

Wrap-up RL

RL II slides

Midterm 2

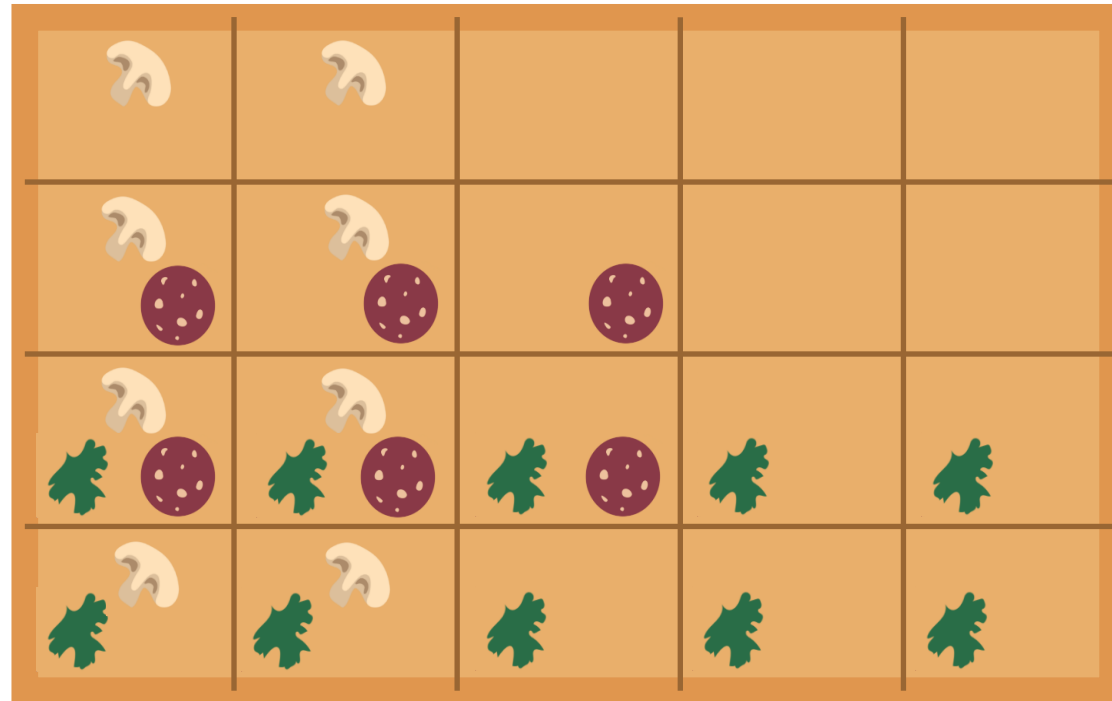
Topics

- Logic
- Planning
- MDPs
- RL
- Probability (but not Bayes Nets)

Watch Ed for details

AI: Representation and Problem Solving

Probability



Instructor: Pat Virtue

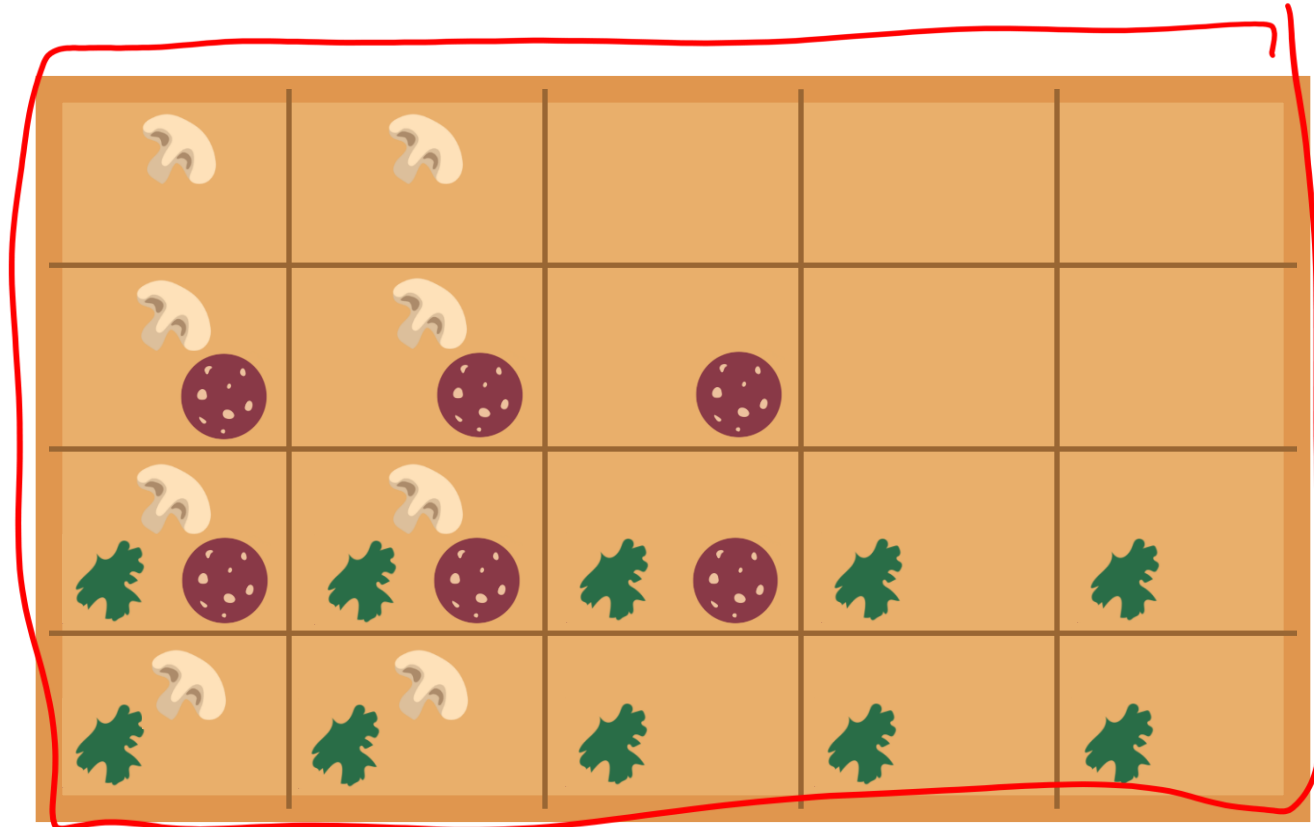
Slide credits: CMU AI and <http://ai.berkeley.edu>

Icons: CC, <https://openclipart.org/detail/296791/pizza-slice>

Omega Pizzeria!

