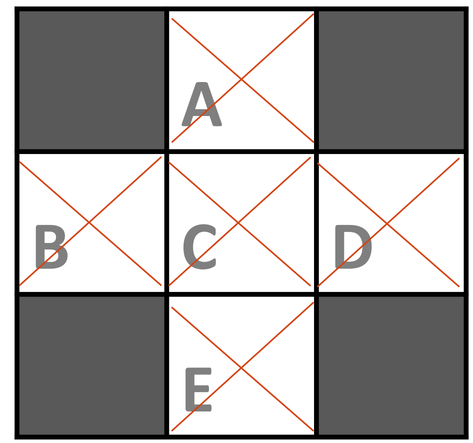
Output Q-Values



$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[r + \gamma \max_{a'} Q(s',a')\right]$$

The Story So Far: MDPs and RL

Known MDP: Offline Solution

Goal	Technique
Compute V^*, Q^*, π^*	Value / policy iteration
Evaluate a fixed policy π	Policy evaluation

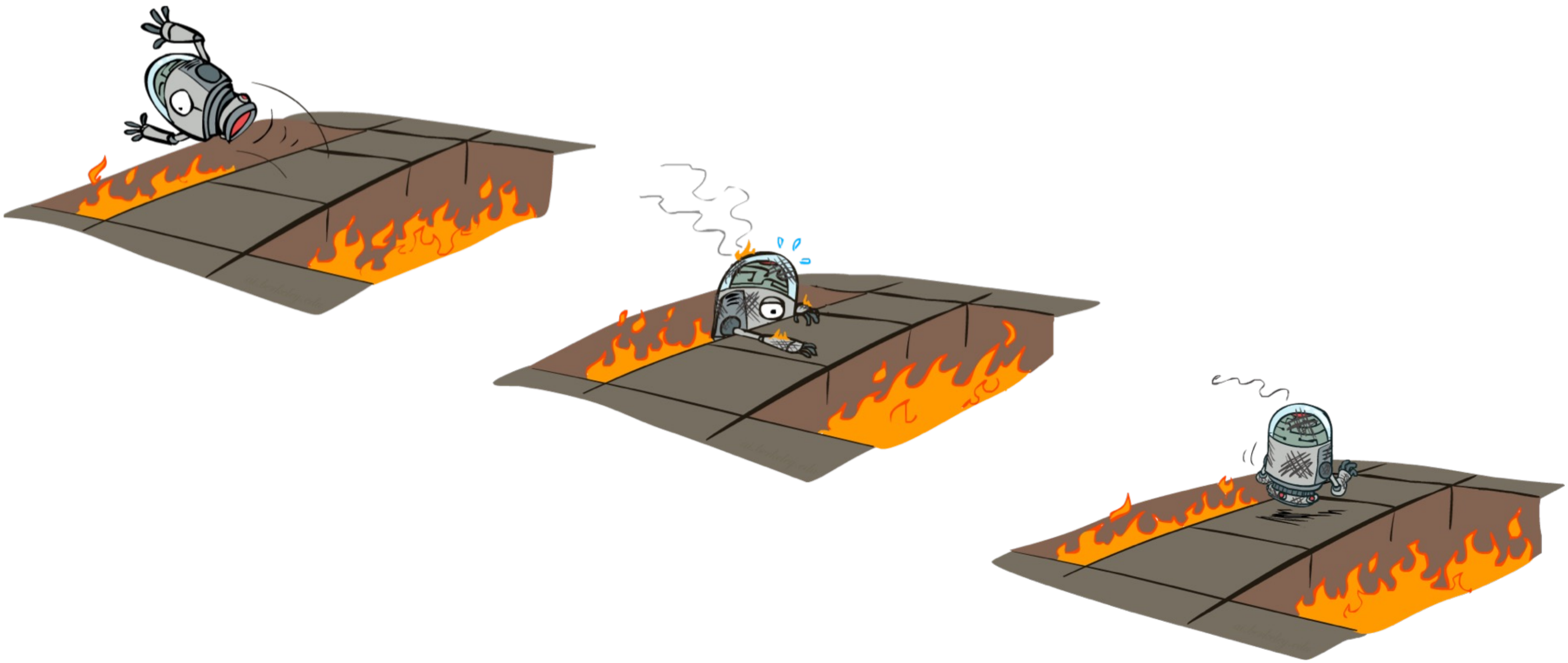
Model-Based RL

Goal	Technique
Compute V^*, Q^*, π^*	VI/PI on approx. MDP
Evaluate a fixed policy π	PE on approx. MDP

Model-Free RL

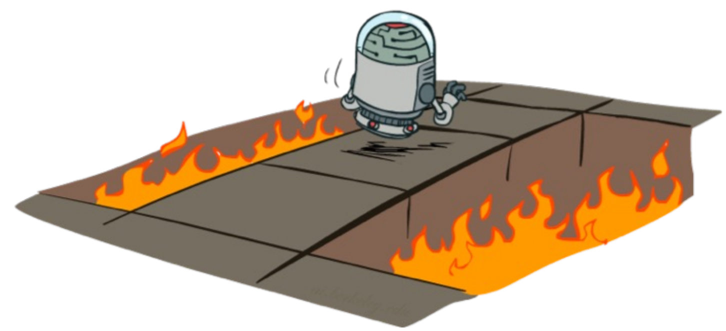
Goal	Technique
Compute V^*, Q^*, π^*	Q-learning
Evaluate a fixed policy π	TD Value Learning

