Announcements

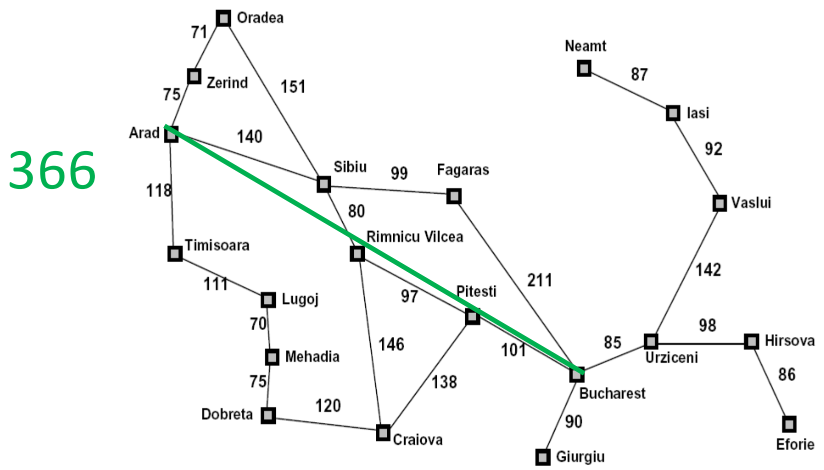
Assignments:

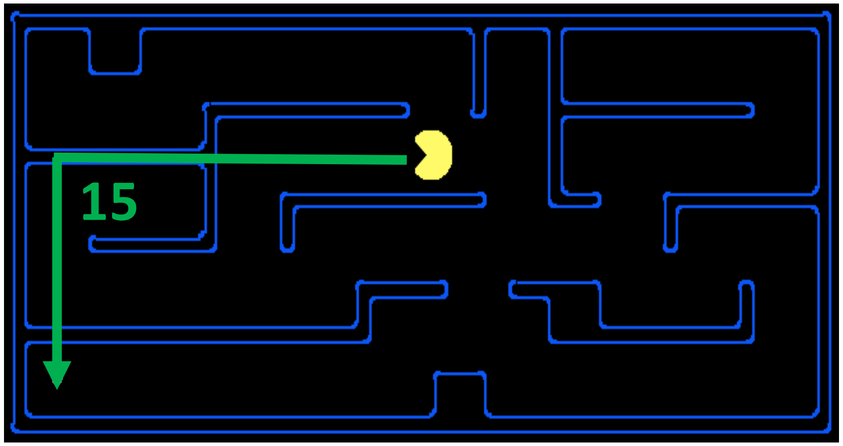
- P1: Search and Games
  - Due Thu 2/6, 10 pm
  - Recommended to work in pairs
  - Submit to Gradescope early and often
- HW2 (written) – Search and Heuristics
  - Due tomorrow Tue 1/28, 10 pm
- HW3 (online) – Adversarial Search and CSPs
  - Out tomorrow
  - Due Tue 2/4, 10 pm

Creating Admissible Heuristics

Most of the work in solving hard search problems optimally is in coming up with admissible heuristics

Often, admissible heuristics are solutions to ***relaxed problems***, where new actions are available





Combining heuristics

Dominance: $h_a \geq h_c$ if

$$\forall n \;\; h_a(n) \geq h_c(n)$$

- Roughly speaking, larger is better as long as both are admissible
- The zero heuristic is pretty bad (what does A* do with h=0?)
- The exact heuristic is pretty good, but usually too expensive!

What if we have two heuristics, neither dominates the other?

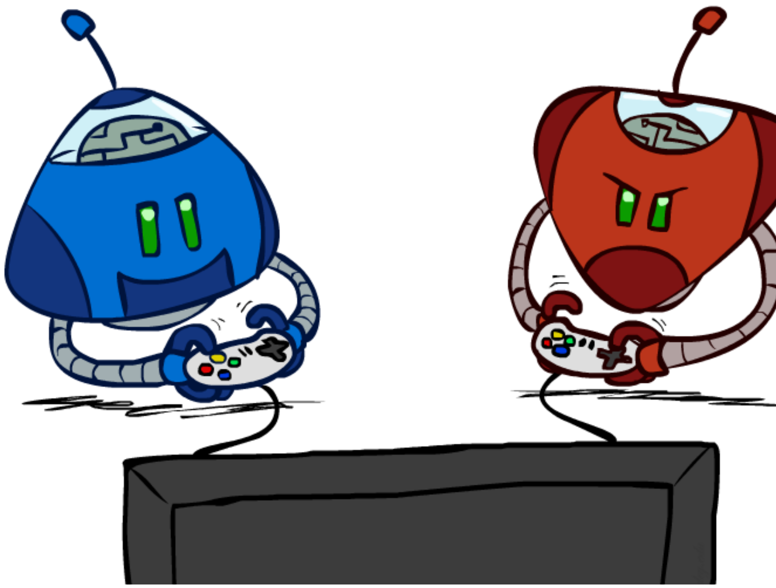
- Form a new heuristic by taking the max of both:

$$h(n) = \max(\, h_a(n), h_b(n) \,)$$

- Max of admissible heuristics is admissible and dominates both!

AI: Representation and Problem Solving

Adversarial Search



Instructors: Pat Virtue & Stephanie Rosenthal

Slide credits: CMU AI, http://ai.berkeley.edu

Outline

History / Overview

Zero-Sum Games (Minimax)

Evaluation Functions

Search Efficiency (α-β Pruning)

Games of Chance (Expectimax)

Game Playing State-of-the-Art

Checkers:

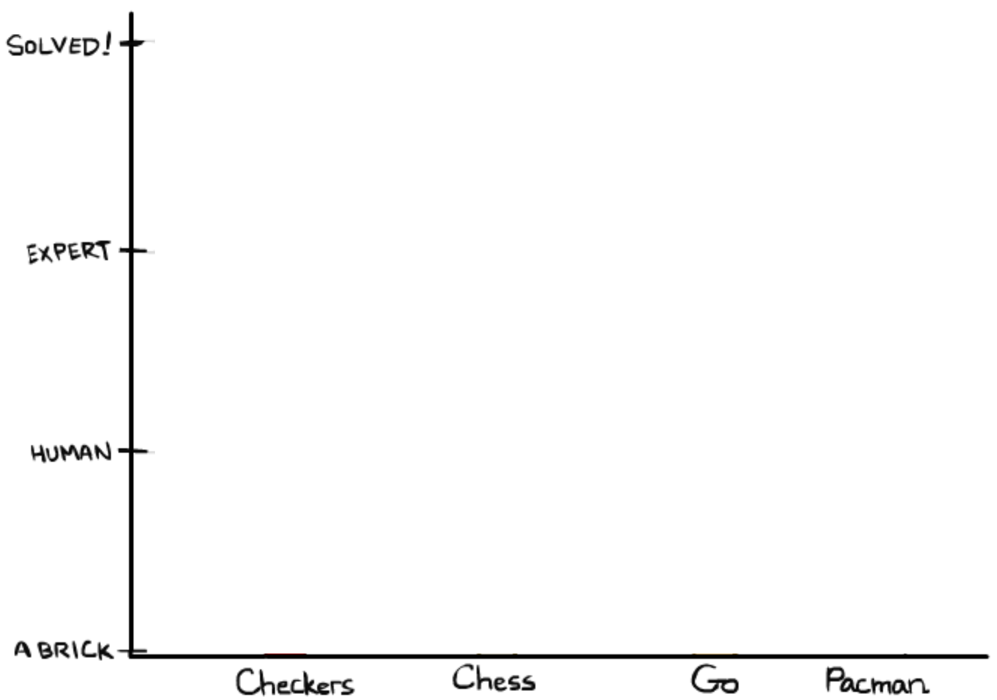
- 1950: First computer player.
- 1959: Samuel's self-taught program.
- 1994: First computer world champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame.
- 2007: Checkers solved! Endgame database of 39 trillion states

Chess:

- 1945-1960: Zuse, Wiener, Shannon, Turing, Newell & Simon, McCarthy.
- 1960s onward: gradual improvement under "standard model"
- 1997: special-purpose chess machine Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second and extended some lines of search up to 40 ply. Current programs running on a PC rate > 3200 (vs 2870 for Magnus Carlsen).

Go:

- 1968: Zobrist's program plays legal Go, barely (b>300!)
- 2005-2014: Monte Carlo tree search enables rapid advances: current programs beat strong amateurs, and professionals with a 3-4 stone handicap.



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Go:

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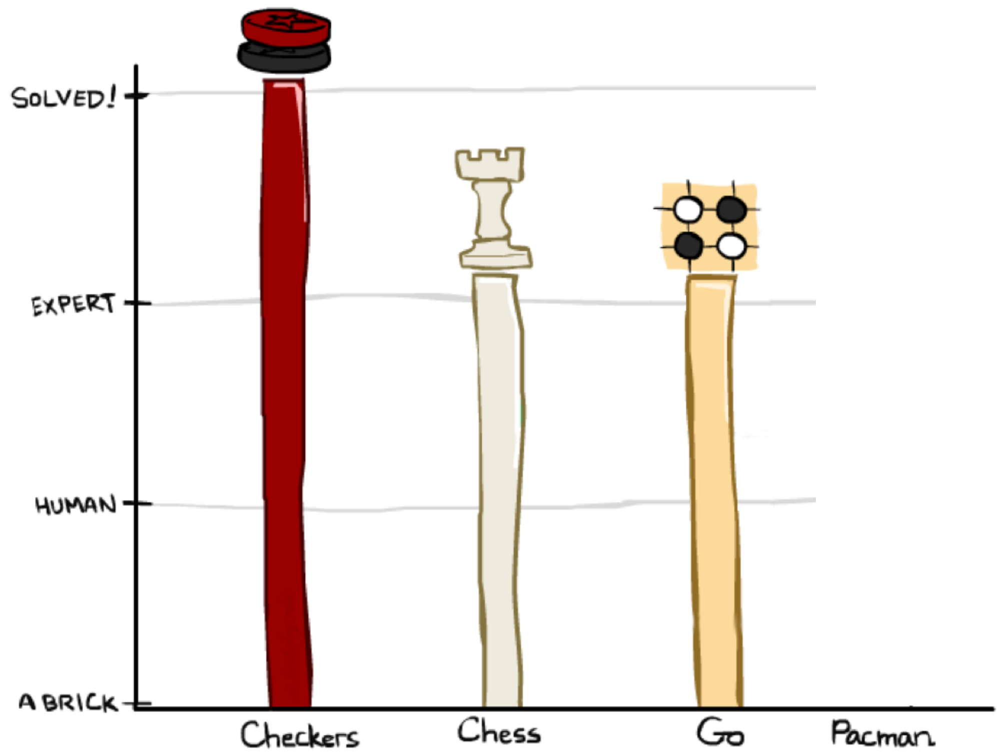
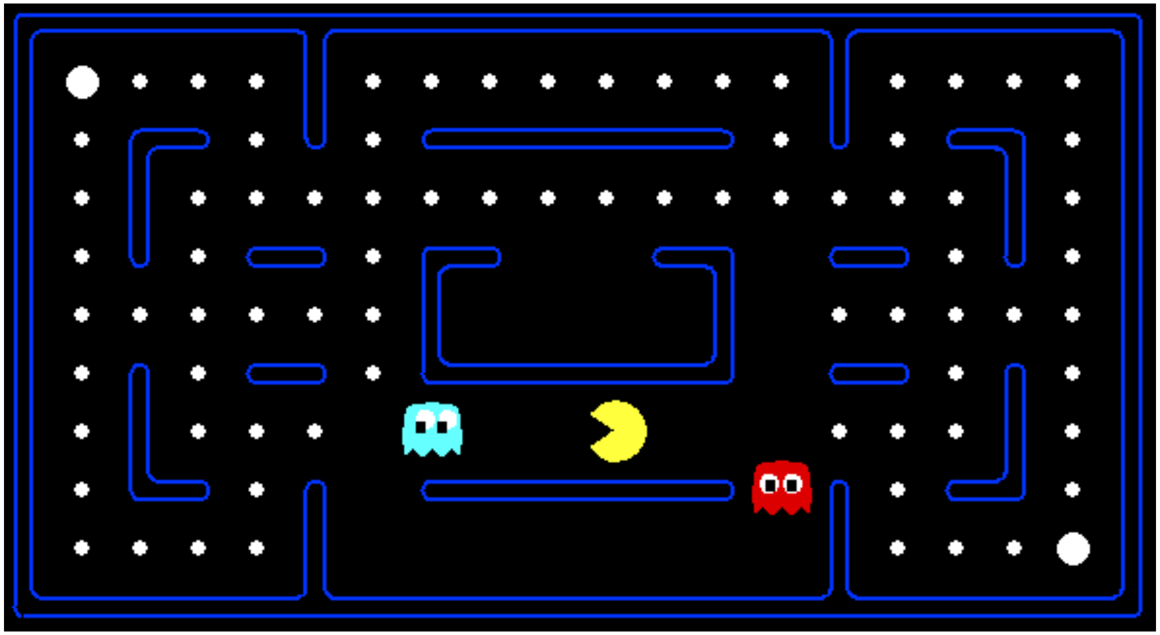
- 2005-2014: Monte Carlo tree search enables rapid advances: current programs beat strong amateurs, and professionals with a 3-4 stone handicap.
- 2015: AlphaGo from DeepMind beats Lee Sedol



Behavior from Computation



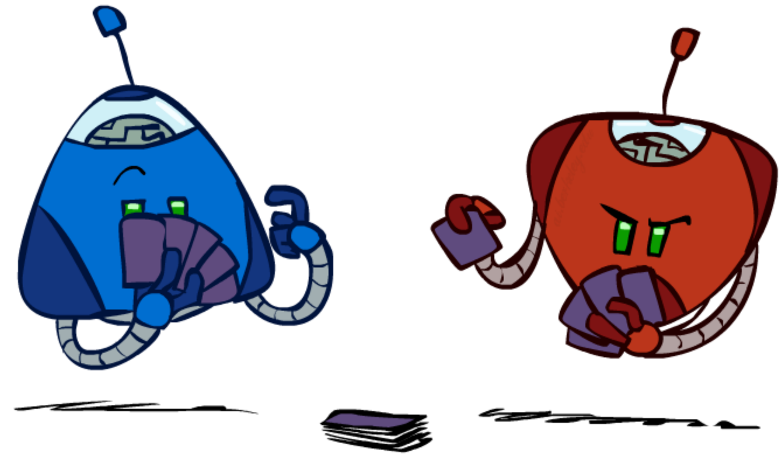
[Demo: mystery pacman (L6D1)]

Types of Games

Many different kinds of games!