What is the probability of getting a slice with:

- 1) No mushrooms
 - 2) Spinach and no mushrooms
 - 3) Spinach, when asking for slice with no mushrooms
- Mushrooms
 - Spinach
 - No spinach
 - No spinach and mushrooms
 - No spinach when asking for no mushrooms
 - No spinach when asking for mushrooms
 - Spinach when asking for mushrooms
 - No mushrooms and no spinach



Icons: CC, <https://openclipart.org/detail/296791/pizza-slice>

Probability Notation

Notation and conventions in this course

$$P(B = +b, C) = \sum_{a \in \{a_1, a_2, a_3\}} P(\underline{A} = a, B = +b, C) \leftarrow \sum_a P(\underset{\uparrow}{a}, \underset{\uparrow}{+b}, \underset{\uparrow}{C})$$
$$P(+b, C) = \sum_{a \in \{\underline{a}_1, \underline{a}_2, \underline{a}_3\}} P(\underset{\uparrow}{a}, +b, C)$$

■ Random variables:

- ■ Capitalized
 - Represents all potential outcomes
 - e.g. C

■ Outcomes (values):

- ■ lower case
 - e.g. $+b, a_1, a_2, a_3$

$+b$ $-b$

■ Variables for values:

- lower case
- ■ E.g. a

Probability Notation

Notation and conventions in this course

$$P(B = +b, C) = \sum_{a \in \{a_1, a_2, a_3\}} P(A = a, B = +b, C)$$

$$P(+b, C) = \sum_{a \in \{a_1, a_2, a_3\}} P(a, +b, C)$$

Discrete Probability Distributions

Partitions

For each random variable

- Discrete outcomes
- Disjoint outcomes
- Accounts for entire event space
- Not always binary

Discrete Random Variables

(and their domains)

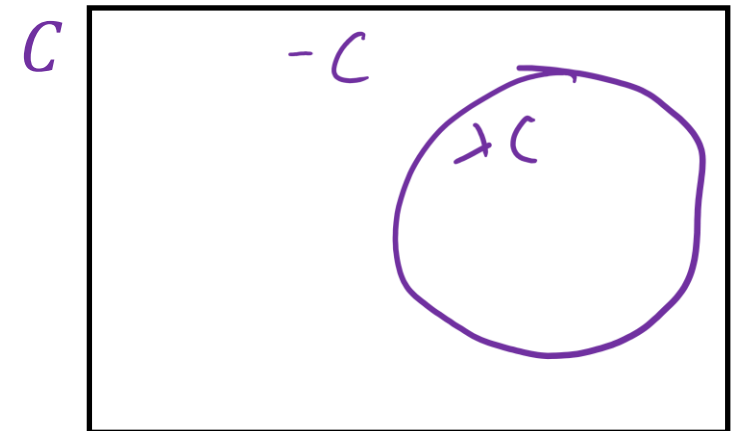
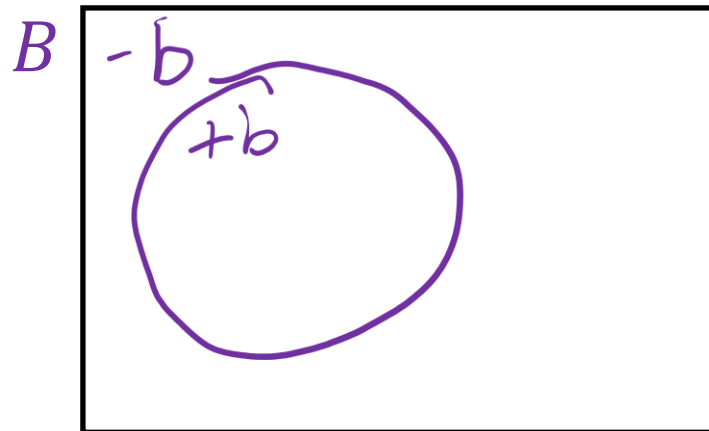
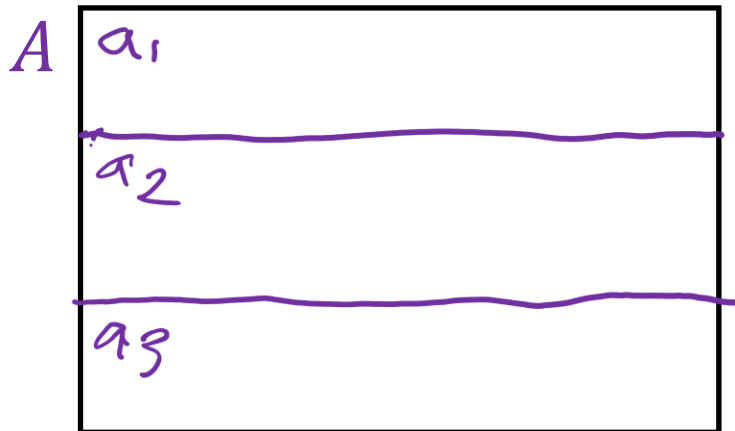
$$A \in \{a_1, a_2, a_3\}$$

$$B \in \{+b, -b\}$$

$$C \in \{+c, -c\}$$

Categorical
Bernoulli

Event space Ω



Discrete Probability Distributions

Marginal distribution

$P(A)$

$P(a_1)$
$P(a_2)$
$P(a_3)$

$P(B)$

$P(+b)$	$P(-b)$
---------	---------

$P(C)$

$P(-c)$	$P(+c)$
---------	---------

Discrete Random Variables
(and their domains)

