Announcements

Assignments:

- HW7 (online)
  - Due Tue 3/24, 10 pm
- HW8 (online)
  - Also due Tue 3/24, 10 pm
- P4
  - Due Thu 4/2, 10 pm
- Head's up: HW9 (written)
  - Released soon
  - Due Tue 3/31, 10 pm

Announcements

Coronavirus – COVID-19

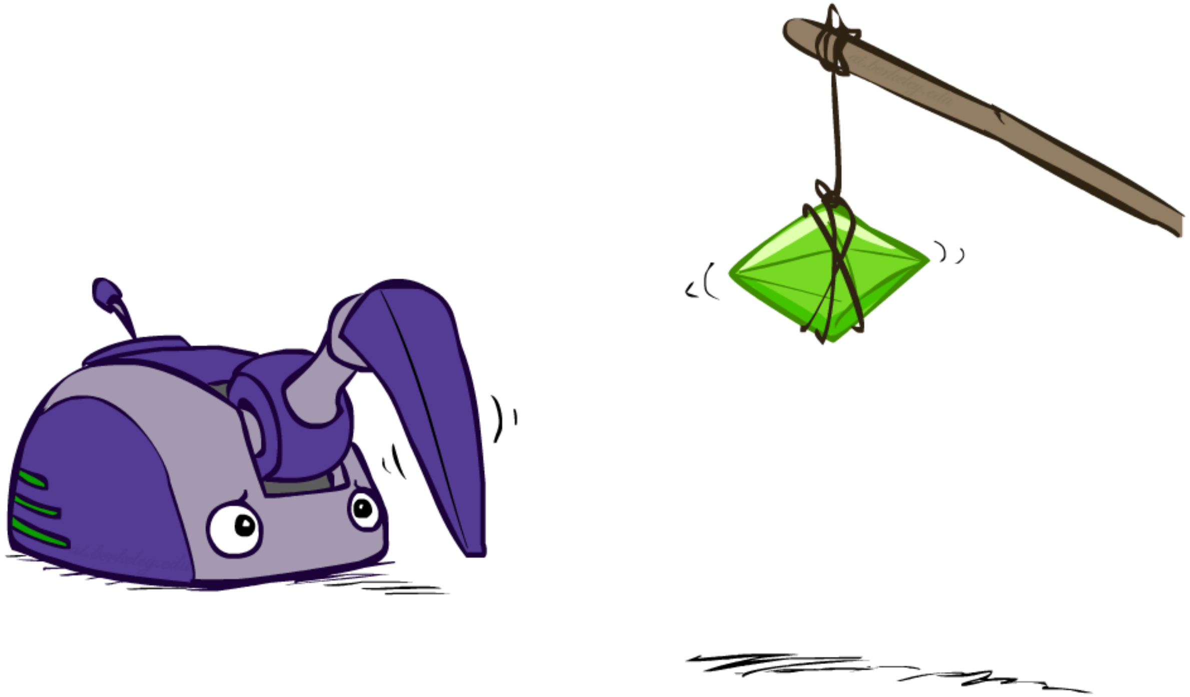
- Take care of yourself and others around you
- Follow CMU and government guidelines
- We’re “here” to help in any capacity that we can
- Use tools like zoom to communicate with each other too!

Zoom

- Let us know if you have issues
- Etiquette: Turn on video when:
  - Talking
  - Your turn in OH
  - In recitation

AI: Representation and Problem Solving

Reinforcement Learning



Instructors: Pat Virtue & Stephanie Rosenthal

Slide credits: CMU AI and http://ai.berkeley.edu

MDP Notation

Standard expectimax: $V(s) = \max_a \sum_{s'} P(s'|s,a)V(s')$

Bellman equations: $V^*(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^*(s')]$

Value iteration: $V_{k+1}(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')], \quad \forall\, s$

Q-iteration: $Q_{k+1}(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')], \quad \forall\, s,a$

Policy extraction: $\pi_V(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')], \quad \forall\, s$

Policy evaluation: $V_{k+1}^{\pi}(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma V_k^{\pi}(s')], \quad \forall\, s$

Policy improvement: $\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')], \quad \forall\, s$

Piazza Poll 1

$R(s, a, s')$ $R(s)$

Rewards may depend on any combination of *state*, *action*, *next state*.

Which of the following are valid formulations of the Bellman equations?

~~A.~~ $\cancel{V^*(s) = \max_{s'} \prod_a P(s'|s,a)[R(s,a,s') + \gamma V^*(s')]}$

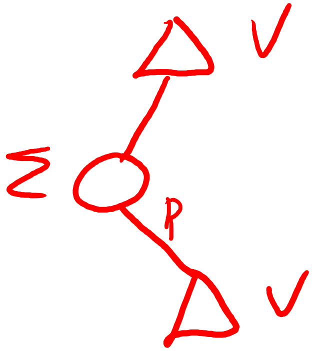
✓ B. $V^*(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,\cancel{a,s'}) + \gamma V^*(s')]$

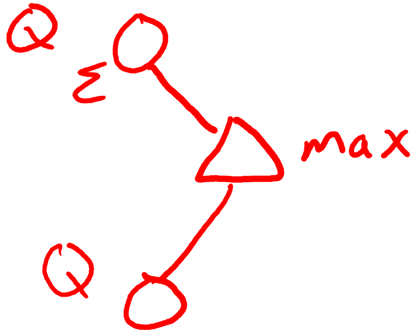
✓ C. $V^*(s) = R(s) + \gamma \max_a \sum_{s'} P(s'|s,a) V^*(s')$

✓ D. $V^*(s) = \max_a [R(s,a) + \gamma \sum_{s'} P(s'|s,a) [V^*(s')]]$

✓ E. $Q^*(s,a) = R(s,a) + \gamma \sum_{s'} P(s'|s,a) \max_{a'} Q^*(s',a')$

$Q^*(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,\cancel{s'}) + \gamma \max_{a'} Q^*(s',a')]$





Piazza Poll 2

do until π stops changing

1

2

Which of the following are used in policy iteration?

1 A. Value iteration: $V_{k+1}(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')]\,, \quad \forall\, s$

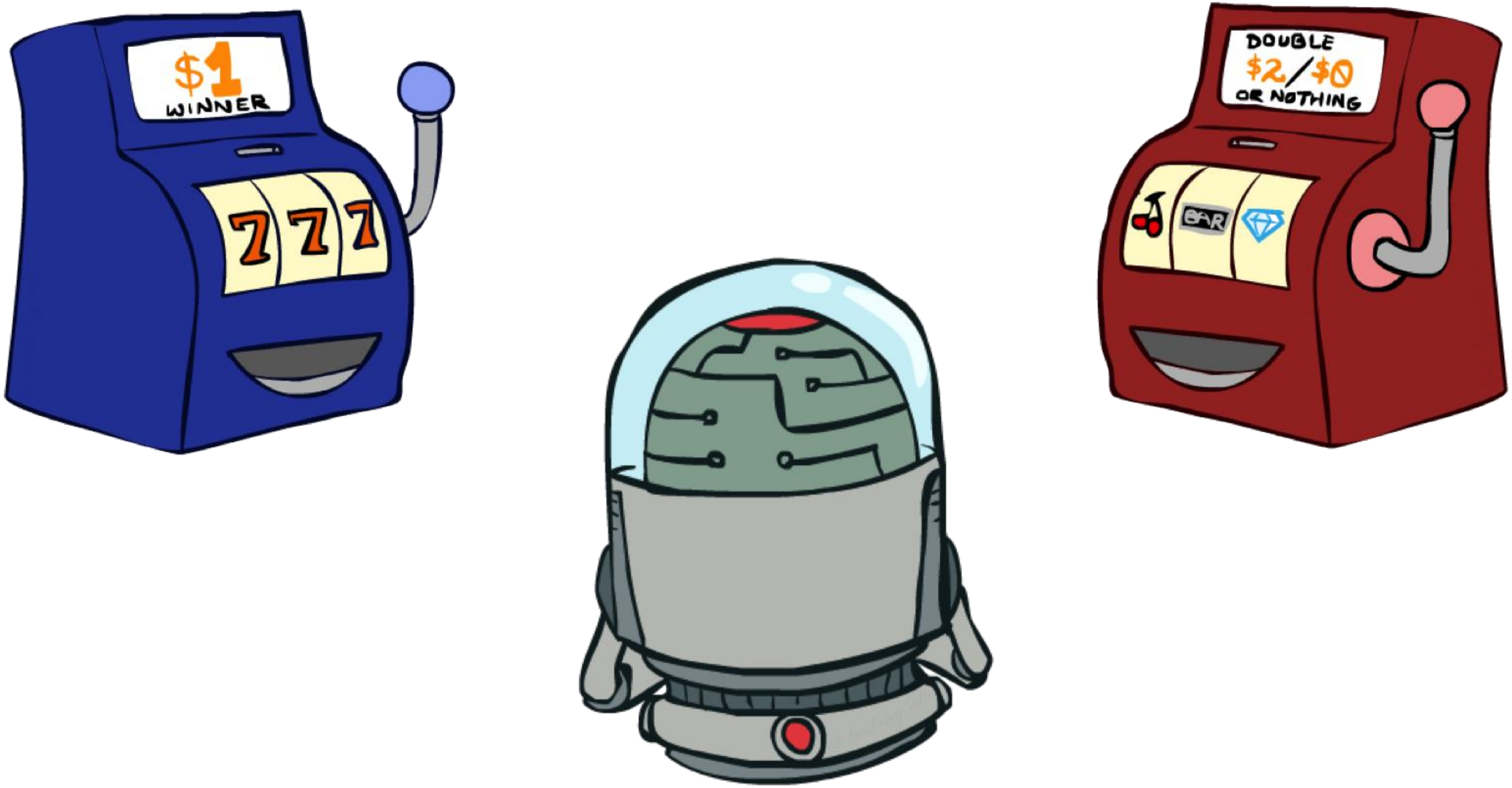
~~B. Q-iteration:~~ $\cancel{Q_{k+1}(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')]\,, \quad \forall\, s,a}$