Active Reinforcement Learning



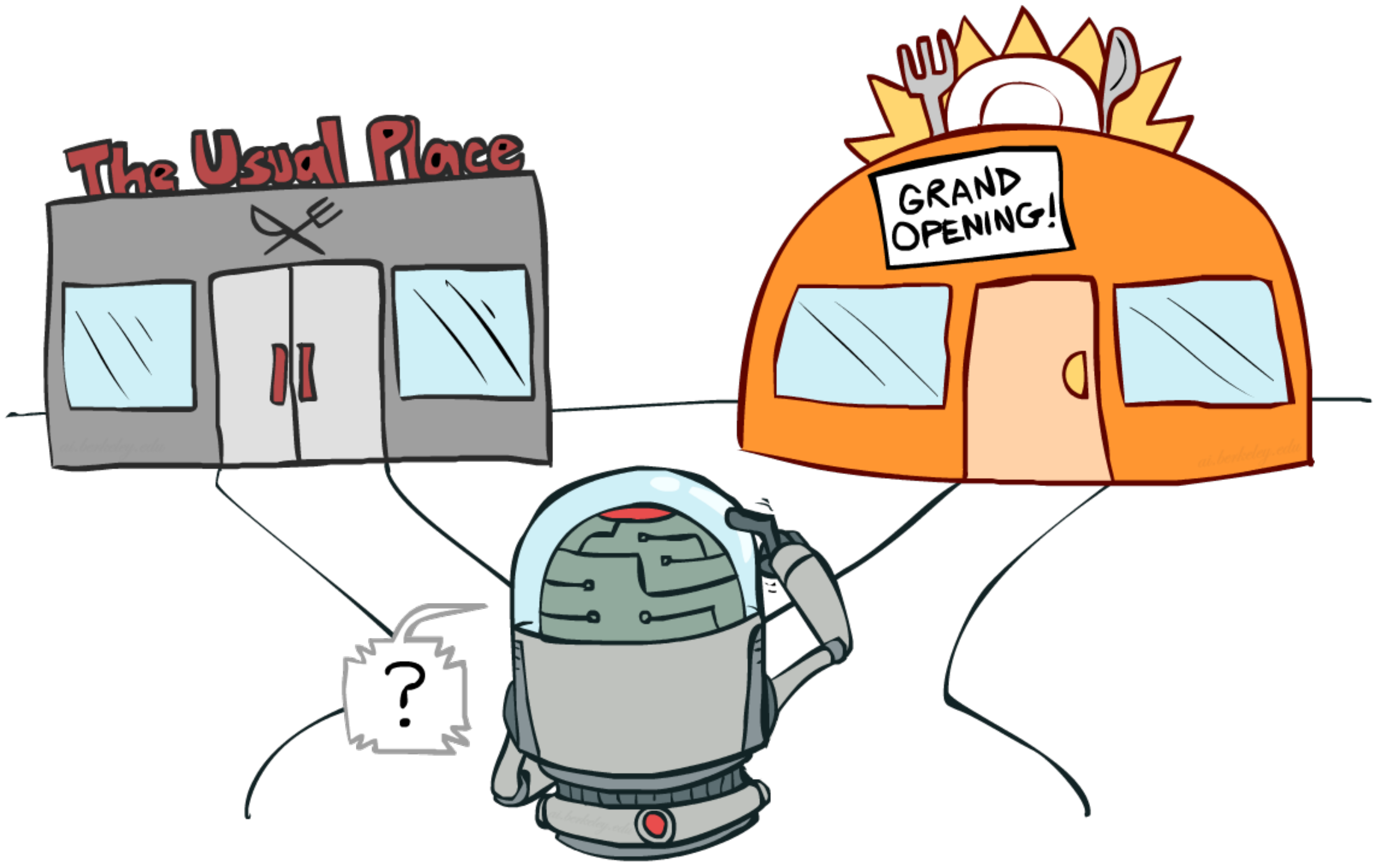
Active Reinforcement Learning

- Full reinforcement learning: optimal policies (like value iteration)
  - You don't know the transitions T(s,a,s')
  - You don't know the rewards R(s,a,s')
  - You choose the actions now
  - Goal: learn the optimal policy / values



- In this case:
  - Learner makes choices!
  - Fundamental tradeoff: exploration vs. exploitation
  - This is NOT offline planning! You actually take actions in the world and find out what happens…

Exploration vs. Exploitation



How to Explore?

- Several schemes for forcing exploration
  - Simplest: random actions (ε-greedy)
    - Every time step, flip a coin
    - With (small) probability ε, act randomly
    - With (large) probability 1-ε, act on current policy
  - Problems with random actions?
    - You do eventually explore the space, but keep thrashing around once learning is done
    - One solution: lower ε over time
    - Another solution: exploration function …

Exploration Functions

- When to explore?
  - Random actions: explore a fixed amount
  - Better idea: explore areas whose badness is not (yet) established, eventually stop exploring
- Exploration function
  - Takes a value estimate u and a visit count n, and returns an optimistic utility, e.g.

$$f(u, n) = u + k/(n+1)$$

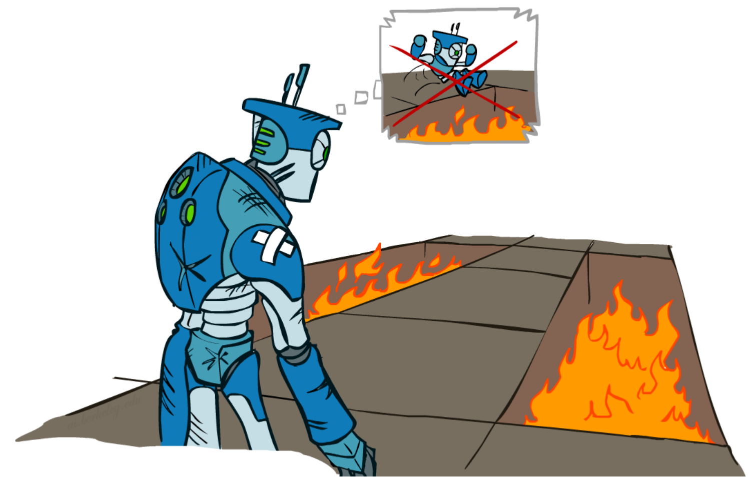
Regular Q-Update: $Q(s,a) = (1-\alpha)Q(s,a) + \alpha\,[r + \gamma \max_{a'} Q(s',a')]$