$A \in \{a_1, a_2, a_3\}$

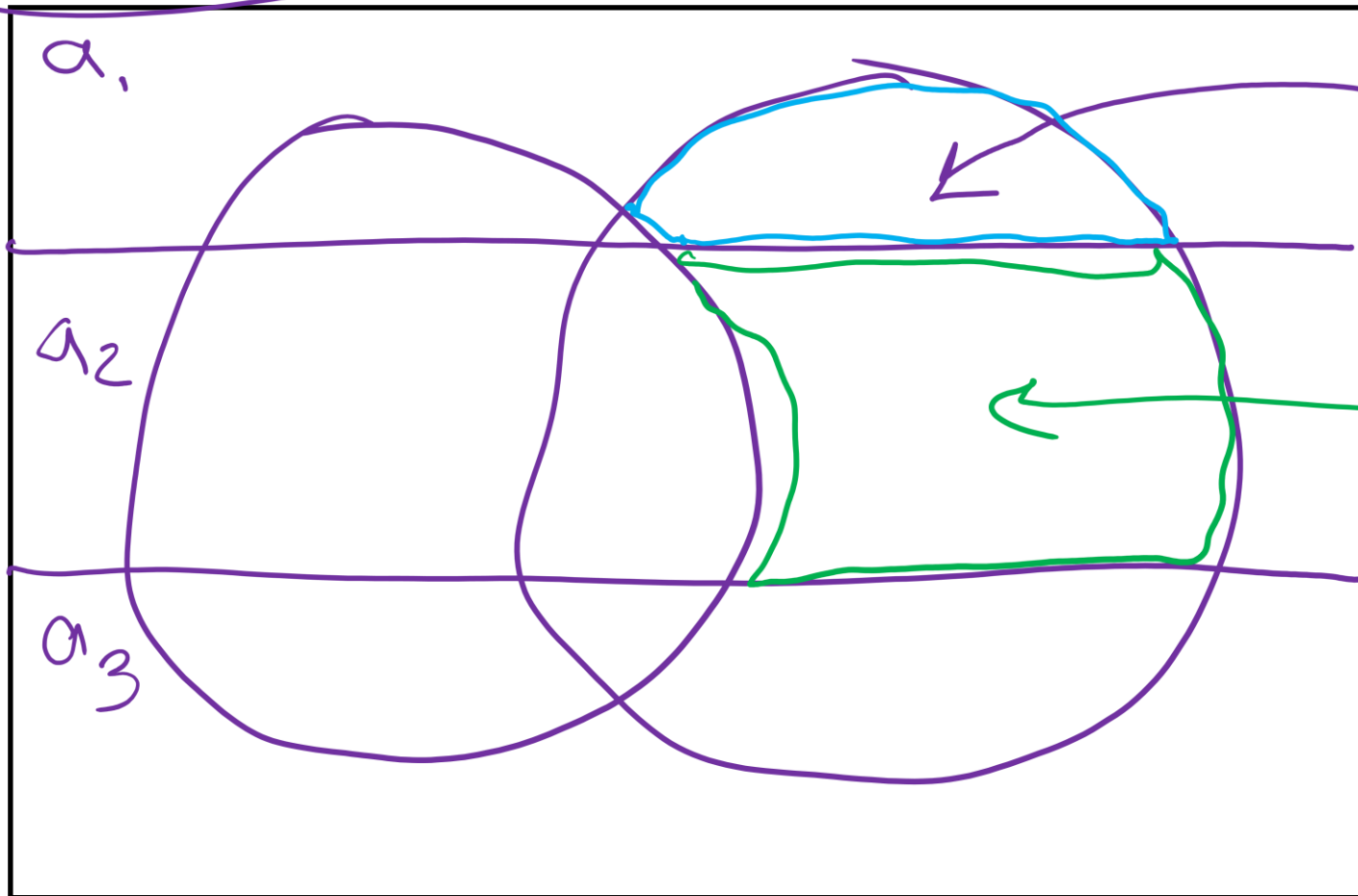
$B \in \{+b, -b\}$

$C \in \{+c, -c\}$

Discrete Probability Distributions

Joint distribution

$P(A, B, C)$



Discrete Random Variables
(and their domains)

$A \in \{a_1, a_2, a_3\}$

$B \in \{+b, -b\}$

$C \in \{+c, -c\}$

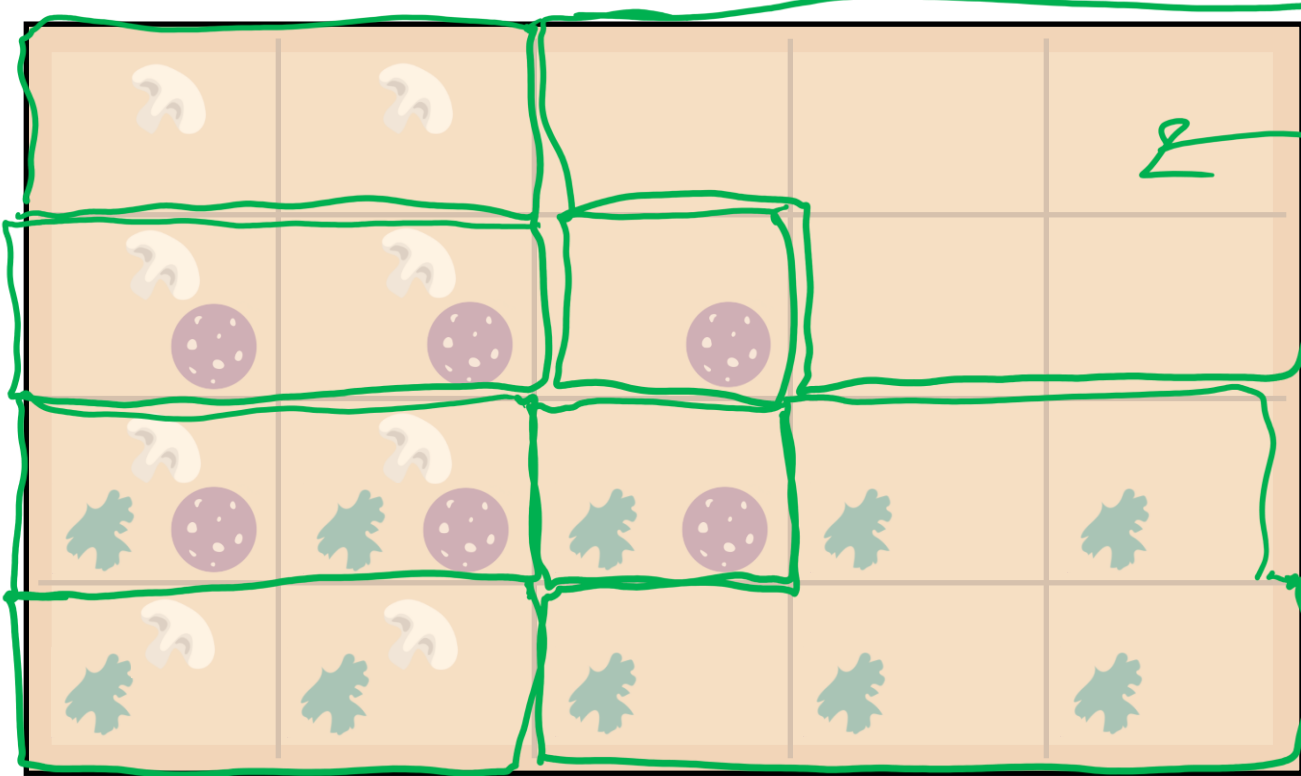
$P(a_1, -b, +c)$

$P(a_1, -b, +c)$

Discrete Probability Distributions

Joint distribution

$P(M, S, R)$



Discrete Random Variables
(and their domains)

