

## Warm-up: What to eat?

We are trying healthy by finding the optimal amount of food to purchase.

We can choose the amount of **stir-fry** (ounce) and **boba** (fluid ounces).

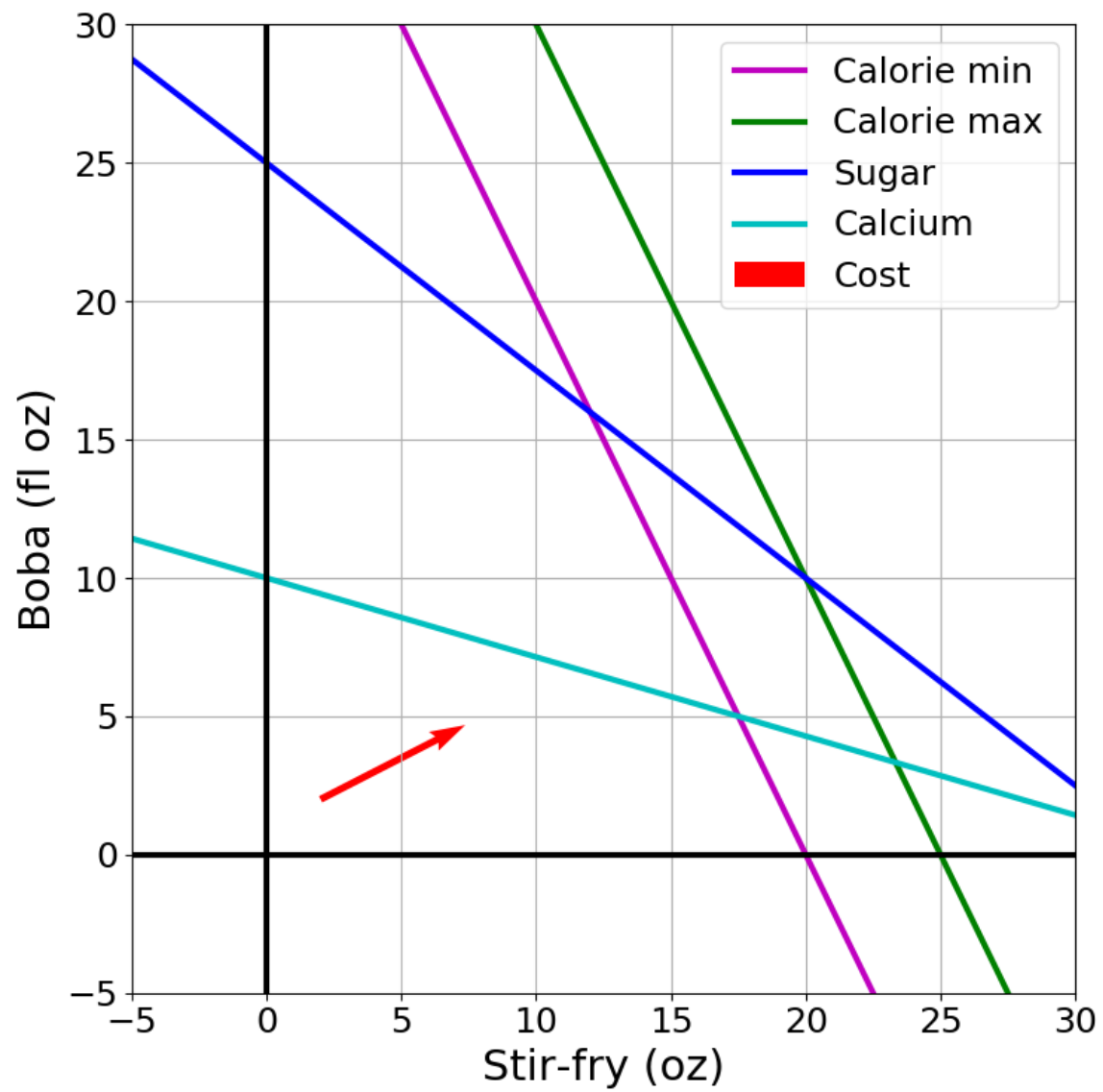
### Healthy Squad Goals

- $2000 \leq \text{Calories} \leq 2500$
- $\text{Sugar} \leq 100 \text{ g}$
- $\text{Calcium} \geq 700 \text{ mg}$

Food	Cost	Calories	Sugar	Calcium
<b>Stir-fry</b> (per oz)	1	100	3	20
<b>Boba</b> (per fl oz)	0.5	50	4	70

What is the cheapest way to stay “healthy” with this menu?

How much **stir-fry** (ounce) and **boba** (fluid ounces) should we buy?