Axes:

- Deterministic or stochastic?
- Perfect information (fully observable)?
- One, two, or more players?
- Turn-taking or simultaneous?
- Zero sum?



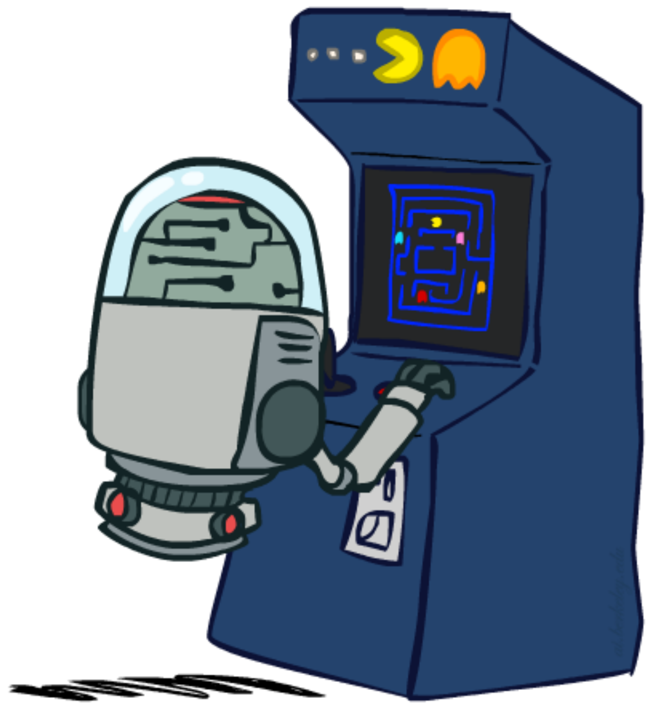
Want algorithms for calculating a ***contingent plan*** (a.k.a. strategy or policy) which recommends a move for every possible eventuality

“Standard” Games

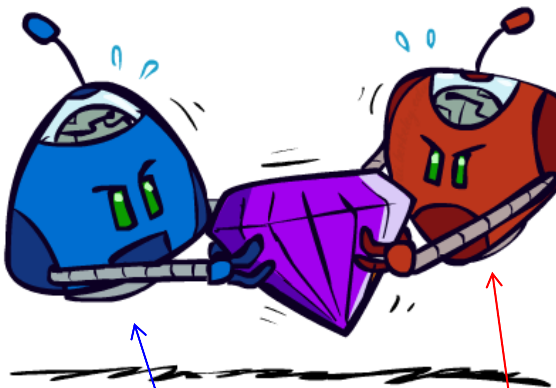
Standard games are deterministic, observable, two-player, turn-taking, zero-sum

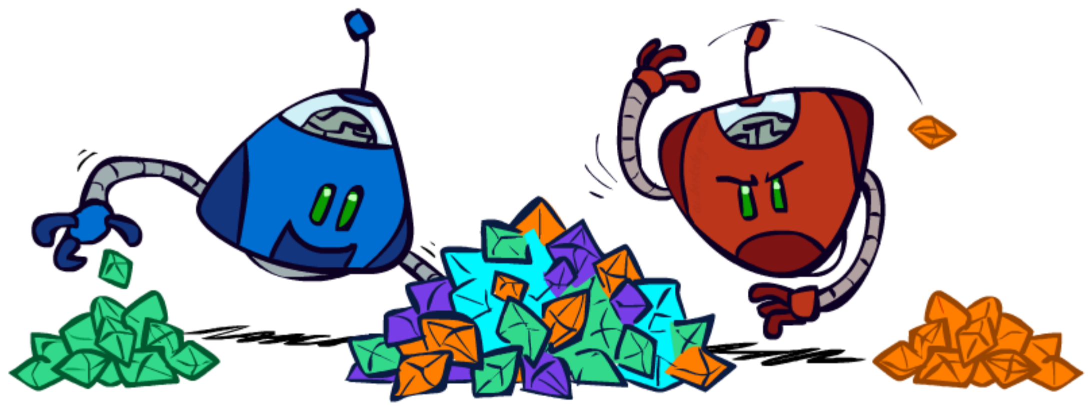
Game formulation:

- Initial state: s_0
- Players: Player(s) indicates whose move it is
- Actions: Actions(s) for player on move
- Transition model: Result(s,a)
- Terminal test: Terminal-Test(s)
- Terminal values: Utility(s,p) for player p
  - Or just Utility(s) for player making the decision at root



Zero-Sum Games

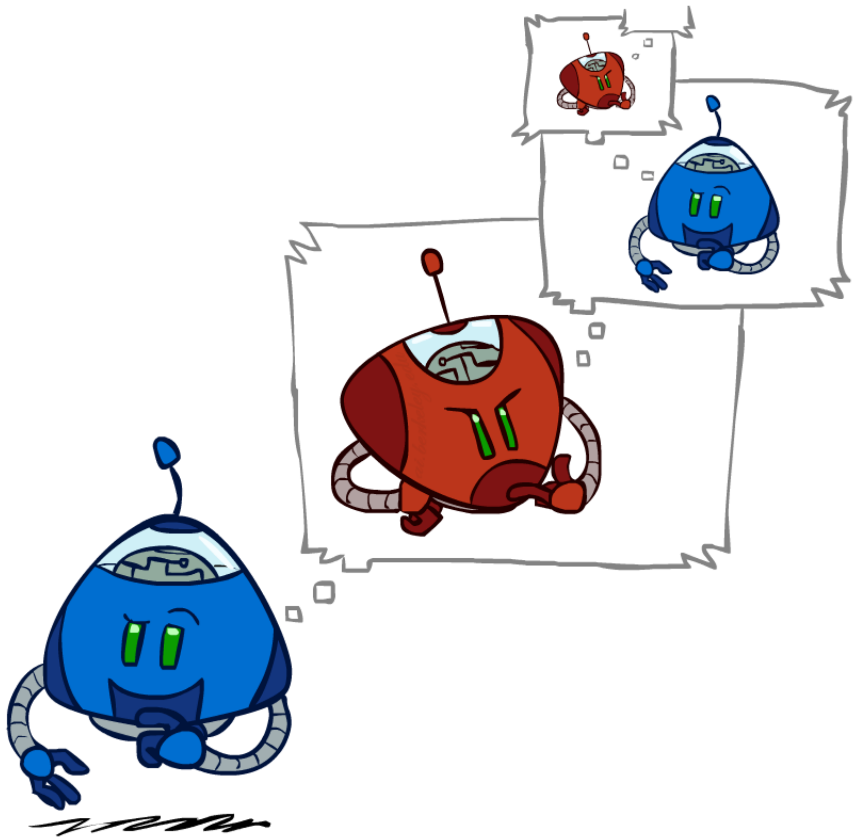




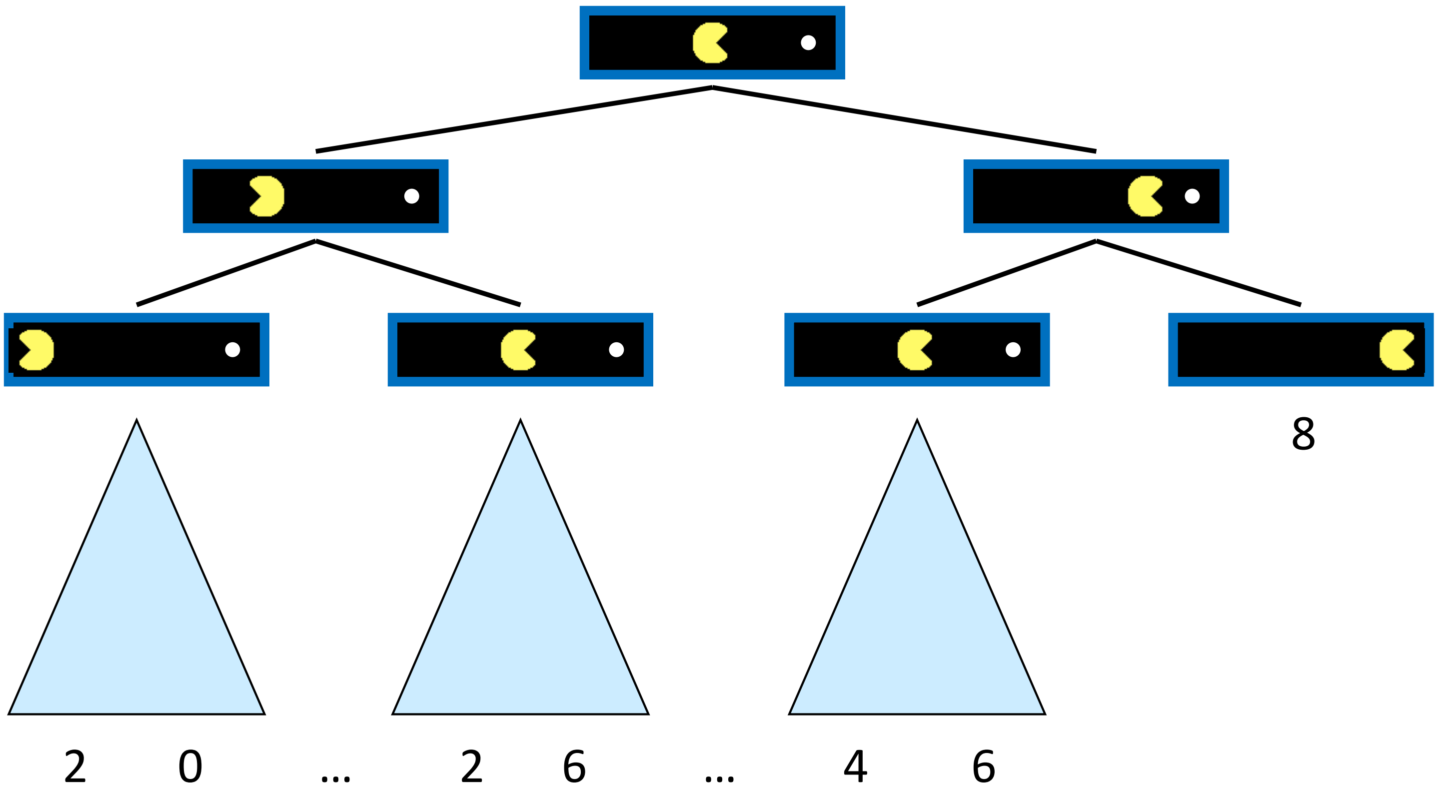
- Zero-Sum Games
  - Agents have ***opposite*** utilities
  - Pure competition:
    - One ***maximizes***, the other ***minimizes***

- General Games
  - Agents have ***independent*** utilities
  - Cooperation, indifference, competition, shifting alliances, and more are all possible

