

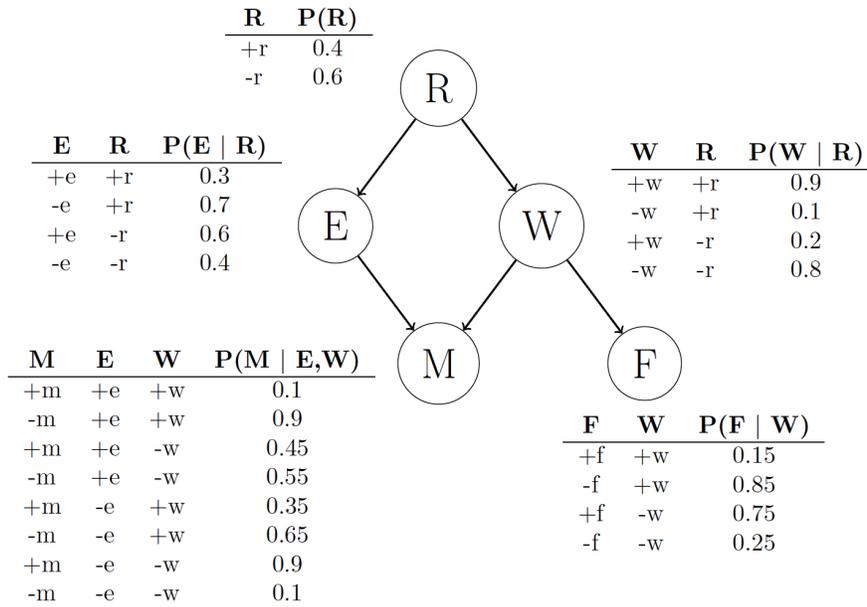
INSTRUCTIONS

- **Due: Monday, April 22, 2024 at 10:00 PM EDT.** Remember that you may use up to 2 slip days for the written homework making the last day to submit **Wednesday, April 24, 2024 at 10:00 PM EDT.**
- **Format:** Write your answers in the `yoursolution.tex` file and compile a pdf (preferred) or you can type directly on the blank pdf. Make sure that your answers are within the dedicated regions for each question/part. If you do not follow this format, we may deduct points. Handwritten solutions are not acceptable and may lead to lost points.
- **How to submit:** Submit a pdf with your answers on Gradescope. Log in and click on our class 15-281, click on the HW10 assignment, and upload your pdf containing your answers. **Misaligned submissions will have at least 5% taken off their score.**
- **Policy:** See the course website for homework policies and academic integrity.

Name	
Andrew ID	
Hours to complete?	<input type="radio"/> (0, 2] hours <input type="radio"/> (2, 3] hours <input type="radio"/> (3, 4] hours <input type="radio"/> (4, 5] hours <input type="radio"/> (5, 6] hours <input type="radio"/> (6, 7] hours <input type="radio"/> (7, 8] hours <input type="radio"/> > 8 hours

Q1. [8 pts] Bayes Nets Sampling

Consider the following Bayes Net and corresponding probability tables.



Consider the case where we are sampling to approximate the query $P(R \mid +f, +m)$.

- (a) [6 pts] Fill in the following table with the probabilities of *drawing* each respective sample given that we are using each of the following sampling techniques.

Method	$\langle +r, +e, -w, +m, +f \rangle$	$\langle +r, -e, +w, -m, +f \rangle$
Prior sampling		
Rejection sampling		
Likelihood weighting		

- (b) [2 pts]

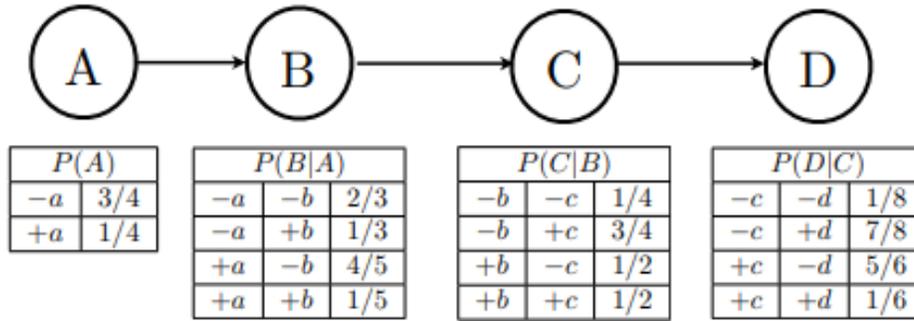
"Sampling is important." – Gibby Gibson, *iCarly*, S2 Ep. 81

We are going to use Gibbs sampling to estimate the probability of getting the sample $\langle +r, +e, -w, +m, +f \rangle$. We will start from the sample $\langle -r, -e, -w, +m, +f \rangle$ and resample E first then R . What is the probability of drawing sample $\langle +r, +e, -w, +m, +f \rangle$?



Q2. [19 pts] More Sampling

Consider the following Bayes Net and corresponding probability tables.



