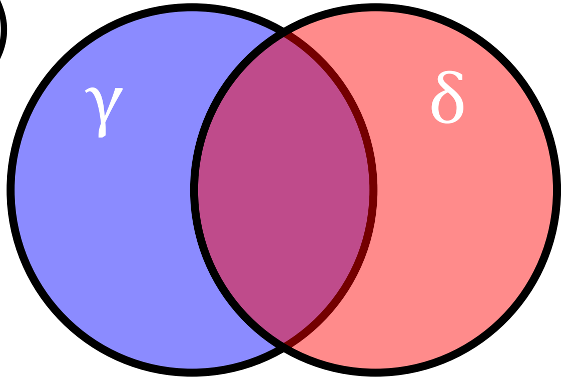
Warm-up

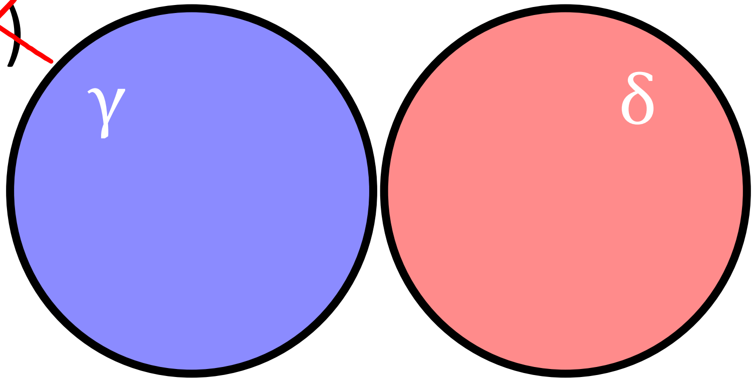
$\gamma \models \delta$ iff if a model satisfies γ, then it satisfies δ

- The regions below visually enclose the set of models that satisfy the respective sentence γ or δ. For which of the following diagrams does γ entail δ. Select all that apply.

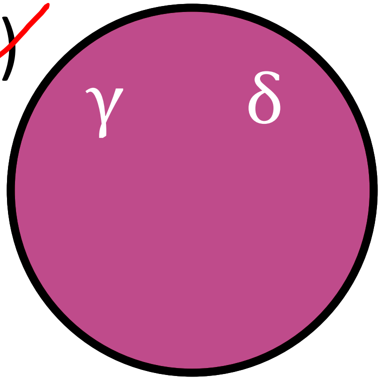
A)



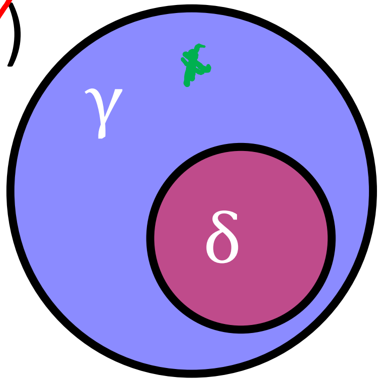
B)



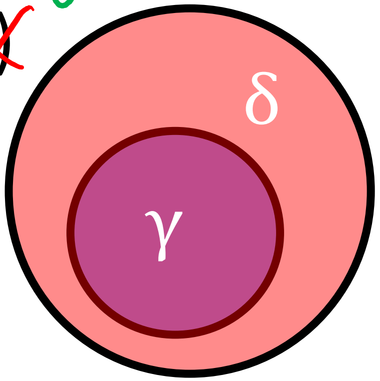
C)



D)



E)