2 C. Policy extraction: $\pi_V(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')]\,, \quad \forall\, s$

✓1 D. Policy evaluation: $V_{k+1}^{\pi}(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma V_k^{\pi}(s')]\,, \quad \forall\, s$

✓2 E. Policy improvement: $\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')]\,, \quad \forall\, s$

Double Bandits



Double-Bandit MDP

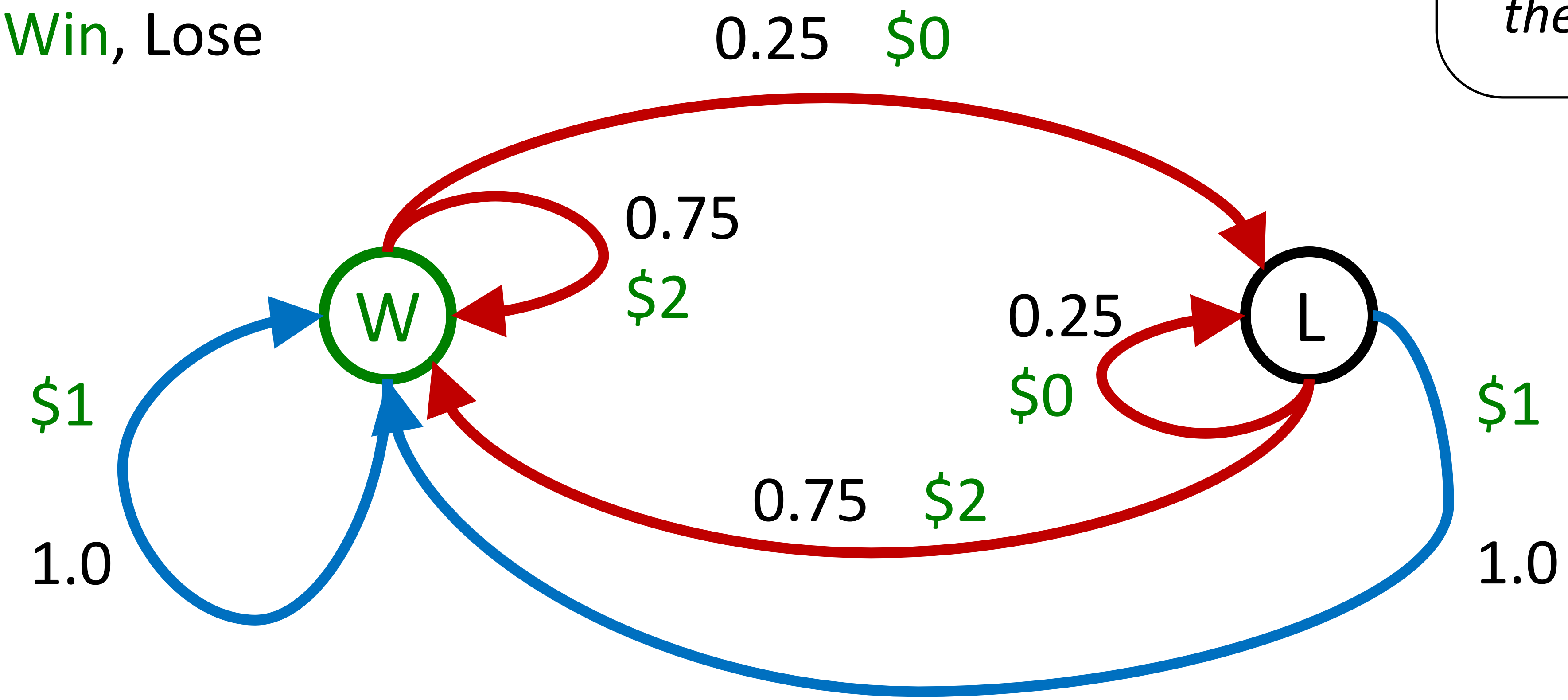
Actions: *Blue*, *Red*

States: Win, Lose

No discount

100 time steps

Both states have the same value



Offline Planning

$100[.25 \cdot 0 + .75 \cdot 2] = 150$

No discount

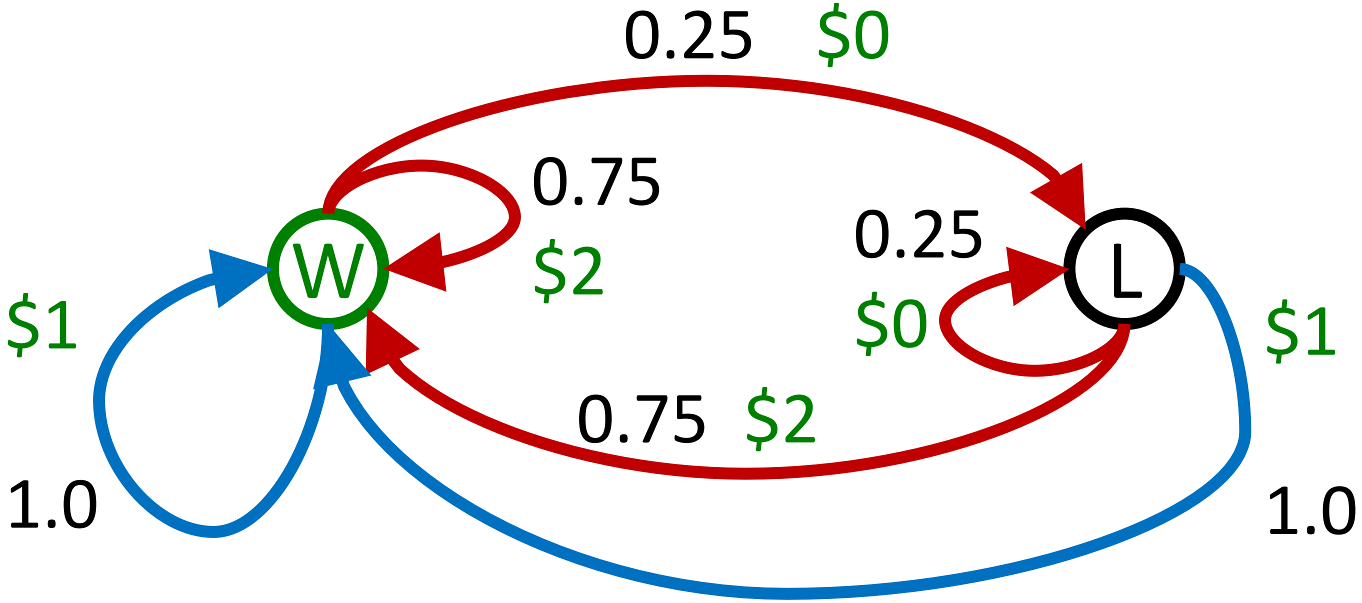
100 time steps

Both states have the same value

Solving MDPs is offline planning

- You determine all quantities through computation
- You need to know the details of the MDP
- You do not actually play the game!

	Value
Play Red	150
Play Blue	100



Let’s Play!