(a) [3 pts] You are given the following samples:

$(-a, +b, +c, -d)$	$(-a, -b, +c, -d)$
$(-a, -b, -c, +d)$	$(-a, -b, +c, -d)$
$(-a, +b, +c, +d)$	$(-a, -b, -c, -d)$
$(+a, +b, -c, +d)$	$(+a, +b, +c, -d)$

(i) [1 pt] Assume that these samples came from performing prior sampling. Estimate $P(-c)$ using the samples.

Answer:

(ii) [2 pts] Now we will sample $P(-c \mid -a, -d)$ using rejection sampling. List out the samples that would not be used when doing rejection sampling for this task, and write the estimate for $P(-c \mid -a, -d)$.

Rejected Samples:

Estimate:

(b) [4 pts] We will now use likelihood weighted sampling to estimate $P(-c \mid -a, -d)$. Give the weights for the following samples.

(i) [1 pt] $(-a, +b, +c, -d)$

Answer:

(ii) [1 pt] $(-a, +b, -c, -d)$

Answer:

(iii) [1 pt] $(-a, -b, +c, -d)$

Answer:

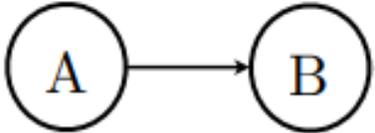
(iv) [1 pt] $(-a, -b, -c, -d)$

Answer:

(c) [1 pt] Using the samples, estimate $P(-c \mid -a, -d)$. Round your answer to 4 decimal places.

Answer:

(d) [11 pts] Consider the following Bayes net with binary variables A, B .



A	$P(A)$	A	B	$P(B A)$
$+a$.01	$+a$	$+b$.99
$-a$.99	$+a$	$-b$.01
		$-a$	$+b$.0001
		$-a$	$-b$.9999

(i) [2 pts] Use inference to find the actual probability distribution $P(A \mid +b)$.

$P(+a \mid +b)$:

$P(-a \mid +b)$:

(ii) [2 pts] We are trying to estimate $P(A \mid +b)$ using likelihood weighted sampling. What is the first sample most likely to be, and what is the corresponding weight of the sample?

Sample:

Weight:

(iii) [3 pts] Suppose we only draw 5 samples, how many of them will we expect to be $(-a, +b)$? (Round to the nearest integer). What distribution $P(A \mid +b)$ will be estimated?

Number $(-a, +b)$:

$P(+a \mid +b)$:

$P(-a \mid +b)$:

(iv) [3 pts] How about 1000 samples?

Number $(-a, +b)$:

$P(+a \mid +b)$:

$P(-a \mid +b)$:

(v) [1 pt] In some Bayes net $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E$, will likelihood weighted sampling provide more accurate distributions for $P(A \mid E)$ or $P(E \mid A)$ with a relatively low sample count? Explain why (you may use previous parts as an example).

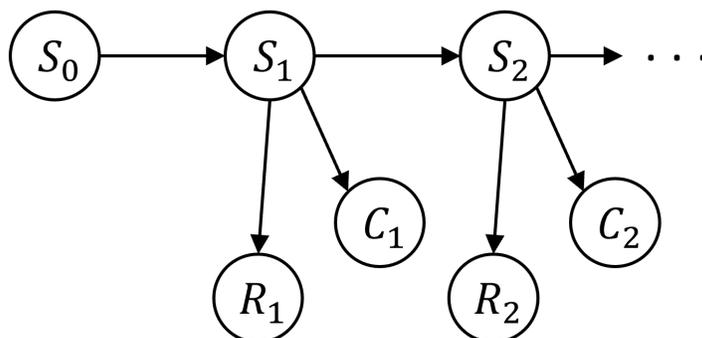
Hint: what happens if the evidence can be reached only by unlikely values of upstream variables?

Answer:

Q3. [30 pts] Dynamic Bayes Net and Hidden Markov Model

A professor wants to know if students are getting enough sleep. Each day, the professor observes whether the students sleep in class, and whether they have red eyes. Let S_t be the random variable of the student having enough sleep, R_t be the random variable for the student having red eyes, and C_t be the random variable of the student sleeping in class on day t . The professor has the following theory:

- The prior probability of getting enough sleep at time t , with no observations, is 0.6
- The probability of getting enough sleep on night t is 0.9 given that the student got enough sleep the previous night, and 0.2 if not
- The probability of having red eyes is 0.1 if the student got enough sleep, and 0.7 if not
- The probability of sleeping in class is 0.2 if the student got enough sleep, and 0.4 if not



S_0	$P(S_0)$	S_{t+1}	S_t	$P(S_{t+1} S_t)$	R_t	S_t	$P(R_t S_t)$	C_t	S_t	$P(C_t S_t)$
$+s$	0.6	$+s_{t+1}$	$+s_t$	0.9	$+r$	$+s$	0.1	$+c$	$+s$	0.2
$-s$	0.4	$-s_{t+1}$	$+s_t$	0.1	$-r$	$+s$	0.9	$-c$	$+s$	0.8
		$+s_{t+1}$	$-s_t$	0.2	$+r$	$-s$	0.7	$+c$	$-s$	0.4
		$-s_{t+1}$	$-s_t$	0.8	$-r$	$-s$	0.3	$-c$	$-s$	0.6