Adversarial Search

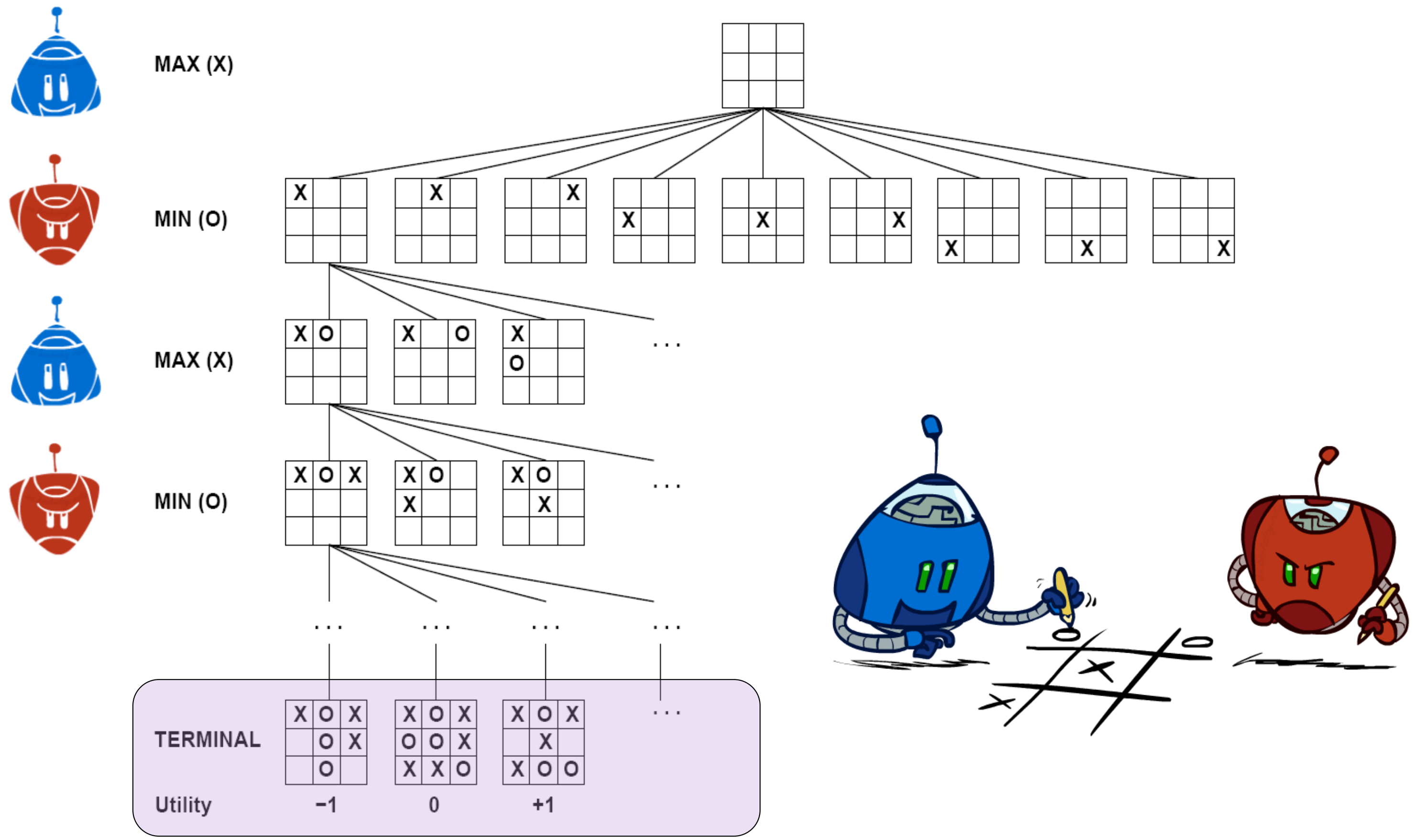


Single-Agent Trees



Minimax

- States
- Actions
- Values

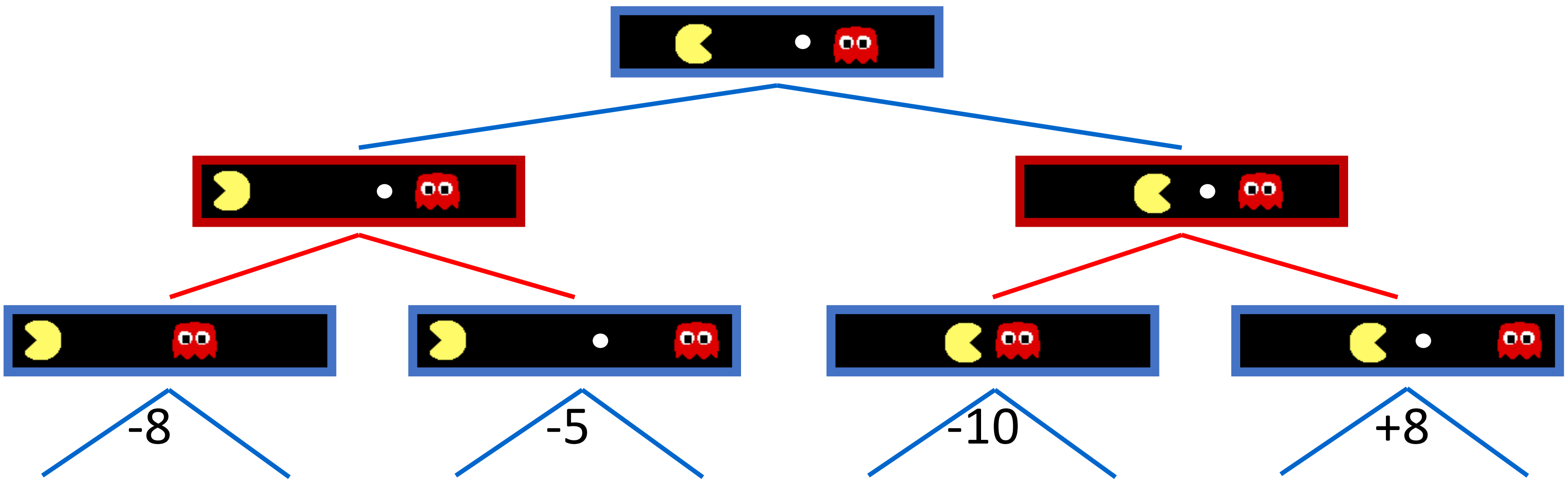


Minimax

States

Actions

Values



Piazza Poll 1

What is the minimax value at the root?

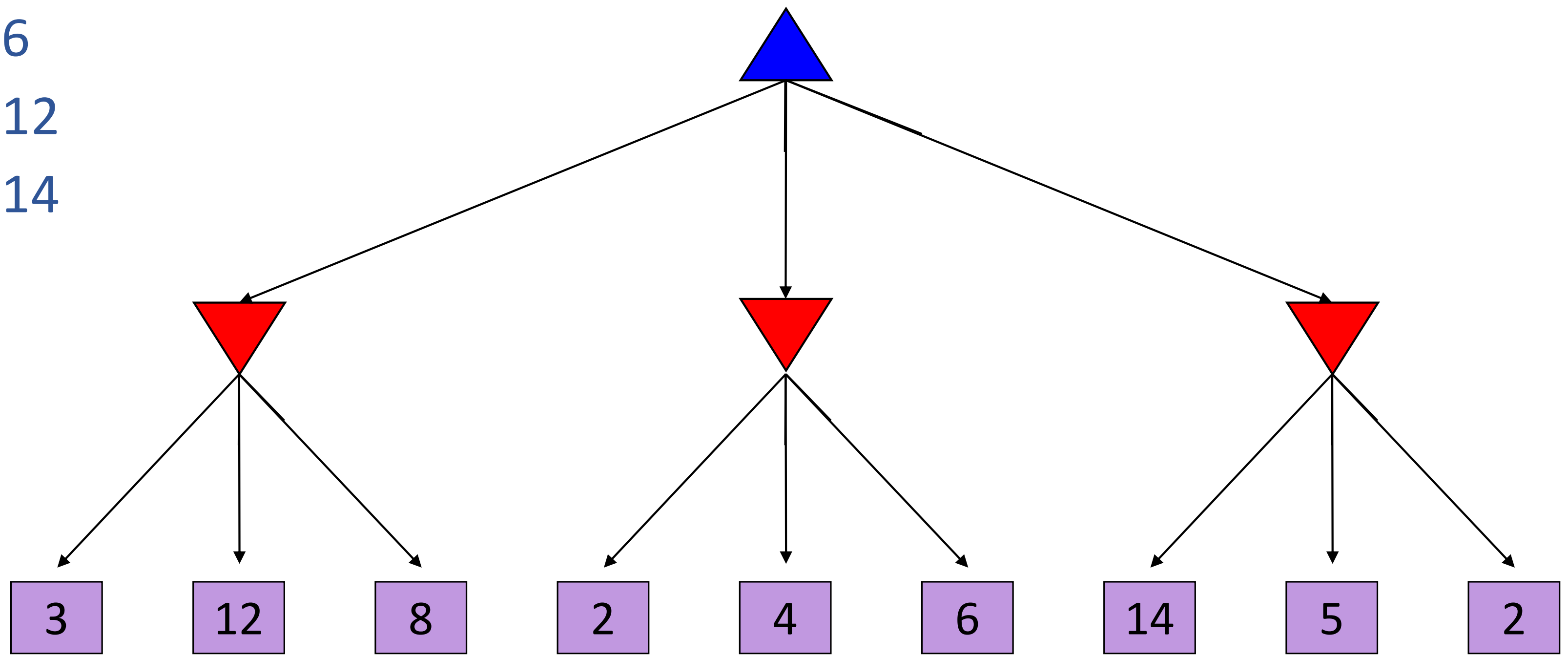
A) 2

B) 3

C) 6

D) 12

E) 14



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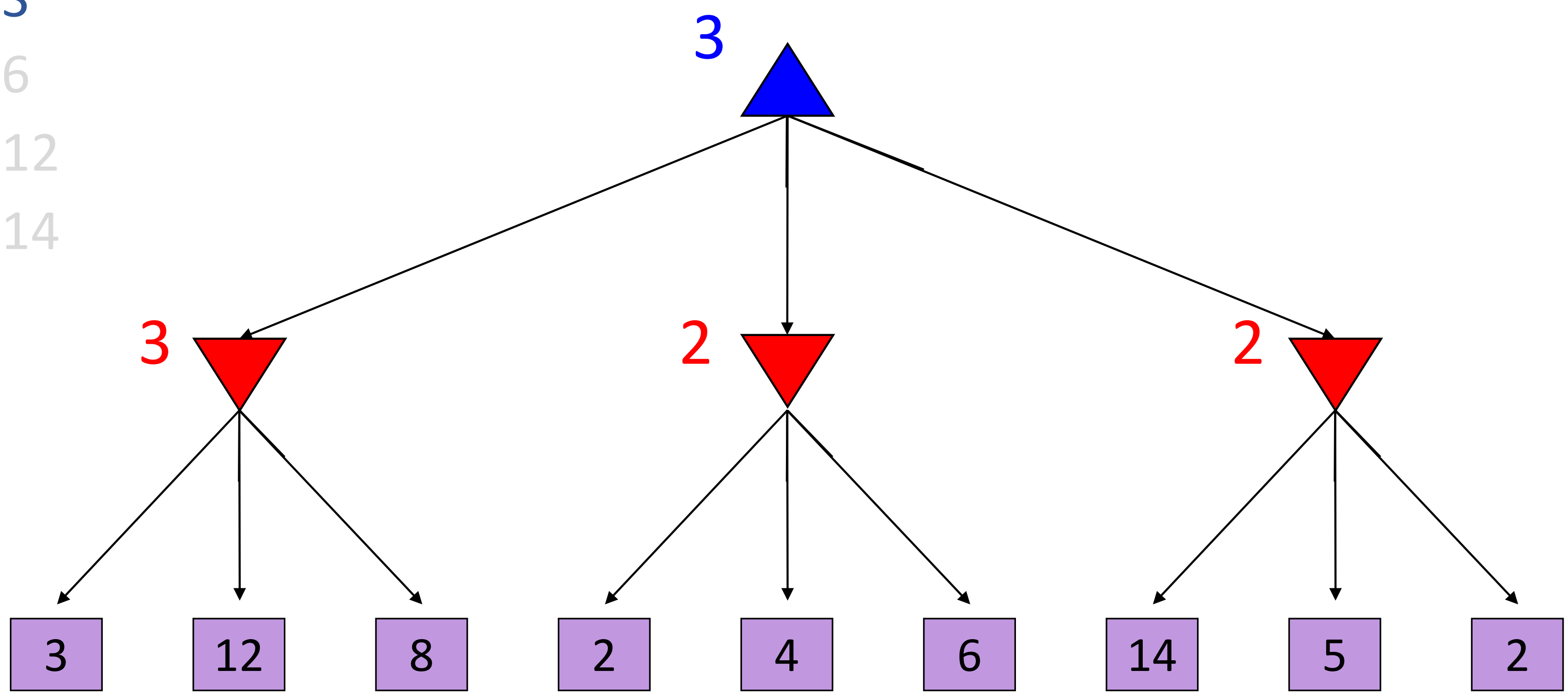
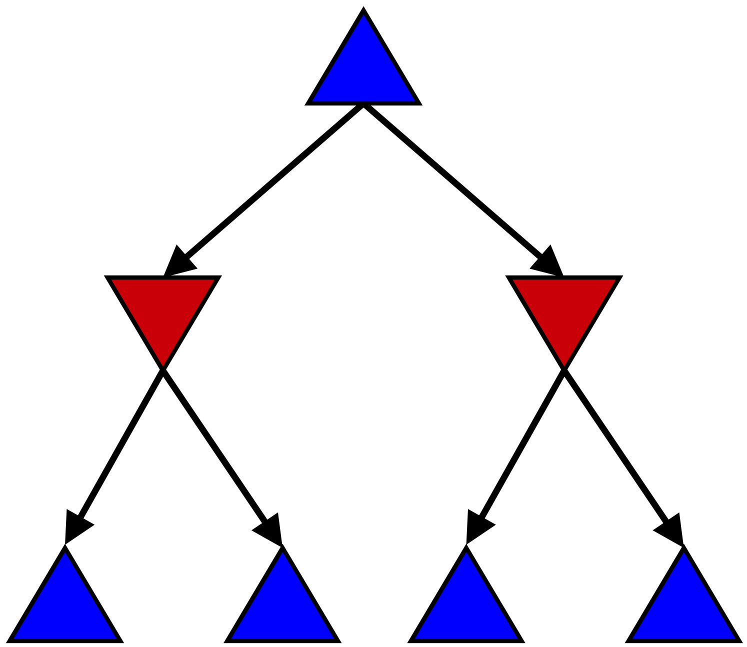
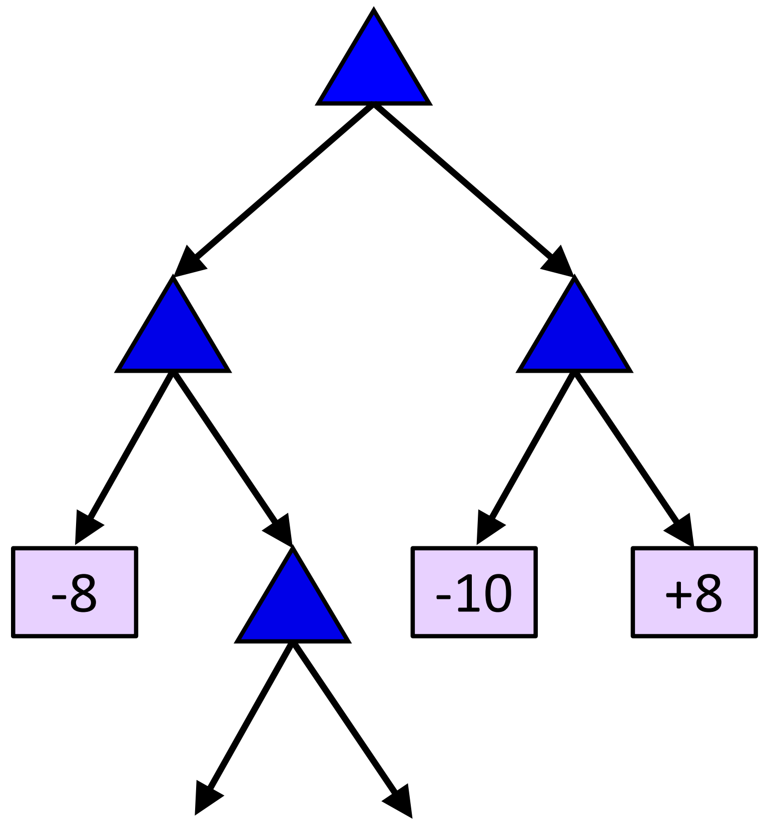


Minimax Code



Max Code



Max Code

```
def max_value(state):

    if state.is_leaf:
        return state.value
    # TODO Also handle depth limit

    best_value = -10000000

    for action in state.actions:
        next_state = state.result(action)

        next_value = max_value(next_state)

        if next_value > best_value:
            best_value = next_value

    return best_value
```

