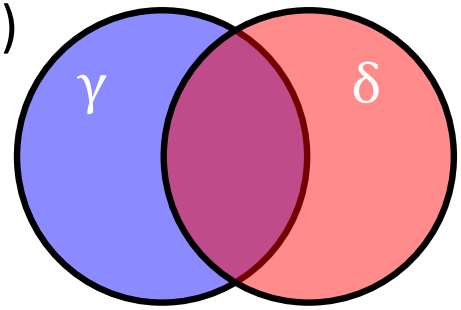
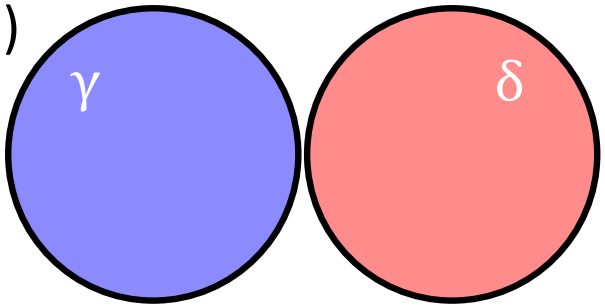
Warm-up:

The regions below visually enclose the set of models that satisfy the respective sentence γ or δ. For which of the following diagrams does γ entail δ. Select all that apply.

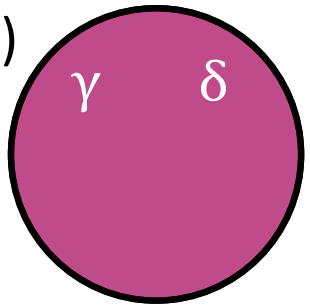
A)



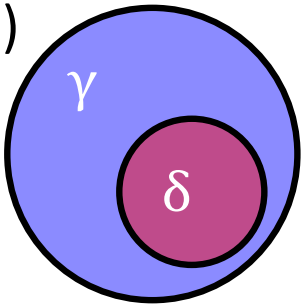
B)



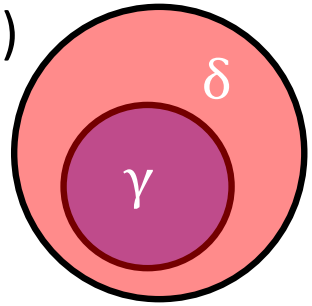
C)



D)



E)



Announcements

Midterm 1 Exam

- Grading should be finished Friday night
- We'll let you know as soon as Canvas reflects your current grade

Assignments:

- P2: Optimization
  - Due Sat 2/22, 10 pm
- HW5
  - Due Tue 2/25, 10pm
- P3: Logic and Classical Planning
  - Out 2/22, Due Thu 3/5 10 pm before spring break!

AI: Representation and Problem Solving

Logical Agents

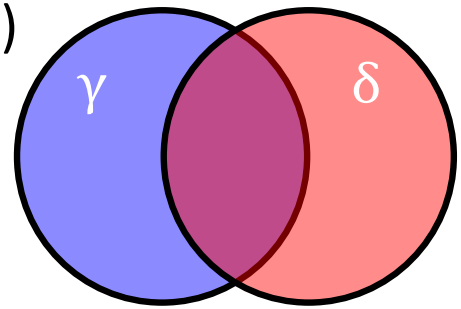
Instructors: Pat Virtue & Stephanie Rosenthal

Slide credits: CMU AI, http://ai.berkeley.edu

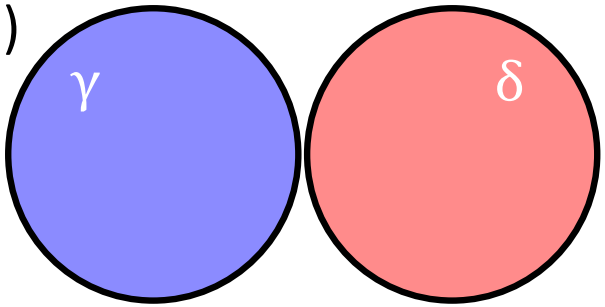
Piazza Poll 1

The regions below visually enclose the set of models that satisfy the respective sentence γ or δ. For which of the following diagrams does γ entail δ. Select all that apply.