# Announcements

## Assignments:

- P1: Search & Games
  - Due Thu 2/6, 10 pm
- HW4 (written)
  - Due Tue 2/11, 10 pm
- P2: Optimization
  - Released later this week
  - Due Thu 2/20, 10 pm

## Slip days

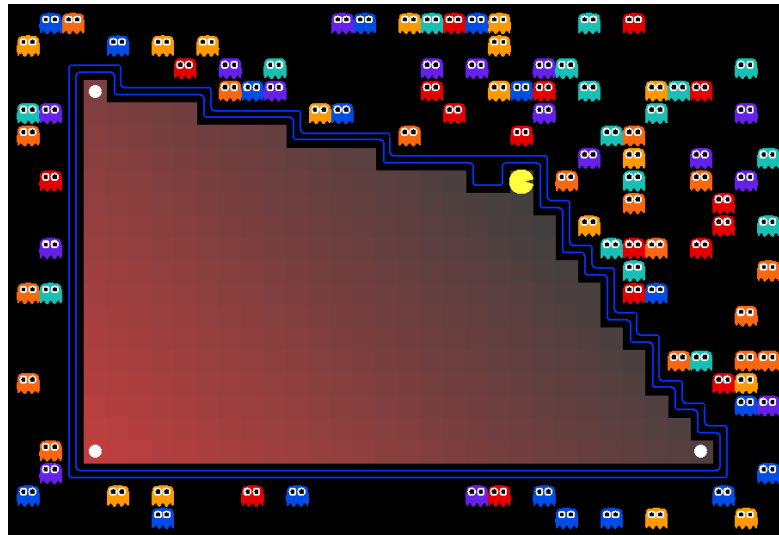
# Announcements

## Midterm 1 Exam

- Mon 2/17, in class

# AI: Representation and Problem Solving

## Linear Programming



Instructors: Pat Virtue & Stephanie Rosenthal

Slide credits: CMU AI, <http://ai.berkeley.edu>

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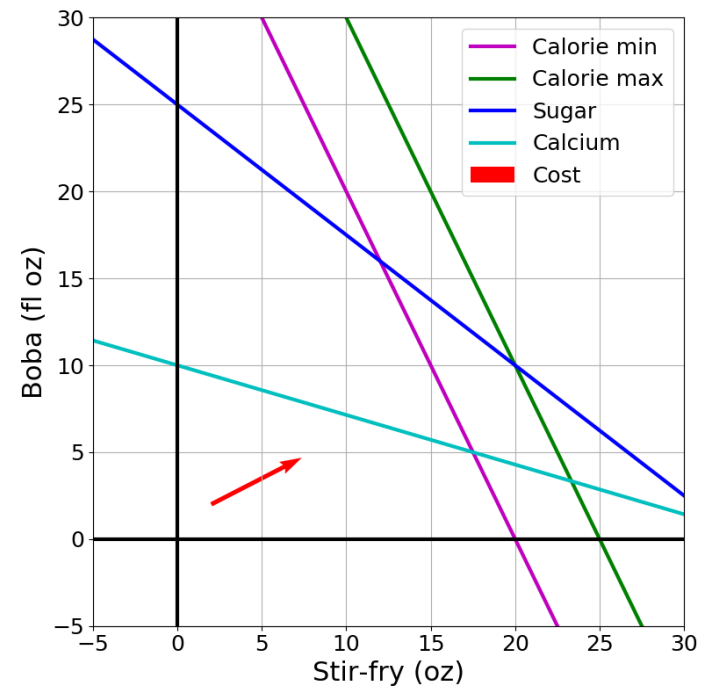
# Optimization

## Problem Description

## Optimization Representation

$$\begin{array}{ll}\min & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b}\end{array}$$

## Graphical Representation



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# Constraint Satisfaction Problems

## Map coloring

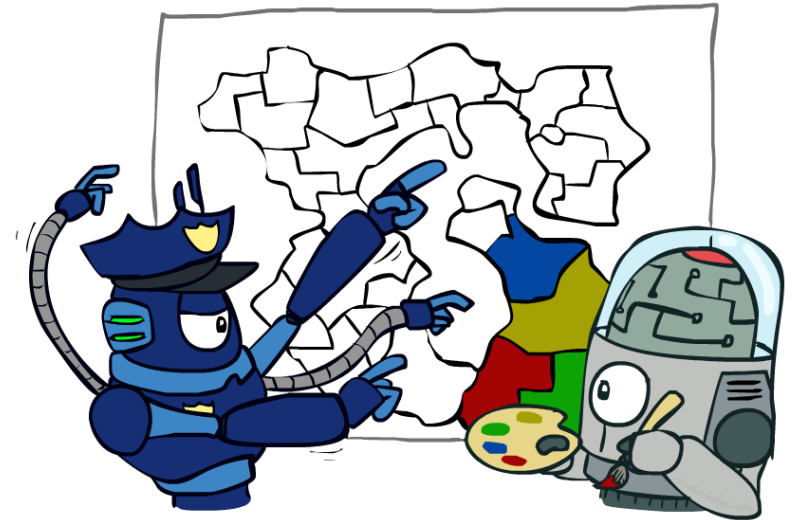
*Any*      $x$   
s.t.      $x$  satisfies constraints



# Constraint Satisfaction Problems

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Notation Alert!