$M \in \{\underline{m_1}, m_2\}$

$S \in \{\underline{s_1}, s_2\}$

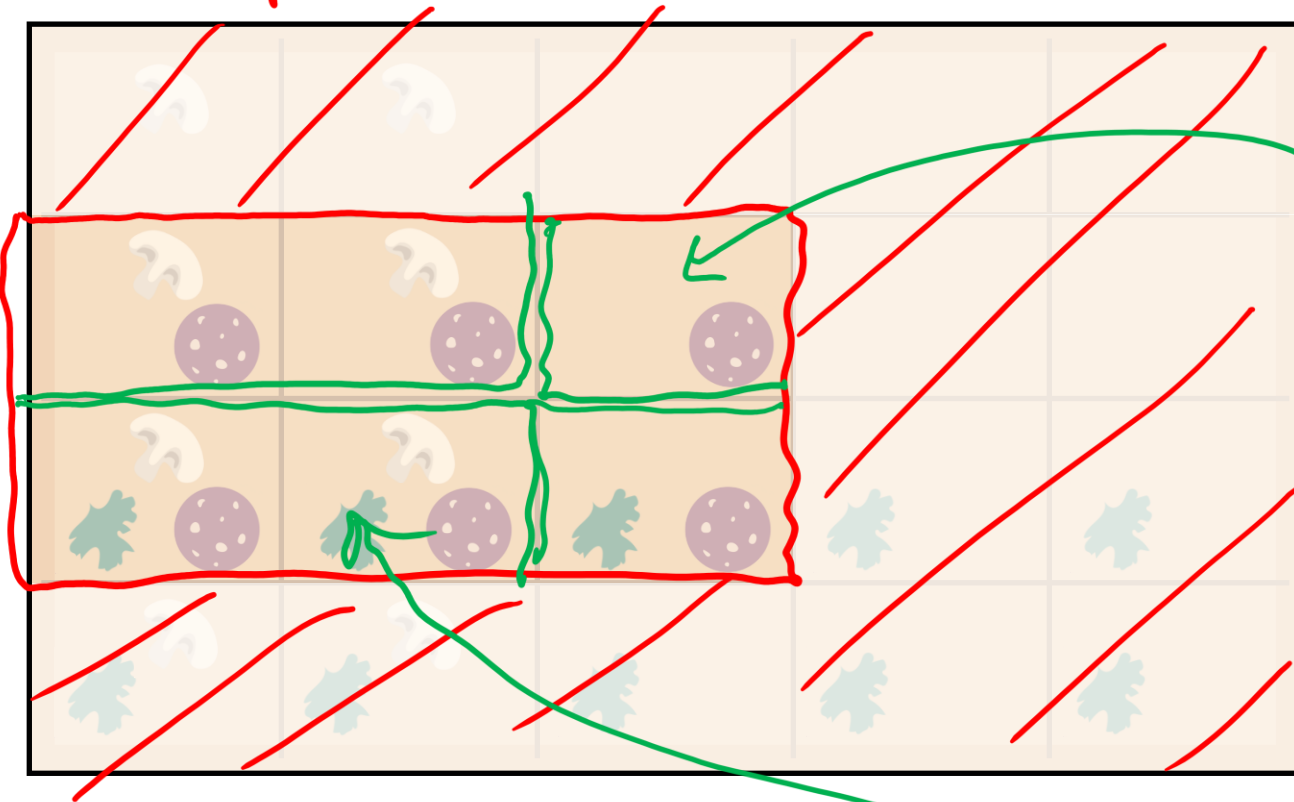
$R \in \{\underline{r_1}, r_2\}$

$$P(m_1, s_1, r_1) = \frac{5}{20}$$

Discrete Probability Distributions

Conditional distribution

$$P(M, S \mid r_2)$$



Discrete Random Variables
(and their domains)

$$M \in \{m_1, m_2\}$$

$$S \in \{s_1, s_2\}$$

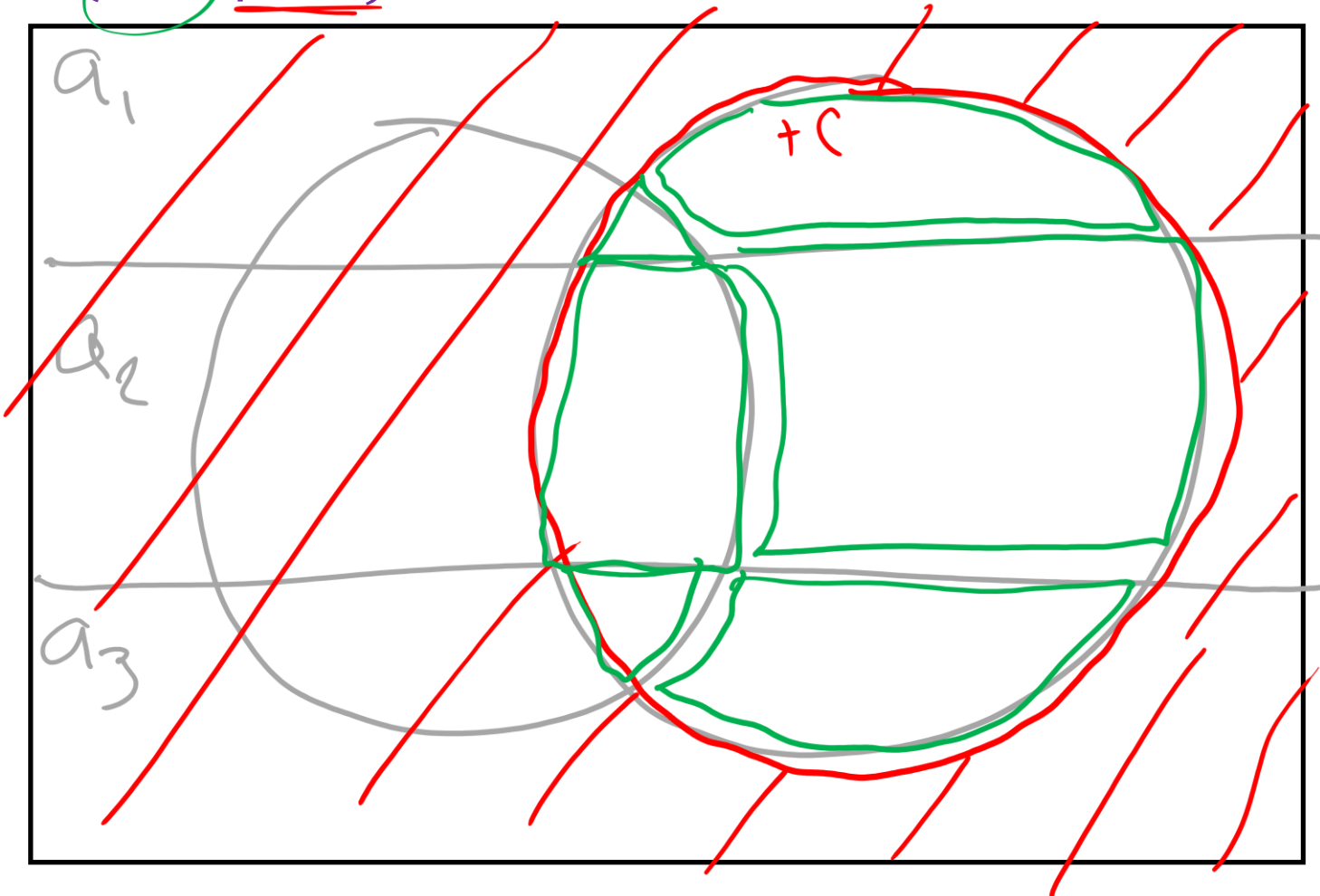
$$R \in \{r_1, r_2\}$$

$$\begin{aligned} P(m_1, s_1 \mid r_2) &= \frac{1}{6} \\ &= \frac{1}{6} \\ P(m_2, s_2 \mid r_2) &= \frac{2}{6} \\ &= \frac{2}{6} \end{aligned}$$