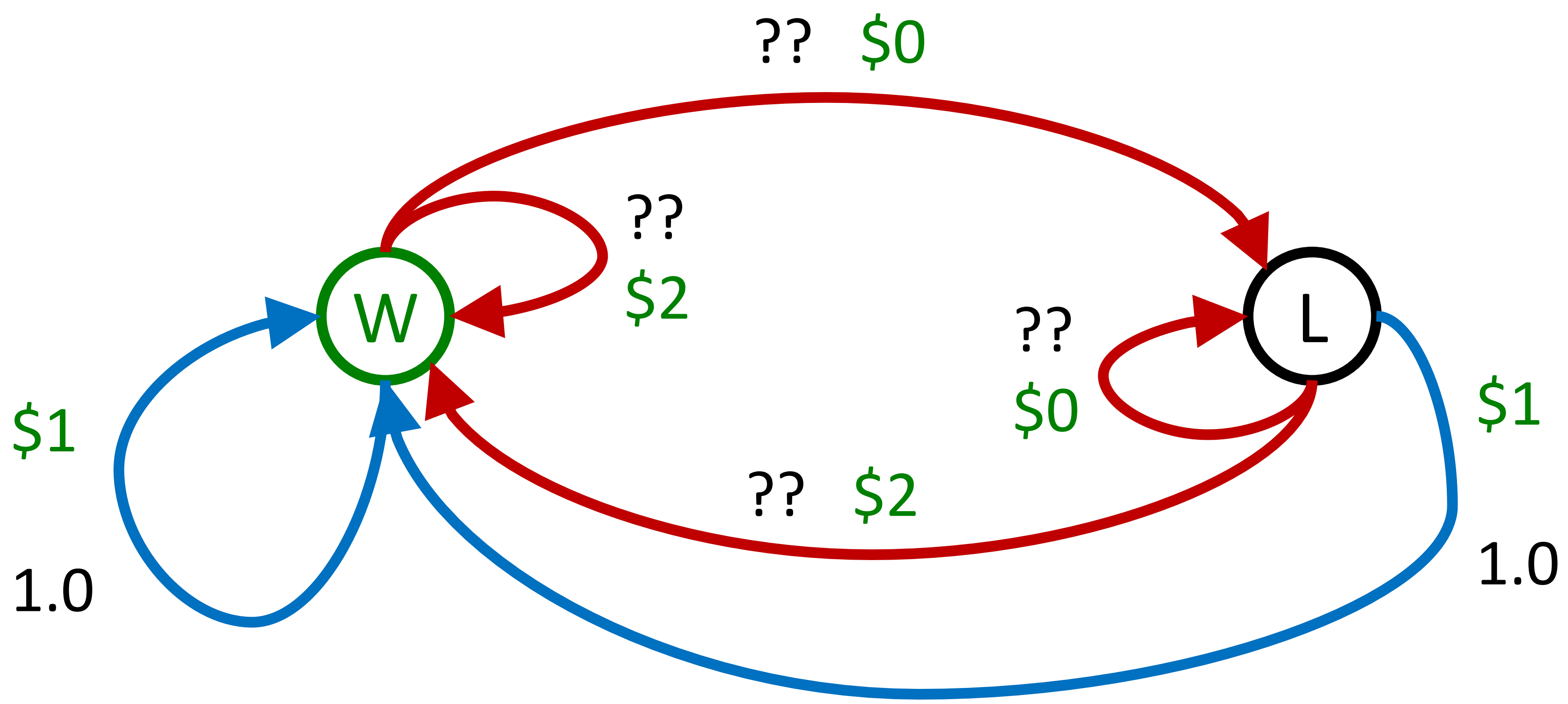
$2 $2 $0 $2 $2

$2 $2 $0 $0 $0

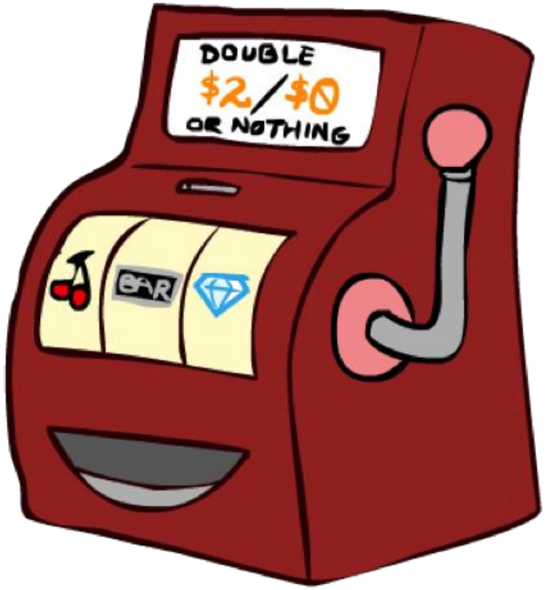
6/10

Online Planning

Rules changed! Red's win chance is different.



Let's Play!



$0 $0 $0 $2 $0

$2 $0 $0 $0 $0

2/10

What Just Happened?

That wasn't planning, it was learning!

- Specifically, reinforcement learning
- There was an MDP, but you couldn't solve it with just computation
- You needed to actually act to figure it out

Important ideas in reinforcement learning that came up

- Exploration: you have to try unknown actions to get information
- Exploitation: eventually, you have to use what you know
- Regret: even if you learn intelligently, you make mistakes
- Sampling: because of chance, you have to try things repeatedly
- Difficulty: learning can be much harder than solving a known MDP

Reinforcement learning

What if we didn't know $P(s'|s,a)$ and $R(s,a,s')$?

Value iteration: $$V_{k+1}(s) = \max_a \sum_{s'} \cancel{P(s'|s,a)}[\cancel{R(s,a,s')} + \gamma V_k(s')], \quad \forall\, s$$

Q-iteration: $$Q_{k+1}(s,a) = \sum_{s'} \cancel{P(s'|s,a)}[\cancel{R(s,a,s')} + \gamma \max_{a'} Q_k(s',a')], \quad \forall\, s,a$$

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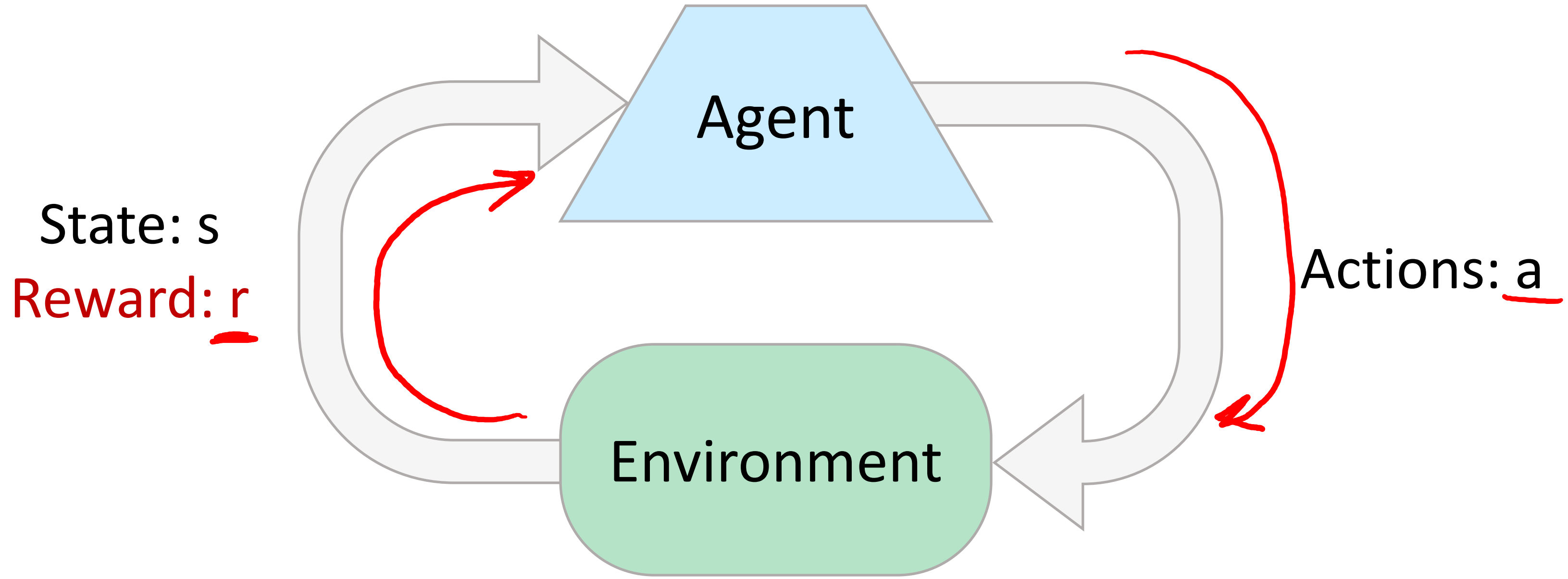
Policy evaluation: $$V_{k+1}^{\pi}(s) = \sum_{s'} \cancel{P(s'|s,\pi(s))}[\cancel{R(s,\pi(s),s')} + \gamma V_k^{\pi}(s')], \quad \forall\, s$$

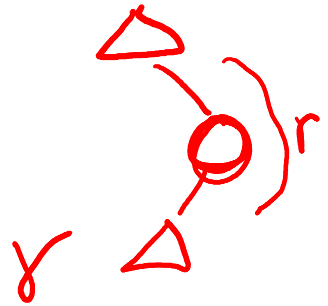
Policy improvement: $$\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} \cancel{P(s'|s,a)}[\cancel{R(s,a,s')} + \gamma V^{\pi_{old}}(s')], \quad \forall\, s$$

Reinforcement Learning



Basic idea:

- Receive feedback in the form of rewards
- Agent's utility is defined by the reward function
- Must (learn to) act so as to maximize expected rewards
- All learning is based on observed samples of outcomes!



Example: Learning to Walk

Initial

A Learning Trial

After Learning [1K Trials]

[Kohl and Stone, ICRA 2004]

Example: Learning to Walk

Initial

[Kohl and Stone, ICRA 2004]

[Video: AIBO WALK – initial]

Example: Learning to Walk

Training

[Kohl and Stone, ICRA 2004]

[Video: AIBO WALK – training]

Example: Learning to Walk

Finished

[Kohl and Stone, ICRA 2004]

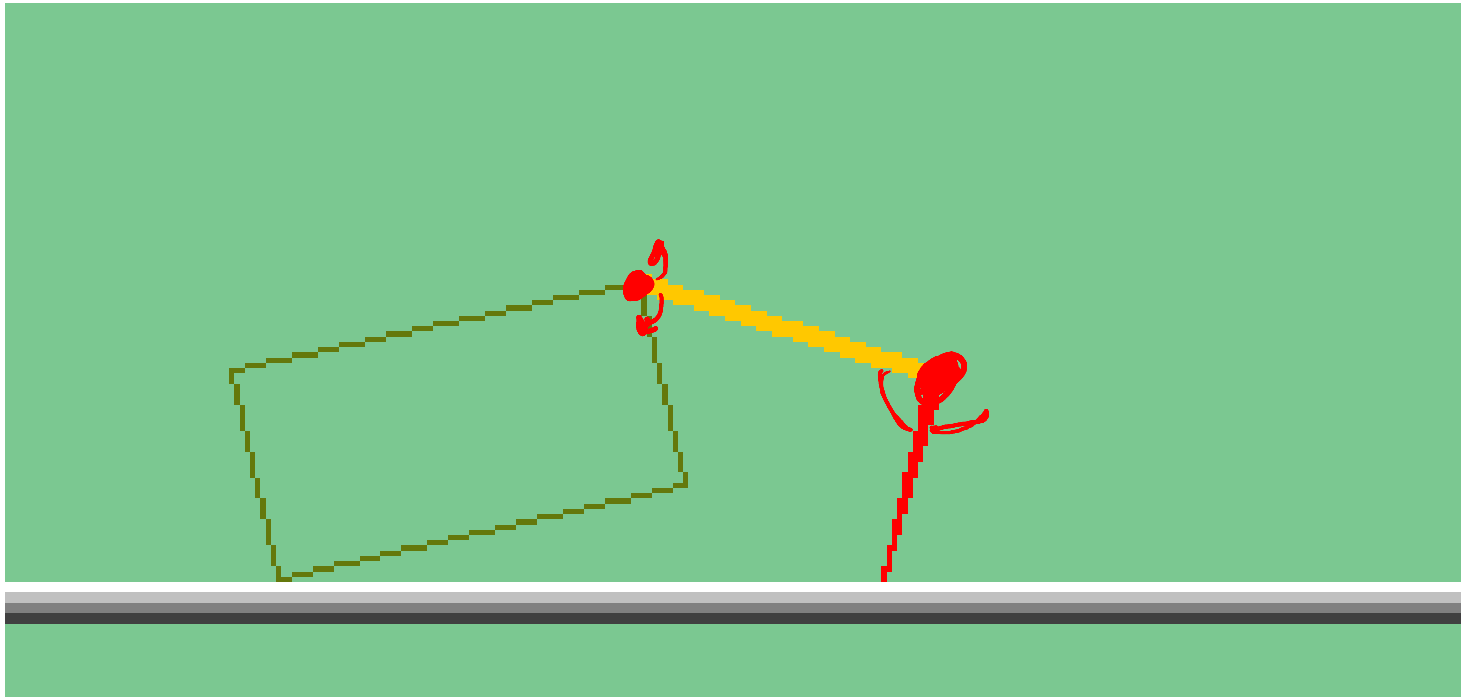
[Video: AIBO WALK – finished]

Example: Toddler Robot

[Tedrake, Zhang and Seung, 2005]

[Video: TODDLER – 40s]

The Crawler!