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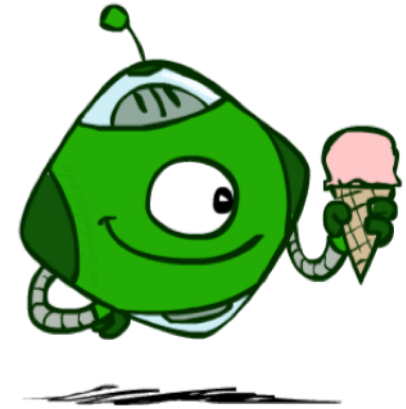
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# Optimization Formulation

## Diet Problem

Any  $x$

s.t.  $x$  satisfies constraints



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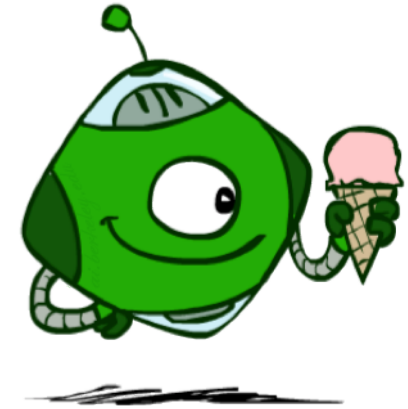
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Food	Cost	Calories	Sugar	Calcium
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# Optimization Formulation

## Diet Problem

$$\begin{array}{ll} \min_{\mathbf{x}} & \text{cost}(\mathbf{x}) \quad \text{Objective} \\ \text{s.t.} & \mathbf{x} \text{ satisfies constraints} \end{array}$$



### Healthy Squad Goals

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# Optimization Formulation

## Diet Problem

$$\begin{array}{ll}\min_{\mathbf{x}} & cost(\mathbf{x}) \\ \text{s.t.} & calories(\mathbf{x}) \text{ contained} \\ & sugar(\mathbf{x}) \leq limit \\ & calcium(\mathbf{x}) \geq limit\end{array}$$



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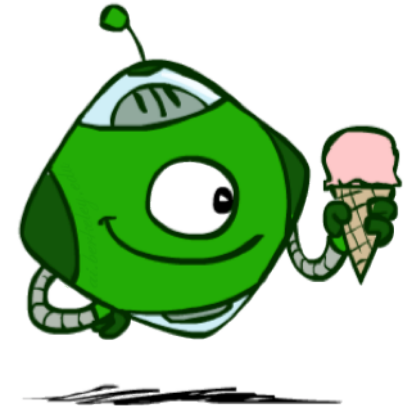
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# Optimization Formulation

## Diet Problem

$$\begin{array}{ll}\min_{x_1, x_2} & 1 x_1 + 0.5 x_2 \\ \text{s.t.} & 100 x_1 + 50 x_2 \geq 2000 \\ & 100 x_1 + 50 x_2 \leq 2500 \\ & 3 x_1 + 4 x_2 \leq 100 \\ & 20 x_1 + 70 x_2 \geq 700\end{array}$$

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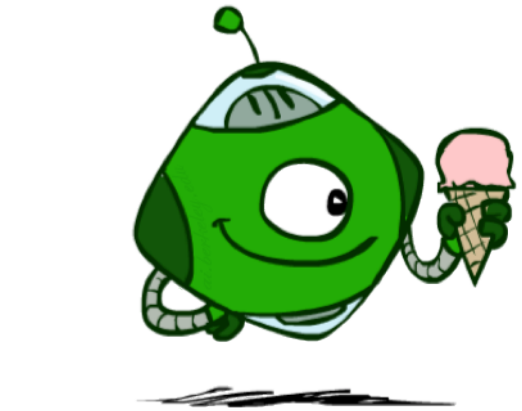
# Optimization Formulation

## Diet Problem

$$\begin{array}{ll}\min_{x_1, x_2} & c_1 x_1 + c_2 x_2 \\ \text{s.t.} & a_{1,1} x_1 + a_{1,2} x_2 \geq b_1 \\ & a_{2,1} x_1 + a_{2,2} x_2 \leq b_2 \\ & a_{3,1} x_1 + a_{3,2} x_2 \leq b_3 \\ & a_{4,1} x_1 + a_{4,2} x_2 \geq b_4\end{array}$$

Notation Alert!

$$A = \begin{array}{cc} & \text{Stir-fry} & \text{Boba} \\ \begin{bmatrix} 100 & 50 \\ 100 & 50 \\ 3 & 4 \\ 20 & 70 \end{bmatrix} \end{array}$$



Cost

$$c = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

Limit

$$b = \begin{bmatrix} 2000 \\ 2500 \\ 100 \\ 700 \end{bmatrix} \begin{array}{l} \text{Calorie min} \\ \text{Calorie max} \\ \text{Sugar} \\ \text{Calcium} \end{array}$$