Discrete Probability Distributions

Conditional distribution

$$P(\underbrace{A, B}_{+c})$$



Discrete Random Variables
(and their domains)

$$A \in \{a_1, a_2, a_3\}$$

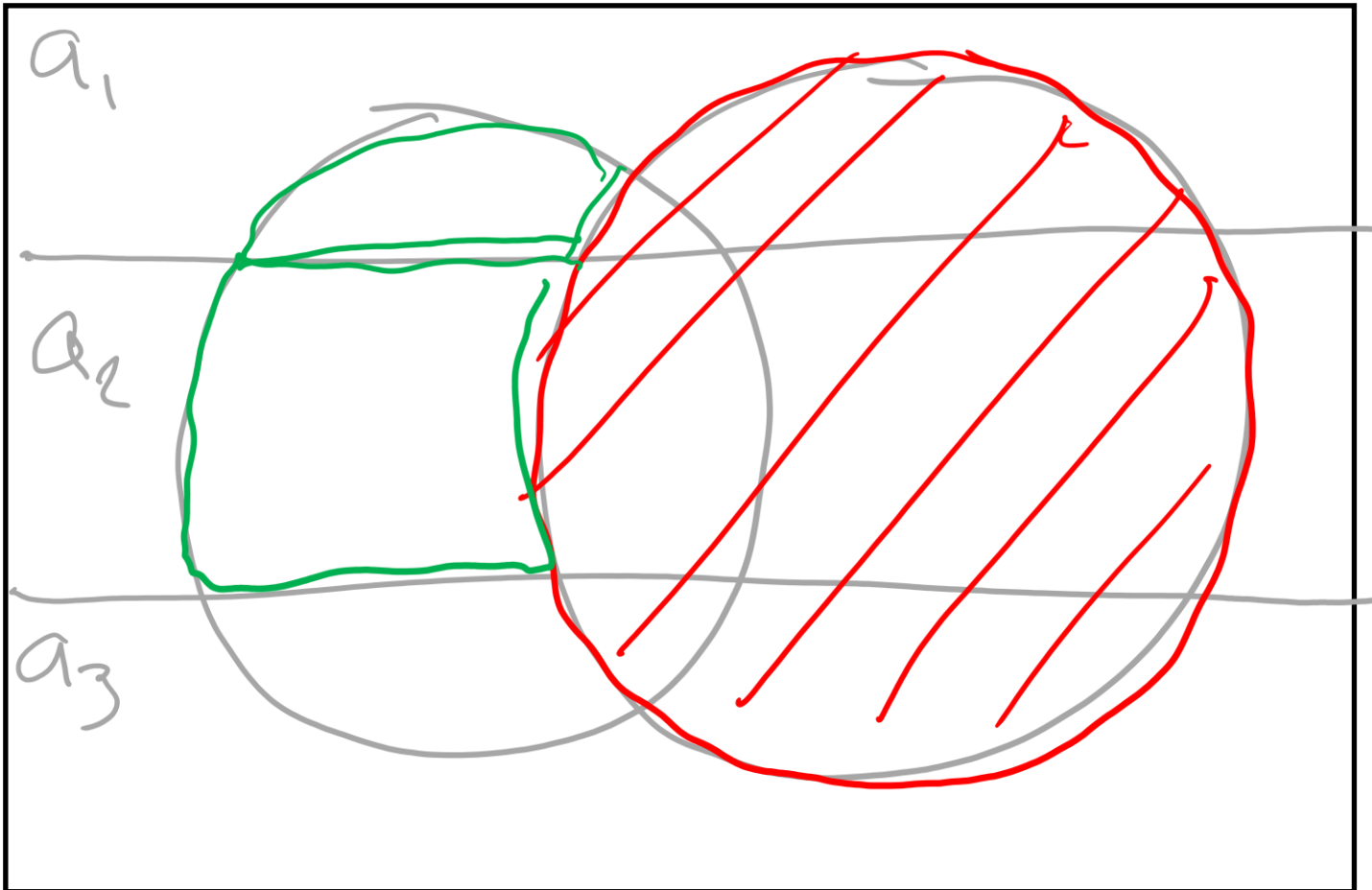
$$B \in \{+b, -b\}$$

$$C \in \{+c, -c\}$$

Discrete Probability Distributions

Conditional distribution

$$P(A, B \mid \underline{-c})$$



Discrete Random Variables
(and their domains)

$$A \in \{a_1, a_2, a_3\}$$

$$B \in \{+b, -b\}$$

$$C \in \{+c, -c\}$$

Discrete Probability Distributions

Conditional distribution

$$P(A, B \mid C)$$

$$P(a_1, +b \mid +c)$$

$$P(a_1, -b \mid +c)$$

$$P(a_2, +b \mid +c)$$

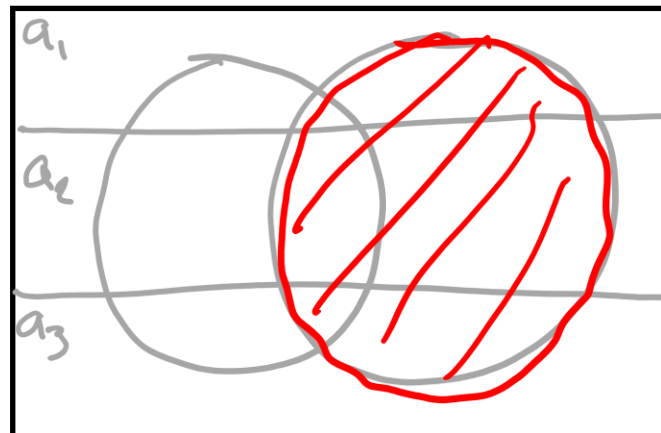
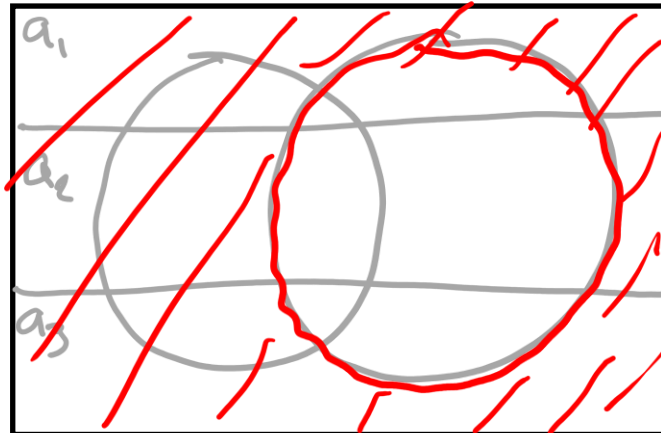
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$$P(a_1, +b \mid -c)$$

$$P(a_1, -b \mid -c)$$

$$P(a_2, +b \mid -c)$$

...



Discrete Random Variables
(and their domains)

$$A \in \{a_1, a_2, a_3\}$$

$$B \in \{+b, -b\}$$

$$C \in \{+c, -c\}$$

2

Poll 1

Which of the following probability tables sum to one?