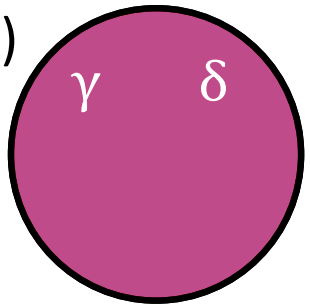
A)



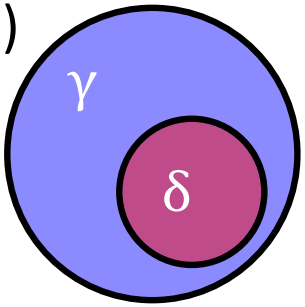
B)



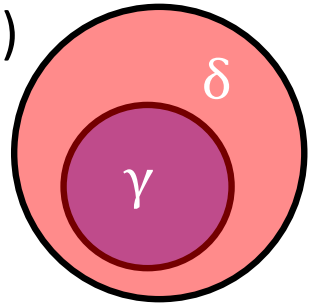
C)



D)



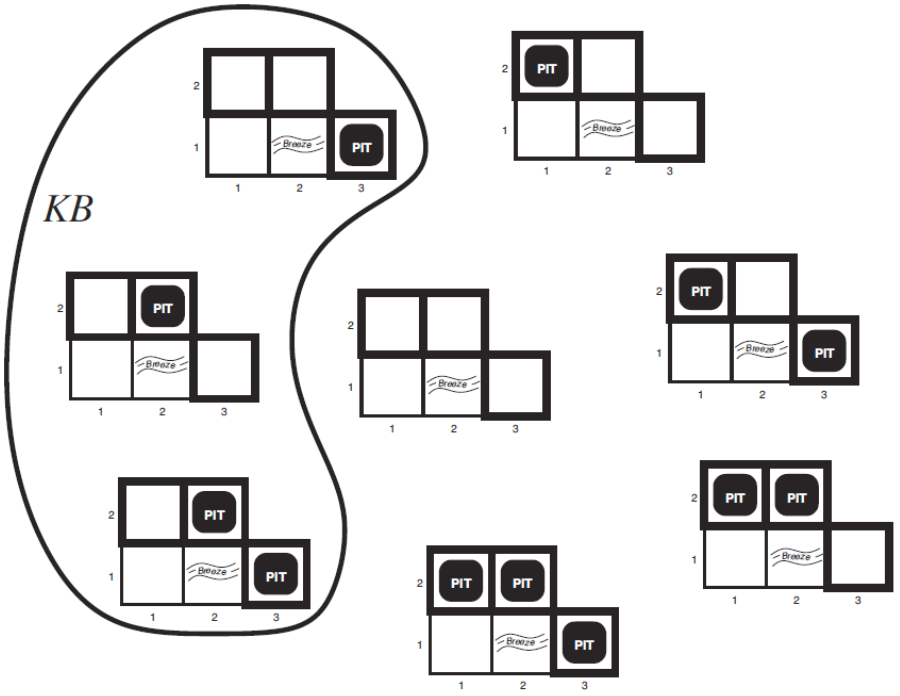
E)



Entailment

Does the knowledge base entail my query?

- Query 1: $\neg P[1,2]$
- Query 2: $\neg P[2,2]$



Propositional Logic Vocab

Literal

- Atomic sentence: True, False, Symbol, ¬Symbol

Clause

- Disjunction of literals: $A \vee B \vee \neg C$

Definite clause

- Disjunction of literals, *exactly one* is positive
- $\neg A \vee B \vee \neg C$

Horn clause

- Disjunction of literals, *at most one* is positive
- All definite clauses are Horn clauses

Logical Agent Vocab

Model

- Complete assignment of symbols to True/False

Sentence

- Logical statement
- Composition of logic symbols and operators

KB

- Collection of sentences representing facts and rules we know about the world

Query

- Sentence we want to know if it is *probably* True, *provably* False, or *unsure*.

Logical Agent Vocab

Entailment

- Input: sentence1, sentence2
- Each model that satisfies sentence1 must also satisfy sentence2
- "If I know 1 holds, then I know 2 holds"
- Simple model checking TT-ENTAILS, Forward chaining FC-ENTAILS

Satisfy

- Input: model, sentence
- Is this sentence true in this model?
- Does this model satisfy this sentence
- "Does this particular state of the world work?'
- PL-TRUE

Today: Logical Agent Vocab

Satisfiable

- Input: sentence
- Can find at least one model that satisfies this sentence
  - (We often want to know what that model is)
- "Is it possible to make this sentence true?"
- DPLL

