

## 1 Baymax's Factory

Baymax and the 281 TAs have opened a factory to produce special medicine and bandages. These are really difficult to produce and require the collaboration of robots and humans.

To produce an ounce of medicine, it takes 0.2 hours of human labor and 4 hours of robot labor. To produce an inch of bandage, it takes 0.5 hours of human labor and 2 hours of robot labor. An ounce of medicine sells for \$30 and an inch of bandages sells for \$30. Medicine and bandages can be sold in fractions of an ounce or inch.

We want to maximize our profit so we can buy gifts for all the students. However, the TAs are really busy so they can only devote 90 human hours. In addition, Baymax can only devote 800 robot hours because he has other obligations to tend to. How can we maximize our profit?

(a) Is this a linear, mixed or integer programming problem? Formulate and solve it.

Now suppose the items can only be sold in whole units (by ounce/inch).

(b) Is this a linear, mixed, or integer programming problem? Perform branch and bound for one branch level. You do not have to evaluate; writing out the constraints will suffice.

(c) Now assume medicine can be sold in fractions but bandages can only be sold in whole units. What kind of a programming problem would this be, and how would our evaluation process differ from the problem type in part b?

(d) How many optimal solutions can a LP have? How about IP?

## 2 4-Queens (CSP to IP)

Recall our formulation of the 4-Queens Problem as a CSP. Our goal was to place 4 chess queens on a 4x4 board such that no two queens are on the same row, column, or diagonal.

Now, let's try formulating the 4-Queens problem as an integer programming problem. We have defined the variables for you as a first step.

**Variables:** Let our variables be  $x_{ij}$  for  $0 \leq i \leq 3$ ,  $0 \leq j \leq 3$ , representing whether there is a queen in row  $i$ , column  $j$ .

(a) What is the goal?

*Hint:* Think about how we can write the goal by summing over the variables defined above.

(b) What are the constraints?

*Hint:* How many queens can we have in any given row, column, or diagonal? How can we write these as summations over the variables?

### 3 Discussion Based Warm-ups

(a) Vocabulary check: Are you familiar with the following terms?

- Symbols:
  
  
  
  
  
  
  
  
  
  
- Operators:
  
  
  
  
  
  
  
  
  
  
- Sentences:
  
  
  
  
  
  
  
  
  
  
- Equivalence:
  
  
  
  
  
  
  
  
  
  
- Literals:
  
  
  
  
  
  
  
  
  
  
- Knowledge Base:
  
  
  
  
  
  
  
  
  
  
- Entailment:
  
  
  
  
  
  
  
  
  
  
- Clauses (Definite vs. Horn Clauses):
  
  
  
  
  
  
  
  
  
  
- Model Checking:
  
  
  
  
  
  
  
  
  
  
- Theorem Proving:
  
  
  
  
  
  
  
  
  
  
- Modus Ponens:

(b) Determine which of the following are correct, and explain your reasoning.

- $(A \vee B) \models (A \Rightarrow B)$
- $A \iff B \models A \vee \neg B$
- $(A \vee B) \wedge \neg(A \Rightarrow B)$  is satisfiable