Select all that apply.

i. $P(A | \underline{b})$

ii. $P(A, b, C)$

iii. $P(A, C | \underline{b})$

iv. $P(a, c | b)$

v. $P(a | B, C)$

vi. $P(c | A)$

$P(A | A)$

$P(a | B) = P(a | +b) + P(a | -b) \times$

$P(\underline{UZ} | \underline{xyz})$

all possible exactly 1

definitely

Poll 1

Which of the following probability tables sum to one?

Select all that apply.

i. $P(A \mid b)$

ii. $P(A, b, C)$

iii. $P(A, C \mid b)$

iv. $P(a, b \mid c)$

v. $P(a \mid B, C)$

vi. $P(c \mid A)$

Probability Tools Summary

Our toolbox

1. Definition of conditional probability

$$P(A|B) = \frac{P(A, B)}{P(B)}$$

2. Product Rule

$$P(B|A)P(A) = P(A, B) = \underline{P(A|B)P(B)}$$

3. Bayes' theorem

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

4. Chain Rule

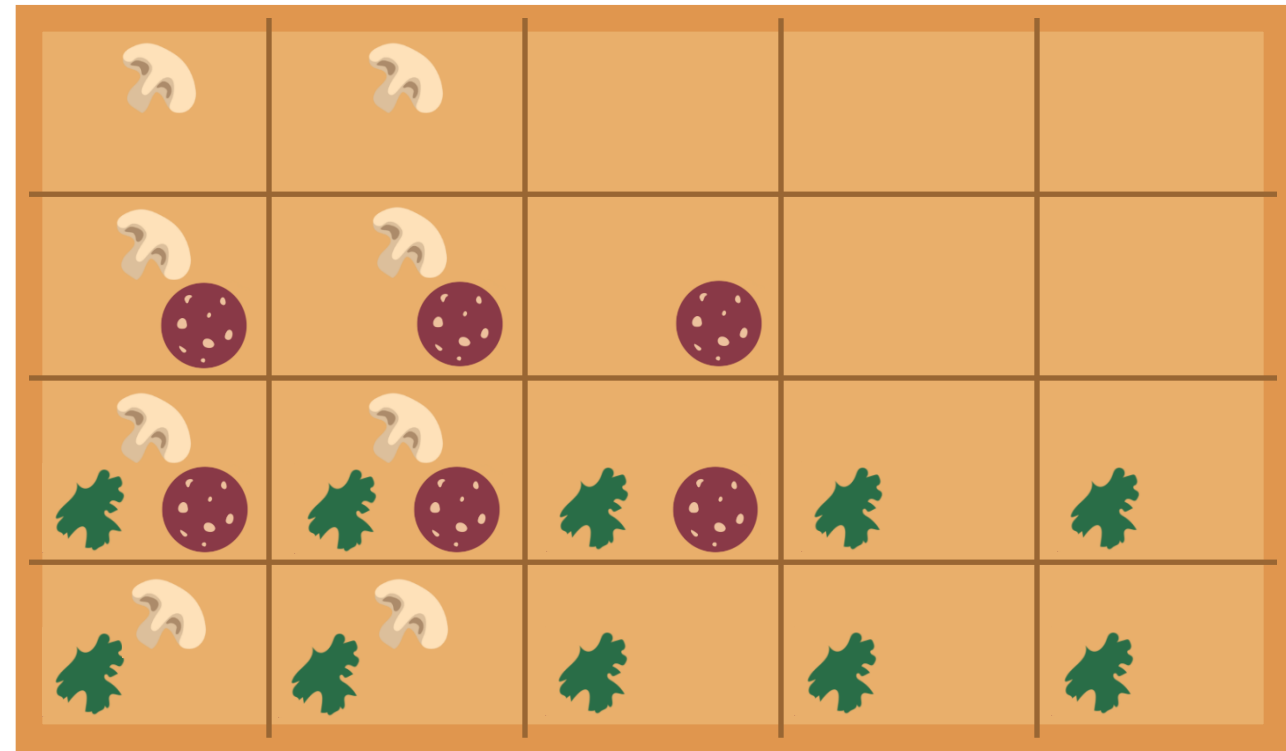
$$P(A, B, C) = P(A)P(B|A)P(C|A, B)$$

$$P(X_1, \dots, X_N) = \prod_{n=1}^N P(X_n | X_1, \dots, X_{n-1})$$

Answer Any Query from Joint Distribution

What is the probability of getting a slice with:

- 1) No mushrooms
 - 2) Spinach and no mushrooms
 - 3) Spinach, when asking for slice with no mushrooms
- Mushrooms
 - Spinach
 - No spinach
 - No spinach and mushrooms
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Icons: CC, <https://openclipart.org/detail/296791/pizza-slice>

Answer Any Query from Joint Distribution

You can answer all of these questions:

$P(M)$

m_1	12/20
m_2	

$P(S)$

s_1	
s_2	

$P(M, S)$

m_1	s_1	
m_1	s_2	6/20
m_2	s_1	
m_2	s_2	

$P(M|s_1)$

m_1	
m_2	

$P(M|s_2)$

m_1	
m_2	

$P(S|m_1)$

s_1	
s_2	6/12

$P(S|m_2)$

s_1	
s_2	

Answer Any Query from Joint Distribution

$P(\text{Weather})?$

$P(\text{Weather} \mid \text{winter})?$

$P(\text{Weather} \mid \text{winter, hot})?$

Season	Temp	Weather	P(S, T, W)
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20

Answer Any Query from Joint Distribution

P(Weather)?

$$P(W=\text{sun}) = [.3 + .1 + .1 + .15] = .65$$

$$P(W=\text{sun}) = \sum_s \sum_t P(s, t, \text{sun})$$

$$P(W=\text{rain})$$



Season	Temp	Weather	P(S, T, W)
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20

Answer Any Query from Joint Distribution

$P(\text{Weather} \mid \text{winter})?$

$$\begin{aligned} P(W=\text{sun} \mid \text{winter}) &= \frac{P(\text{sun}, \text{winter})}{P(\text{winter})} \\ &= \frac{\sum_t P(\text{winter}, t, \text{sun})}{\sum_w \sum_t P(\text{winter}, t, w)} \\ &= \frac{0.1 + 0.15}{0.1 + 0.05 + 0.15 + 0.20} \end{aligned}$$

Season	Temp	Weather	$P(S, T, W)$
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
<u>winter</u>	hot	<u>sun</u>	0.10
<u>winter</u>	hot	rain	0.05
<u>winter</u>	cold	<u>sun</u>	0.15
<u>winter</u>	cold	rain	0.20

$P(s, t, w)$

Answer Any Query from Joint Distribution

(Using normalization trick)

$P(\text{Weather} \mid \text{winter, hot})?$

$$P(\text{sun} \mid \text{winter, hot}) = \frac{0.1}{0.1 + 0.05}$$

$$P(\text{winter, hot, sun}) = 0.1$$

$$P(\text{rain} \mid \text{winter, hot}) = \frac{0.05}{0.1 + 0.05}$$

$$P(\text{winter, hot, rain}) = 0.05$$

Season	Temp	Weather	P(S, T, W)
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20



