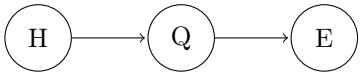
1 Conceptual Review

1. When would we want to use inference?

2. Suppose we are given binary random variables Q, H, E (query, hidden, evidence).
We want to query $P(q \mid e)$.



(a) **Enumeration**
Perform inference on a joint distribution. Use the Bayes net above to break down joint into CPT factors.

Note: You may use a proportionality constant α in your answer.

(b) **Variable Elimination**
Rewrite your answer to enumeration by moving summations inwards as far as possible.

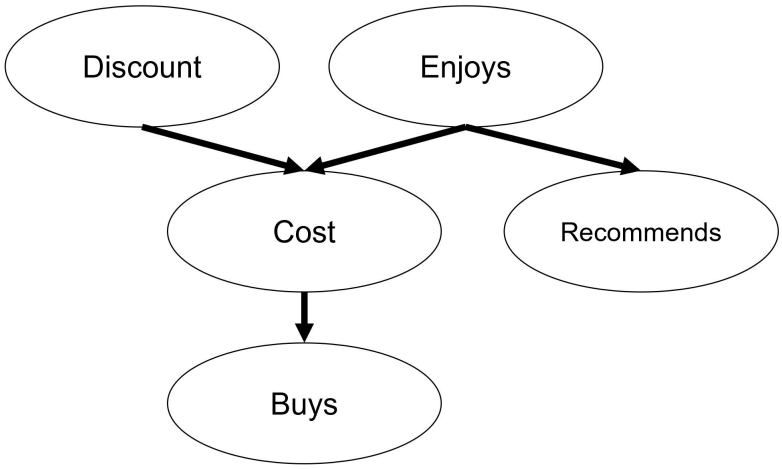
Note: You may use a proportionality constant α in your answer.

(c) Based on 2a and 2b, why is variable elimination more efficient than enumeration?

2 Inference

Realizing that students aren't particularly fond of reading the textbook, the 281 course staff have developed a software that automatically scans the textbook and outputs key points for each individual chapter. However, since the development of the software requires time and computational resources, the 281 staff decides to offer a free one month trial to students, after which a paid subscription is necessary to keep using the software. The following network and variables are used to represent the problem:

- $Discount(D)$: $+d$ if a discount is offered, $-d$ otherwise
- $Enjoys(E)$: $+e$ if a student enjoys the software, $-e$ otherwise
- $Cost(C)$: $+c$ if the software cost is < 20, $-c$ otherwise
- $Recommends(R)$: $+s$ if the student recommends the software to a friend, $-s$ otherwise
- $Buys(B)$: $+b$ if the student buys a software subscription, $-b$ otherwise



1. How can we represent the probability that a student buys and recommends the software using the conditional probabilities at each node?

2. The staff has surveyed students and collected data on whether the students enjoyed the software or not. With this information, we want to perform inference on a joint distribution where the query variable is $Buys$ (B).
   (a) How can we represent the probability expression in terms of conditional probabilities from the network?

   (b) What are the hidden and evidence variable(s)?

3. Using the probability expression from the previous part, we want to compute the query B given evidence that the student enjoys the software. Assume the variable ordering is in alphabetical order.
   (a) How many factors are there, and what are the dimensions of each factor?

   (b) Run the variable elimination algorithm to eliminate repeated computations for the expression $P(B|+e)$.

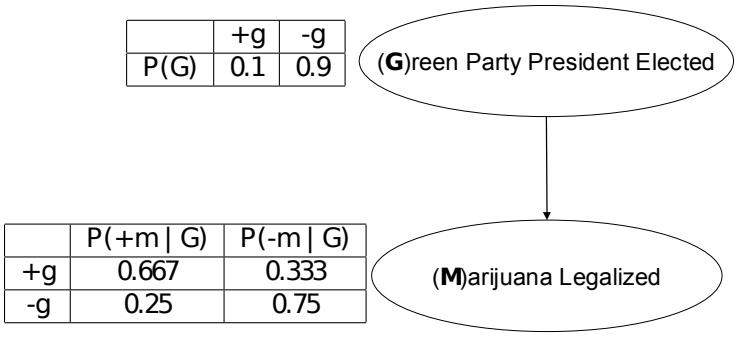
(c) How does the resulting expression change if the variable ordering is instead in reverse alphabetical order?

(d) How do the two orderings compare with respect to time and space complexity?

(e) Describe a heuristic that could be useful in determining a variable ordering to minimize the size of the largest factor.