Valid

- Input: sentence
- sentence is true in all possible models

Simple Model Checking

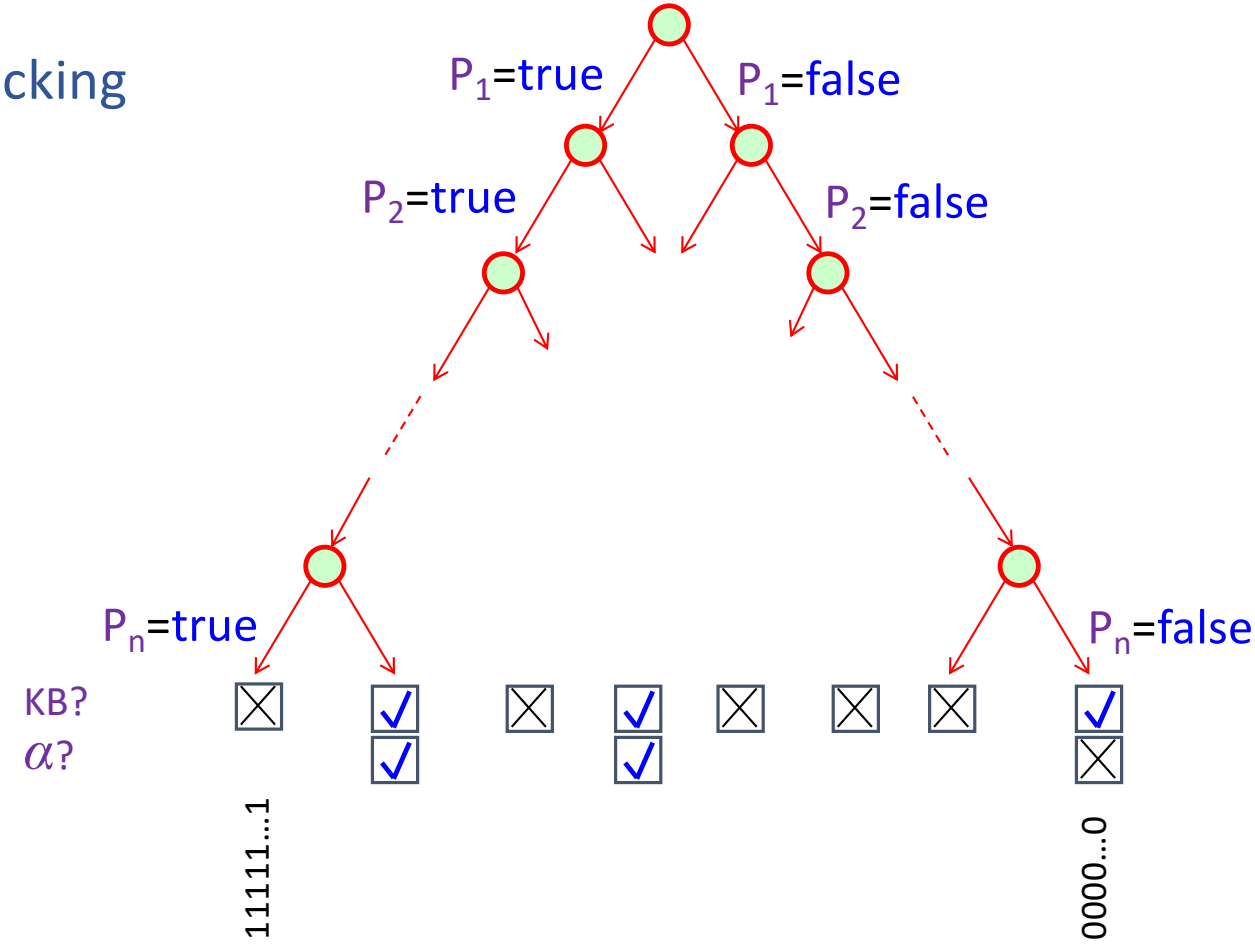
function TT-ENTAILS?(KB, α) returns true or false

Simple Model Checking, contd.

Same recursion as backtracking

$O(2^n)$ time, linear space

We can do much better!



Simple Model Checking

```
function TT-ENTAILS?(KB, α) returns true or false
    return TT-CHECK-ALL(KB, α, symbols(KB) U symbols(α),{})

function TT-CHECK-ALL(KB, α, symbols,model) returns true or false
    if empty?(symbols) then
        if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
        else return true
    else
        P ← first(symbols)
        rest ← rest(symbols)
        return  and (TT-CHECK-ALL(KB, α, rest, model U {P = true})
                     TT-CHECK-ALL(KB, α, rest, model U {P = false }))
```

Propositional Logic

Check if sentence is true in given model

In other words, does the model *satisfy* the sentence?

function PL-TRUE?(α,model) returns true or false

if α is a symbol then return Lookup(α, model)

if Op(α) = $\neg$ then return not(PL-TRUE?(Arg1(α),model))

if Op(α) = $\wedge$ then return and(PL-TRUE?(Arg1(α),model),
PL-TRUE?(Arg2(α),model))

etc.