Modified Q-Update: $Q(s,a) = (1-\alpha)Q(s,a) + \alpha\,[r + \gamma \max_{a'} f(Q(s',a'), N(s',a'))]$

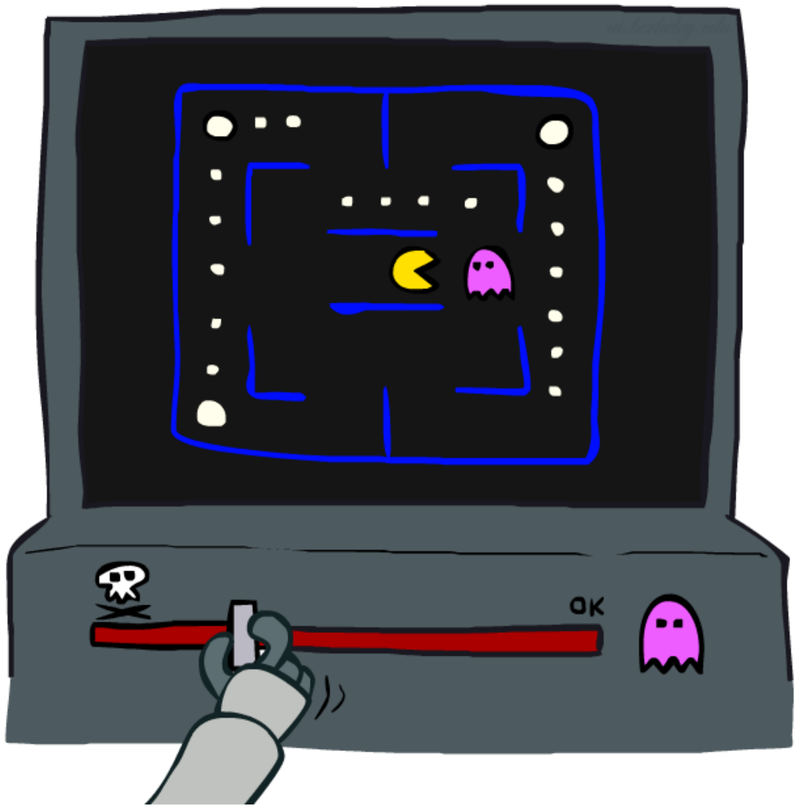
  - *Note: this propagates the "bonus" back to states that lead to unknown states as well!*

Regret

- Even if you learn the optimal policy, you still make mistakes along the way!
- Regret is a measure of your total mistake cost: the difference between your (expected) rewards, including youthful suboptimality, and optimal (expected) rewards
- Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal
- Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret

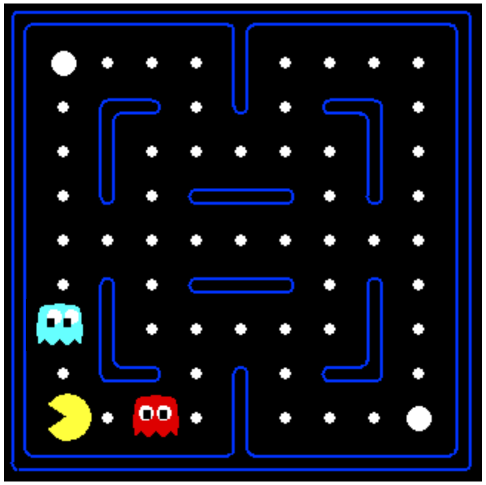


Approximate Q-Learning

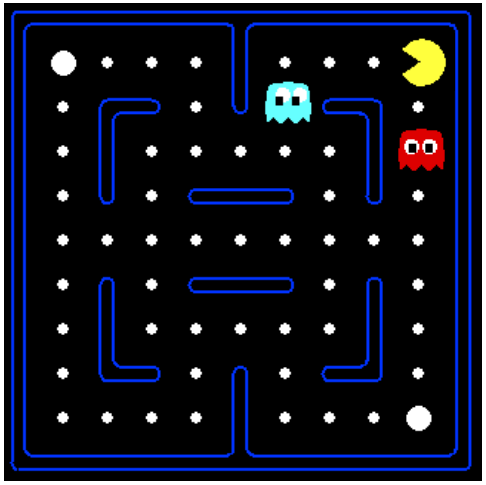


Example: Pacman

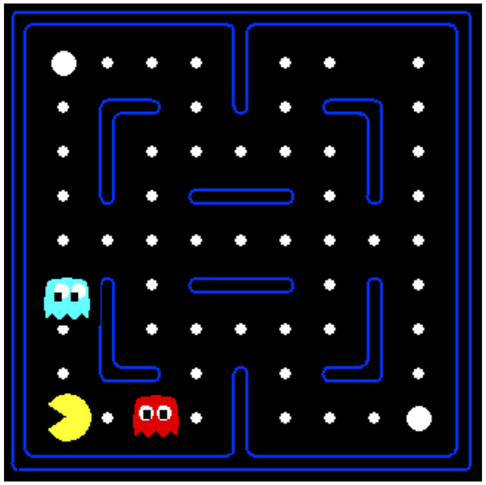
Let's say we discover through experience that this state is bad:



In naïve q-learning, we know nothing about this state:

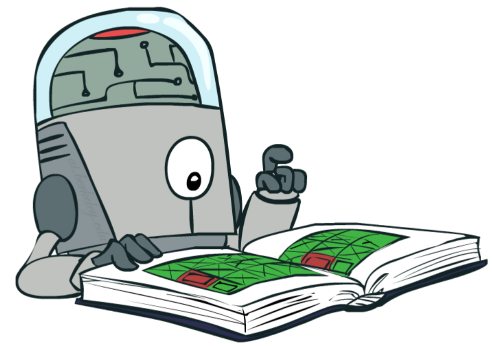


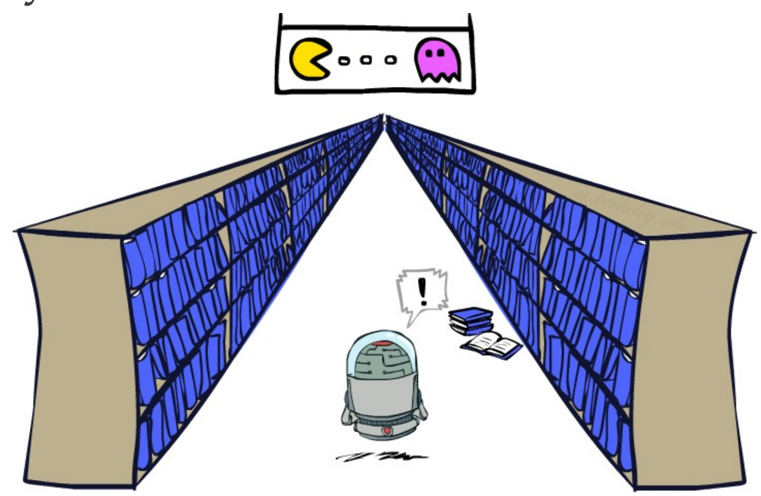
Or even this one!



Generalizing Across States

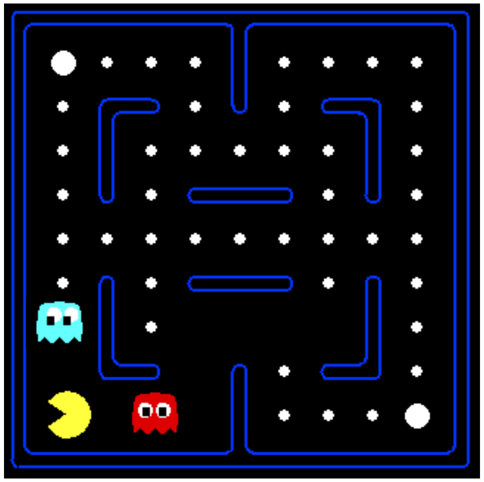
- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations





Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Can also describe a q-state (s, a) with features (e.g., action moves closer to food)
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)^2
    - Is Pacman in a tunnel? (0/1)



Linear value functions

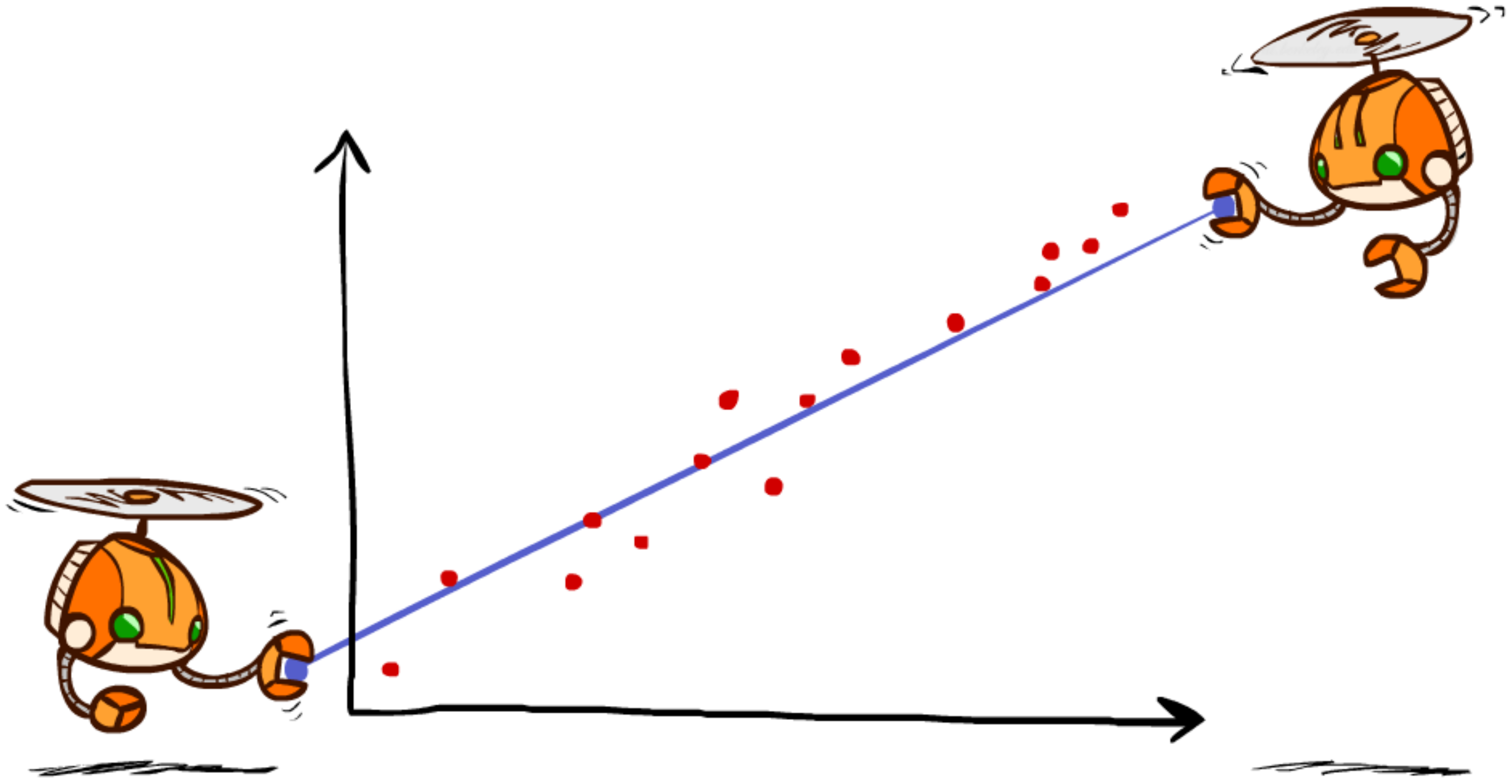
- Using a feature representation, we can write a q function (or value function) for any state using a few weights:
  - $V_w(s) = w_1 f_1(s) + w_2 f_2(s) + \ \dots + w_n f_n(s)$
  - $Q_w(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ \dots + w_n f_n(s,a)$
- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but actually be very different in value!