Using the DBN above and these evidence values

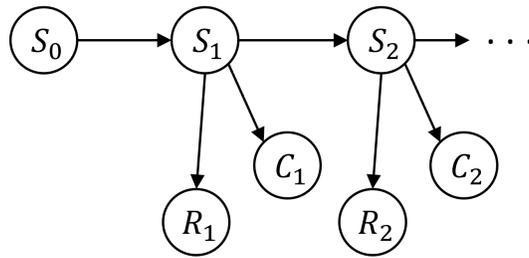
- $[-r_1, -c_1]$ = not red eyes, not sleeping in class
- $[+r_2, -c_2]$ = red eyes, not sleeping in class
- $[+r_3, +c_3]$ = red eyes, sleeping in class

we want to compute $P(S_t | r_{1:t}, c_{1:t})$ for each of $t = 1, 2, 3$ as well as perform smoothing to get $P(S_2 | r_{1:3}, c_{1:3})$.

In order to do so, we will compute intermediate values which will correspond to the predict and update steps of our forward algorithm as well as finding the value of α (the normalization constant) in each case.

Note: Please round your answers to 3 decimal places at the end of each calculation. That is, if you need to compute multiple intermediate values to get your answer, do not round until you get your final answer. Please also note we will only be able to award partial credit if work is shown.

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S_0	$P(S_0)$	S_{t+1}	S_t	$P(S_{t+1} S_t)$	R_t	S_t	$P(R_t S_t)$	C_t	S_t	$P(C_t S_t)$
$+s$	0.6	$+s_{t+1}$	$+s_t$	0.9	$+r$	$+s$	0.1	$+c$	$+s$	0.2
$-s$	0.4	$-s_{t+1}$	$+s_t$	0.1	$-r$	$+s$	0.9	$-c$	$+s$	0.8
		$+s_{t+1}$	$-s_t$	0.2	$+r$	$-s$	0.7	$+c$	$-s$	0.4
		$-s_{t+1}$	$-s_t$	0.8	$-r$	$-s$	0.3	$-c$	$-s$	0.6

Round all numerical answers to 3 decimal places. Please also note we will only be able to award partial credit if work is shown. Evidence values: $[-r_1, -c_1]$, $[+r_2, -c_2]$, $[+r_3, +c_3]$

(a) [7 pts] State Estimation: $t = 1$

(i) [3 pts] Predict:

Use the Bayes' Net's CPTs to write an equivalent expression for $P(+s_1)$:

$P(+s_1) =$

$P(-s_1) =$

(ii) [3 pts] Update:

Use the Bayes' Net's CPTs and α to find an equivalent expression for $P(+s_1 | -r_1, -c_1)$:

When solving for probabilities, **don't** leave α in your answer.

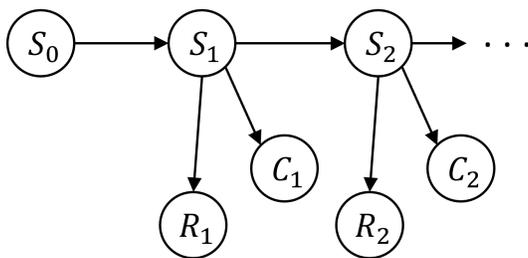
$P(+s_1 | -r_1, -c_1) =$

$P(-s_1 | -r_1, -c_1) =$

(iii) [1 pt] What was the value for α ?

$\alpha =$

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S_0	$P(S_0)$	S_{t+1}	S_t	$P(S_{t+1} S_t)$	R_t	S_t	$P(R_t S_t)$	C_t	S_t	$P(C_t S_t)$
$+s$	0.6	$+s_{t+1}$	$+s_t$	0.9	$+r$	$+s$	0.1	$+c$	$+s$	0.2
$-s$	0.4	$-s_{t+1}$	$+s_t$	0.1	$-r$	$+s$	0.9	$-c$	$+s$	0.8
		$+s_{t+1}$	$-s_t$	0.2	$+r$	$-s$	0.7	$+c$	$-s$	0.4
		$-s_{t+1}$	$-s_t$	0.8	$-r$	$-s$	0.3	$-c$	$-s$	0.6

Round all numerical answers to 3 decimal places. Please also note we will only be able to award partial credit if work is shown. Evidence values: $[-r_1, -c_1]$, $[+r_2, -c_2]$, $[+r_3, +c_3]$

(b) [9 pts] State Estimation: $t = 2$

(i) [4 pts] Predict:

$$P(+s_2 | -r_1, -c_1) =$$

$$P(-s_2 | -r_1, -c_1) =$$

(ii) [4 pts] Update:

Use the Bayes' Net's CPTs, previous probabilities, and α to find an equivalent expression for $P(+s_2 | r_{1:2}, c_{1:2})$:

When solving for probabilities, **don't** leave α in your answer.