(Sometimes called “recursion over syntax”)

Inference: Proofs

A proof is a *demonstration* of entailment between α and β

Method 1: *model-checking*

- For every possible world, if α is true make sure that is β true too
- OK for propositional logic (finitely many worlds); not easy for first-order logic

Method 2: *theorem-proving*

- Search for a sequence of proof steps (applications of *inference rules*) leading from α to β
- E.g., from $P \wedge (P \Rightarrow Q)$, infer Q by *Modus Ponens*

Simple Theorem Proving: Forward Chaining

Forward chaining applies Modus Ponens to generate new facts:

- Given $X_1 \wedge X_2 \wedge \ldots X_n \Rightarrow Y$ and $X_1, X_2, \ldots, X_n$
- Infer Y

Forward chaining keeps applying this rule, adding new facts, until nothing more can be added

Requires KB to contain only *definite clauses*:

- (Conjunction of symbols) $\Rightarrow$ symbol; or
- A single symbol (note that X is equivalent to True $\Rightarrow X$)

Forward Chaining Algorithm

function PL-FC-ENTAILS?(KB, q) returns true or false

count ← a table, where count[c] is the number of symbols in c's premise

inferred ← a table, where inferred[s] is initially false for all s

agenda ← a queue of symbols, initially symbols known to be true in KB

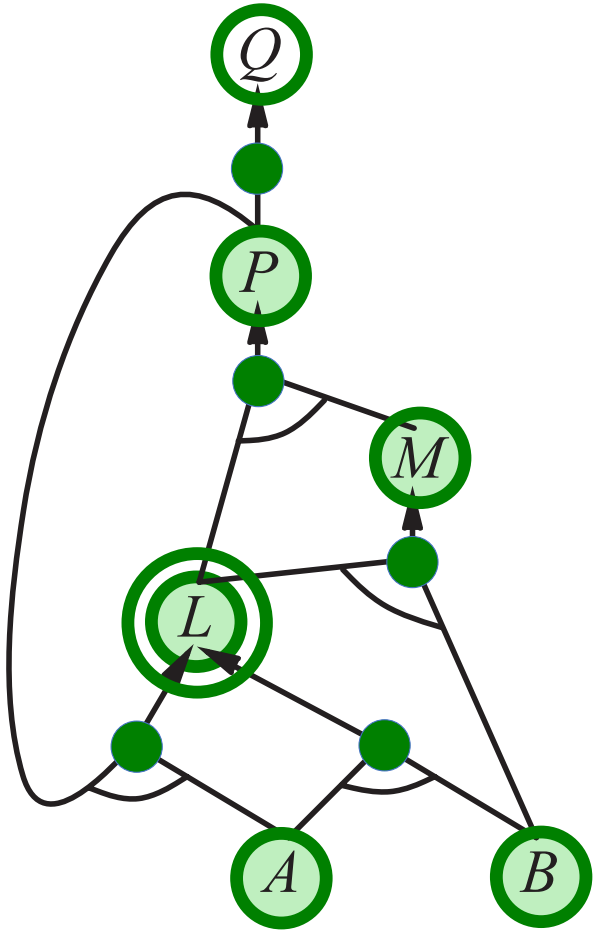
CLAUSES	*COUNT*	*INFERRED*	*AGENDA*
$P \Rightarrow Q$	1	A false	
$L \wedge M \Rightarrow P$	2	B false	
$B \wedge L \Rightarrow M$	2	L false	
$A \wedge P \Rightarrow L$	2	M false	
$A \wedge B \Rightarrow L$	2	P false	
A	0	Q false	
B	0		

Forward Chaining Example: Proving Q

Clauses	*Count*	*Inferred*
$P \Rightarrow Q$	1 / 0	A ~~false~~ true
$L \wedge M \Rightarrow P$	2 / 1 / 0	B ~~false~~ true
$B \wedge L \Rightarrow M$	2 / 1 / 0	L ~~false~~ true
$A \wedge P \Rightarrow L$	2 / 1 / 0	M ~~false~~ true
$A \wedge B \Rightarrow L$	2 / 1 / 0	P ~~false~~ true
A	0	Q ~~false~~ true
B	0	