Updating a linear value function

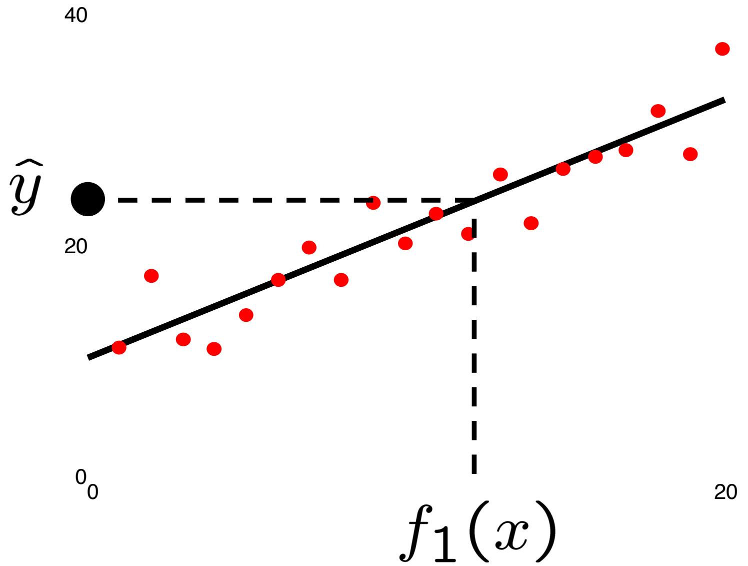
- Original Q learning rule tries to reduce prediction error at s, a:
  - $Q(s,a) \leftarrow (1 - \alpha)Q(s,a) + \alpha[R(s,a,s') + \gamma \max_{a'} Q(s',a')]$
  - $Q(s,a) \leftarrow Q(s,a) + \alpha[R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)]$

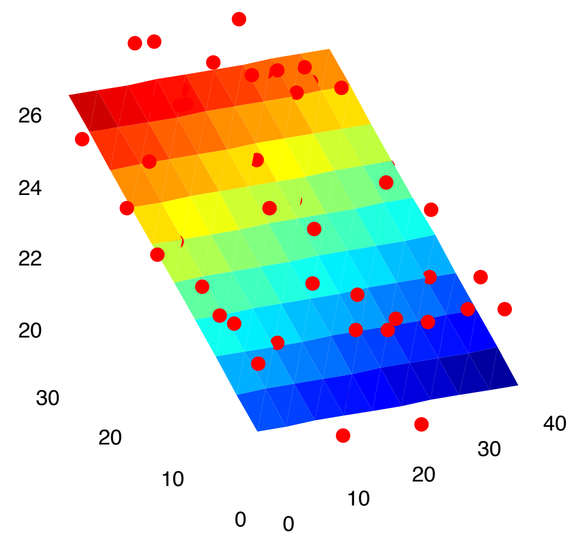
- Instead, we update the weights to try to reduce the error at s, a:
  - $w_i \leftarrow ?$

Detour: Minimizing Error and Least Squares



Linear Approximation: Regression





Prediction:

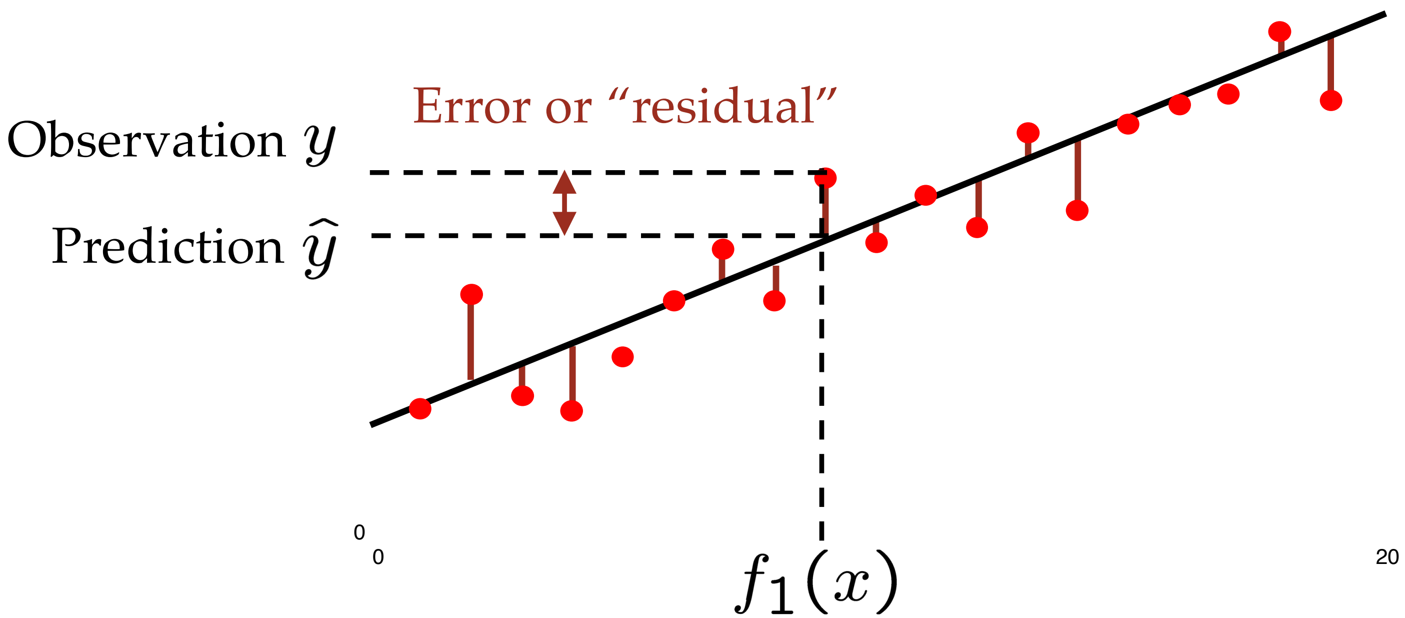
$$\hat{y} = w_0 + w_1 f_1(x)$$

Prediction:

$$\hat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

Optimization: Least Squares

$$\text{total error} = \sum_i (y_i - \hat{y}_i)^2 = \sum_i \left(y_i - \sum_k w_k f_k(x_i) \right)^2$$