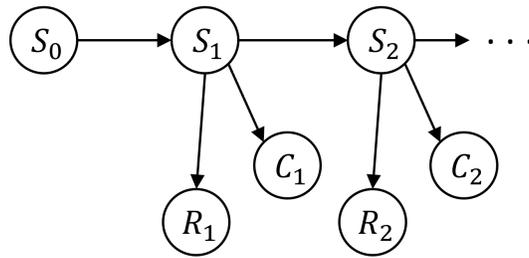
$$P(+s_2 | r_{1:2}, c_{1:2}) =$$

$$P(-s_2 | r_{1:2}, c_{1:2}) =$$

(iii) [1 pt] What was the value for α ?

$$\alpha =$$

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S_0	$P(S_0)$	S_{t+1}	S_t	$P(S_{t+1} S_t)$	R_t	S_t	$P(R_t S_t)$	C_t	S_t	$P(C_t S_t)$
$+s$	0.6	$+s_{t+1}$	$+s_t$	0.9	$+r$	$+s$	0.1	$+c$	$+s$	0.2
$-s$	0.4	$-s_{t+1}$	$+s_t$	0.1	$-r$	$+s$	0.9	$-c$	$+s$	0.8
		$+s_{t+1}$	$-s_t$	0.2	$+r$	$-s$	0.7	$+c$	$-s$	0.4
		$-s_{t+1}$	$-s_t$	0.8	$-r$	$-s$	0.3	$-c$	$-s$	0.6

Round all numerical answers to 3 decimal places. Please also note we will only be able to award partial credit if work is shown. Evidence values: $[-r_1, -c_1]$, $[+r_2, -c_2]$, $[+r_3, +c_3]$

(c) [9 pts] State Estimation: $t = 3$

(i) [4 pts] Predict:

$$P(+s_3 | r_{1:2}, c_{1:2}) =$$

$$P(-s_3 | r_{1:2}, c_{1:2}) =$$

(ii) [4 pts] Update:

Use the Bayes' Net's CPTs, previous probabilities, and α to find an equivalent expression for $P(+s_3 | r_{1:3}, c_{1:3})$:

When solving for probabilities, **don't** leave α in your answer.

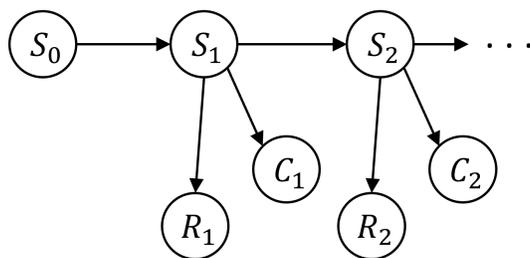
$$P(+s_3 | r_{1:3}, c_{1:3}) =$$

$$P(-s_3 | r_{1:3}, c_{1:3}) =$$

(iii) [1 pt] What was the value for α ?

$$\alpha =$$

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		S_{t+1}	S_t	$P(S_{t+1} S_t)$	R_t	S_t	$P(R_t S_t)$	C_t	S_t	$P(C_t S_t)$
S_0	$P(S_0)$	$+s_{t+1}$	$+s_t$	0.9	$+r$	$+s$	0.1	$+c$	$+s$	0.2
$+s$	0.6	$-s_{t+1}$	$+s_t$	0.1	$-r$	$+s$	0.9	$-c$	$+s$	0.8
$-s$	0.4	$+s_{t+1}$	$-s_t$	0.2	$+r$	$-s$	0.7	$+c$	$-s$	0.4
		$-s_{t+1}$	$-s_t$	0.8	$-r$	$-s$	0.3	$-c$	$-s$	0.6

Round all numerical answers to 3 decimal places. Please also note we will only be able to award partial credit if work is shown. Evidence values: $[-r_1, -c_1], [+r_2, -c_2], [+r_3, +c_3]$

(d) [5 pts] We can build upon the previous three parts and use smoothing to compute $P(S_2 | r_{1:3}, c_{1:3})$.

(i) [2 pts] **Backward message:** $P(+r_3, +c_3 | S_2) = \sum_{s_3} P(+r_3, +c_3 | s_3)P(s_3 | S_2)$

$P(+r_3, +c_3 | +s_2) =$

$P(+r_3, +c_3 | -s_2) =$

(ii) [2 pts] **Smoothing:** $P(S_2 | r_{1:3}, c_{1:3}) = \alpha P(S_2 | r_{1:2}, c_{1:2})P(+r_3, +c_3 | S_2)$
When solving for probabilities, **don't** leave α in your answer.

$P(+s_2 | r_{1:3}, c_{1:3}) =$

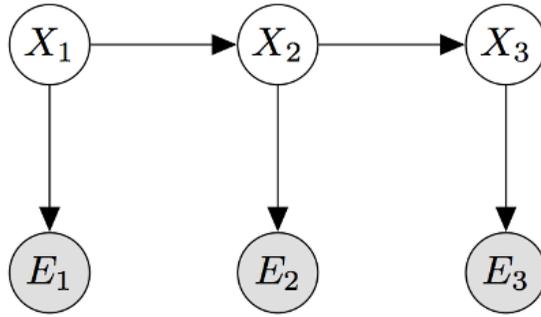
$P(-s_2 | r_{1:3}, c_{1:3}) =$

(iii) [1 pt] What was the value for α ?

$\alpha =$

Q4. [12 pts] Modified HMM Updated Equations

Consider the HMM graph structure shown below.