[Demo: Crawler Bot (L10D1)] [You, in Project ~~2~~ 4]

Demo Crawler Bot

Reinforcement Learning

Still assume a Markov decision process (MDP):

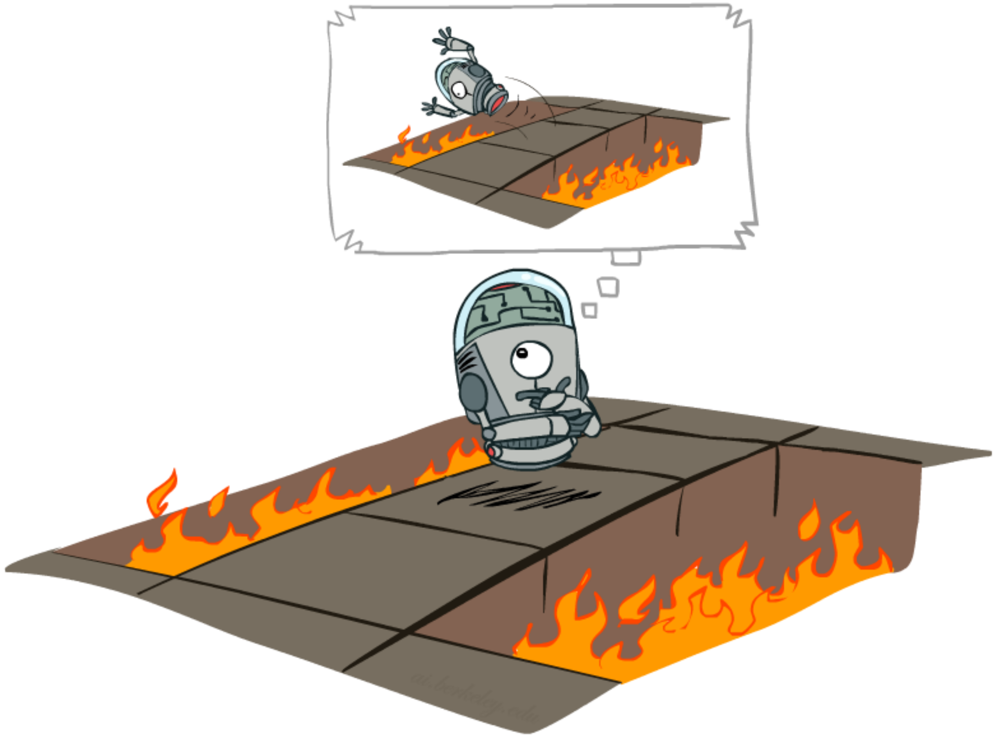
- A set of states $s \in S$
- A set of actions (per state) A
- A model $T(s,a,s')$ $\leftrightarrow P(s'|s,a)$
- A reward function $R(s,a,s')$

Still looking for a policy $\pi(s)$

New twist: don't know T or R

- I.e. we don't know which states are good or what the actions do
- Must actually try actions and states out to learn

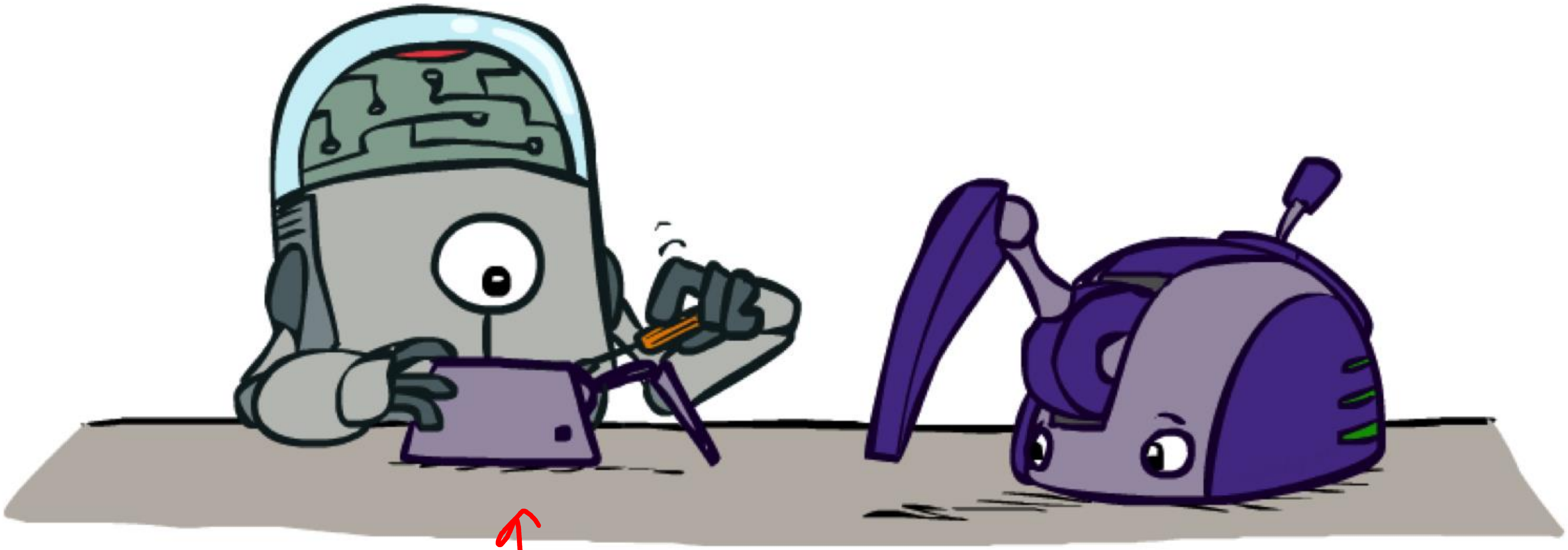
Offline (MDPs) vs. Online (RL)



Offline Solution

Online Learning

Model-Based Learning



T R

Model-Based Learning

Model-Based Idea:

- Learn an approximate model based on experiences
- Solve for values as if the learned model were correct

Step 1: Learn empirical MDP model

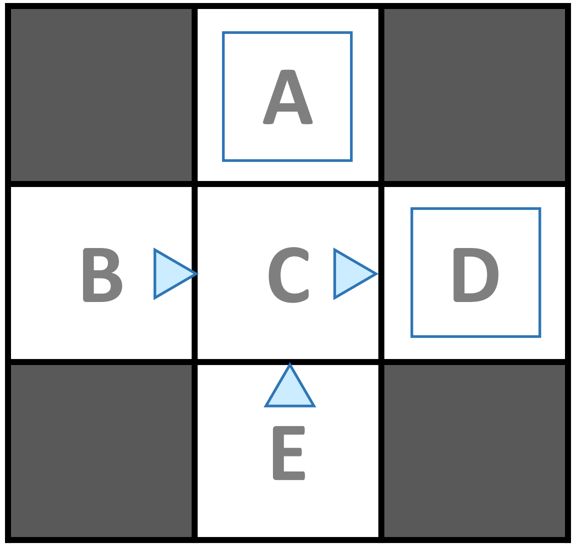
- Count outcomes s’ for each s, a
- Normalize to give an estimate of $\hat{T}(s, a, s')$
- Discover each $\hat{R}(s, a, s')$ when we experience (s, a, s’)

Step 2: Solve the learned MDP

- For example, use value iteration, as before

Example: Model-Based Learning

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

$$\hat{T}(s, a, s')$$

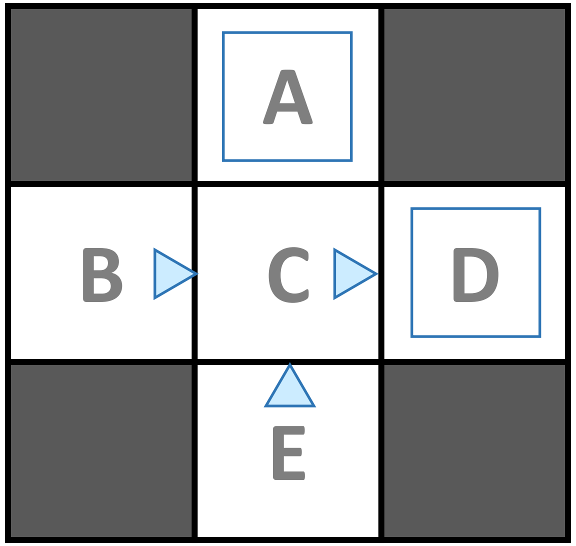
T(B, east, C) = 1.00
T(C, east, D) = 0.75
T(C, east, A) = 0.25
…

$$\hat{R}(s, a, s')$$

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
…

Example: Model-Based Learning

Input Policy π



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T(B, east, C) = 1.00
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…

$\widehat{R}(s, a, s')$

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
…

Example: Expected Age

Goal: Compute expected age of 15-281 students

Known P(A)

$$E[A] = \sum_a P(a) \cdot a \qquad = 0.35 \times 20 + \ldots$$

Without P(A), instead collect samples [a_1, a_2, … a_N]

Unknown P(A): "Model Based"

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

Why does this work? Because eventually you learn the right model.