Gradient Descent

Goal: find x that minimizes $f(x)$

1. Start with initial guess, x_0
2. Update x by taking a step in the direction that $f(x)$ is changing fastest (in the negative direction) with respect to x:

   $x \leftarrow x - \alpha \nabla_x f$, where α is the step size or learning rate
3. Repeat until convergence

Gradient Descent and Q learning

- Gradient descent on $f(x) = \frac{1}{2}(y - x)^2$
- We know that $\frac{df}{dx} = -(y - x)$; so $x \leftarrow x + \alpha\,(y - x)$
- Q-learning: find values Q(s, a) that minimizes difference between samples and Q(s, a)
  - $Error(Q(s,a)) = \frac{1}{2}\,(sample - Q(s,a))^2$
  - $Q(s,a) \leftarrow Q(s,a) - \alpha \nabla_{Q(s,a)} Error$
  - $Q(s,a) \leftarrow Q(s,a) + \alpha[(R(s,a,s') + \gamma \max_{a'} Q(s',a')) - Q(s,a)]$

"target" (sample) *"prediction"*

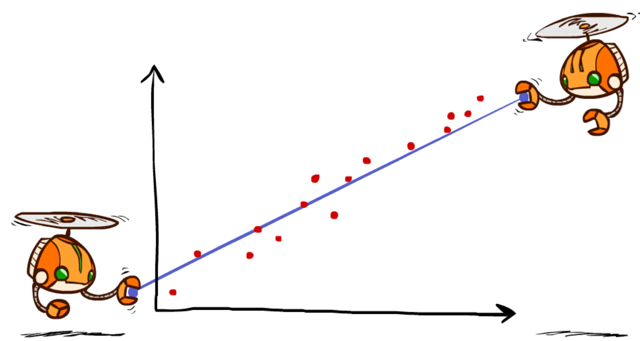
Approximate Q-learning and gradient descent

Imagine we had only one point x, with features f(x), target value y, and weights w:

$$\text{error}(w) = \frac{1}{2}\left(y - \sum_k w_k f_k(x)\right)^2$$

$$\frac{\partial\ \text{error}(w)}{\partial w_m} = -\left(y - \sum_k w_k f_k(x)\right) f_m(x)$$

$$w_m \leftarrow w_m + \alpha \left(y - \sum_k w_k f_k(x)\right) f_m(x)$$



Approximate Q-update:

$$w_m \leftarrow w_m + \alpha \left[r + \gamma \max_a Q(s', a') - Q(s,a)\right] f_m(s,a)$$

"target" (sample) *"prediction"*

Updating a linear value function

- Original Q learning rule tries to reduce prediction error at s, a:
  - $Q(s,a) \leftarrow Q(s,a) + \alpha[(R(s,a,s') + \gamma \max_{a'} Q(s',a')) - Q(s,a)]$

- Instead, we update the weights to try to reduce the error at s, a:
  - $w_i \leftarrow w_i + \alpha * f_i(s,a) * [(R(s,a,s') + \gamma \max_{a'} Q(s',a')) - Q(s,a)]$

Approximate Q-Learning summary

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

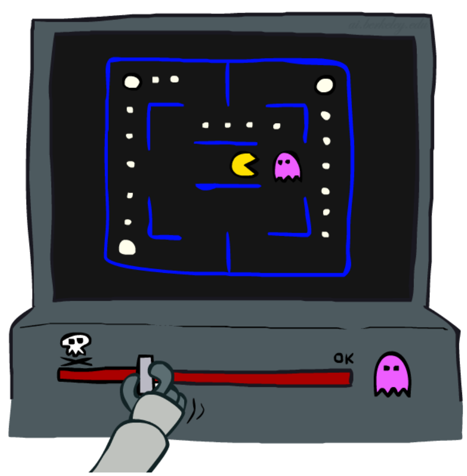
- Q-learning with linear Q-functions:

$$\text{transition} = (s, a, r, s')$$

$$\text{difference} = \left[r + \gamma \max_{a'} Q(s', a') \right] - Q(s,a)$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha \, [\text{difference}] \qquad \text{Exact Q's}$$