Agenda

~~A~~ ~~B~~ ~~L~~ ~~M~~ ~~P~~ ~~L~~ ~~Q~~



Forward Chaining Algorithm

```
function PL-FC-ENTAILS?(KB, q) returns true or false
    count ← a table, where count[c] is the number of symbols in c's premise
    inferred ← a table, where inferred[s] is initially false for all s
    agenda ← a queue of symbols, initially symbols known to be true in KB
    while agenda is not empty do
        p ← Pop(agenda)
        if p = q then return true
        if inferred[p] = false then
            inferred[p]←true
            for each clause c in KB where p is in c.premise do
                decrement count[c]
                if count[c] = 0 then add c.conclusion to agenda
    return false
```

Properties of forward chaining

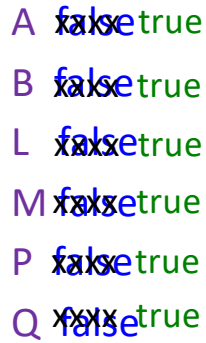
Theorem: FC is sound and complete for definite-clause KBs

Soundness: follows from soundness of Modus Ponens (easy to check)

Completeness proof:

1. FC reaches a fixed point where no new atomic sentences are derived
2. Consider the final *inferred* table as a model ***m***, assigning true/false to symbols
3. Every clause in the original KB is true in ***m***

   Proof: Suppose a clause $a_1 \wedge \ldots \wedge a_k \Rightarrow b$ is false in ***m***
   Then $a_1 \wedge \ldots \wedge a_k$ is true in ***m*** and b is false in ***m***
   Therefore the algorithm has not reached a fixed point!
4. Hence ***m*** is a model of KB
5. If KB |= q, q is true in every model of KB, including ***m***



Inference Rules

Modus Ponens

$$\frac{\alpha \Rightarrow \beta, \quad \alpha}{\beta}$$

Notation Alert!

Unit Resolution

$$\frac{a \vee b, \quad \neg b \vee c}{a \vee c}$$

General Resolution

$$\frac{a_1 \vee \cdots \vee a_m \vee b, \quad \neg b \vee c_1 \vee \cdots \vee c_n}{a_1 \vee \cdots \vee a_m \vee c_1 \vee \cdots \vee c_n}$$

Resolution

Algorithm Overview

function PL-RESOLUTION?(KB, α) returns true or false

We want to prove that KB entails α

In other words, we want to prove that we cannot satisfy (KB and **not** α)

1. Start with a set of CNF clauses, including the KB as well as $\neg\alpha$
2. Keep resolving pairs of clauses until
   A. You resolve the empty clause
      Contradiction found!
      KB $\wedge \neg\alpha$ cannot be satisfied
      Return true, KB entails α
   B. No new clauses added
      Return false, KB does not entail α

Resolution

General Resolution

$$\frac{a_1 \vee \cdots \vee a_m \vee b, \quad \neg b \vee c_1 \vee \cdots \vee c_n}{a_1 \vee \cdots \vee a_m \vee c_1 \vee \cdots \vee c_n}$$

