

1 Game Theory: Equilibrium Warm-Up

- (a) What is a Nash Equilibrium?
- (b) Does a Nash Equilibrium always exist?
- (c) What is another example of a solution concept that might be useful? What is the difference between this concept and Nash Equilibrium?
- (d) What are some drawbacks to following the strategy of Nash equilibrium as a solution concept?
- (e) What is a strategy? What is the difference between a pure and a mixed strategy?
- (f) What is the difference between a weakly dominant and strictly dominant strategy?
- (g) Consider rock paper scissors where Player 1's strategy is to always play rock, and Player 2's strategy is to play scissors or paper with equal probability. Is this a Nash Equilibrium?
- (h) Recursively remove dominated strategies to find the Nash Equilibrium of the following game. The order of utilities in each cell is the roman numeral player then the alphabet player.

	A	B	C
i	3,0	0,-5	0,-4
ii	1,-1	3,3	-2,4
iii	2,4	4,1	-1,8

2 Game Theory: Pacman Hunt!

Let us define the Pacman Hunt game for Ahana and Tina (derivative of the well-known Stag Hunt game!). Suppose Ahana and Tina are both hunters, they can choose to hunt Pacman or ghosts. If they hunt a ghost, they will always be successful and gain the modest payoff of 1. If they hunt Pacman, they are successful only if they both chose to hunt Pacman because then they can cooperate. In this case, they gain payoff of 2 each. However, if one hunts Pacman and one hunts ghosts, the one that hunted Pacman gets a payoff of 0. We can formulate the game in the below utility table:

Ahana, Tina	Pacman	Ghost
Pacman	2,2	0,1
Ghost	1,0	1,1

- (a) What are the pure Nash Equilibria of this problem?

- (b) We will now investigate the possibility of a mixed Nash equilibrium. Recall that in a mixed Nash Equilibrium, the utilities of the weighted actions are equal. Let p be the probability that Tina picks Pacman.
 - (i) What is the expected value of action Pacman for Ahana?

 - (ii) What is the expected value of action Ghost for Ahana?

 - (iii) What value of p makes these two expected values the same?

 - (iv) Since the table is symmetric, the probability that equalizes the value of action Pacman and Ghost for both players is the same. What is the expected utility for both Tina and Ahana if they play according to the mixed Nash Equilibria? How does this utility compare to the equilibria from (a)?

- (c) What if the game changed and somehow the Pacmen in the wild got larger or more profitable so the utility table then became

Ahana, Tina	Pacman	Ghost
Pacman	3,3	0,1
Ghost	1,0	1,1

Calculate the mixed Nash Equilibrium for this game. Are the results surprising?

3 More Game Theory

(a) What are the differences between extensive form and normal form games?

(b) We can practice computing utilities using this example that was given in lecture:

CRAM	DO HW	PLAY GAME	
98	100	85	P(EASY) = .2
97	90	65	P(HARD) = .8

- What is the utility of the pure strategy: cram?
- What is the utility of the pure strategy: do HW?
- What is the utility of the mixed strategy: $\frac{1}{2}$ cram, $\frac{1}{2}$ do HW?

- (c) Consider a two player game, where each player must simultaneously choose a number from $\{2, 3, \dots, 99, 100\}$. Let x_1 represent the value chosen by player 1, and x_2 represent the value chosen by player 2. The rules of the game are such that the utility for a player 1 can be given as:

$$u(p_1) = \begin{cases} x_1 & x_1 = x_2 \\ x_2 - 2 & x_1 > x_2 \\ x_2 + 2 & x_1 < x_2 \end{cases}$$

Because the rules of the game for everyone are the same, the utility function for player 2 is symmetric to $u(p_1)$. Does there exist a pure Nash Equilibrium for this game? It may help to try to play a few rounds of this game with someone next to you.