# Optimization Formulation

## Diet Problem

$$\begin{array}{ll}\min_x & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & a_{1,1} x_1 + a_{1,2} x_2 \geq b_1 \\ & a_{2,1} x_1 + a_{2,2} x_2 \leq b_2 \\ & a_{3,1} x_1 + a_{3,2} x_2 \leq b_3 \\ & a_{4,1} x_1 + a_{4,2} x_2 \geq b_4\end{array}$$

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$$\mathbf{b} = \begin{array}{cc} \begin{array}{c} \text{Limit} \\ 2000 \\ 2500 \\ 100 \\ 700 \end{array} & \begin{array}{c} \text{Calorie min} \\ \text{Calorie max} \\ \text{Sugar} \\ \text{Calcium} \end{array} \end{array}$$

$$\mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

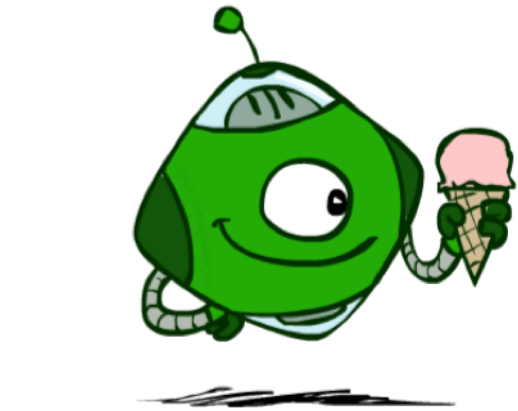


# Optimization Formulation

## Diet Problem

$$\begin{array}{ll}\min_x & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & -a_{1,1} x_1 - a_{1,2} x_2 \leq -b_1 \\ & a_{2,1} x_1 + a_{2,2} x_2 \leq b_2 \\ & a_{3,1} x_1 + a_{3,2} x_2 \leq b_3 \\ & -a_{4,1} x_1 - a_{4,2} x_2 \leq -b_4\end{array}$$

$$A = \begin{array}{cc} & \begin{array}{c} \text{Stir-fry} \\ \text{Boba} \end{array} \\ \begin{bmatrix} 100 & 50 \\ 100 & 50 \\ 3 & 4 \\ 20 & 70 \end{bmatrix} \end{array}$$



Cost

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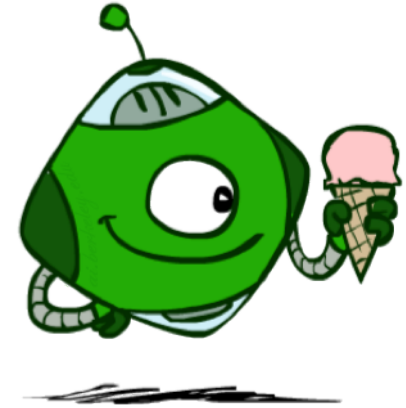
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$$\begin{array}{ll}\min_x & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & a_{1,1} x_1 + a_{1,2} x_2 \leq b_1 \\ & a_{2,1} x_1 + a_{2,2} x_2 \leq b_2 \\ & a_{3,1} x_1 + a_{3,2} x_2 \leq b_3 \\ & a_{4,1} x_1 + a_{4,2} x_2 \leq b_4\end{array}$$



Cost

$$\mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

$$A = \begin{array}{cc} & \text{Stir-fry} & \text{Boba} \\ \begin{bmatrix} -100 & -50 \\ 100 & 50 \\ 3 & 4 \\ -20 & -70 \end{bmatrix} & \mathbf{b} = \begin{bmatrix} -2000 \\ 2500 \\ 100 \\ -700 \end{bmatrix} & \begin{array}{l} \text{Limit} \\ \text{Calorie min} \\ \text{Calorie max} \\ \text{Sugar} \\ \text{Calcium} \end{array} \end{array}$$

# Optimization Formulation

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$$\begin{array}{ll}\min_x & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{A}\mathbf{x} \leq \mathbf{b}\end{array}$$



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Limit

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Notation Alert!

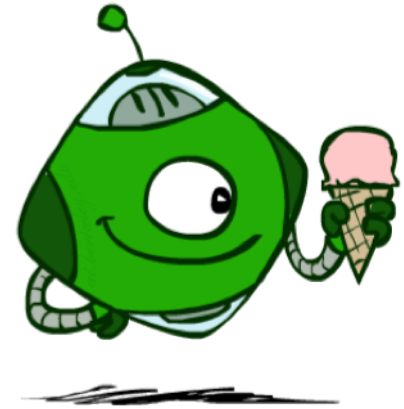
## Piazza Poll 1

What has to increase to add more nutrition constraints?

$$\begin{array}{ll}\min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{A} \mathbf{x} \leq \mathbf{b}\end{array}$$

Select all that apply

- A) length  $\mathbf{x}$
- B) length  $\mathbf{c}$
- C) height  $\mathbf{A}$
- D) width  $\mathbf{A}$
- E) length  $\mathbf{b}$