Example trying to prove $\neg P_{1,2}$

Knowledge Base

$\neg P_{2,1} \vee B_{1,1}$

$\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}$

$\neg P_{1,2} \vee B_{1,1}$

$\neg B_{1,1}$

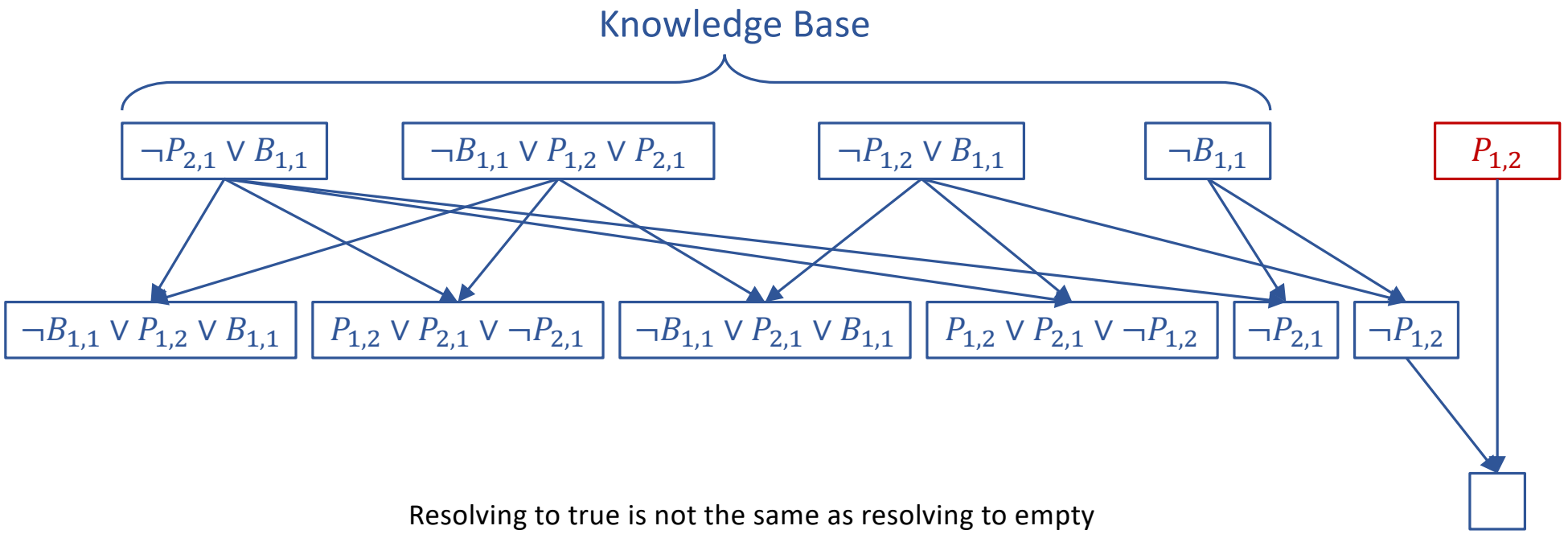
$\neg\neg P_{1,2}$

Resolution

General Resolution

$$\frac{a_1 \vee \cdots \vee a_m \vee b, \quad \neg b \vee c_1 \vee \cdots \vee c_n}{a_1 \vee \cdots \vee a_m \vee c_1 \vee \cdots \vee c_n}$$

Example trying to prove $\neg P_{1,2}$



Resolving to true is not the same as resolving to empty

Resolution

function PL-RESOLUTION?(KB, α) returns true or false

clauses ← the set of clauses in the CNF representation of KB $\wedge \neg\alpha$

new ← { }

loop do

for each pair of clauses C_i, C_j in clauses do

resolvents ← PL-RESOLVE(C_i, C_j)

if resolvents contains the empty clause then

return true

new ← new ∪ resolvants

if new ⊆ clauses then

return false

clauses ← clauses ∪ new

Inference: Proofs

A proof is a *demonstration* of entailment between α and β

Method 1: *model-checking*

- For every possible world, if α is true make sure that is β true too
- OK for propositional logic (finitely many worlds); not easy for first-order logic

Method 2: *theorem-proving*

- Search for a sequence of proof steps (applications of *inference rules*) leading from α to β
- from $P \wedge (P \Rightarrow Q)$, infer Q by *Modus Ponens*
- by resolving clauses in the KB by *General Resolution*

Piazza Polls 2 and 3

Properties

- *Sound* algorithm: everything it claims to prove is in fact entailed
- *Complete* algorithm: every sentence that is entailed can be proved

Which of the following algorithms are sound and which are complete?

- Simple Model Checking
- Forward Chaining (Modus Ponens)
- Resolution

Properties

Simple Model Checking is:

- Sound and complete for any PL KBs!
- Complexity: exponential time ☹

Forward Chaining is:

- Sound and complete for definite-clause KBs
- Complexity: linear time

Resolution is:

- Sound and complete for any PL KBs!
- Complexity: exponential time ☹

Inference: Proofs

A proof is a *demonstration* of entailment between α and β

Method 1: *model-checking*

- For every possible world, if α is true make sure that is β true too
- OK for propositional logic (finitely many worlds); not easy for first-order logic

Method 2: *theorem-proving*

- Search for a sequence of proof steps (applications of *inference rules*) leading from α to β
- from $P \wedge (P \Rightarrow Q)$, infer Q by *Modus Ponens, Forward Chaining*
- by resolving clauses in the KB by *General Resolution*

Method 3: *SAT solvers*

Satisfiability and Entailment

A sentence is *satisfiable* if it is true in at least one world (cf CSPs!)

Suppose we have a hyper-efficient SAT solver; how can we use it to test entailment?