Additional Probability Tools

- Marginalization (law of total probability) (summing out)

$$P(A) = \sum_b \sum_c P(A, b, c)$$

- Normalization

$$P(B | a) = \frac{P(a, B)}{P(a)} \quad \leftarrow z$$

$$P(B | a) \propto P(a, B)$$

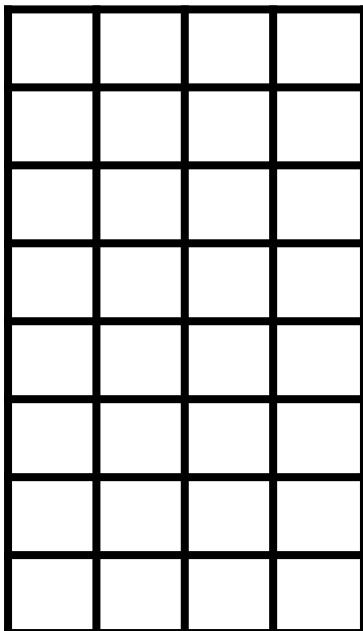
$$P(B | a) = \frac{1}{z} P(a, B)$$

$$z = P(a) = \sum_b P(a, b)$$

Answer Any Query from Joint Distribution

Joint distributions are the best!

Joint





Query

$$P(q_1, q_2 \mid e_1, e_2, e_3)$$

Answer Any Query from Joint Distribution

Two tools to go from joint to query

1. Definition of conditional probability

$$P(A|B) = \frac{P(A, B)}{P(B)}$$

2. Law of total probability (marginalization, summing out)

$$P(A) = \sum_b P(A, b)$$

$$P(Y \mid U, V) = \sum_x \sum_z P(x, Y, z \mid U, V)$$

Answer Any Query from Joint Distribution

Two tools to go from joint to query

Joint: $P(H_1, H_2, Q, E)$

Query: $P(Q | e)$

1. Definition of conditional probability

$$P(Q|e) = \frac{P(Q, e)}{P(e)}$$

2. Law of total probability (marginalization, summing out)

$$P(Q, e) = \sum_{h_1} \sum_{h_2} P(h_1, h_2, Q, e)$$

$$P(e) = \sum_q \sum_{h_1} \sum_{h_2} P(h_1, h_2, q, e)$$

Answer Any Query from Joint Distribution

$P(\text{Weather})?$

$P(\text{Weather} \mid \text{winter})?$

$P(\text{Weather} \mid \text{winter, hot})?$

Season	Temp	Weather	P(S, T, W)
summer	hot	sun	0.30
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winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20

Answer Any Query from Joint Distribution

Joint distributions are the best!

Problems with joints

- We aren't given the joint table
 - Usually some set of conditional probability tables

Joint

of
num
in
 $P(\text{---})$

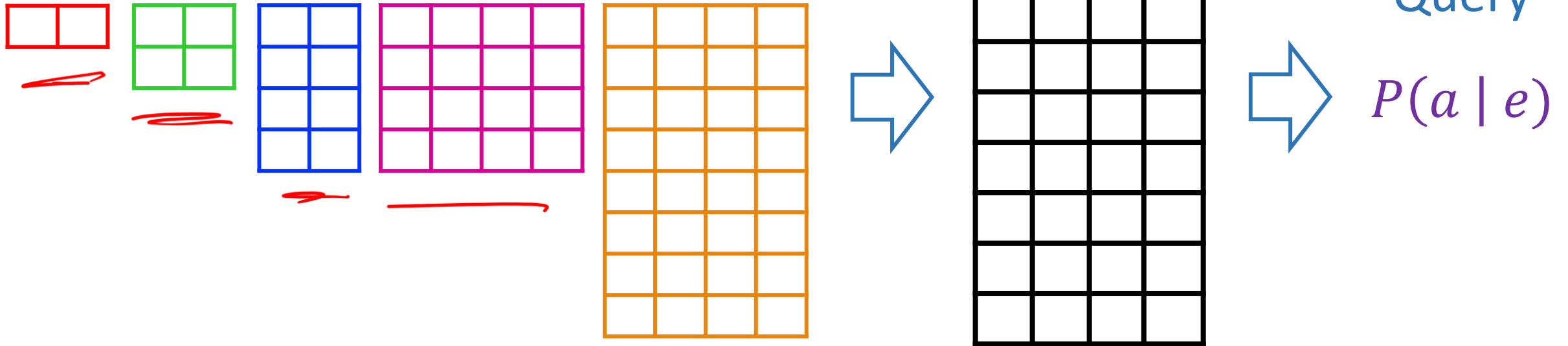
Query

$P(a | e)$



Build Joint Distribution Using Chain Rule

Conditional Probability Tables
and Chain Rule



$$\underbrace{P(A) \ P(B|A) \ P(C|A, B) \ P(D|A, B, C) \ P(E|A, B, C, D)}$$

Build Joint Distribution Using Chain Rule

Two tools to construct joint distribution

1. Product rule

$$\begin{aligned}P(A, B) &= P(A | B)P(B) \\P(A, B) &= P(B | A)P(A)\end{aligned}$$

2. Chain rule

$$P(X_1, X_2, \dots, X_n) = \prod_i P(X_i | X_1, \dots, X_{i-1})$$

$$P(A, B, C) = P(A)P(B | A)P(C | A, B) \quad \text{for ordering } A, B, C$$

$$P(A, B, C) = P(A)P(C | A)P(B | A, C) \quad \text{for ordering } A, C, B$$

$$P(A, B, C) = P(C)P(B | C)P(A | C, B) \quad \text{for ordering } C, B, A$$

CAUTION: Multiplying Probability Tables!

We're going to write things like this quite a bit:

$$P(A, B, C) = P(A) P(B | A) P(C | A, B)$$

Is this regular multiplication?

We have tables... is this matrix multiplication?