## Piazza Poll 1

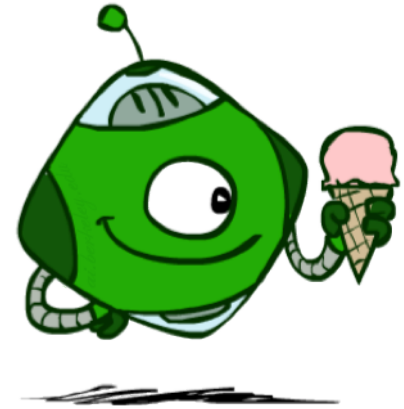
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$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

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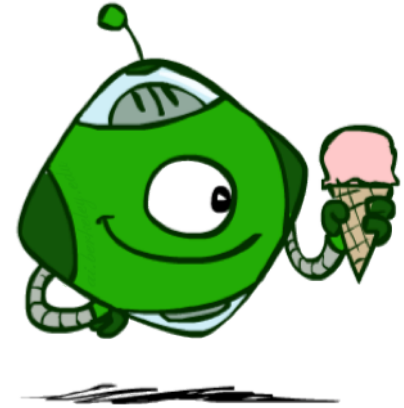
## Piazza Poll 2

What has to increase to add more menu items?

$$\begin{array}{ll}\min_x & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{A} \mathbf{x} \leq \mathbf{b}\end{array}$$

Select all that apply

- A) length  $\mathbf{x}$
- B) length  $\mathbf{c}$
- C) height  $\mathbf{A}$
- D) width  $\mathbf{A}$
- E) length  $\mathbf{b}$



## Piazza Poll 2

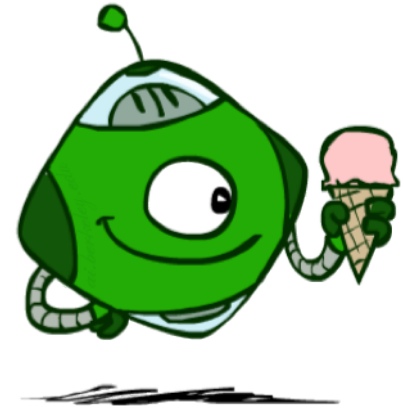
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## Question

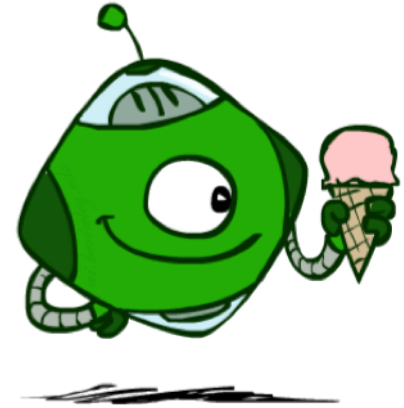
If  $A \in \mathbb{R}^{M \times N}$ , which of the following also equals  $N$ ?

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b} \end{array}$$

Select all that apply

- A) length  $\mathbf{x}$
- B) length  $\mathbf{c}$
- C) length  $\mathbf{b}$

Notation Alert!



# Linear Programming

Linear objective with linear constraints

$$\begin{array}{ll}\min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b}\end{array}$$

As opposed to general optimization

$$\begin{array}{ll}\min. & f_0(\mathbf{x}) \\ \text{s.t.} & f_i(\mathbf{x}) \leq 0, \quad i = 1 \dots M \\ & \mathbf{a}_i^T \mathbf{x} = \mathbf{b}_i, \quad i = 1 \dots P\end{array}$$

# Linear Programming

## Different formulations

### Inequality form

$$\begin{array}{ll}\min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{Ax} \leq \mathbf{b}\end{array}$$

### General form

$$\begin{array}{ll}\min. & \mathbf{c}^T \mathbf{x} + \mathbf{d} \\ \text{s.t.} & \mathbf{Gx} \leq \mathbf{h} \\ & \mathbf{Ax} = \mathbf{b}\end{array}$$

### Standard form

$$\begin{array}{ll}\min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{Ax} = \mathbf{b} \\ & \mathbf{x} \geq 0\end{array}$$

Important to pay attention to form!

# Linear Programming

## Different formulations

### Inequality form

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$$\begin{array}{ll}\min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{Ax} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}\end{array}$$

Can switch between formulations!

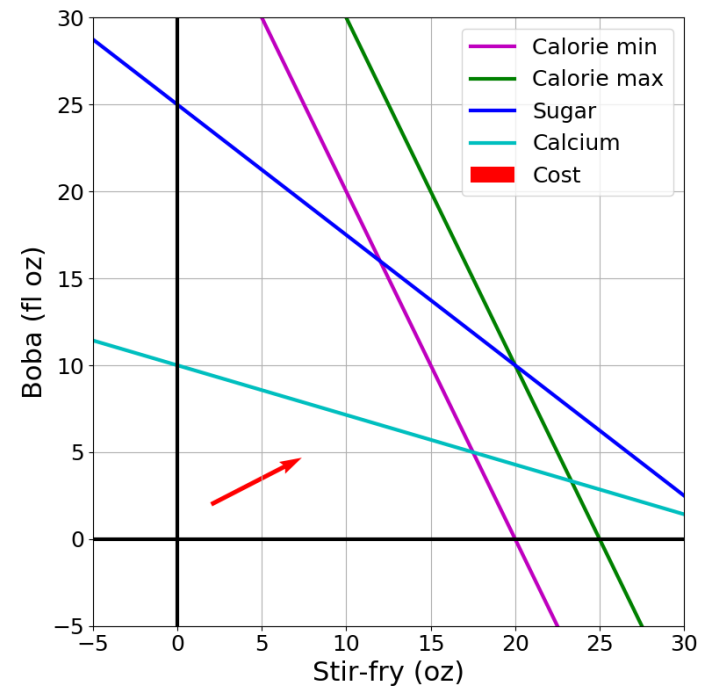
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## Graphical Representation