- Suppose $\alpha \models \beta$
- Then $\alpha \Rightarrow \beta$ is true in all worlds
- Hence $\neg(\alpha \Rightarrow \beta)$ is false in all worlds
- Hence $\alpha \wedge \neg\beta$ is false in all worlds, i.e., unsatisfiable

So, add the negated conclusion to what you know, test for (un)satisfiability; also known as *reductio ad absurdum*

Efficient SAT solvers operate on ***conjunctive normal form***

Conjunctive Normal Form (CNF)

Every sentence can be expressed as a conjunction of clauses

Each clause is a disjunction of literals

Each literal is a symbol or a negated symbol

Conversion to CNF by a sequence of standard transformations

Conjunctive Normal Form (CNF)

Original sentence:

- $A \Rightarrow (B \Leftrightarrow C)$

Biconditional Elimination: Replace biconditional by two implications

- $A \Rightarrow ((B \Rightarrow C) \wedge (C \Rightarrow B))$

Implication Elimination: Replace $\alpha \Rightarrow \beta$ by $\neg\alpha \vee \beta$

- $\neg A \vee ((\neg B \vee C) \wedge (\neg C \vee B))$

Distribution: Distribute v over $\wedge$, i.e., replace $\alpha \vee (\beta \wedge \gamma)$ by $(\alpha \vee \beta) \wedge (\alpha \vee \gamma)$

- $(\neg A \vee \neg B \vee C) \wedge (\neg A \vee \neg C \vee B)$

Conjunctive Normal Form (CNF)

Original sentence:

- $(\neg(A \vee B) \vee C) \wedge (\neg C \wedge A)$

De Morgan's Law: Replace $\neg(\alpha \vee \beta)$ by $\neg\alpha \wedge \neg\beta$, and $\neg(\alpha \wedge \beta)$ by $\neg\alpha \vee \neg\beta$

- $((\neg A \wedge \neg B) \vee C) \wedge (\neg C \wedge A)$

Distribution: Distribute v over $\wedge$, i.e., replace $\alpha \vee (\beta \wedge \gamma)$ by $(\alpha \vee \beta) \wedge (\alpha \vee \gamma)$

- $(\neg A \vee C) \wedge (\neg B \vee C) \wedge (\neg C \wedge A)$

Other Logical Equivalences

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) && \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) && \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) && \text{De Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) && \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) && \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Efficient SAT solvers

DPLL (Davis-Putnam-Logemann-Loveland) is the core of modern solvers

Essentially a backtracking search over models with some extras:

- *Early termination*: stop if
  - all clauses are satisfied; e.g., $(A \vee B) \wedge (A \vee \neg C)$ is satisfied by {A=true}
  - any clause is falsified; e.g., $(A \vee B) \wedge (A \vee \neg C)$ is satisfied by {A=false, B=false}

- *Pure literals*: if all occurrences of a symbol in as-yet-unsatisfied clauses have the same sign, then give the symbol that value
  - E.g., A is pure and positive in $(A \vee B) \wedge (A \vee \neg C) \wedge (C \vee \neg B)$ so set it to true

- *Unit clauses*: if a clause is left with a single literal, set symbol to satisfy clause
  - E.g., if A=false, $(A \vee B) \wedge (A \vee \neg C)$ becomes $(\text{false} \vee B) \wedge (\text{false} \vee \neg C)$, i.e. $(B) \wedge (\neg C)$
  - Satisfying the unit clauses often leads to further propagation, new unit clauses, etc.

DPLL algorithm

```
function DPLL(clauses, symbols, model) returns true or false
    if every clause in clauses is true in model then return true
    if some clause in clauses is false in model then return false

    P, value ←FIND-PURE-SYMBOL(symbols, clauses, model)
    if P is non-null then return DPLL(clauses, symbols–P, modelU{P=value})

    P, value ←FIND-UNIT-CLAUSE(clauses, model)
    if P is non-null then return DPLL(clauses, symbols–P, modelU{P=value})

    P ← First(symbols)
    rest ← Rest(symbols)

    return or(DPLL(clauses, rest, modelU{P=true}),
                  DPLL(clauses, rest, modelU{P=false}))
```