Unknown P(A): "Model Free"

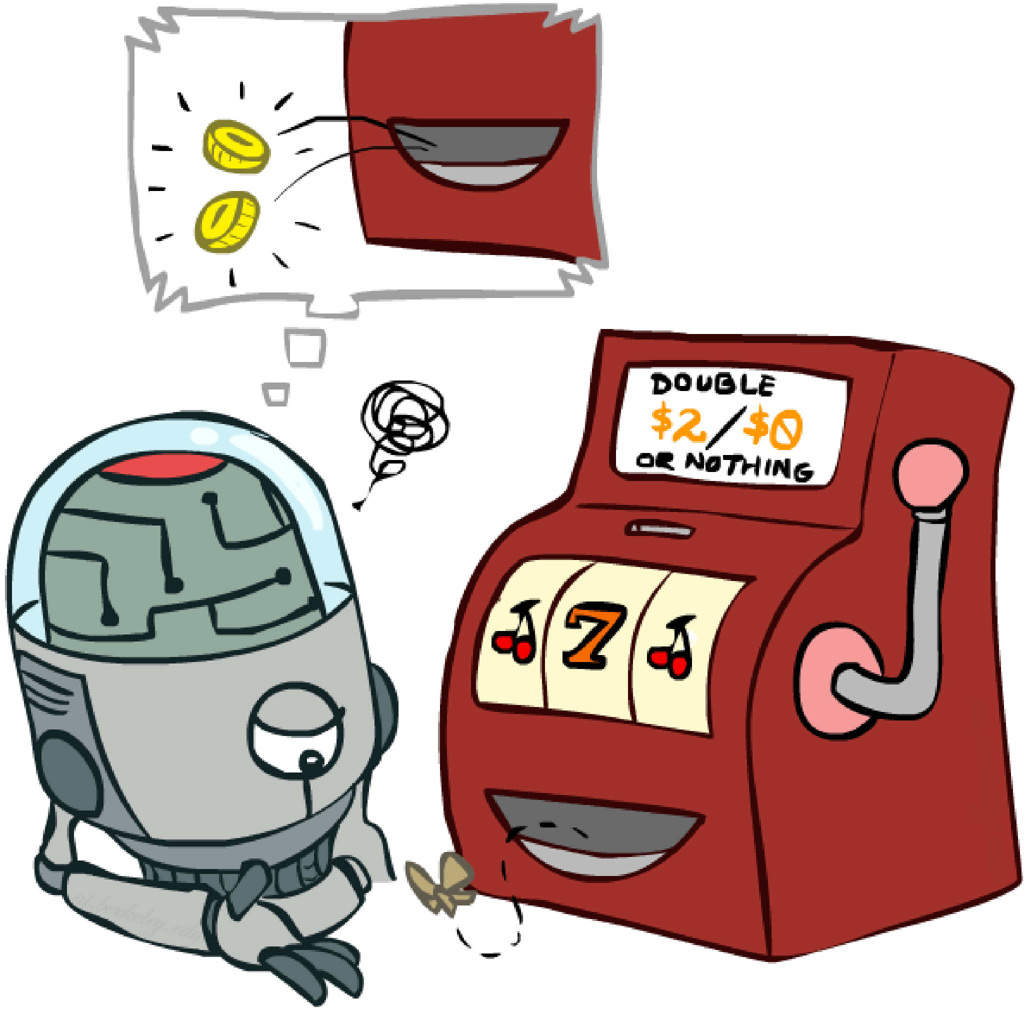
$$E[A] \approx \frac{1}{N} \sum_i a_i$$

V(s)

r_i

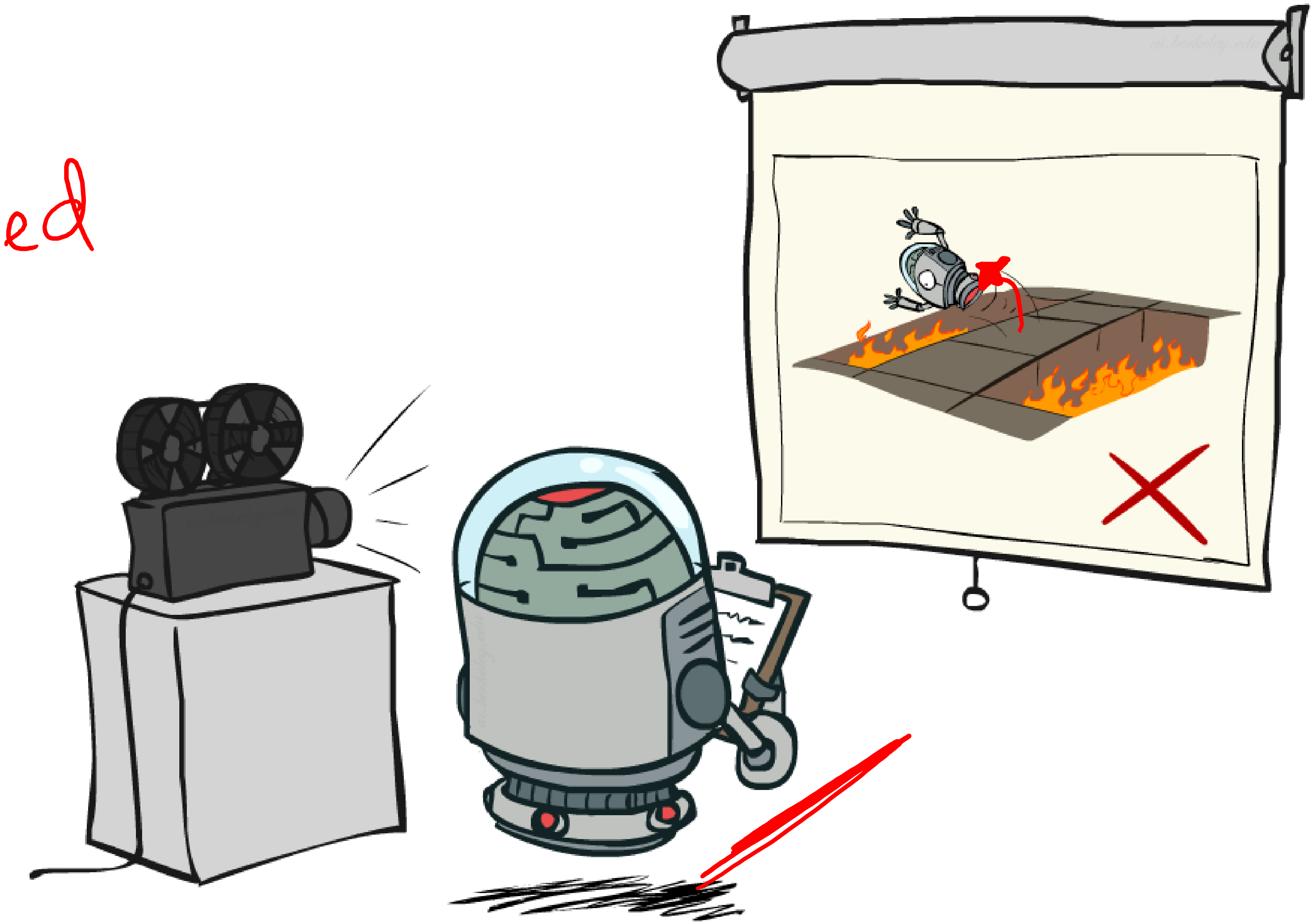
Why does this work? Because samples appear with the right frequencies.

Model-Free Learning



Passive Reinforcement Learning

π fixed

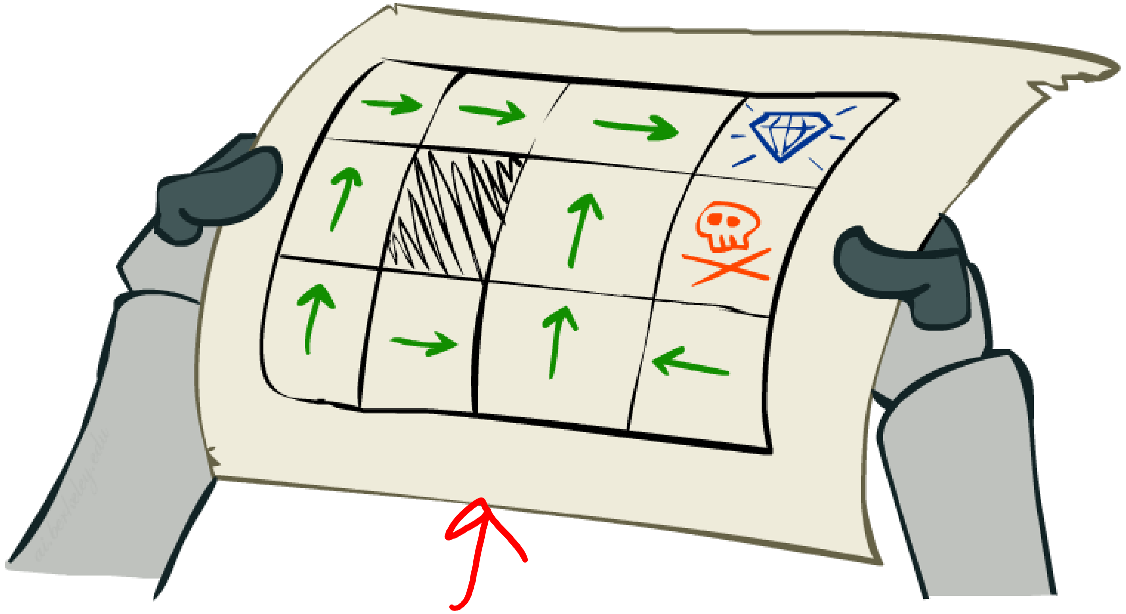


Passive Reinforcement Learning

Simplified task: policy evaluation

- Input: a fixed policy $\pi(s)$
- You don't know the transitions T(s,a,s')
- You don't know the rewards R(s,a,s')
- Goal: learn the state values

$V^\pi(s)$



In this case:

- Learner is "along for the ride"
- No choice about what actions to take
- Just execute the policy and learn from experience
- This is NOT offline planning! You actually take actions in the world.