Minimax Code

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def min_value(state):
```

Minimax Notation

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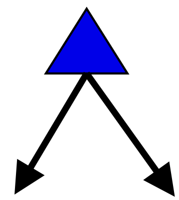
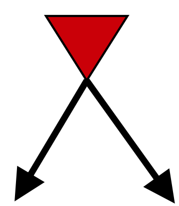


$$V(s) = \max_a V(s'),$$

where $s' = result(s, a)$



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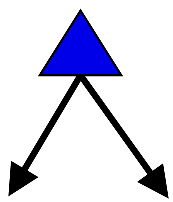

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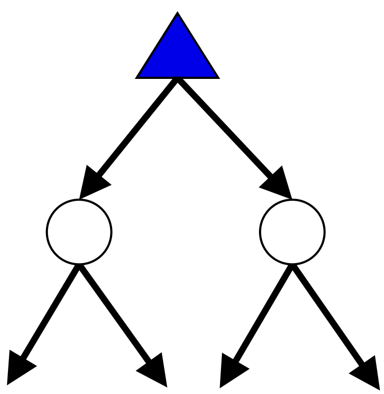


$$\hat{a} = \underset{a}{\operatorname{argmax}}\, V(s'),$$
$$\text{where } s' = result(s, a)$$

Generic Game Tree Pseudocode

```
function  minimax_decision( state )
    return  argmax a in state.actions  value( state.result(a) )

function  value( state )
    if  state.is_leaf
        return state.value

    if  state.player  is  MAX
        return  max a in state.actions  value( state.result(a) )