Recall the Forward algorithm is a two step iterative algorithm used to approximate the probability distribution

$$P(X_t | e_1, \dots, e_t)$$

The two steps of the algorithm are as follows:

1. Predict:

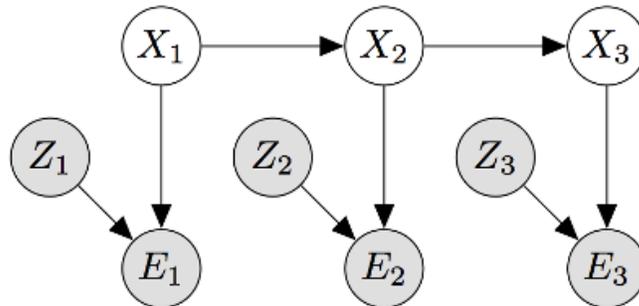
$$P(X_t | e_{1..t-1}) = \sum_{x_{t-1}} P(X_t | x_{t-1})P(x_{t-1} | e_{1..t-1})$$

2. Observe:

$$P(X_t | e_{1..t}) = \frac{P(e_t | X_t)P(X_t | e_{1..t-1})}{\sum_{x_t} P(e_t | x_t)P(x_t | e_{1..t-1})}$$

For this problem we will consider modifying the forward algorithm as the HMM graph structure changes. Our goal will continue to be to create an iterative algorithm which is able to compute the distribution of states, X_t , given all available evidence from time 0 to time t .

(a) [4 pts] Consider the graph below where new observed variables, Z_i , are introduced and influence the evidence.



(i) [2 pts] What will the modified **predict** update be?

$$P(X_t | e_{1..t-1}, z_{1..t-1}) =$$

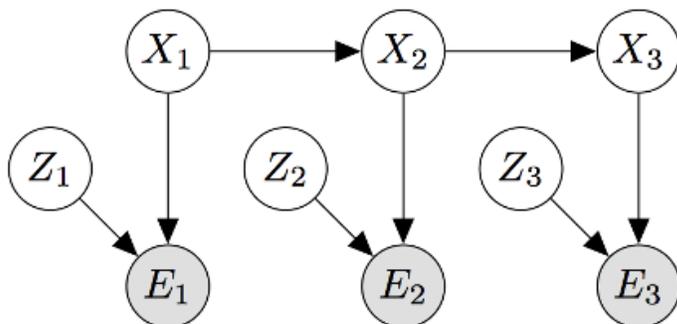
- $\sum_{x_{t-1}} P(X_t | z_{1..t-1})P(x_{t-1} | e_{1..t-1}, z_{1..t-1})$
- $\sum_{x_{t-1}} P(X_t | x_{t-1})P(x_{t-1} | e_{1..t-1}, z_{1..t-1})$
- $\sum_{x_{t-1}} P(X_t | e_{1..t-1}, z_{1..t-1})P(x_{t-1} | x_{t-1}, z_{1..t-1})$
- $\sum_{x_{t-1}} P(X_t | x_{t-1})P(x_{t-1} | e_{1..t-1})$ (no change)

(ii) [2 pts] What will the modified **observed** update be?

$$P(X_t | e_{1...t}, z_{1...t})$$

- $\frac{P(e_t | X_t, z_t) P(X_t | e_{1...t-1}, z_{1...t-1})}{\sum_{x_t, z_t} P(e_t | x_t, z_t) P(x_t | e_{1...t-1}, z_{1...t-1})}$
 $\frac{P(e_t | X_t, z_t) P(X_t | e_{1...t-1}, z_{1...t-1})}{\sum_{x_t} P(e_t | x_t, z_t) P(x_t | e_{1...t-1}, z_{1...t-1})}$
 $\frac{P(e_t | X_t) P(X_t | e_{1...t-1})}{\sum_{z_t} P(e_t | x_t, z_t) P(x_t | e_{1...t-1}, z_{1...t-1})}$
 $\frac{P(e_t | X_t, z_t) P(X_t | e_{1...t-1}, z_{1...t-1})}{\sum_{z_t} P(e_t | x_t, z_t) P(x_t | e_{1...t-1}, z_{1...t-1})}$
 $\frac{P(e_t | X_t) P(X_t | e_{1...t-1})}{\sum_{x_t} P(e_t | x_t) P(x_t | e_{1...t-1})}$ (no change)

(b) [4 pts] Next, consider the graph below where the Z_i variables are unobserved.



(i) [2 pts] What will the modified **predict** update be?

$$P(X_t | e_{1...t-1}) =$$

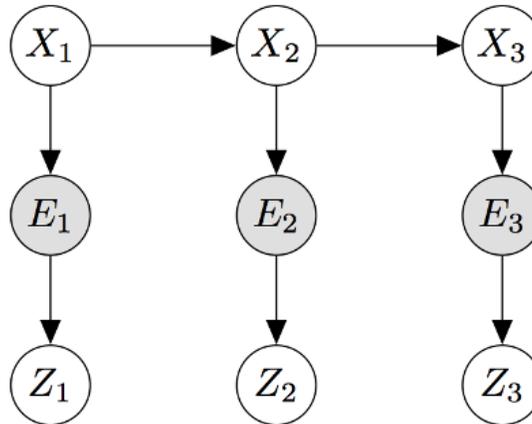
- $\sum_{x_{t-1}} P(X_t | z_{1...t-1}) P(x_{t-1} | e_{1...t-1}, z_{1...t-1})$
 $\sum_{x_{t-1}} P(X_t | x_{t-1}) P(x_{t-1} | e_{1...t-1}, z_{1...t-1})$
 $\sum_{x_{t-1}} P(X_t | e_{1...t-1}, z_{1...t-1}) P(x_{t-1} | x_{t-1}, z_{1...t-1})$
 $\sum_{x_{t-1}} P(X_t | x_{t-1}) P(x_{t-1} | e_{1...t-1})$ (no change)