Direct Evaluation

Goal: Compute values for each state under π

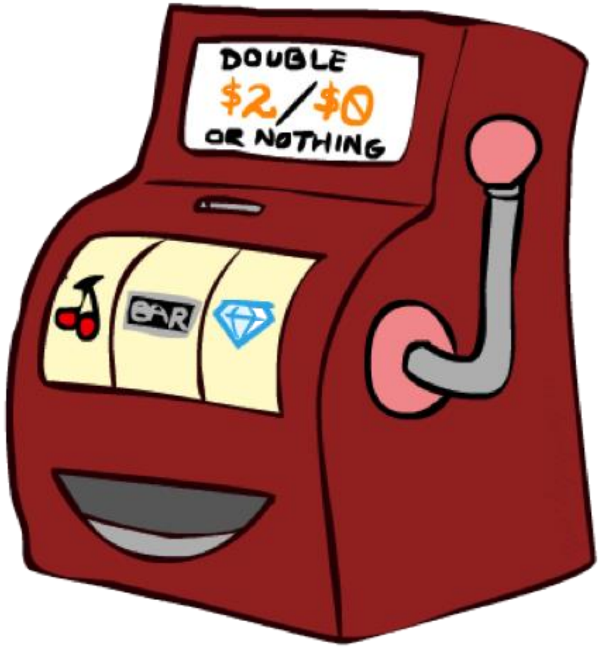
Idea: Average together observed sample values

- Act according to π $\rightarrow V^{\pi}(s)$
- Every time you visit a state, write down what the sum of discounted rewards turned out to be
- Average those samples

This is called direct evaluation

$$S \xrightarrow[r]{} \xrightarrow[\gamma r]{} \xrightarrow[\gamma^2 r]{} \|$$

$$S \xrightarrow[r']{} \xrightarrow[\gamma r']{} \xrightarrow[\gamma^2 r']{} \xrightarrow[\gamma^3 r']{} \|$$



Direct Evaluation

Goal: Compute values for each state under π

Idea: Average together observed sample values

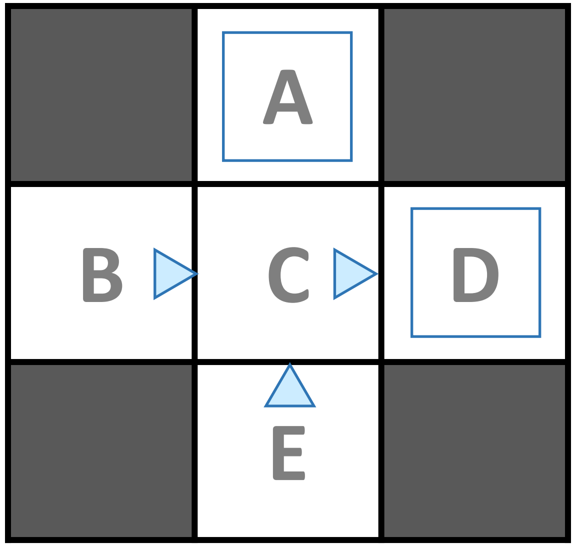
- Act according to π
- Every time you visit a state, write down what the sum of discounted rewards turned out to be
- Average those samples

This is called direct evaluation

Pieces Available	Take 1	Take 2
2	0%	**100%**
3	2%	0%
4	**75%**	2%
5	4%	**68%**
6	5%	6%
7	**60%**	5%

Example: Direct Evaluation

Input Policy π



Assume: $\gamma = 1$

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D, exit, x, +10

+9 +8

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Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

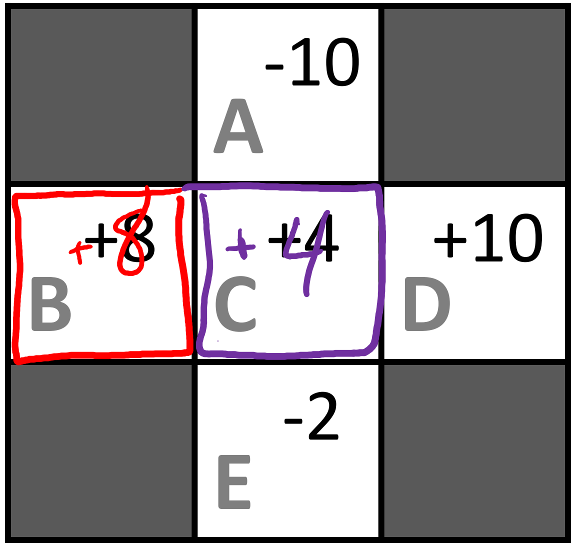
+9

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

-11

Output Values



Problems with Direct Evaluation

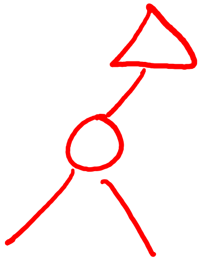
$V^{\pi}(s)$

What's good about direct evaluation?

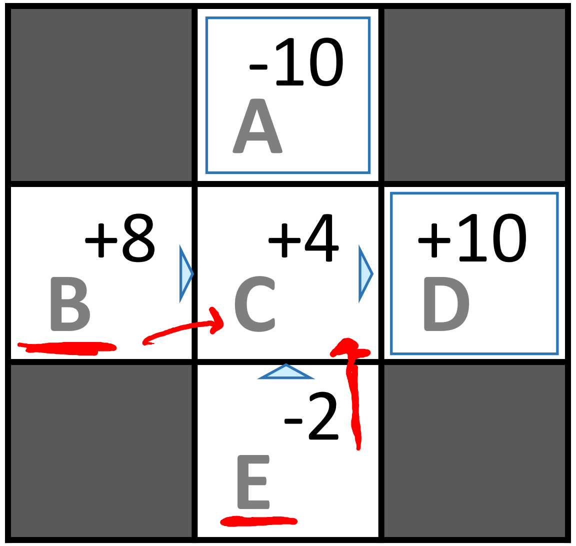
- It's easy to understand
- It doesn't require any knowledge of T, R
- It eventually computes the correct average values, using just sample transitions

What bad about it?

- It wastes information about state connections
- Each state must be learned separately
- So, it takes a long time to learn



Output Values



If B and E both go to C under this policy, how can their values be different?

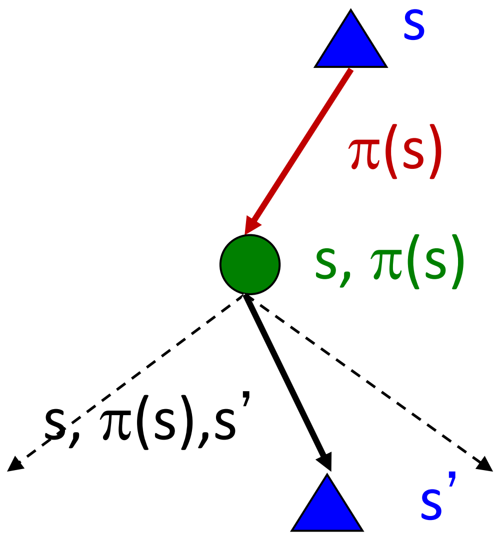
Why Not Use Policy Evaluation?

Simplified Bellman updates calculate V for a fixed policy:

- Each round, replace V with a one-step-look-ahead layer over V

$$V_0^\pi(s) = 0$$

$$\underline{V_{k+1}^\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$



- This approach fully exploited the connections between the states
- Unfortunately, we need T and R to do it!

Key question: how can we do this update to V without knowing T and R?

- In other words, how to we take a weighted average without knowing the weights?

Sample-Based Policy Evaluation?

We want to improve our estimate of V by computing these averages:

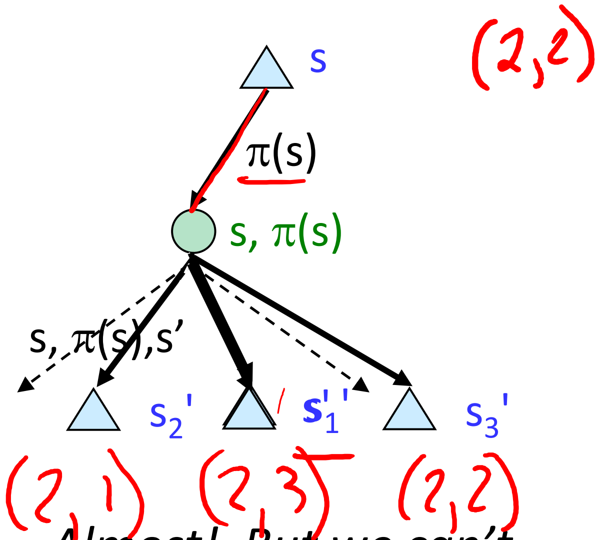
$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s,\pi(s),s')[R(s,\pi(s),s') + \gamma V_k^{\pi}(s')]$$

r γV

Idea: Take samples of outcomes s' (by doing the action!) and average

$$sample_1 = R(s,\pi(s),s_1') + \gamma V_k^{\pi}(s_1')$$
$$sample_2 = R(s,\pi(s),s_2') + \gamma V_k^{\pi}(s_2')$$
$$\ldots$$
$$sample_n = R(s,\pi(s),s_n') + \gamma V_k^{\pi}(s_n')$$

$$V_{k+1}^{\pi}(s) \leftarrow \frac{1}{n}\sum_i sample_i$$

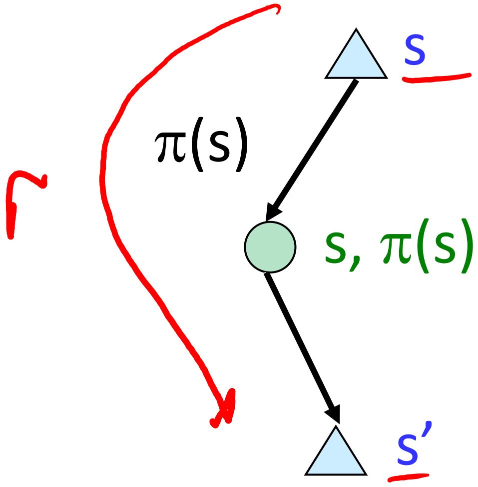


Almost! But we can't rewind time to get sample after sample from state s.