    if  state.player  is  MIN
        return  min a in state.actions  value( state.result(a) )
```

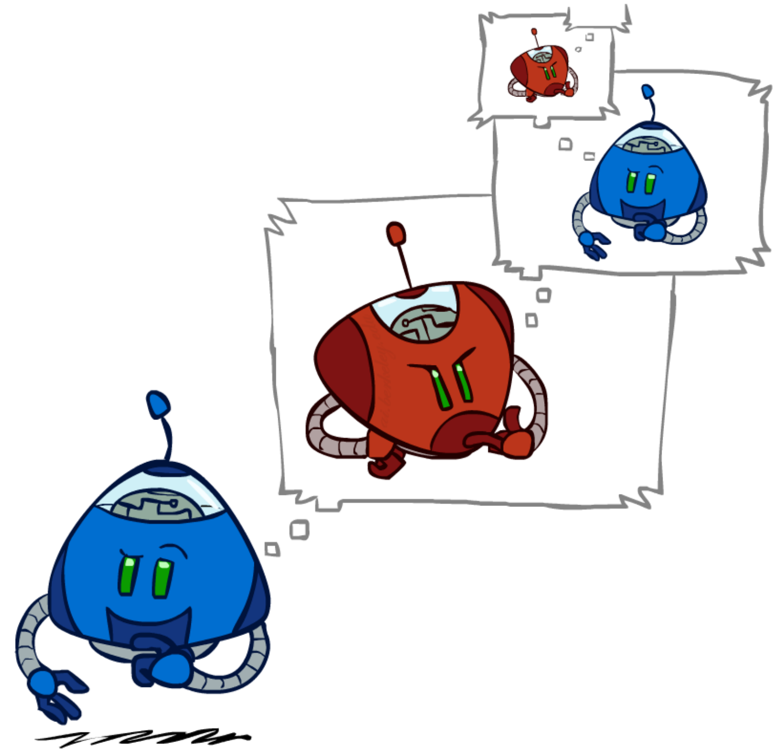
Minimax Efficiency

How efficient is minimax?

- Just like (exhaustive) DFS
- Time: $O(b^m)$
- Space: $O(bm)$

Example: For chess, $b \approx 35$, $m \approx 100$

- Exact solution is completely infeasible
- Humans can't do this either, so how do we play chess?
- Bounded rationality – Herbert Simon



Resource Limits

Resource Limits

Problem: In realistic games, cannot search to leaves!

Solution 1: Bounded lookahead

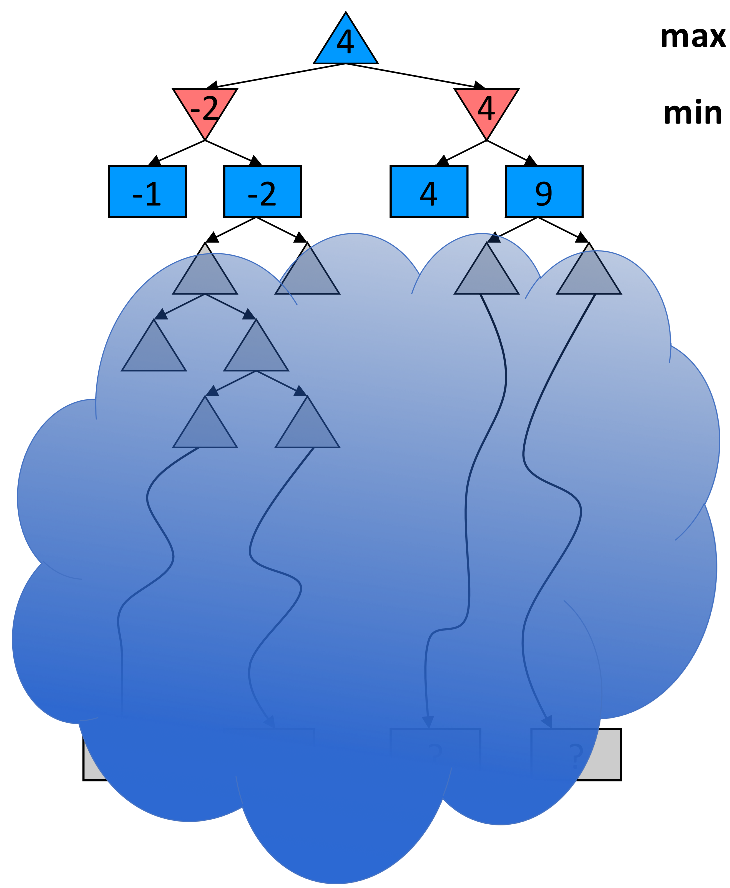
- Search only to a preset ***depth limit*** or ***horizon***
- Use an ***evaluation function*** for non-terminal positions

Guarantee of optimal play is gone

More plies make a BIG difference

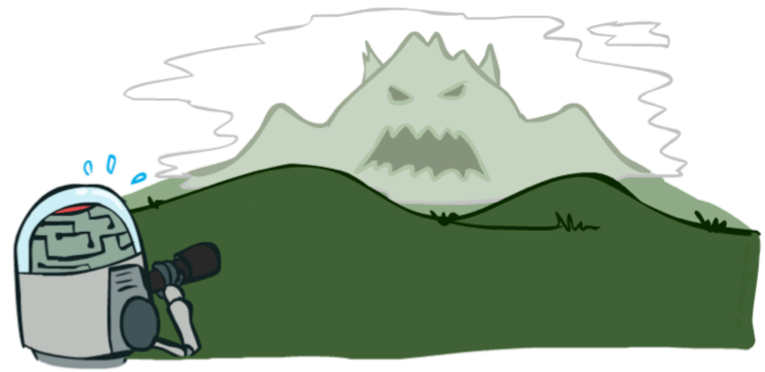
Example:

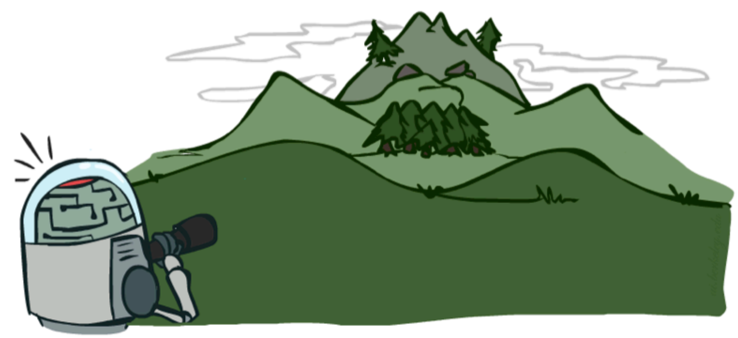
- Suppose we have 100 seconds, can explore 10K nodes / sec
- So can check 1M nodes per move
- For chess, b=~35 so reaches about depth 4 – not so good



Depth Matters

- Evaluation functions are always imperfect
- Deeper search => better play (usually)
- Or, deeper search gives same quality of play with a less accurate evaluation function
- An important example of the tradeoff between complexity of features and complexity of computation



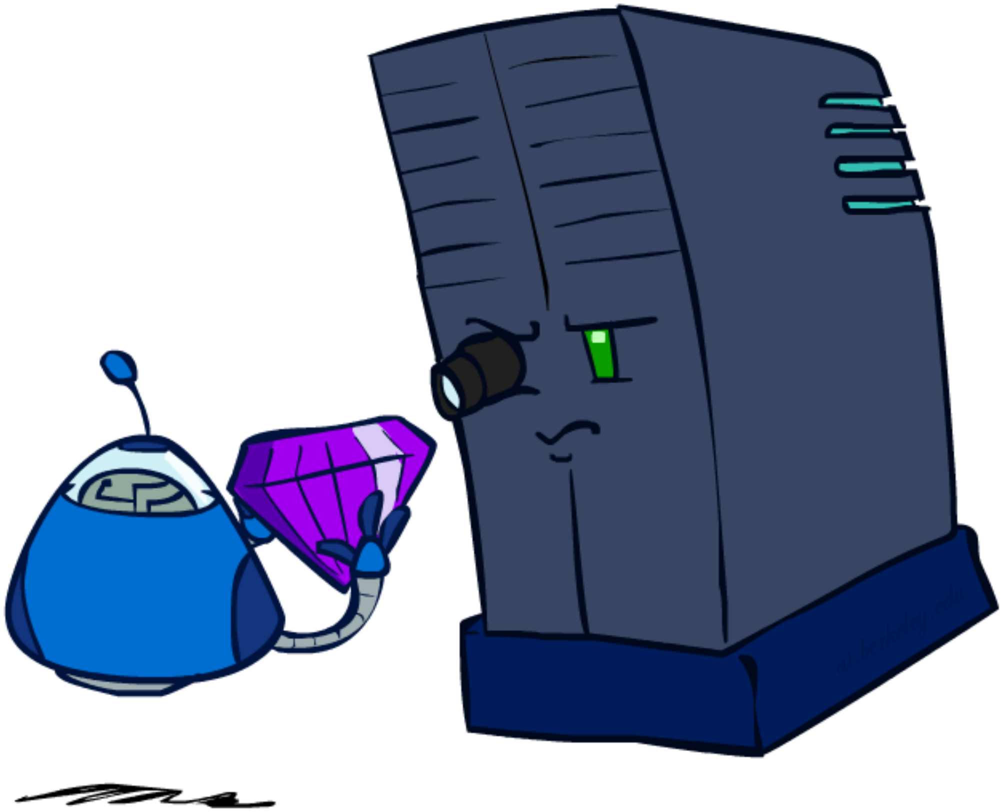


[Demo: depth limited (L6D4, L6D5)]

Demo Limited Depth (2)

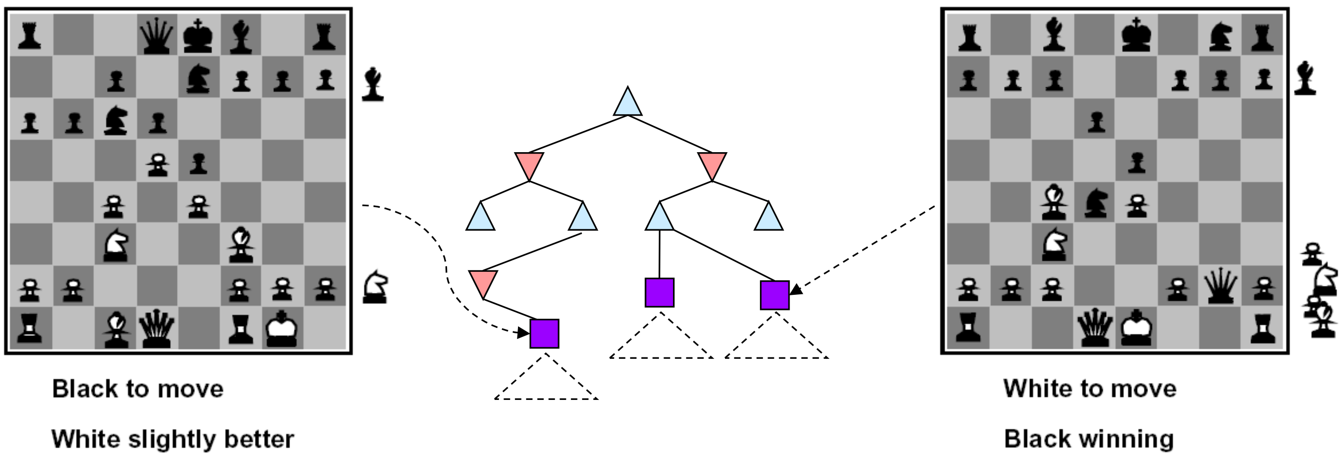
Demo Limited Depth (10)

Evaluation Functions



Evaluation Functions

Evaluation functions score non-terminals in depth-limited search

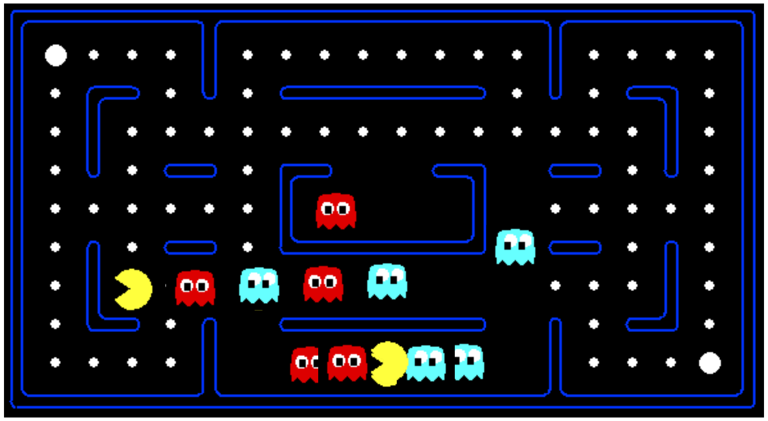


Ideal function: returns the actual minimax value of the position

In practice: typically weighted linear sum of features:

- EVAL(s) = $w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$
- E.g., $w_1 = 9$, $f_1(s)$ = (num white queens – num black queens), etc.