# Graphics Representation

## Geometry / Algebra I Quiz

What shape does this inequality represent?

$$a_1 x_1 + a_2 x_2 \leq b_1$$

# Graphics Representation

## Geometry / Algebra I Quiz

What shape does this inequality represent?

$$a_1 x_1 + a_2 x_2 = b_1$$

$$a_1 x_1 + a_2 x_2 \leq b_1$$

$$a_{1,1} x_1 + a_{1,2} x_2 \leq b_1$$

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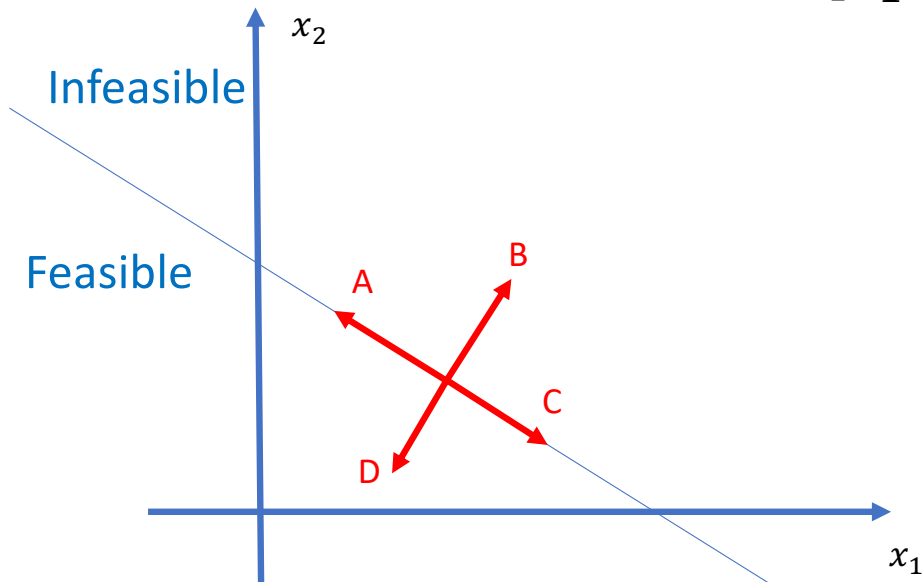
## Piazza Poll 3

What is the relationship between the half plane:

$$a_1 x_1 + a_2 x_2 \leq b_1$$

and the vector:

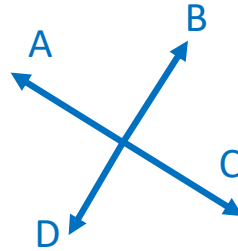
$$[a_1, a_2]^T$$





## Piazza Poll 4

Given the cost vector  $[c_1, c_2]^T$  and initial point  $\mathbf{x}^{(0)}$ ,  
Which unit vector step  $\Delta \mathbf{x}$  will cause  $\mathbf{x}^{(1)} = \mathbf{x}^{(0)} + \Delta \mathbf{x}$   
to have the lowest cost  $\mathbf{c}^T \mathbf{x}^{(1)}$ ?



Notation Alert!

## Cost Contours

Given the cost vector  $[c_1, c_2]^T$  where will  $\mathbf{c}^T \mathbf{x} = 0$  ?

## Cost Contours

Given the cost vector  $[c_1, c_2]^T$  where will

$$\mathbf{c}^T \mathbf{x} = 0 ?$$

$$\mathbf{c}^T \mathbf{x} = 1 ?$$

$$\mathbf{c}^T \mathbf{x} = 2 ?$$

$$\mathbf{c}^T \mathbf{x} = -1 ?$$

$$\mathbf{c}^T \mathbf{x} = -2 ?$$

# LP Graphical Representation

Inequality form

$$\begin{array}{ll}\min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{Ax} \leq \mathbf{b}\end{array}$$

# LP Graphical Representation

Inequality form, with no constraints

$$\min_{\mathbf{x}} \quad \mathbf{c}^T \mathbf{x}$$

# LP Graphical Representation

Inequality form, with no constraints

$$\begin{array}{ll}\min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & a_1 x_1 + a_2 x_2 \leq b\end{array}$$

## Piazza Poll 5

True or False: An minimizing LP with exactly one constraint, will always have a minimum objective at  $-\infty$ .