Temporal Difference Learning

Big idea: learn from every experience!

- Update V(s) each time we experience a transition (s, a, s', r)
- Likely outcomes s' will contribute updates more often



Temporal difference learning of values

- Policy still fixed, still doing evaluation!
- Move values toward value of whatever successor occurs: running average

Sample of V(s): $$sample = \overset{r}{R(s, \pi(s), s')} + \gamma V^{\pi}(s')$$

Update to V(s): $$V^{\pi}(s) \leftarrow \underbrace{(1-\alpha)}_{.9} V^{\pi}(s) + \underbrace{\alpha}_{.1}\ sample$$

Temporal Difference Learning

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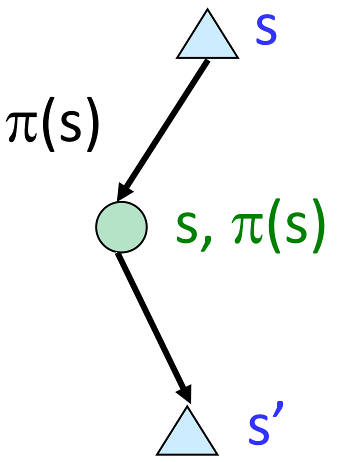


Temporal difference learning of values

- Policy still fixed, still doing evaluation!
- Move values toward value of whatever successor occurs: running average

Sample of V(s): $sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$

Update to V(s): $V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + (\alpha)sample$

Same update: $V^{\pi}(s) \leftarrow \underline{V^{\pi}(s)} + \underset{0.1}{\alpha}(\underline{sample} - V^{\pi}(s))$

Exponential Moving Average

Exponential moving average

- The running interpolation update: $$\bar{x}_n = (1-\alpha)\cdot\bar{x}_{n-1} + \alpha\cdot x_n$$
- Makes recent samples more important:

$$\bar{x}_n = \frac{x_n + (1-\alpha)\cdot x_{n-1} + (1-\alpha)^2\cdot x_{n-2} + \ldots}{1 + (1-\alpha) + (1-\alpha)^2 + \ldots}$$

$(1-\alpha)^{n-1} x_0$

- Forgets about the past (distant past values were wrong anyway)

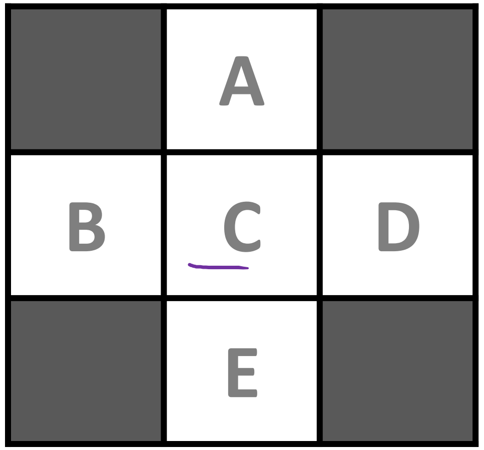
Decreasing learning rate (alpha) can give converging averages

sample: $r + \gamma V(s')$

$-2 + 1 \cdot 0$

Example: Temporal Difference Learning

States

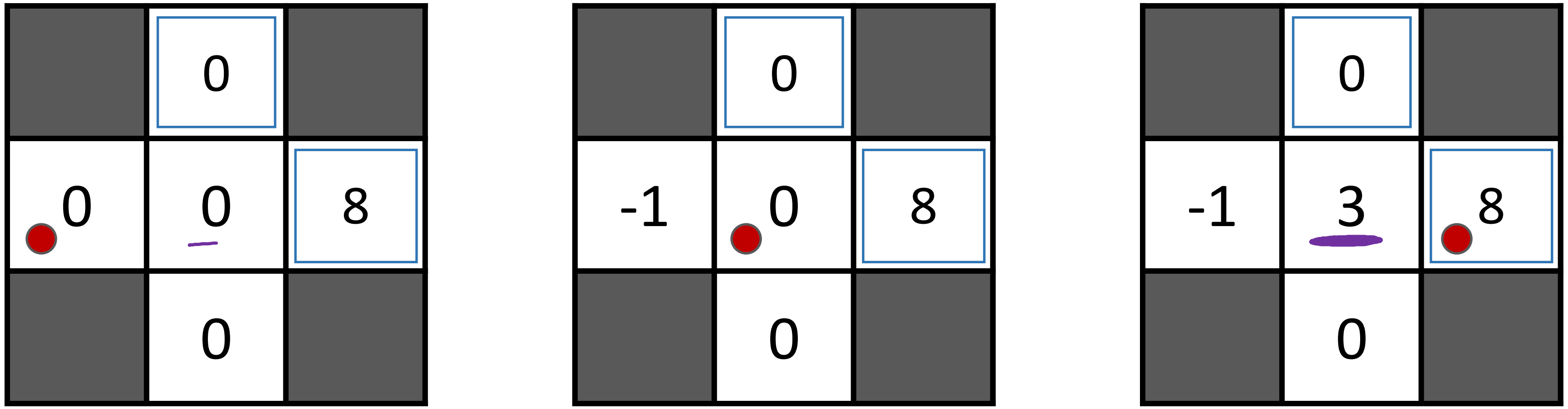


Assume: $\gamma = 1$, $\alpha = 1/2$

Observed Transitions

B, east, C, -2

C, east, D, -2



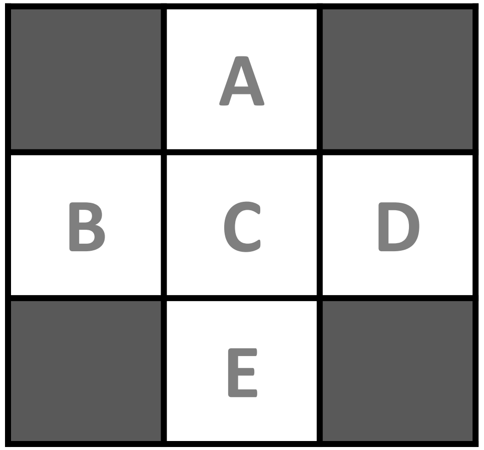
$V(s) \leftarrow (1-\alpha) V(s) + \alpha \text{ sample}$

$V(B) \leftarrow (1-\tfrac{1}{2})\, 0 + \tfrac{1}{2}(-2) = -1$

$V(C) \leftarrow$

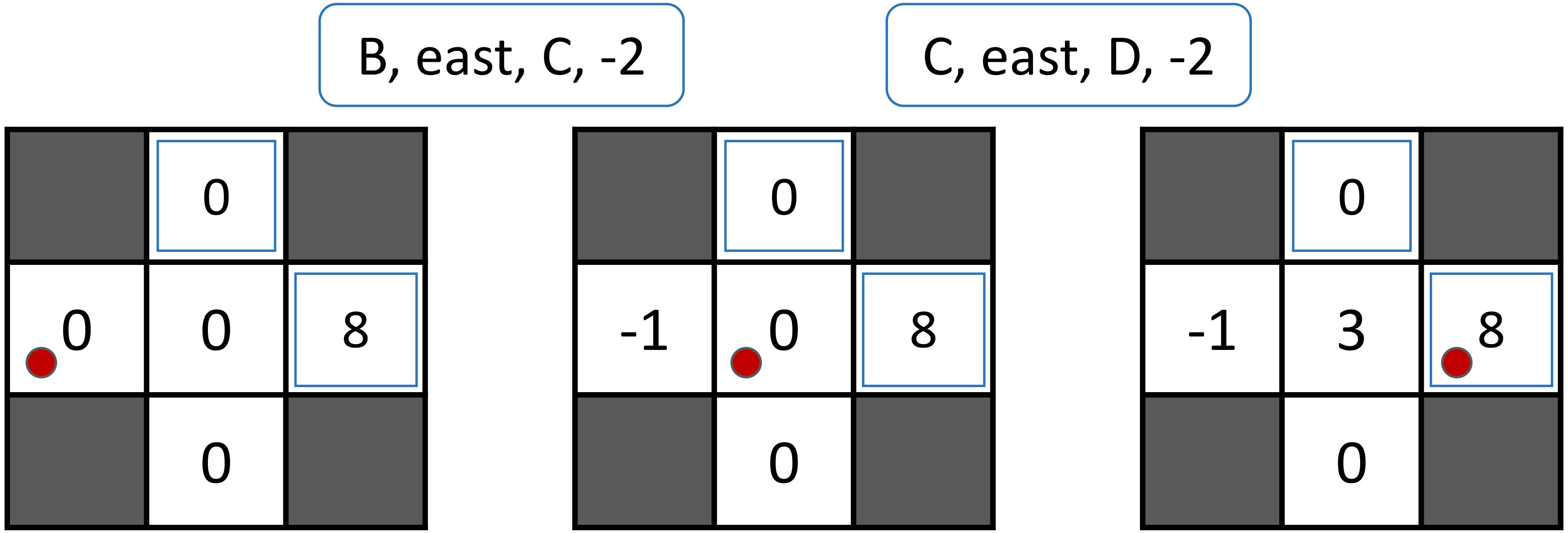
Example: Temporal Difference Learning

States



Assume: $\gamma = 1$,
$\alpha = 1/2$

Observed Transitions



$$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + \alpha\left[R(s,\underline{\pi(s)},s') + \gamma V^{\pi}(s')\right]$$