(ii) [2 pts] What will the modified **observed** update be?

$$P(X_t | e_{1...t}) =$$

- $\frac{P(X_t | e_{1...t-1}) P(z_t) P(e_t | X_t, z_t)}{\sum_{z_t} P(x_t | e_{1...t-1}) P(e_t | x_t, z_t) P(z_t)}$
 $\frac{P(X_t | e_{1...t-1}) \sum_{z_t} P(z_t) P(e_t | X_t, z_t)}{P(x_t | e_{1...t-1}) \sum_{z_t} P(e_t | x_t, z_t) P(z_t)}$
 $\frac{P(X_t | e_{1...t-1}) P(z_t) P(e_t | X_t, z_t)}{\sum_{x_t} P(x_t | e_{1...t-1}) P(e_t | x_t, z_t) P(z_t)}$
 $\frac{P(X_t | e_{1...t-1}) \sum_{z_t} P(z_t) P(e_t | X_t, z_t)}{\sum_{x_t} P(x_t | e_{1...t-1}) \sum_{z_t} P(e_t | x_t, z_t) P(z_t)}$
 $\frac{P(e_t | X_t) P(X_t | e_{1...t-1})}{\sum_{x_t} P(e_t | x_t) P(x_t | e_{1...t-1})}$ (no change)

- (c) [4 pts] Finally, consider a graph where the newly introduced variables are unobserved and influenced by the evidence nodes.



- (i) [2 pts] What will the modified **predict** update be?

$$P(X_t | e_{1...t-1}) =$$

- $\sum_{x_{t-1}} P(X_t | z_{1...t-1})P(x_{t-1} | e_{1...t-1}, z_{1...t-1})$
 $\sum_{x_{t-1}} P(X_t | x_{t-1})P(x_{t-1} | e_{1...t-1}, z_{1...t-1})$
 $\sum_{x_{t-1}} P(X_t | e_{1...t-1}, z_{1...t-1})P(x_{t-1} | x_{t-1}, z_{1...t-1})$
 $\sum_{x_{t-1}} P(X_t | x_{t-1})P(x_{t-1} | e_{1...t-1})$ (no change)

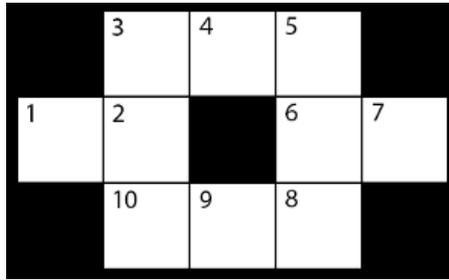
- (ii) [2 pts] What will the modified **observed** update be?

$$P(X_t | e_{1...t}) =$$

- $\frac{P(e_t | X_t, z_t)P(X_t | e_{1...t-1}, z_{1...t-1})}{\sum_{z_t} P(e_t | x_t, z_t)P(x_t | e_{1...t-1}, z_{1...t-1})}$
 $\frac{P(e_t | X_t, z_t)P(X_t | e_{1...t-1}, z_{1...t-1})}{\sum_{x_t} P(e_t | x_t, z_t)P(x_t | e_{1...t-1}, z_{1...t-1})}$
 $\frac{P(e_t | X_t)P(X_t | e_{1...t-1})}{\sum_{x_t, z_t} P(e_t | x_t, z_t)P(x_t | e_{1...t-1}, z_{1...t-1})}$
 $\frac{P(e_t | X_t)P(X_t | e_{1...t-1})}{\sum_{x_t} P(e_t | x_t)P(x_t | e_{1...t-1})}$ (no change)

Q5. [31 pts] Particle Filtering

In this question, we will use a particle filter to track the state of a robot that is lost in the small map below:



As we walk through this problem, there are many values to compute, so you may want to download and print the following worksheet and fill it in as you go.

NOTE: For your convenience, we've created a supplemental gradescope assignment to help you check your work for the first couple answers of parts (a) - (g). Like other online homeworks, entering in the correct answers will prompt a solution explanation. That being said, no credit is given for completing this gradescope assignment (ignore the tiny point values that's required to appease gradescope), so please still put your answers in the writeup.

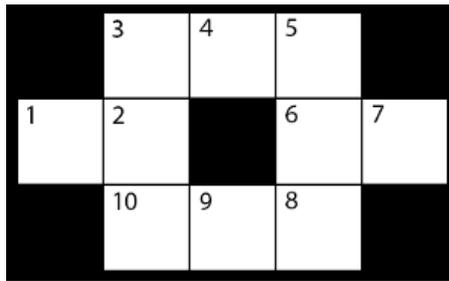
The robot's state is represented by an integer $1 \leq X_t \leq 10$ corresponding to its location in the map at time t . We will approximate our belief over this state with $N = 8$ particles.