Evaluation for Pacman

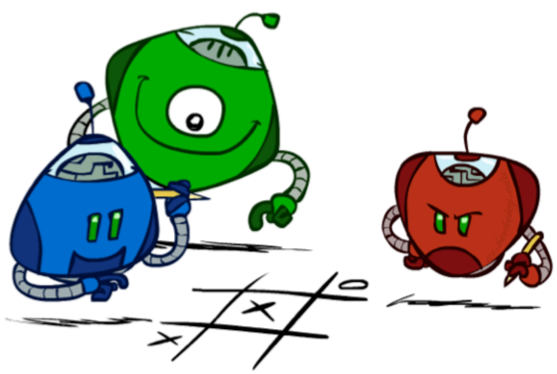


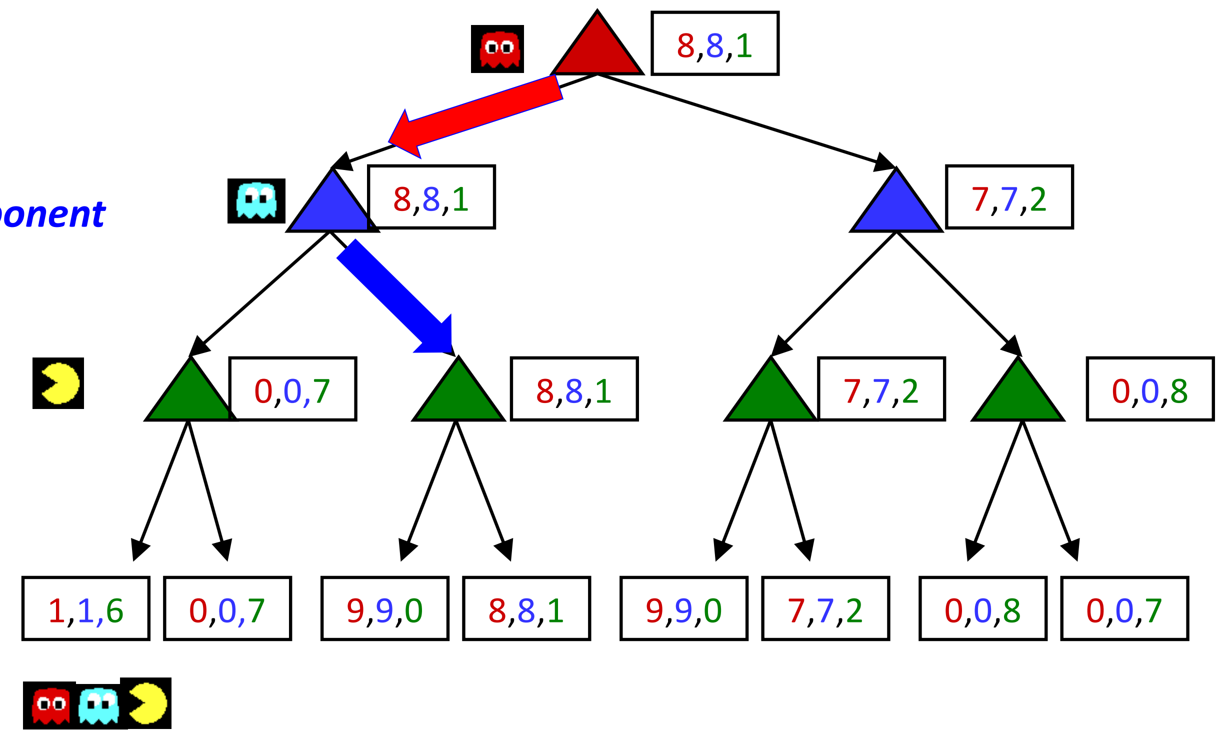
Generalized minimax

What if the game is not zero-sum, or has multiple players?

Generalization of minimax:

- Terminals have ***utility tuples***
- Node values are also utility tuples
- ***Each player maximizes its own component***
- Can give rise to cooperation and competition dynamically…





Bias in Evaluation Functions and Heuristics

Bias is the phenomenon of systematically analyzing results with faulty assumptions

Evaluation functions and heuristics are human-generated.

They are potentially subject to the same biases as people.

Bias in Evaluation Functions and Heuristics

Discussion – Road navigation with A*

What heuristics could we generate that may be biased?

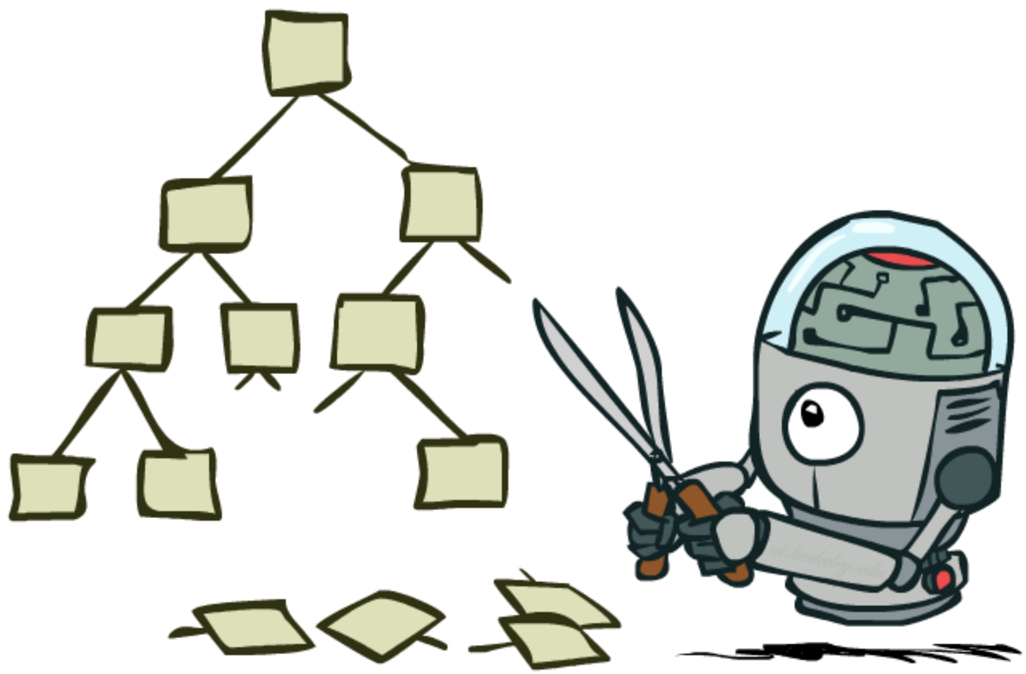
What should we consider to reduce those biases?

Discussion – International negotiations (e.g., climate deals)

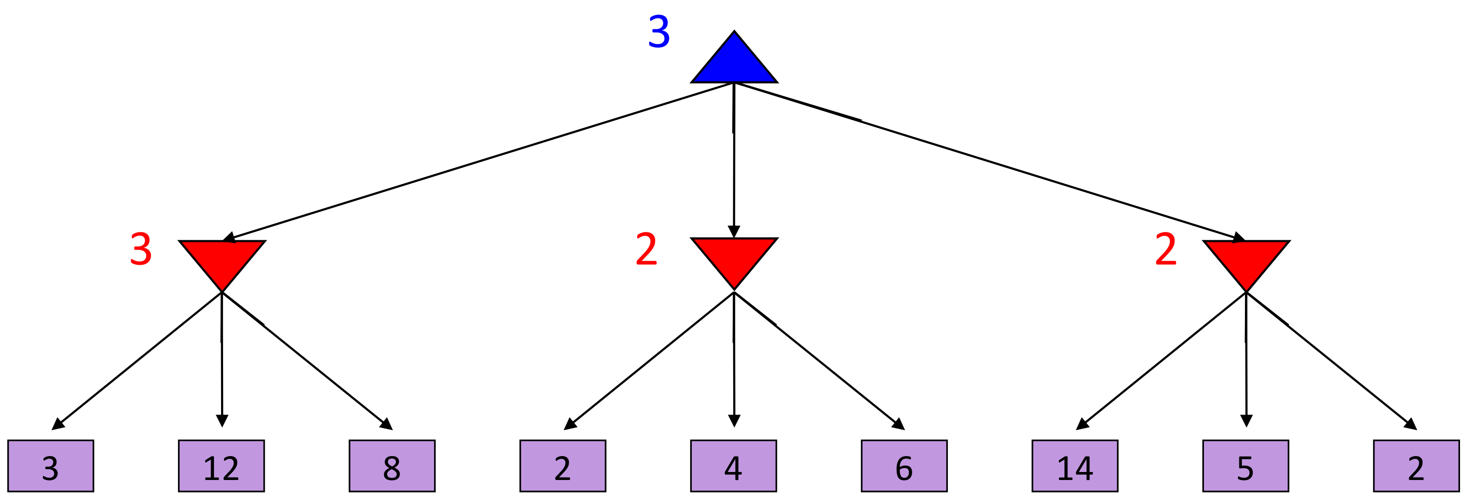
In bounded lookahead, how could one nation's evaluation function be biased and affect the outcome of the negotiation?

What are some strategies for potentially reducing those biases?

Game Tree Pruning

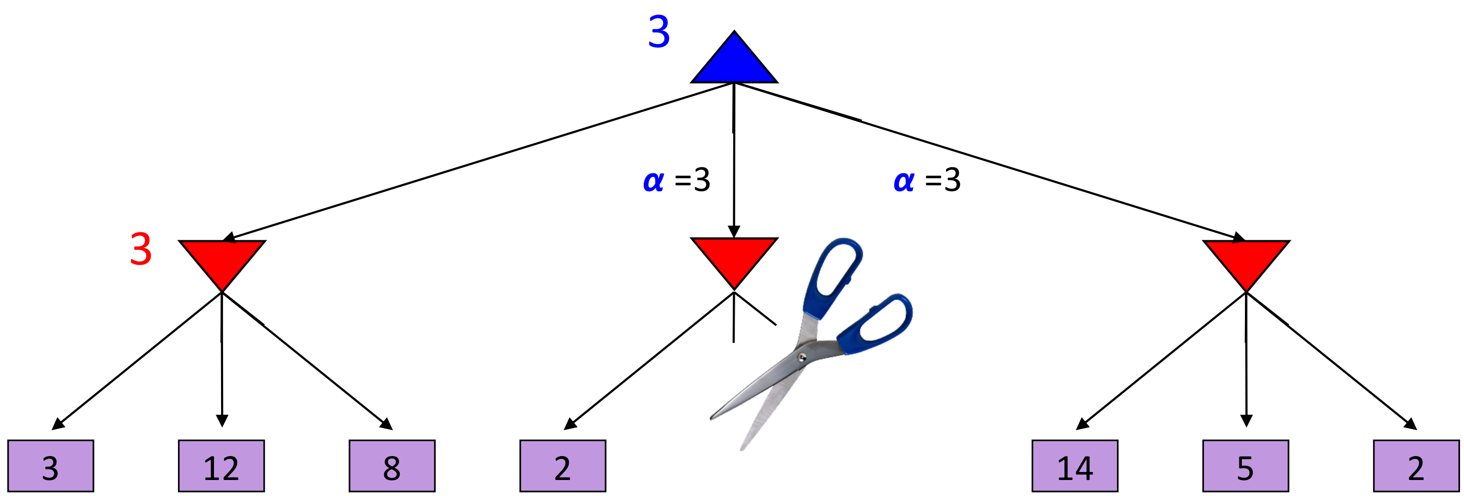


Minimax Example



Alpha-Beta Example

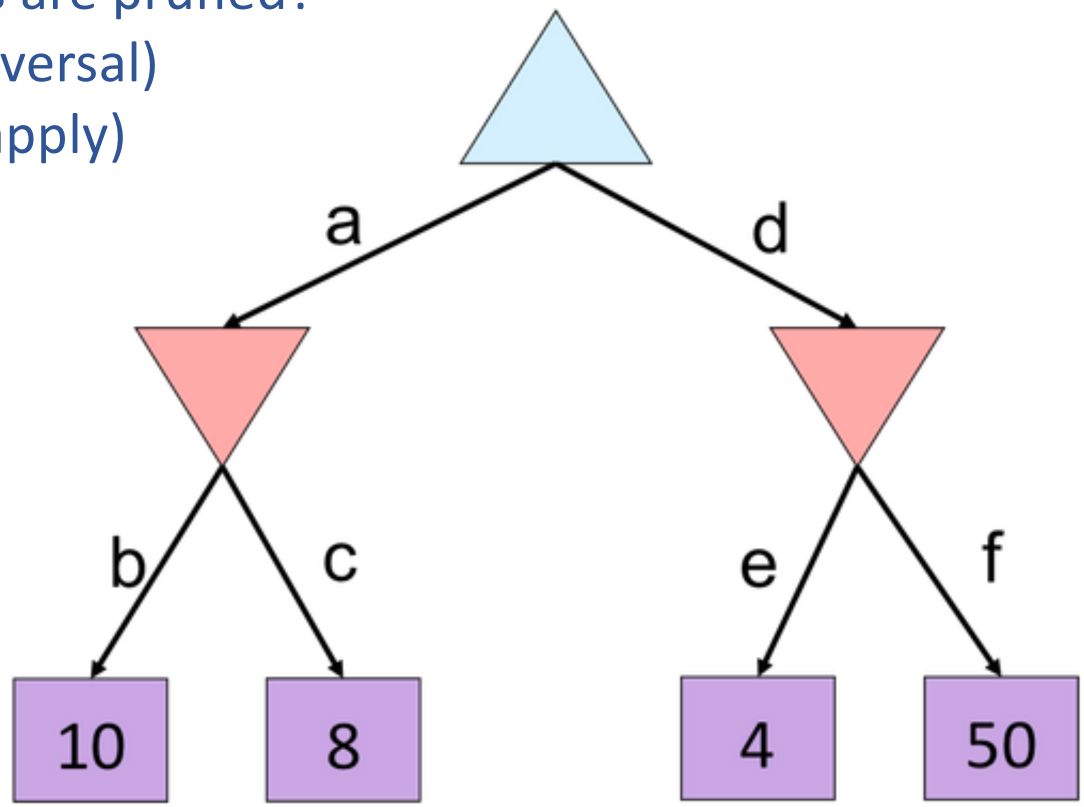
α = best option so far from any MAX node on this path



The order of generation matters: more pruning is possible if good moves come first

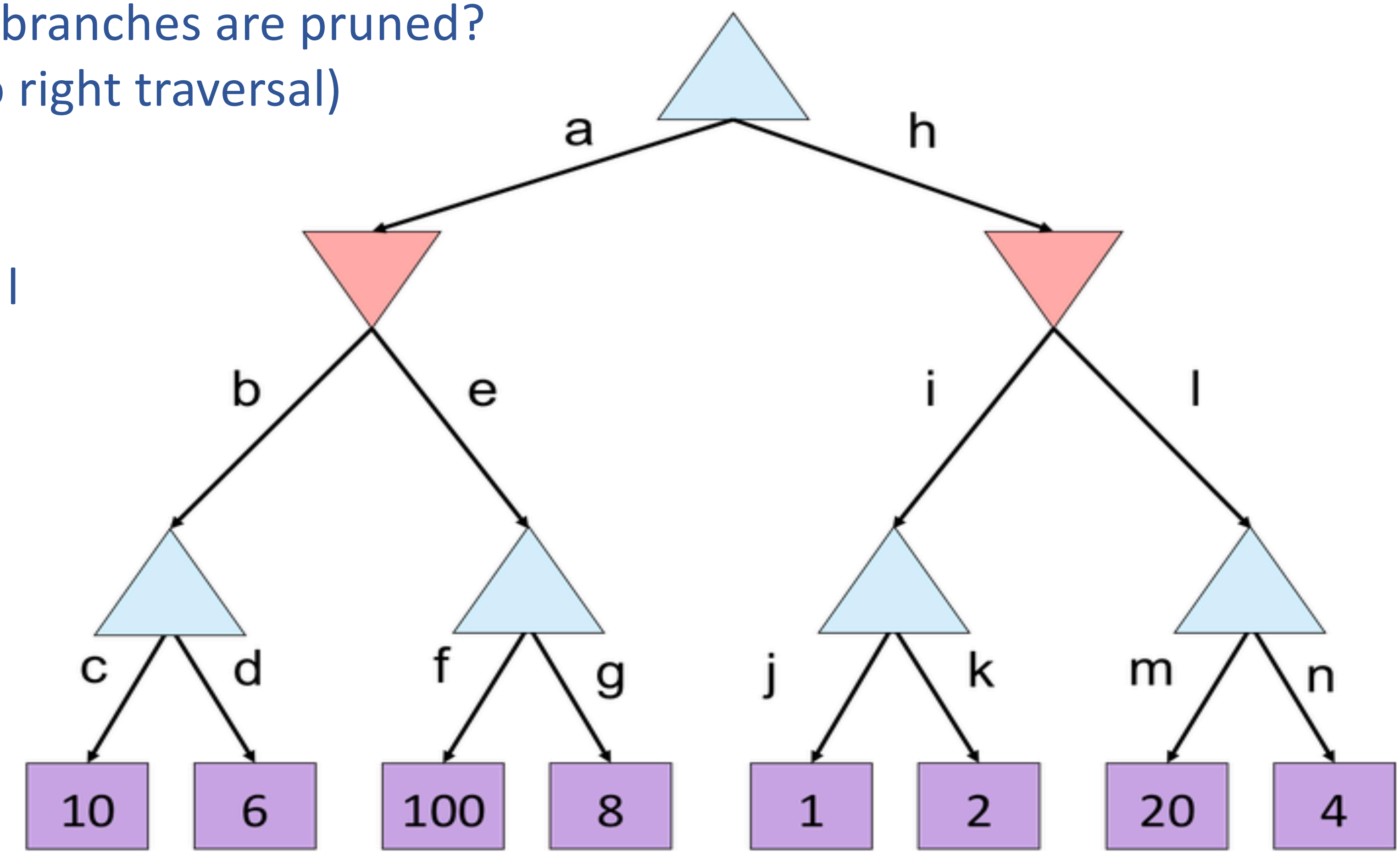
Piazza Poll 2

Which branches are pruned?
(Left to right traversal)
(Select all that apply)

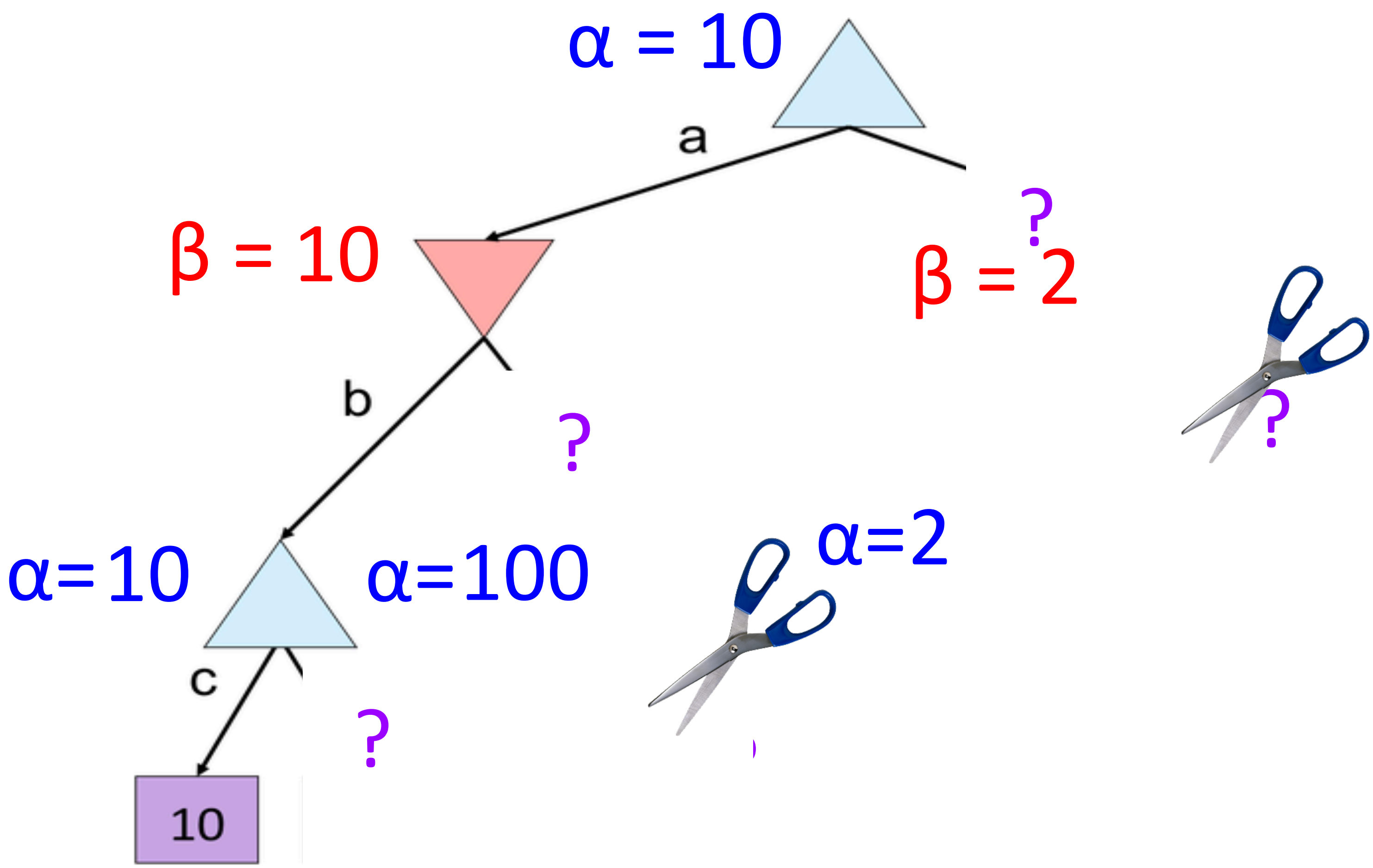


Piazza Poll 3

Which branches are pruned?
(Left to right traversal)
A) e, l
B) g, l
C) g, k, l
D) g, n



Alpha-Beta Quiz 2



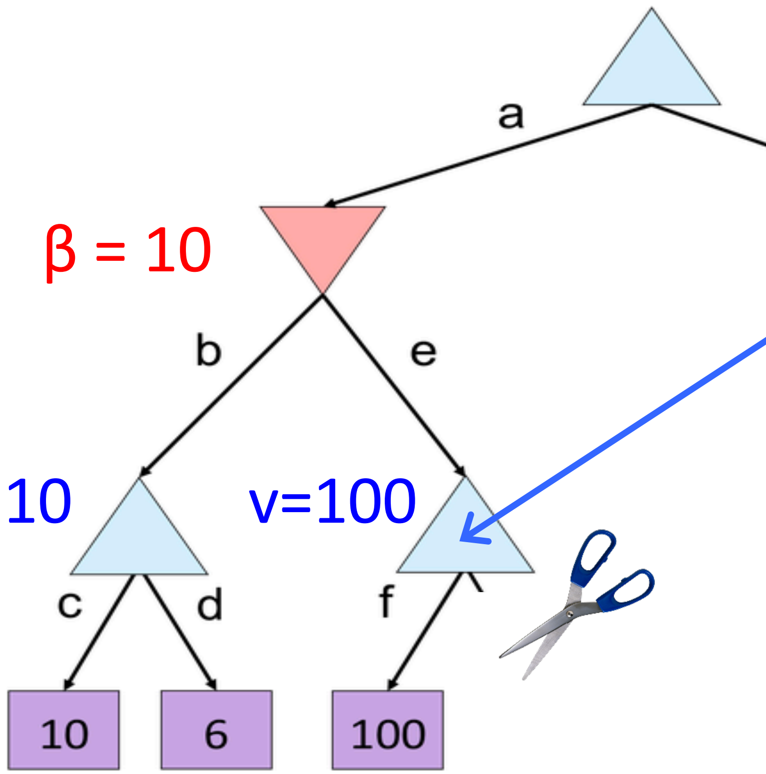
Alpha-Beta Implementation

α: MAX's best option on path to root
β: MIN's best option on path to root