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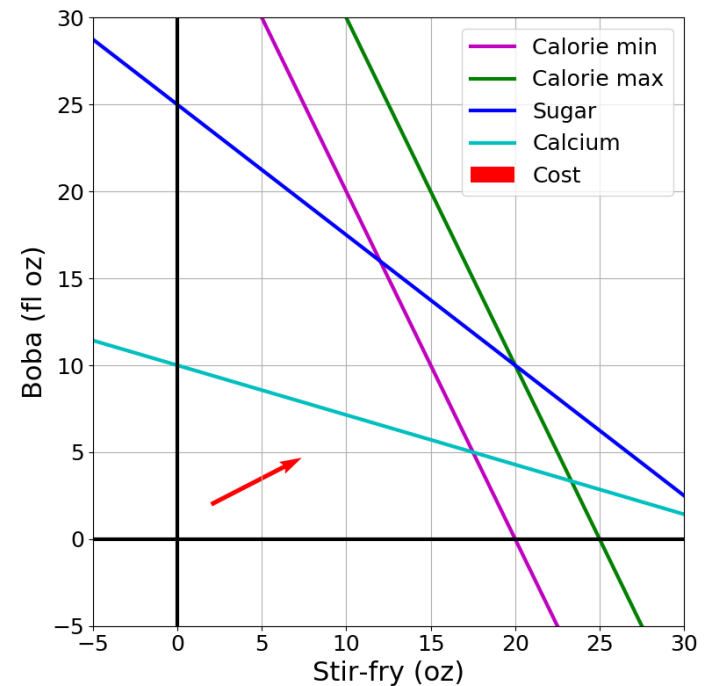
# Optimization

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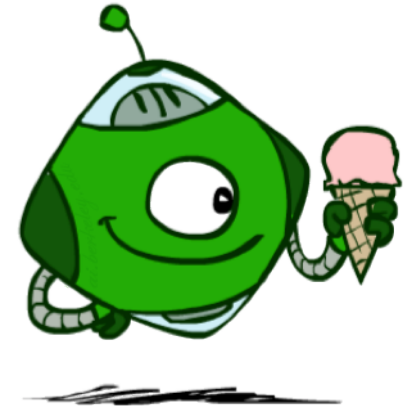
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# Optimization Formulation

## Diet Problem

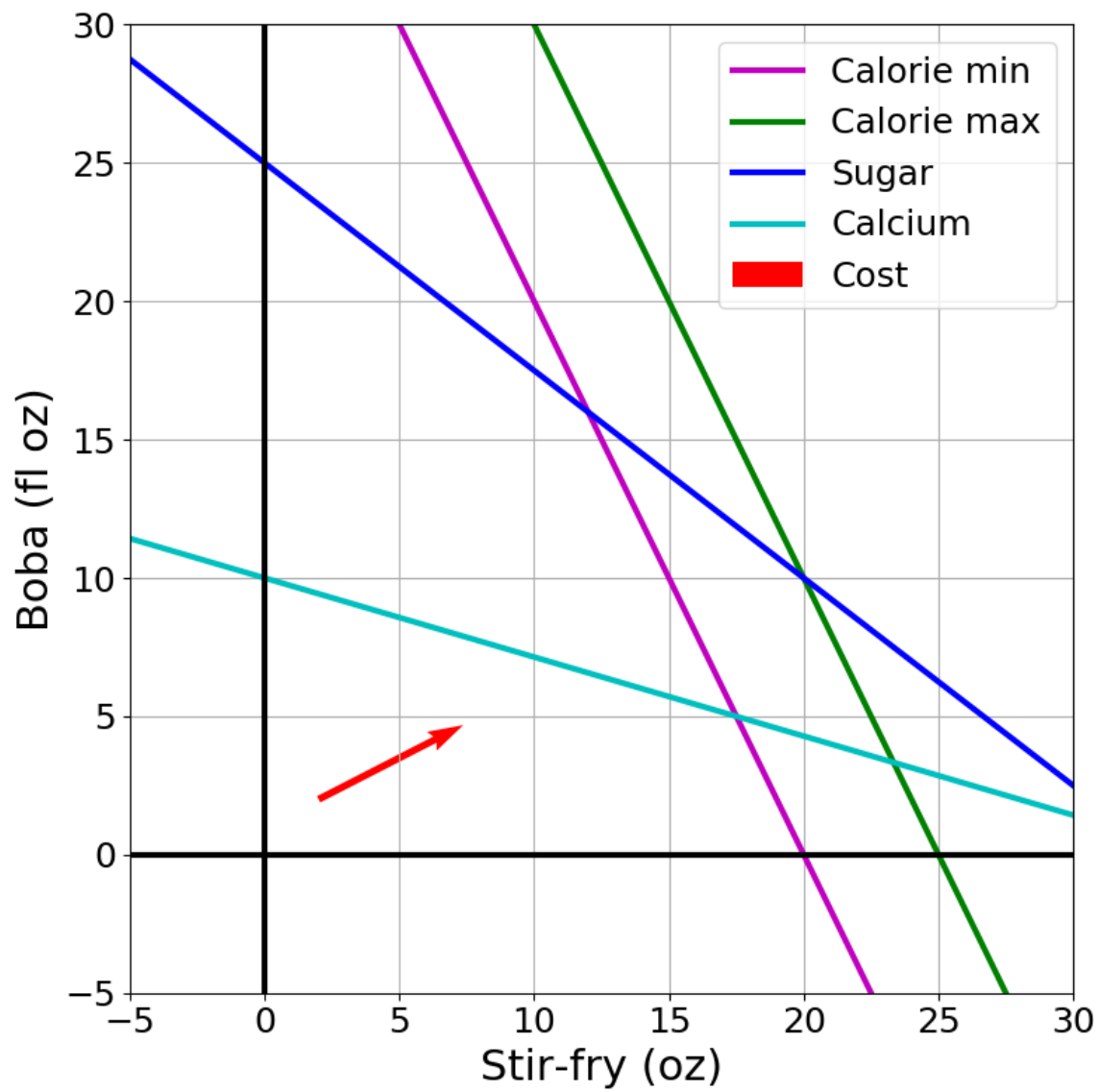
$$\begin{array}{ll}\min_x & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{A}\mathbf{x} \leq \mathbf{b}\end{array}$$