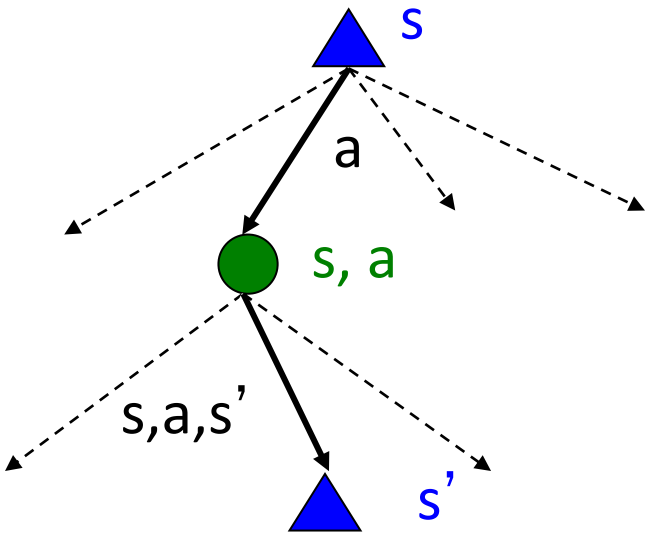
Problems with TD Value Learning

TD value leaning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages

However, if we want to turn values into a (new) policy, we're sunk:

$$\pi(s) = \arg\max_a Q(s,a)$$

$$Q(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V(s') \right]$$

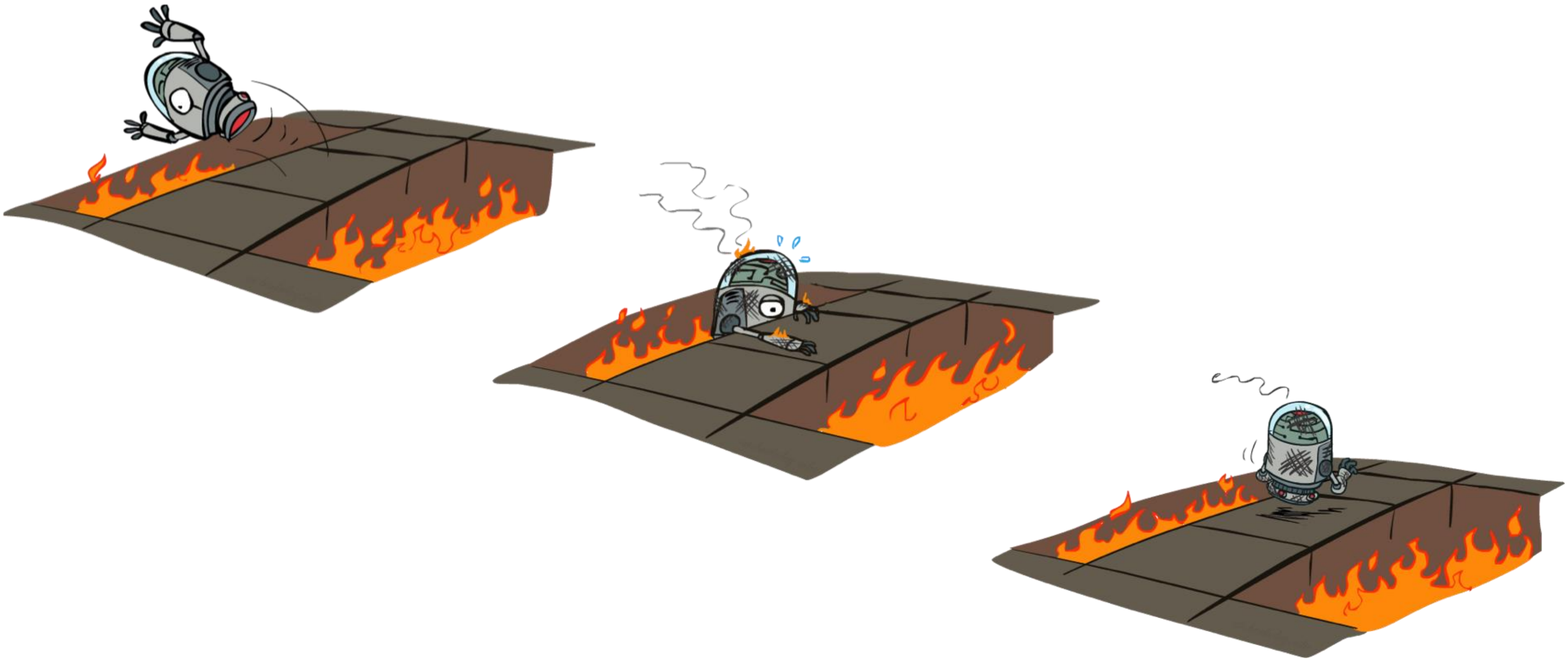


Idea: learn Q-values, not values

Makes action selection model-free too!

Active Reinforcement Learning

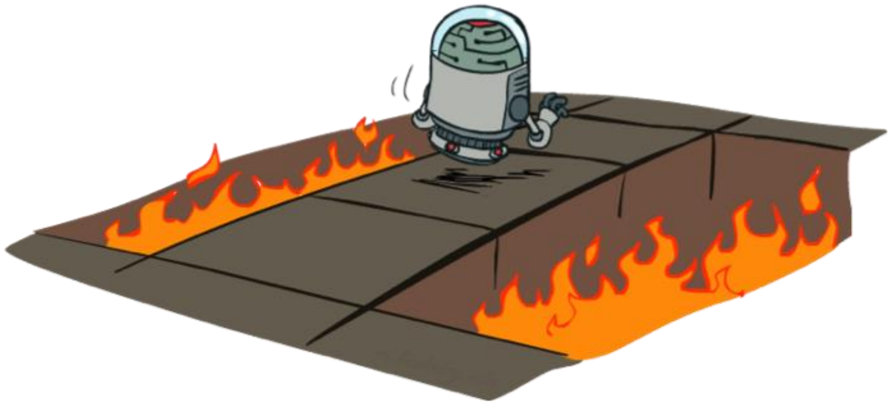
Q-learning



Active Reinforcement Learning

Full reinforcement learning: optimal policies (like value iteration)

- You don't know the transitions T(s,a,s')
- You don't know the rewards R(s,a,s')
- You choose the actions now
- Goal: learn the optimal policy / values



In this case:

- Learner makes choices!
- Fundamental tradeoff: exploration vs. exploitation
- This is NOT offline planning! You actually take actions in the world and find out what happens…

Detour: Q-Value Iteration

Value iteration: find successive (depth-limited) values

- Start with $V_0(s) = 0$, which we know is right
- Given V_k, calculate the depth k+1 values for all states:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$

But Q-values are more useful, so compute them instead

- Start with $Q_0(s,a) = 0$, which we know is right
- Given Q_k, calculate the depth k+1 q-values for all q-states:

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

Q-Learning

Q-Learning: sample-based Q-value iteration

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')\right]$$

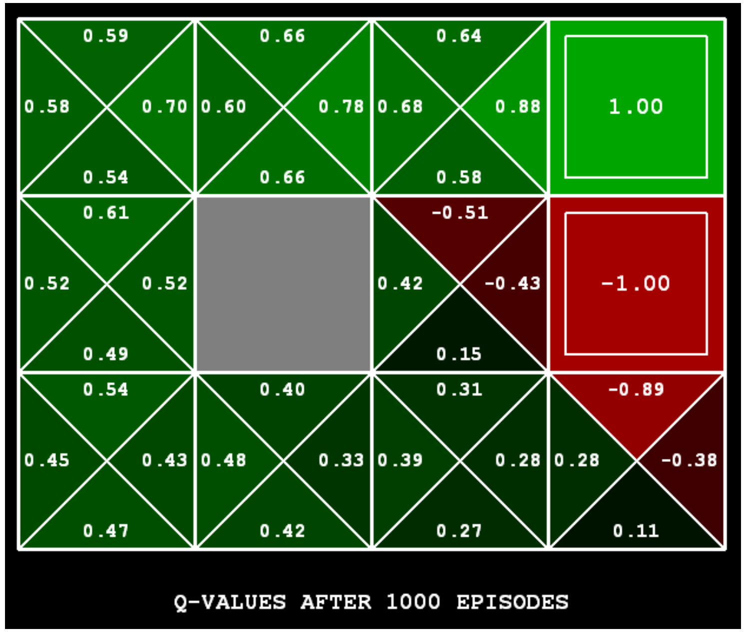
Learn Q(s,a) values as you go

- Receive a sample (s,a,s',r)
- Consider your old estimate: $Q(s,a)$
- Consider your new sample estimate:

$$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

- Incorporate the new estimate into a running average:

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[sample\right]$$



[Demo: Q-learning – gridworld (L10D2)]
[Demo: Q-learning – crawler (L10D3)]

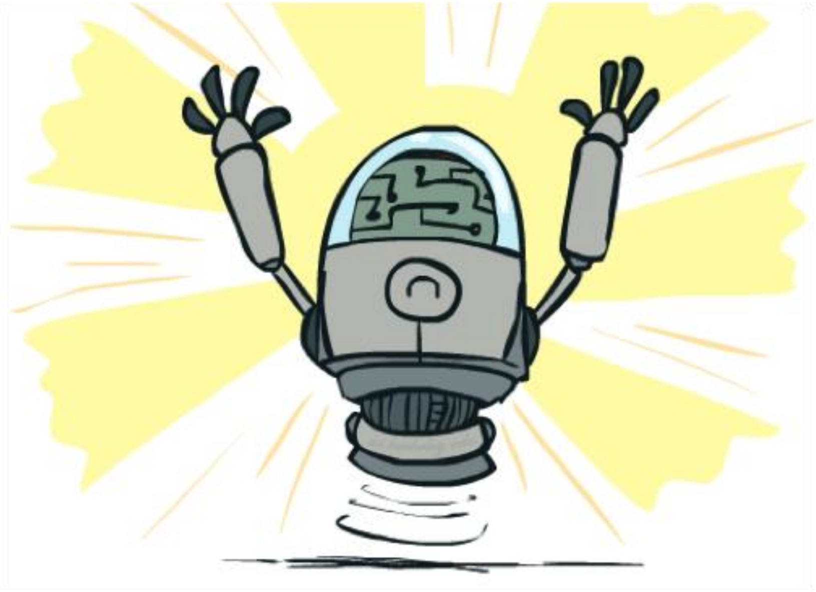
Demo Q-Learning -- Gridworld

Demo Q-Learning -- Crawler

Q-Learning Properties

Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!

This is called off-policy learning



Caveats:

- You have to explore enough
- You have to eventually make the learning rate small enough
- … but not decrease it too quickly
- Basically, in the limit, it doesn't matter how you select actions (!)