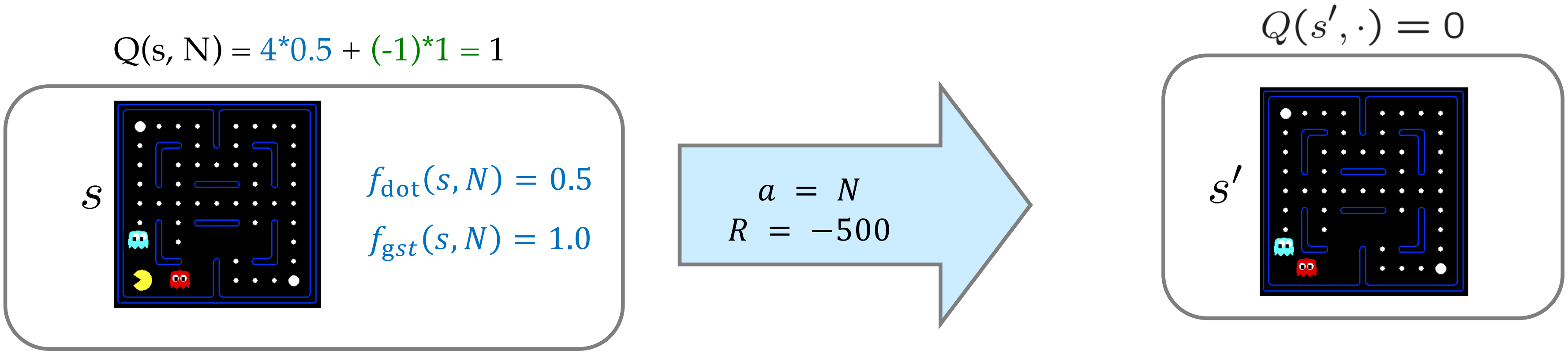
$$w_i \leftarrow w_i + \alpha \, [\text{difference}] \, f_i(s,a) \qquad \text{Approximate Q's}$$



- Intuitive interpretation:
  - Adjust weights of active features
  - E.g., if something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features
- Formal justification: online least squares

Poll: Pacman with approximate Q learning

- Two features: $f_{\text{dot}}(s,a)$ and $f_{\text{gst}}(s,a)$ $\alpha = 0.004$
- Current weights: $w_{\text{dot}} = 4, w_{\text{gst}} = -1$



(A) w_{dot} and w_{gst} both increase by same amount
(B) w_{dot} and w_{gst} both decrease by same amount
(C) w_{dot} and w_{gst} both increase, w_{dot} increases by larger amount
(D) w_{dot} and w_{gst} both increase, w_{gst} increase by larger amount
(E) w_{dot} and w_{gst} both decrease, w_{dot} decreases by larger amount
(F) w_{dot} and w_{gst} both decrease, w_{gst} decreases by larger amount

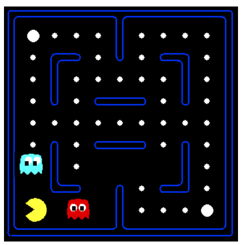
Poll: Pacman with approximate Q learning

- Two features: $f_{\text{dot}}(s,a)$ and $f_{\text{gst}}(s,a)$
- Current weights: $w_{\text{dot}} = 4, w_{\text{gst}} = -1$

$\alpha = 0.004, \gamma = 1.0$

Q(s, N) = 4*0.5 + (-1)*1 = 1

s



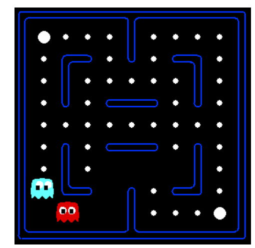
$f_{\text{dot}}(s,N) = 0.5$

$f_{\text{gst}}(s,N) = 1.0$

$a = N$
$R = -500$

$Q(s',a) = 0\ \forall a$

s'



$$w_i \leftarrow w_i + \alpha\,[\text{difference}]\, f_i(s,a)$$

$$sample = R + \gamma \max_{a'} Q(s',a') = -500$$
$$estimate = Q(s,a) = 1$$

$$w_{\text{dot}} \leftarrow 4 + \alpha(-501)\ 0.5 = 3.0$$
$$w_{\text{gst}} \leftarrow -1 + \alpha(-501)\ 1.0 = -3.0$$

All equations we saw so far

Standard expectimax: $V(s) = \max_a \sum_{s'} P(s'|s,a)V(s')$

Bellman equations: $V^*(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^*(s')]$

Value iteration: $V_{k+1}(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')], \quad \forall\, s$

Q-iteration: $Q_{k+1}(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')], \quad \forall\, s, a$

Policy extraction: $\pi_V(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')], \quad \forall\, s$

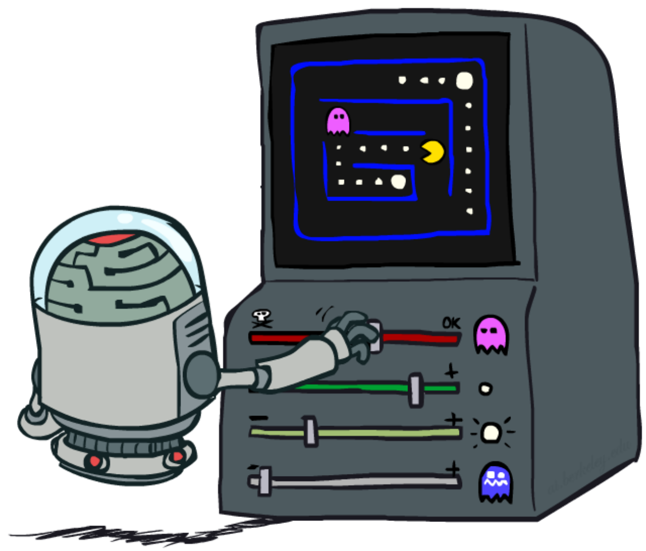
Policy evaluation: $V^{\pi}_{k+1}(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma V^{\pi}_k(s')], \quad \forall\, s$

Policy improvement: $\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')], \quad \forall\, s$

Value (TD) learning: $V^{\pi}(s) = V^{\pi}(s) + \alpha\,[r + \gamma\, V^{\pi}(s') - V^{\pi}(s)]$

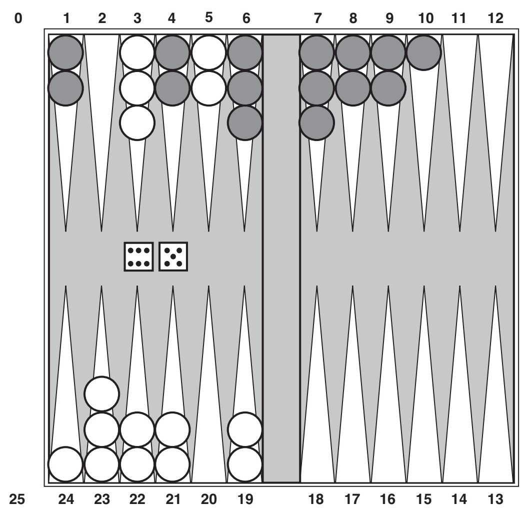
Q-learning: $Q(s,a) = Q(s,a) + \alpha[r + \gamma \max_{a'} Q(s',a') - Q(s,a)]$