Cost

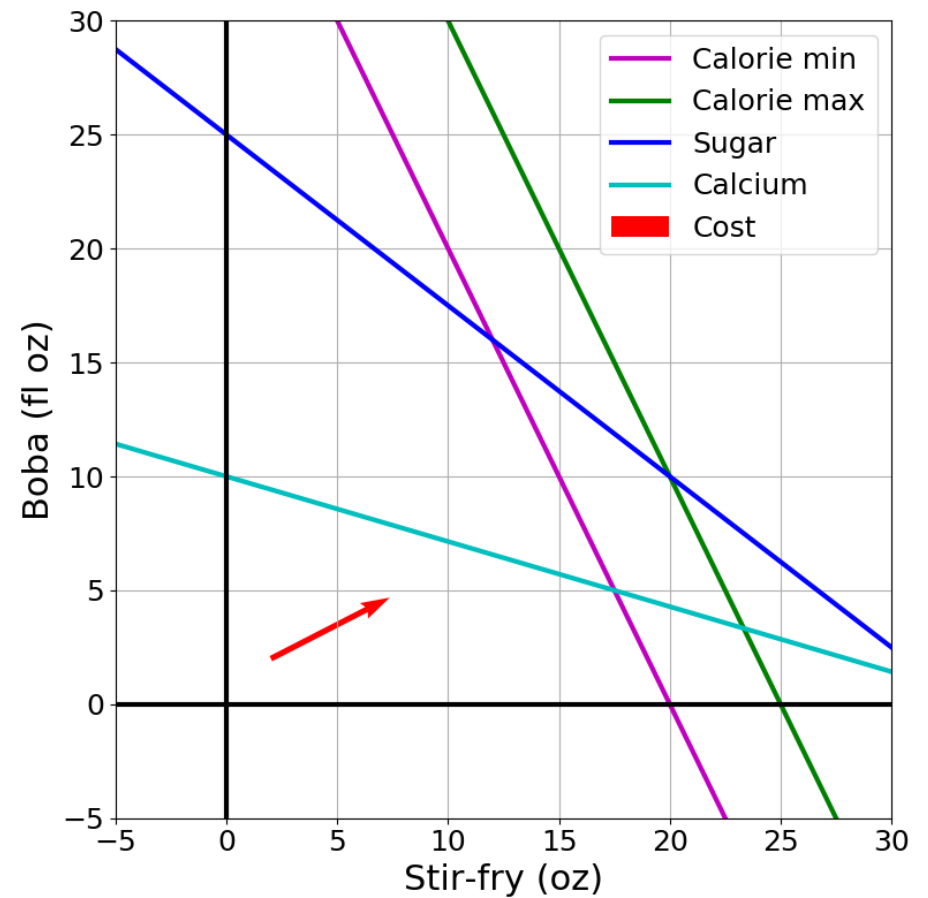
$$\mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

$$\mathbf{A} = \begin{array}{cc} & \text{Stir-fry} & \text{Boba} \\ \begin{bmatrix} -100 & -50 \\ 100 & 50 \\ 3 & 4 \\ -20 & -70 \end{bmatrix} & \end{array} \quad \mathbf{b} = \begin{array}{c} \text{Limit} \\ \begin{bmatrix} -2000 \\ 2500 \\ 100 \\ -700 \end{bmatrix} \end{array} \begin{array}{l} \text{Calorie min} \\ \text{Calorie max} \\ \text{Sugar} \\ \text{Calcium} \end{array}$$



# Solving an LP

Solutions are at feasible intersections of constraint boundaries!!



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