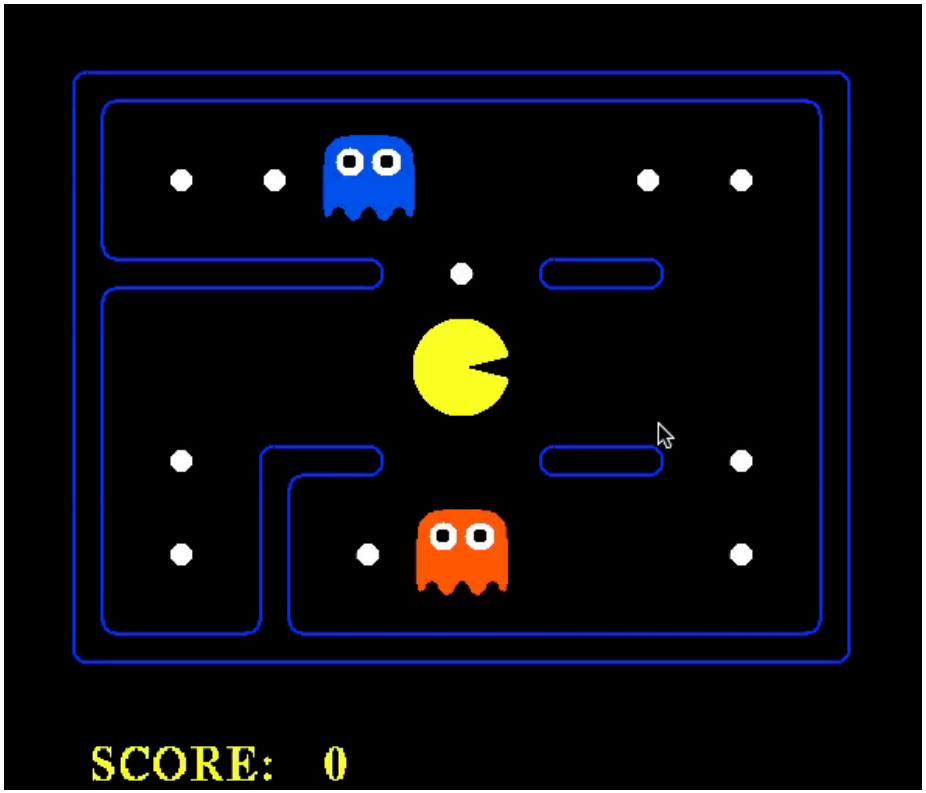
Planning as Satisfiability

Can we represent planning as a satisfiability problem?

Yes, for fully observable, deterministic case: planning problem is solvable iff there is some satisfying assignment for actions etc.

Really what we'd like to know is whether the initial state and a sequence of actions entails a goal state


SCORE: 0


SCORE: 0

Logical Agent Representation

States

Actions

Goal

Logical Agent Representation with Fluents

Fluent – a state variable that changes over time

States

Actions

Goal

Initial State

The agent may know its initial location:

- At_1,1_0

Or, it may not:

- At_1,1_0 v At_1,2_0 v At_1,3_0 v … v At_3,3_0

We also need a *domain constraint* – cannot be in two places at once!

- $\neg$(At_1,1_0 $\wedge$ At_1,2_0) $\wedge$ $\neg$(At_1,1_0 $\wedge$ At_1,3_0) $\wedge$ …
- $\neg$(At_1,1_1 $\wedge$ At_1,2_1) $\wedge$ $\neg$(At_1,1_1 $\wedge$ At_1,3_1) $\wedge$ …
- …

We need to know other important things like walls:

- Are they fluents?

Piazza Poll 4

Without domain constraint, which of the following are true:

1) DPLL will return a model that is satisfiable but not entailed
2) DPLL will return “unsatisfiable”
3) DPLL will return a model that is satisfiable and entailed

Transition Model

How does each *fluent* at each time gets its value?

State variables for PL Pacman are At_x,y_t , e.g., At_3,3_17

A state variable gets its value according to a *successor-state axiom*

- $X_t \Leftrightarrow [X_{t-1} \wedge \neg(\text{some action}_{t-1} \text{ made it false})]$ v
  $[\neg X_{t-1} \wedge (\text{some action}_{t-1} \text{ made it true})]$

For Pacman location:

- At_3,3_17 $\Leftrightarrow$ [At_3,3_16 $\wedge$ $\neg$(($\neg$Wall_3,4 $\wedge$ N_16) v ($\neg$Wall_4,3 $\wedge$ E_16) v …)]
  v [$\neg$At_3,3_16 $\wedge$ ((At_3,2_16 $\wedge$ $\neg$Wall_3,3 $\wedge$ N_16) v
  (At_2,3_16 $\wedge$ $\neg$Wall_3,3 $\wedge$ N_16) v …)]

Planning as Satisfiability

Given a hyper-efficient SAT solver, can we use it to make plans?

Yes, for fully observable, deterministic case: planning problem is solvable iff there is some satisfying assignment for actions etc.

For T = 1 to infinity, set up the KB as follows and run SAT solver:

- Initial state, domain constraints
- Transition model sentences **up to time T**
- Goal is true **at time T**
- *Precondition axioms*: $At\_1,1\_0 \wedge N\_0 \Rightarrow \neg Wall\_1,2$ etc.
- *Action exclusion axioms*: $\neg(N\_0 \wedge W\_0) \wedge \neg(N\_0 \wedge S\_0) \wedge ..$ etc.

Small Example

Initial State at t=0

Goal State at t=T

Successor State Axiom for Right