Recent Reinforcement Learning Milestones



TDGammon

- 1992 by Gerald Tesauro
- 4-ply lookahead using V(s) trained from 1,500,000 games of self-play
- 3 hidden layers, ~100 units each
- Input: contents of each location
  plus several handcrafted features
- Experimental results:
  - Approximately as strong as world champion
  - Led to radical changes in the way humans play backgammon



Deep Q-Networks

- Deep Mind, 2015
- Used a deep learning network to represent Q:
  - Input is last 4 images (84x84 pixel values) plus score
- 49 Atari games, incl. Breakout, Space Invaders, Seaquest, Enduro

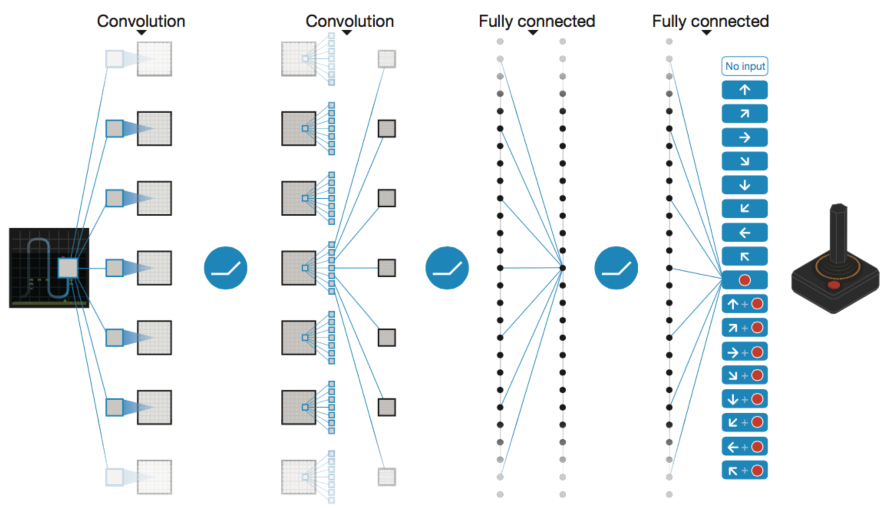
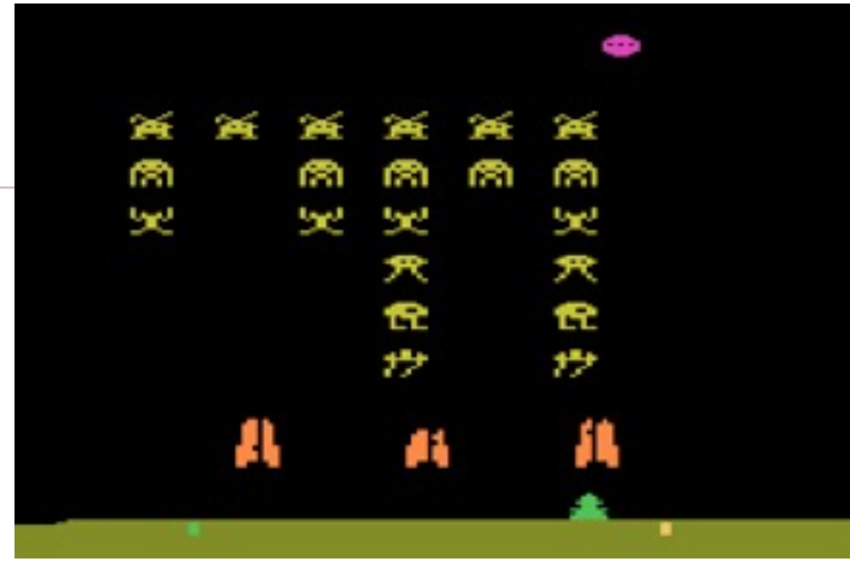


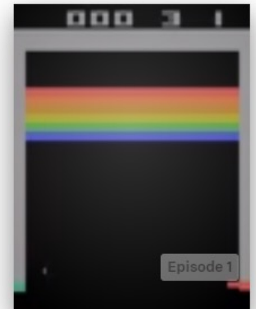
Image: Deep Mind



Images: Open AI, Atari

OpenAI Gym

- 2016+
- Benchmark problems for learning agents
- https://gym.openai.com/envs

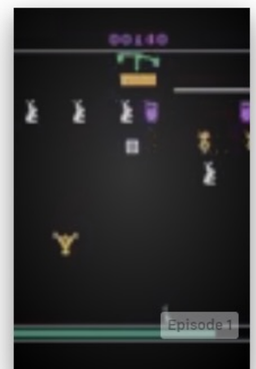


Breakout-ram-v0
Maximize score in the game Breakout, with RAM as input

Acrobot-v1
Swing up a two-link robot.

Ant-v2
Make a 3D four-legged robot walk.

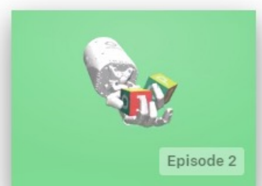
FetchPush-v0
Push a block to a goal position.



Carnival-v0
Maximize score in the game Carnival, with screen images as input

MountainCarContinuous-v0
Drive up a big hill with continuous control.

Humanoid-v2
Make a 3D two-legged robot walk.



HandManipulateBlock-v0
Orient a block using a robot hand.

Images: Open AI

AlphaGo, AlphaZero

- Deep Mind, 2016+

Autonomous Vehicles?