```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β
            return v
        α = max(α, v)
    return v
```

```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α
            return v
        β = min(β, v)
    return v
```

Alpha-Beta Quiz 2



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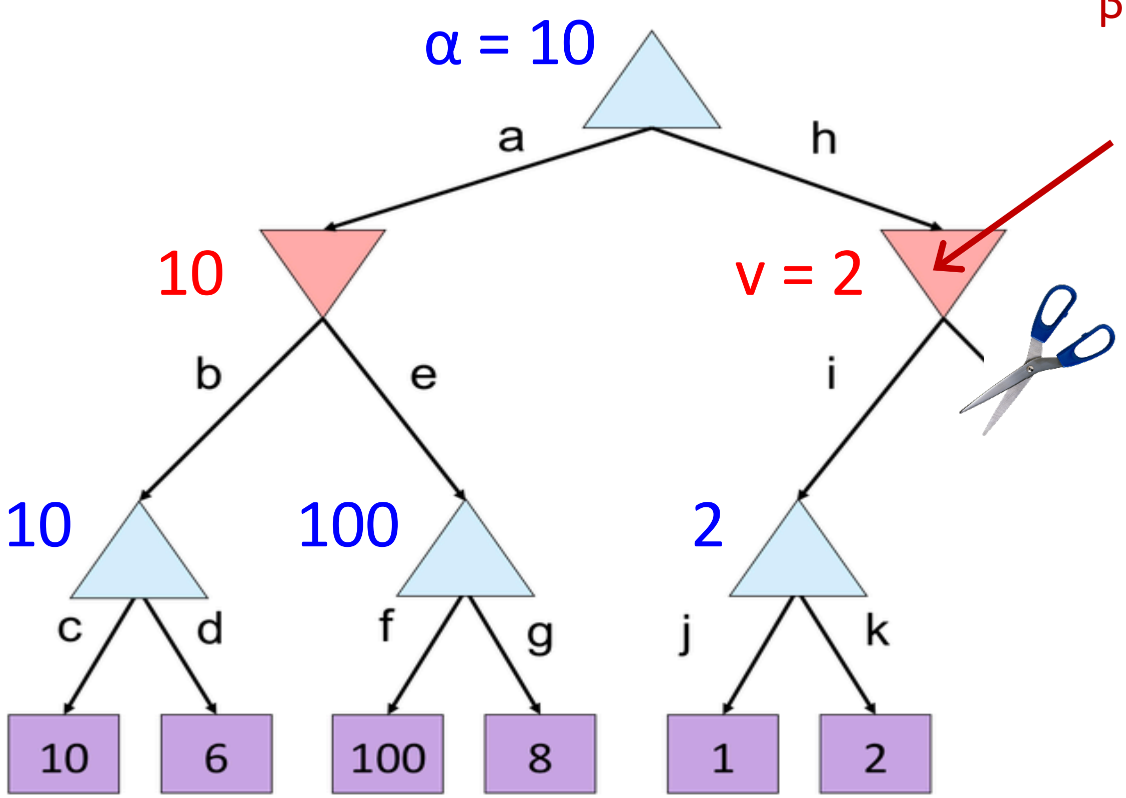
Alpha-Beta Quiz 2



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Alpha-Beta Pruning Properties

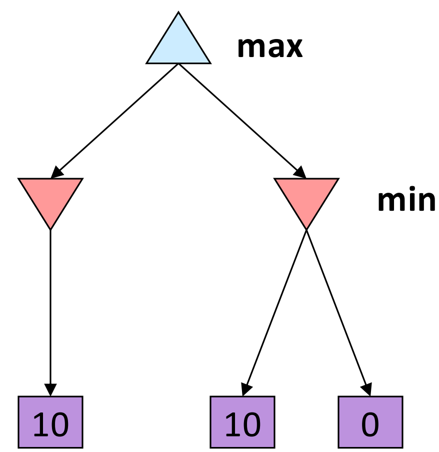
Theorem: This pruning has ***no effect*** on minimax value computed for the root!

Good child ordering improves effectiveness of pruning

- Iterative deepening helps with this

With "perfect ordering":

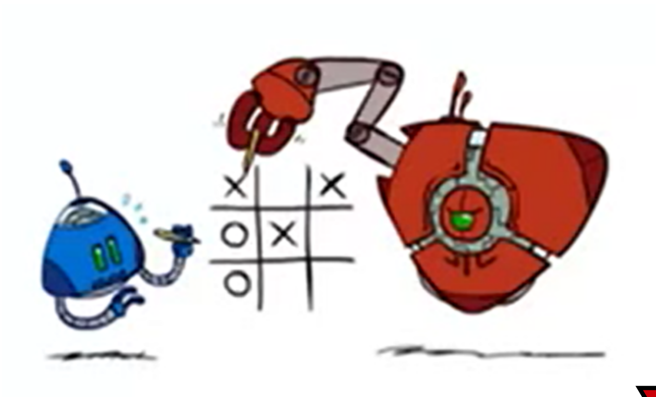
- Time complexity drops to $O(b^{m/2})$
- Doubles solvable depth!
- Chess: 1M nodes/move => depth=8, respectable

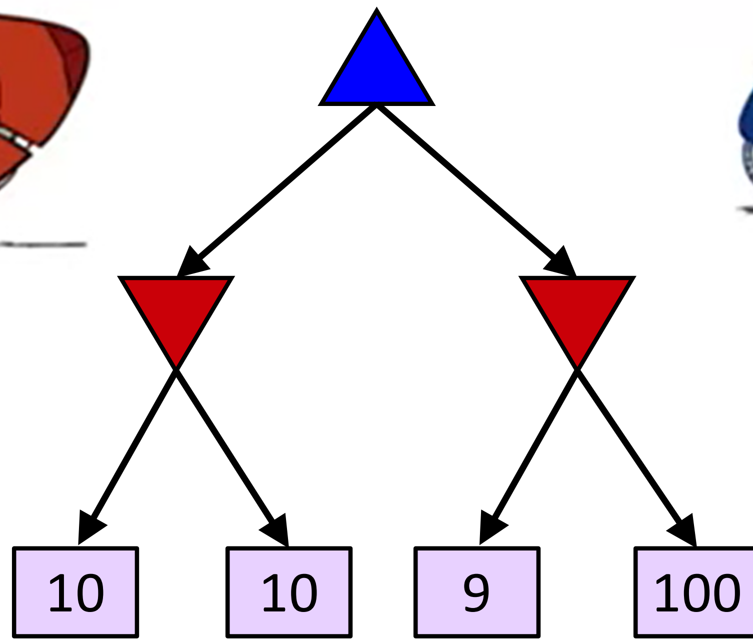


This is a simple example of metareasoning (computing about what to compute)

Modeling Assumptions

Know your opponent – what happens if the other isn't playing optimally?





Modeling Assumptions

Dangerous Pessimism

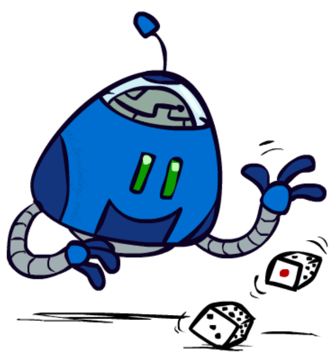
Assuming the worst case when it's not likely

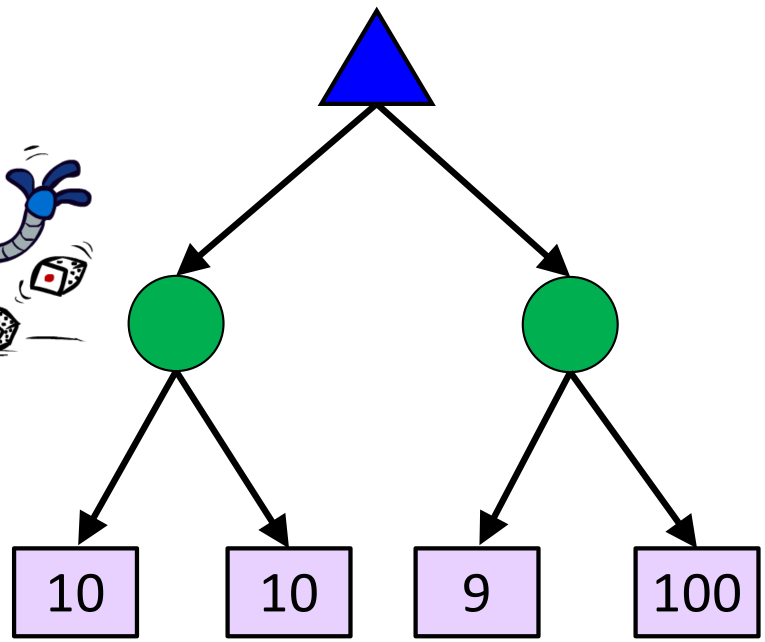
Dangerous Optimism

Assuming chance when the world is adversarial

Modeling Assumptions

Chance nodes: Expectimax

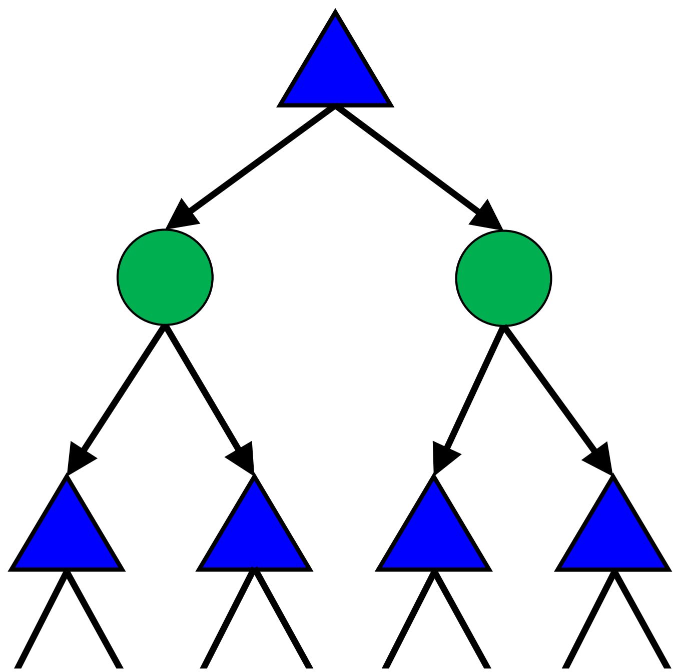




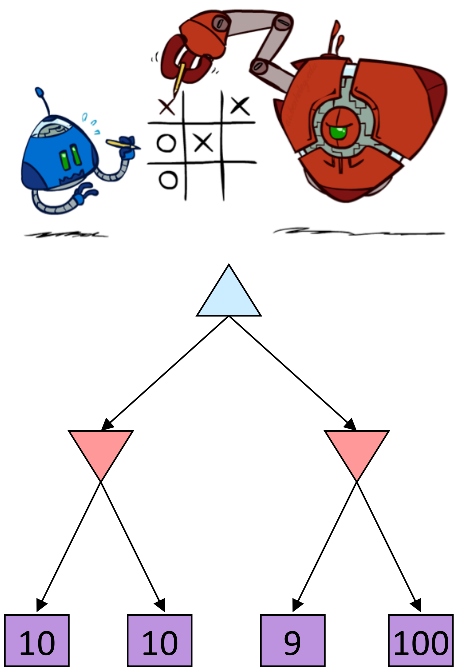
Why Expectimax?

Pretty great model for an agent in the world

Choose the action that has the: highest expected value

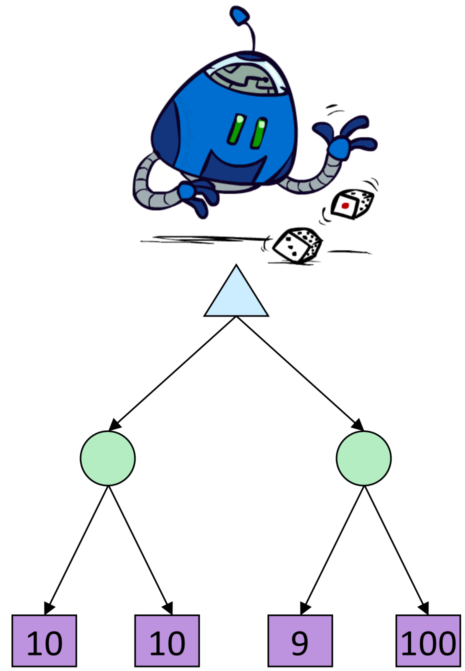


Chance outcomes in trees



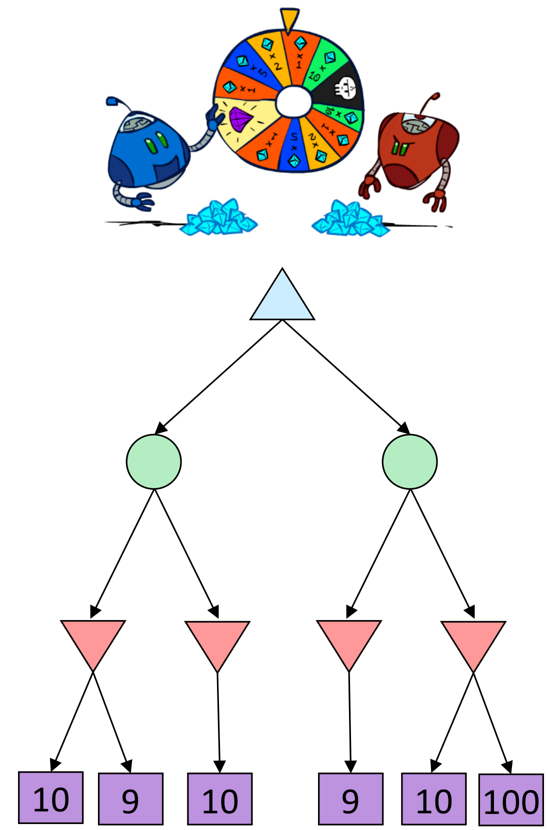
Tictactoe, chess

Minimax



Tetris, investing

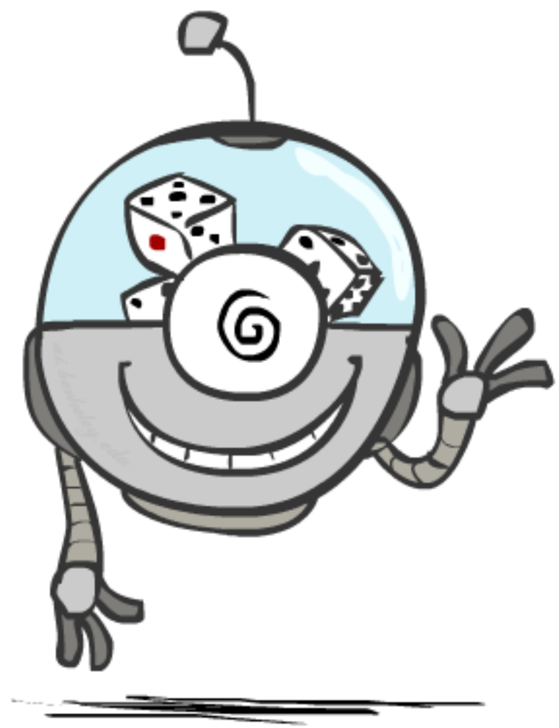
Expectimax



Backgammon, Monopoly

Expectiminimax

Probabilities



Probabilities

A random variable represents an event whose outcome is unknown

A probability distribution is an assignment of weights to outcomes

Example: Traffic on freeway

- Random variable: T = whether there's traffic
- Outcomes: T in {none, light, heavy}
- Distribution:

  P(T=none) = 0.25, P(T=light) = 0.50, P(T=heavy) = 0.25

Probabilities over all possible outcomes sum to one

0.25

0.50

0.25

Expected Value

Expected value of a function of a random variable:

Average the values of each outcome,
weighted by the probability of that outcome

Example: How long to get to the airport?

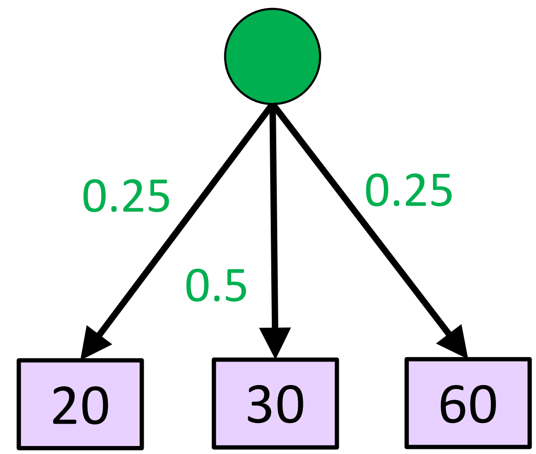
Time:	20 min		30 min		60 min		
	x	+	x	+	x	⇒	35 min
Probability:	0.25		0.50		0.25		



Expectations

Time:	20 min		30 min		60 min
	x	+	x	+	x
Probability:	0.25		0.50		0.25





Max node notation

$$V(s) = \max_{a} V(s'),$$

where $s' = result(s, a)$

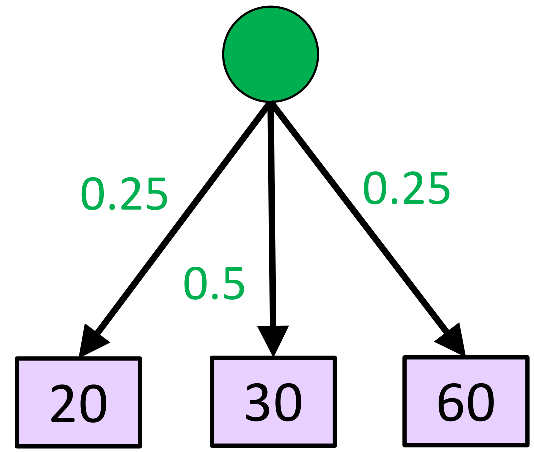
Chance node notation

$$V(s) =$$

Expectations

Time: 20 min x 0.25 + 30 min x 0.50 + 60 min x 0.25

Probability: 0.25, 0.50, 0.25



Max node notation

$$V(s) = \max_a V(s'),$$

where $s' = result(s, a)$

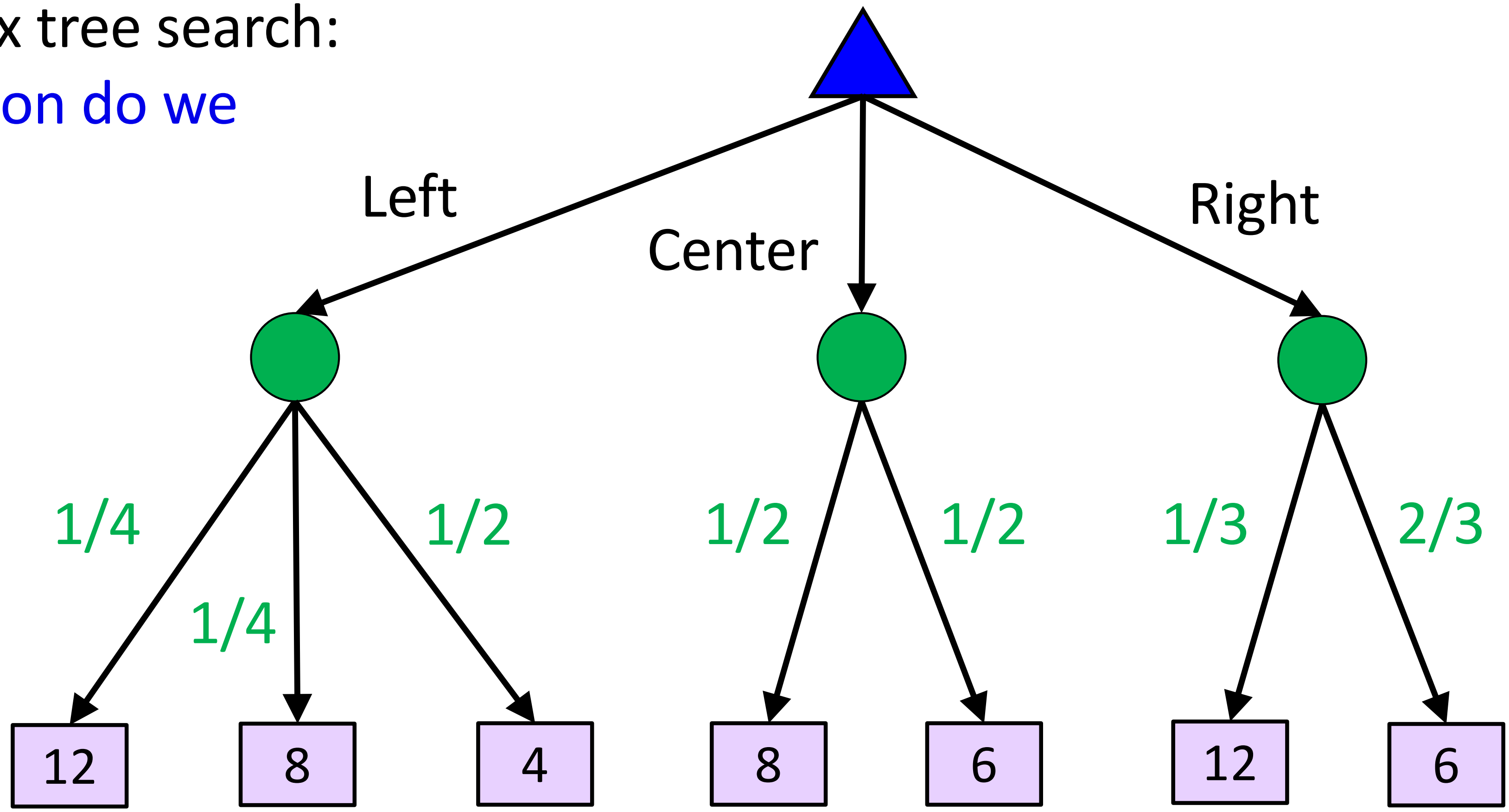
Chance node notation

$$V(s) = \sum_{s'} P(s')\, V(s')$$

Piazza Poll 4

Expectimax tree search:
Which action do we choose?

A: Left
B: Center
C: Right
D: Eight



Piazza Poll 4

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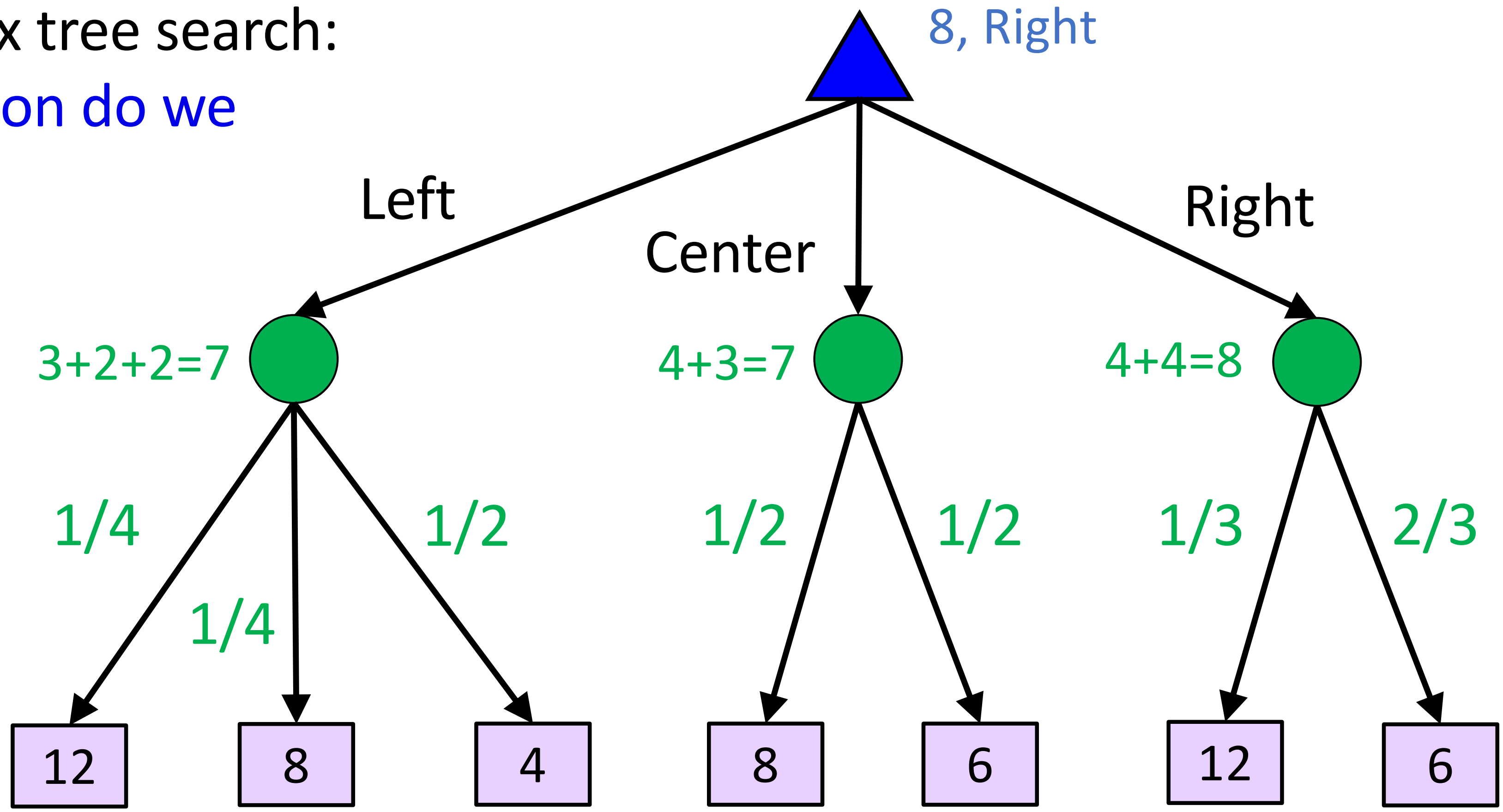
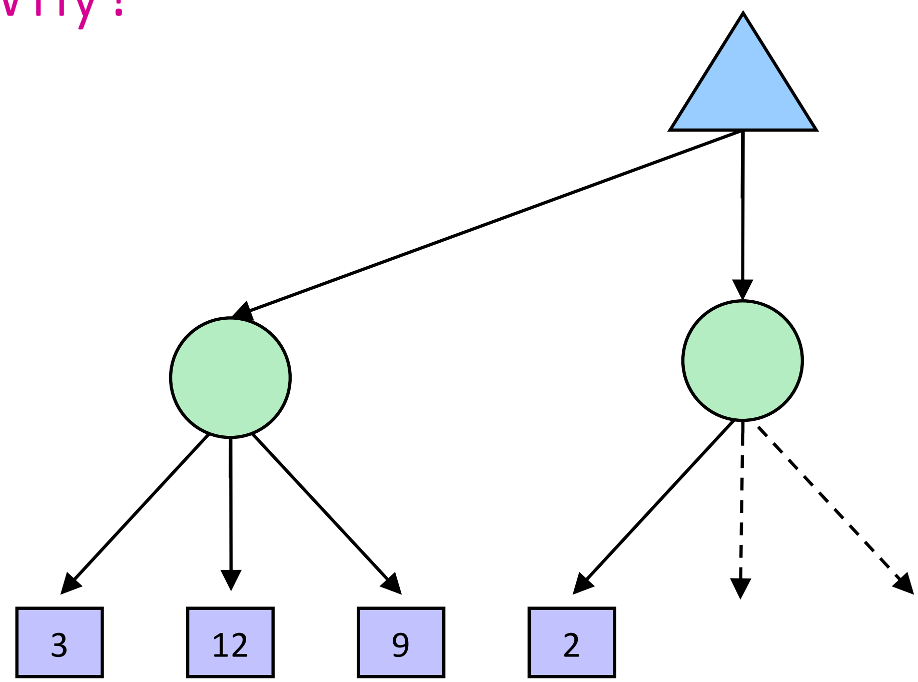
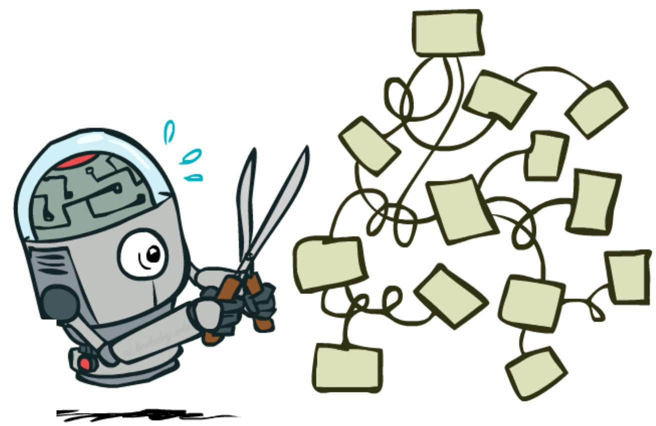


Expectimax Pruning?

No! Why?





Expectimax Code