3 Bayes' Nets: Green Party President

It's election year again! In a parallel universe the Green Party is running for presidency. Pundits believe that Green Party presidents are more likely to legalize marijuana than candidates from other parties, but legalization could occur under any administration. Armed with the power of probability, the analysts model the situation with the Bayes Net below.



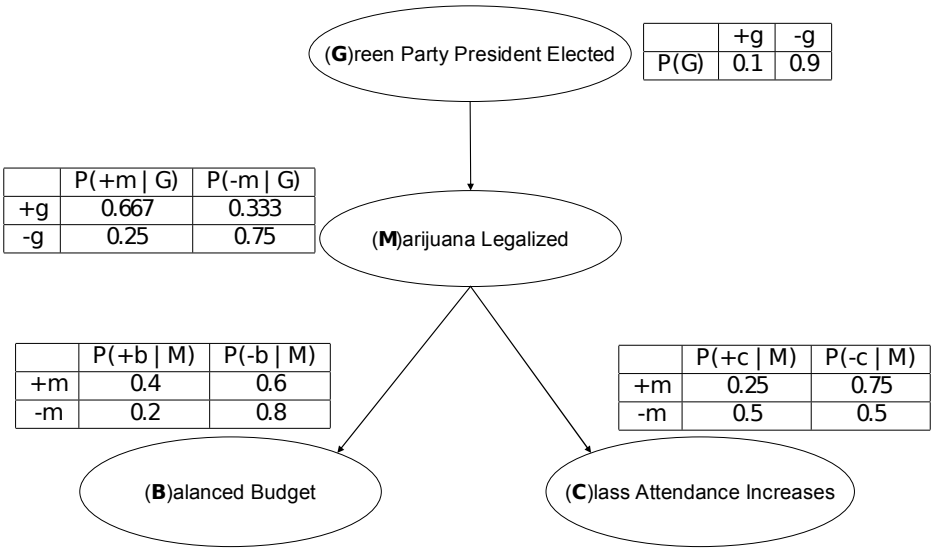
1. Fill in the joint probability table over G and M.

G	M	P(G, M)
+g	+m	
+g	-m	
-g	+m	
-g	-m	

2. What is P(+m), the marginal probability that marijuana is legalized?

3. News agencies air 24/7 coverage of the recent legalization of marijuana (+m), but you can't seem to find out who won the election. What is the conditional probability P(+g | +m) that a Green Party president was elected?

We can make better inferences if we observe more evidence. On the next page, we will expand on the model (Bayes net) by introducing two new random variables: whether the budget is balanced (B), and whether class attendance increases (C). The expanded Bayes net and conditional distributions are shown below.



	+g	-g
P(G)	0.1	0.9

	P(+m | G)	P(-m | G)
+g	0.667	0.333
-g	0.25	0.75

	P(+b | M)	P(-b | M)
+m	0.4	0.6
-m	0.2	0.8

	P(+c | M)	P(-c | M)
+m	0.25	0.75
-m	0.5	0.5

4. The full joint distribution is given below. Fill in the missing values.

G	M	B	C	$P(G, M, B, C)$
+	+	+	+	$1/150$
+	+	+	-	
+	+	-	+	$1/100$
+	+	-	-	$3/100$
+	-	+	+	$1/300$
+	-	+	-	$1/300$
+	-	-	+	
+	-	-	-	$1/75$

G	M	B	C	$P(G, M, B, C)$
-	+	+	+	$9/400$
-	+	+	-	$27/400$
-	+	-	+	$27/800$
-	+	-	-	$81/800$
-	-	+	+	$27/400$
-	-	+	-	$27/400$
-	-	-	+	
-	-	-	-	$27/100$

5. Compute the following quantities. You may use either the full joint distribution or the conditional tables, whichever is more convenient.
   (a) P(+b | +m) =

   (b) P(+b | +m, +g) =

   (c) P(+b) =

(d) P(+c | +b) =

6. Now, add a node S to the Bayes net above that reflects the possibility that a new scientific study could influence the probability that marijuana is legalized. Assume that the study does not directly influence B or C. Which CPT(s) need to be modified?

7. Consider your augmented model. Just based on the structure, which of the following are guaranteed to be true?

(a) $B \perp\!\!\!\perp G$
(b) $C \perp\!\!\!\perp G \mid M$
(c) $G \perp\!\!\!\perp S$
(d) $G \perp\!\!\!\perp S \mid M$
(e) $G \perp\!\!\!\perp S \mid B$
(f) $B \perp\!\!\!\perp C$
(g) $B \perp\!\!\!\perp C \mid G$