You have no control over the robot's actions. At each timestep, the robot either stays in place, or moves to any one of its neighboring locations, all with equal probability. For example, if the robot starts in state $X_t = 7$, it will move to state $X_{t+1} = 6$ with probability $\frac{1}{2}$ or $X_{t+1} = 7$ with probability $\frac{1}{2}$. Similarly, if the robot starts in state $X_t = 2$, the next state X_{t+1} can be any element of $\{1, 2, 3, 10\}$, and each occurs with probability $\frac{1}{4}$.

At each time step, a sensor on the robot gives a reading $E_t \in \{H, C, T, D\}$ corresponding to the *type* of state the robot is in. The possible types are:

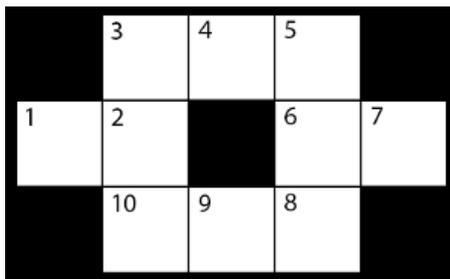
- **Hallway (H)** for states bordered by two parallel walls (4,9).
- **Corner (C)** for states bordered by two orthogonal walls (3,5,8,10).
- **Tee (T)** for states bordered by one wall (2,6).
- **Dead End (D)** for states bordered by three walls (1,7)

The sensor is not very reliable: it reports the correct type with probability $\frac{1}{2}$, but gives erroneous readings the rest of the time, with probability $\frac{1}{6}$ for each of the three other possible readings.



(a) [4 pts] Fill in the sensor model below:

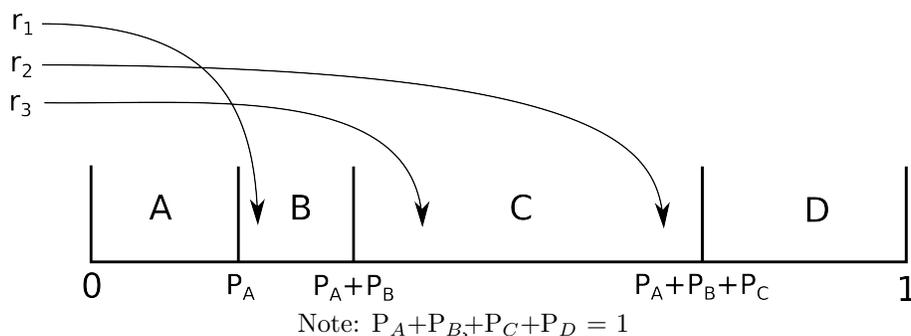
P(Sensor Reading	State Type)	Answer
P(Sensor Reading = H	State Type = H)	
P(Sensor Reading = C	State Type = H)	
P(Sensor Reading = T	State Type = H)	
P(Sensor Reading = D	State Type = H)	
P(Sensor Reading = H	State Type = C)	
P(Sensor Reading = C	State Type = C)	
P(Sensor Reading = T	State Type = C)	
P(Sensor Reading = D	State Type = C)	
P(Sensor Reading = H	State Type = T)	
P(Sensor Reading = C	State Type = T)	
P(Sensor Reading = T	State Type = T)	
P(Sensor Reading = D	State Type = T)	
P(Sensor Reading = H	State Type = D)	
P(Sensor Reading = C	State Type = D)	
P(Sensor Reading = T	State Type = D)	
P(Sensor Reading = D	State Type = D)	



- (b) [4 pts] Suppose that we want to sample from a set of 4 events, $\{A, B, C, D\}$, which occur with corresponding probabilities P_A, P_B, P_C, P_D .

First, we form the set of cumulative weights, given by $\{0, P_A, P_A + P_B, P_A + P_B + P_C, 1\}$. (Note: $P_A + P_B + P_C + P_D = 1$). These weights partition the $[0, 1)$ interval into bins, as shown below. We then draw a number r uniformly at random from $[0, 1)$ and pick A, B, C , or D based on which bin r lands in.

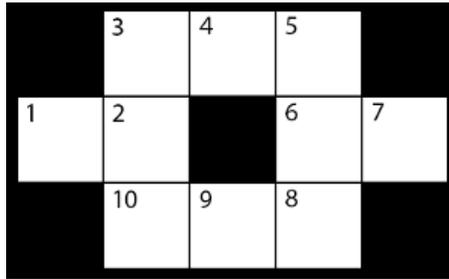
The process is illustrated in the diagram below. If r_1 uniformly chosen from $[0, 1)$ lands in the interval $[P_A, P_A + P_B]$, then the resulting sample would be B . Similarly, if r_2 lands in $[P_A + P_B, P_A + P_B + P_C]$, the sample would be C , and r_3 landing in $[P_A + P_B, P_A + P_B + P_C]$ would also be C .



Now we will sample the starting positions for our particles at time $t = 0$. For each particle p_i , we have generated a random number r_i sampled uniformly from $[0, 1)$.