```
function  value( state )
    if  state.is_leaf
        return  state.value

    if  state.player  is  MAX
        return  max a in state.actions  value( state.result(a) )

    if  state.player  is  MIN
        return  min a in state.actions  value( state.result(a) )

    if  state.player  is  CHANCE
        return  sum s in state.next_states  P( s ) * value( s )
```

Summary

Games require decisions when optimality is impossible

- Bounded-depth search and approximate evaluation functions

Games force efficient use of computation

- Alpha-beta pruning

Game playing has produced important research ideas

- Reinforcement learning (checkers)
- Iterative deepening (chess)
- Rational metareasoning (Othello)
- Monte Carlo tree search (Go)
- Solution methods for partial-information games in economics (poker)

Video games present much greater challenges – lots to do!

- $b = 10^{500}$, $|S| = 10^{4000}$, $m = 10{,}000$

Bonus Question

Let’s say you know that your opponent is actually running a depth 1 minimax, using the result 80% of the time, and moving randomly otherwise

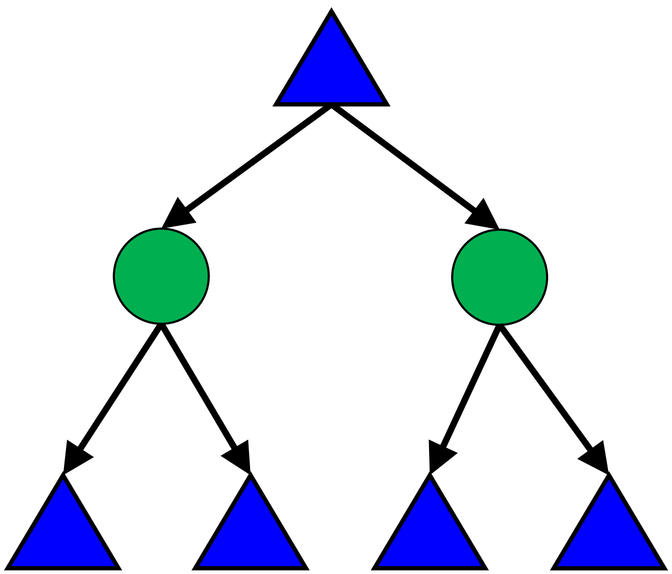
Question: What tree search should you use?

A: Minimax

B: Expectimax

C: Something completely different

Preview: MDP/Reinforcement Learning Notation



$$V(s) = \max_a \sum_{s'} P(s')\, V(s')$$

Preview: MDP/Reinforcement Learning Notation

Standard expectimax: $$V(s) = \max_a \sum_{s'} P(s'|s,a)V(s')$$

Bellman equations: $$V(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')]$$

Value iteration: $$V_{k+1}(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')], \quad \forall s$$

Q-iteration: $$Q_{k+1}(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')], \quad \forall s,a$$

Policy extraction: $$\pi_V(s) = \operatorname*{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')], \quad \forall s$$

Policy evaluation: $$V_{k+1}^{\pi}(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma V_k^{\pi}(s')], \quad \forall s$$

Policy improvement: $$\pi_{new}(s) = \operatorname*{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')], \quad \forall s$$

Preview: MDP/Reinforcement Learning Notation

Standard expectimax: $V(s) = \max_a \sum_{s'} P(s'|s,a)V(s')$

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