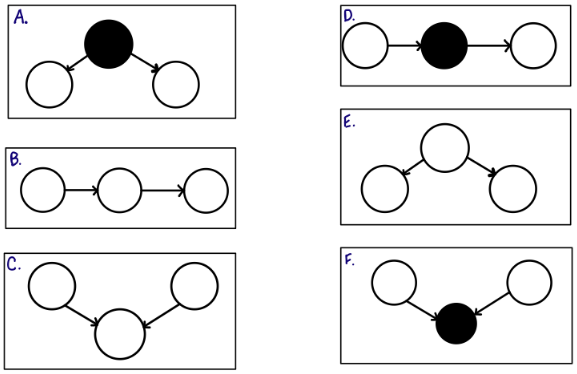
1 Warm Up

1. Bayes Ball Matching: Select the image that exemplifies each of the following Bayes Ball rules.



   (i) Active Causal Chain
   (ii) Active Common Cause
   (iii) Active Common Effect
   (iv) Inactive Causal Chain
   (v) Inactive Common Cause
   (vi) Inactive Common Effect

2. Are the following statements true or false? If it is false, briefly justify why not.

   (a) In a directed acyclic graph, each node can correspond to multiple random variables.

   (b) Bayes Nets allow us to represent conditional independence relationships in a concise way.

   (c) The number of entries in a conditional probability table in a Bayes Net is exponentially larger than in a joint probability table.

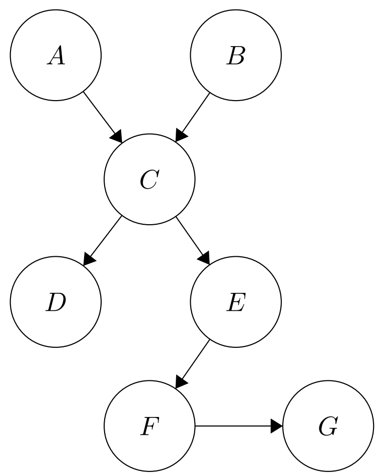
   (d) In a Bayes Net with no edges, the variables are all dependent on each other.

2 Bayes' Nets

Bayesian networks represent our beliefs about the conditional independence relationships between the variables in a system. The joint distribution is factored into conditional distributions, which are represented by the connections between nodes.

1. What would a Bayes net of variables which are all independent of each other look like?

2. Consider the following Bayes net.



(a) Which of the following statements is true? (Hint: d-separation!)

- $P(B \mid A) = P(B)$
- $A \perp\!\!\!\perp B \mid D, F$
- $A \perp\!\!\!\perp B \mid C$
- $D \perp\!\!\!\perp E$
- $P(D \mid C, E) = P(D \mid C)$
- $D \perp\!\!\!\perp E \mid A, B$
- $P(B \mid E) = P(B \mid G, E)$

(b) Suppose all variables have domains of size 2. How many entries are in the full joint probability table $P(A, B, C, D, E, F, G)$?

(c) How many entries are in the conditional probability table at each of the individual variables?

Variable	A	B	C	D	E	F	G
# Entries							

(d) How many CPT entries are needed to represent this Bayes net in total?

(e) How do we represent the full joint probability using the conditional probability tables of each variable?

Now let's look at some common types of queries we might be interested in answering using a Bayes' Net.

Let C = some variable we care about, D = some variable we don't care about (and we also don't know the value of), E = some variable we've observed to have the value e (i.e., current knowledge/evidence). (Suppose for now C, D, E are the only nodes in our BN.)

3. Express each of the following generic queries using probability notation, then write how we would compute it using the full joint distribution, $P(C, D, E)$. The first expression has been written for you as an example to get started.

   (a) What's the probability of the outcome $C = c$ given what I know?

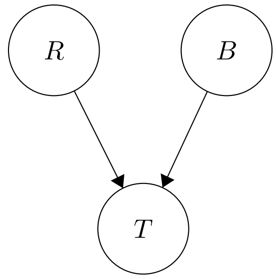
   $$P(c \mid e) = \frac{P(c, e)}{P(e)} =$$

   (b) What are the probabilities of all possible outcomes of C given what I know?

   (c) Which outcome of event C is the least likely outcome given what I know?

3 Common Effect

Recall the common effect example from lecture: both raining (R) and a ballgame (B) can cause traffic (T).



In this example $P(R = 1) = 0.7, P(B = 1) = 0.2$, and the probability table of T given R and B is below:

R	B	$P(T = 1\|R, B)$
1	1	0.9
1	0	0.5
0	1	0.5
0	0	0.1

We will now answer a few queries on this Bayes Net to determine the probability of a ballgame given different sets of knowledge.

1. What is the factorization of the joint distribution? What is the probability that it is not raining and there is no ballgame but there is still traffic?

2. What is the probability of a ballgame if we have not observed any of the variables?

3. What is the probability of a ballgame given that there is traffic? How does it compare to the probability from the previous question? Is ballgame independent of traffic?

4. What is the probability of a ballgame given that there is traffic but no rain? How does it compare to the probability from the previous questions? Is ballgame conditionally independent of rain given traffic?