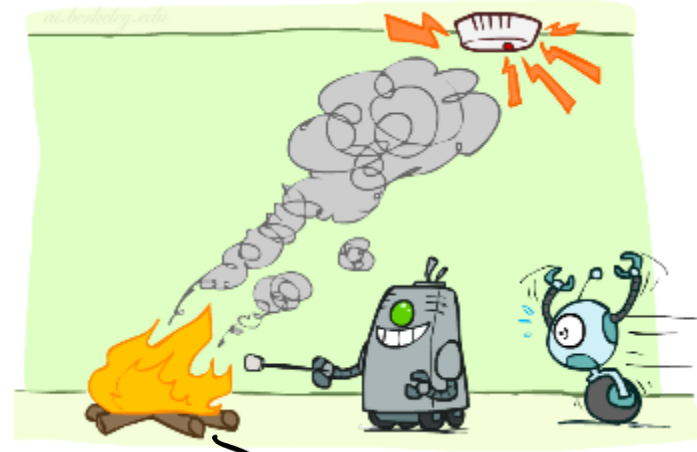
$P(A, B, C)$		$P(A)$		$P(B A)$		$P(C A, B)$		
	$+a$	$-a$	$+a$	$-a$	$+b$	$-b$	$+c$	$-c$
$+b, +c$	0.15		0.1	0.9	0.5	0.5	0.3	0.7
$-b, +c$							0.0	1.0
$+b, -c$							0.7	0.3
$-b, -c$							1.0	0.0

$P(+a, +b, +c)$
 $= P(+a) P(+b | +a) P(+c | +a, +b)$

Build Joint Distribution Using Chain Rule

Binary random variables

- Fire
- Smoke
- Alarm

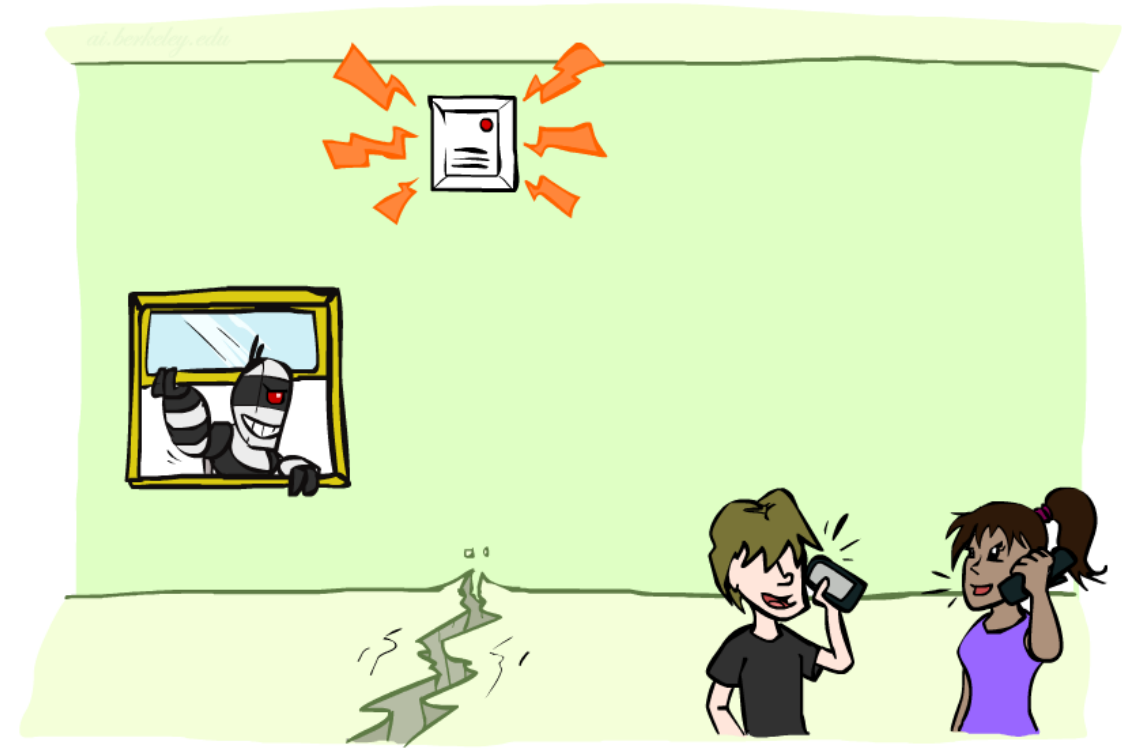


$$P(F, S, A) = P(F)P(S|F)P(A|F, S)$$

Poll 2 (unused)

Variables

- B: Burglary
- A: Alarm goes off
- M: Mary calls
- J: John calls
- E: Earthquake!



How many different ways can we write the chain rule?

A. 1

B. 5

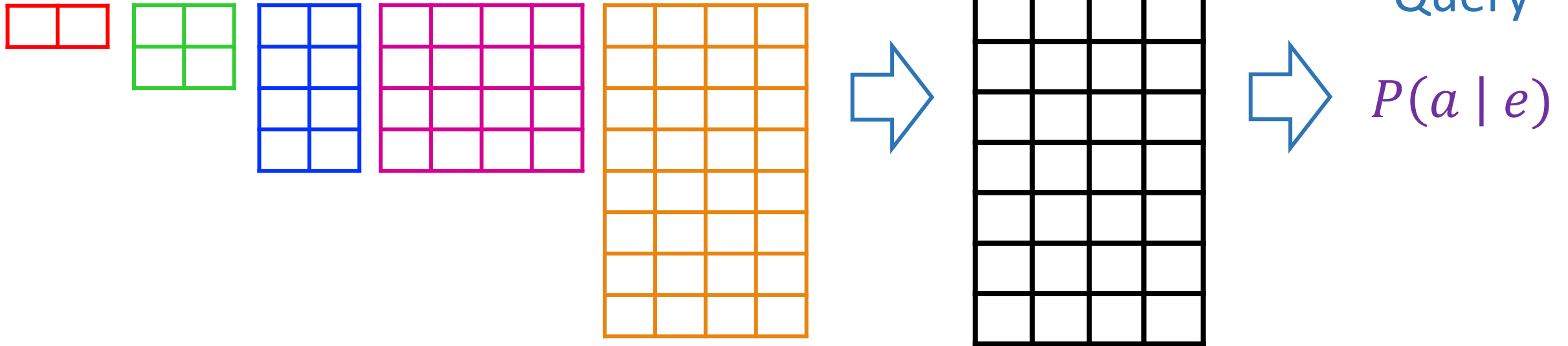
C. 5 choose 5

D. 5!

E. 5^5

Build Joint Distribution Using Chain Rule

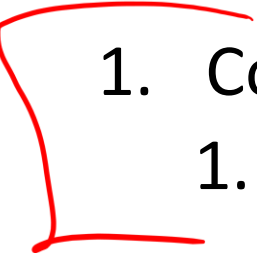
Conditional Probability Tables
and Chain Rule



$$P(A) \ P(B|A) \ P(C|A, B) \ P(D|A, B, C) \ P(E|A, B, C, D)$$

Answer Any Query from Condition Probability Tables

Process to go from (specific) conditional probability tables to query



1. Construct the joint distribution

1. Product Rule or Chain Rule

2. Answer query from joint

1. Definition of conditional probability

2. Law of total probability (marginalization, summing out)

Answer Any Query from Condition Probability Tables

Example

Given: $P(E|H, Q)$, $P(H|Q)$, $P(Q)$ Query: $P(Q | e)$

1. Construct the **joint** distribution

1. Product Rule or Chain Rule

$$P(E, H, Q) = P(E|H, Q) P(H|Q) P(Q)$$

2. Answer **query** from **joint**

1. Definition of conditional probability

$$P(Q | e) = \frac{P(e, Q)}{P(e)}$$

2. Law of total probability (marginalization, summing out)

$$P(Q | e) = \frac{\sum_h P(e, h, Q)}{\sum_h \sum_q P(e, h, q)}$$