Your job is to use these numbers to sample a starting location for each particle. As a reminder, locations are integers from the range $[1, 10]$, as shown in the map. You should assume that the locations go in **ascending order and that each location has equal probability**. The random number generated for particle i , denoted by r_i is provided. Please fill in the locations of the eight particles.

r_i	p_i	Initial Location X_0
$r_1 = 0.139$	p_1	
$r_2 = 0.416$	p_2	
$r_3 = 0.683$	p_3	
$r_4 = 0.825$	p_4	
$r_5 = 0.396$	p_5	
$r_6 = 0.161$	p_6	
$r_7 = 0.554$	p_7	
$r_8 = 0.013$	p_8	



- (c) [4 pts] Now we'll perform a time update from $t = 0$ to $t = 1$ using the transition model. Stated again, the transition model is as follows: At each timestep, the robot either stays in place, or moves to any one of its neighboring locations, **all with equal probability**.

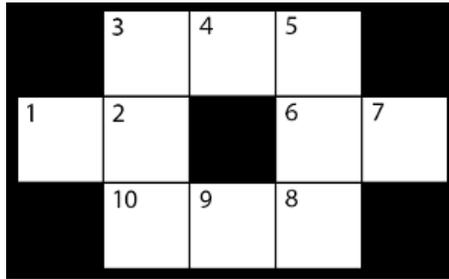
You should again sample from the range $[0, 1)$, where the bins are the possible locations **sorted in ascending numerical order**. As an example, if $X_0 = 3$, the next state X_1 can be one of $\{2, 3, 4\}$, each with equal probability. Thus the $[0, 0.333)$ bin would be for $X_1 = 2$, the $[0.333, 0.667)$ bin would be for $X_1 = 3$, and the $[0.667, 1)$ bin would be for $X_1 = 4$.

Fill in the bin partitioning for the update from particles starting in the $X_0 = 2$ state in the first table below. **Note the first row is given an example (do not use in your answer) and you may not need to use all of the rows provided.**

bin	X_1
$[0, 0.333)$	2

For each particle, take the starting position you found in part (b), and perform the time update for that particle. That is, sample the next position of each of particles.

r_i	p_i	Predicted Location X_1
$r_1 = 0.822$	p_1	
$r_2 = 0.244$	p_2	
$r_3 = 0.686$	p_3	
$r_4 = 0.842$	p_4	
$r_5 = 0.113$	p_5	
$r_6 = 0.314$	p_6	
$r_7 = 0.177$	p_7	
$r_8 = 0.406$	p_8	



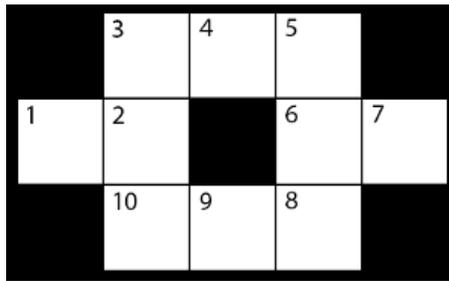
- (d) [5 pts] Recall that a particle filter just keeps track of a list of particles, but at any given time, we can compute a probability distribution from these particles.

Using the current newly updated set of particles (that you found in Part (c)), give the estimated probability that the robot is in each location.

X_1	$\hat{P}(X_1)$
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

- (e) [4 pts] The sensor reading at $t = 1$ is: $E_1 = D$. Using the sensor model you specified in Part (a), incorporate the evidence by weighting the particles. Refer back to Part (c) to get the positions of your particles.

Particle p_i	Weight w_i
p_1	
p_2	
p_3	
p_4	
p_5	
p_6	
p_7	
p_8	



- (f) [5 pts] After incorporating the evidence by weighting the particles, we can compute an updated probability distribution from these particles.

Using the set of particles (Parts (c) and (d)), and the weights (Part (e)), give the estimated probability that the robot is in each location given the evidence, $\hat{P}(X_1 | E_1 = D)$. *Hint:* You will first want to calculate $\hat{P}(X_1, E_1 = D)$ and then normalize.

X_1	$\hat{P}(X_1, E_1 = D)$	$\hat{P}(X_1 E_1 = D)$
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Use this space to show any other work for partial credit

- (g) [4 pts] Finally, we'll resample the particles. This reallocates resources to the most relevant parts of the state space in the next time update step.

Use your $\hat{P}(X_1 | E_1)$ from the previous part to sample eight new particles.

First fill in the bins to partition the $[0, 1)$ interval to be sampled from in the first table. You may not need to use all of the rows provided:

bin	X_1

Using the bins above, now fill in the location of the new particles according to the given samples:

r_i	p_i	Resampled Location X_1
$r_1 = 0.803$	p_1	
$r_2 = 0.712$	p_2	
$r_3 = 0.626$	p_3	
$r_4 = 0.140$	p_4	
$r_5 = 0.559$	p_5	
$r_6 = 0.979$	p_6	
$r_7 = 0.231$	p_7	
$r_8 = 0.847$	p_8	

- (h) [1 pt] We said that the sensor provided a reading $E_1 = D$. What fraction of the particles ended up at a dead end in $t = 1$?

Answer:

This completes everything for the first time step, $t = 0 \rightarrow t = 1$. Of course, we would now continue by repeating the time update, evidence incorporation by weighting, and resampling. We'll leave that to the computers, though.