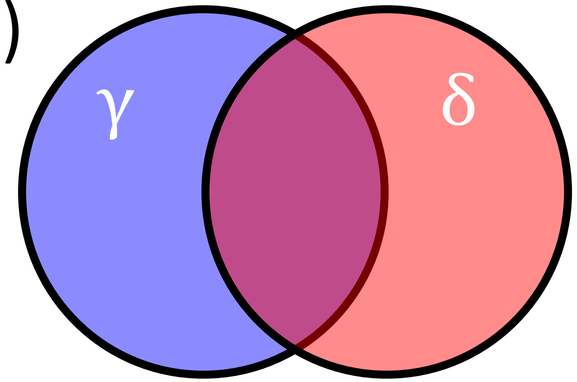


Warm-up

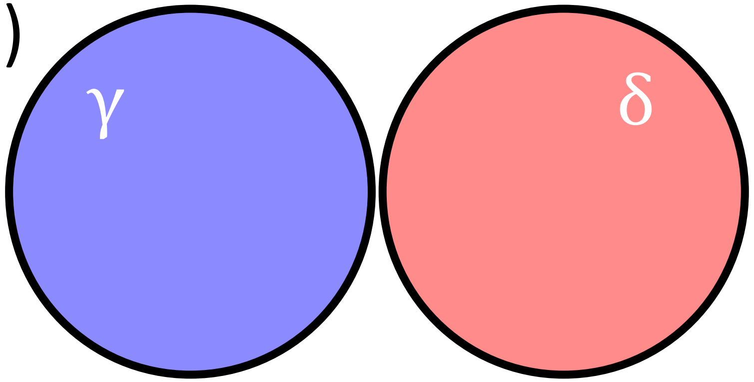
$\gamma \vDash \delta$: iff in every world where γ is true, δ is also true

- The regions below visually enclose the set of models that satisfy the respective sentence γ or δ. For which of the following diagrams does γ entail δ. Select all that apply.

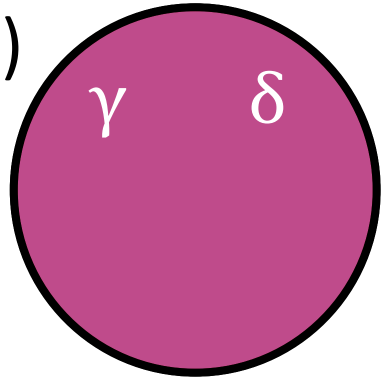
A)



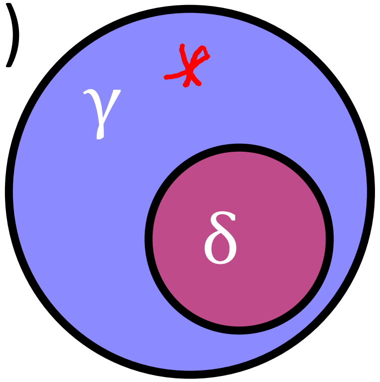
B)



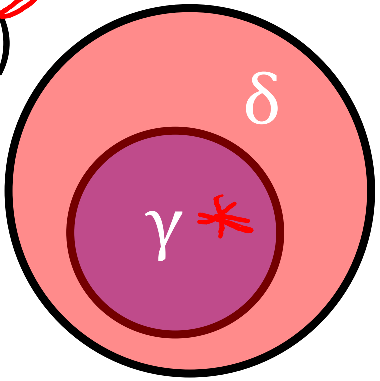
C)



D)



E)



AI: Representation and Problem Solving

Boolean Satisfiability Problem (SAT)

& Logical Agents

Instructors: Fei Fang & Pat Virtue

Slide credits: CMU AI, http://ai.berkeley.edu

Announcements

- Midterm 1 Exam
  - Tue 10/1, in class
- Assignments:
  - HW4
    - Due Tue 9/24, 10 pm
  - P2: Logic and Planning
    - Out today
    - Due Sat 10/5, 10 pm

Learning Objectives

- Describe the definition of (Boolean) Satisfiability Problem (SAT)
- Describe the definition of Conjunctive Normal Form (CNF)
- Describe the following algorithms for solving SAT
  - DPLL, CDCL, WalkSAT, GSAT
- Determine whether a sentence is satisfiable
- Describe Successor-State Axiom
- Describe and implement SATPlan (Planning as Satisfiability)
- (Hybrid Agent)

Logical Agent Vocab: Recap

- Symbol: Variable that can be true or false
- Model: Complete assignment of symbols to True/False
- Operators: $\neg A$ (not), $A \wedge B$ (conjunction), $A \vee B$ (disjunction), $A \Rightarrow B$ (implication), $A \Leftrightarrow B$ (biconditional)
- Sentence: A logical statement composed of logic symbols and operators
- KB: Collection of sentences representing facts and rules we know about the world
- Query: Sentence we want to know if it is *provably* True, *provably* False, or *unsure*.

Logical Agent Vocab: Recap

- Entail
  - Does sentence1 entail sentence2?
  - Input: sentence1, sentence2
  - Output: True if each model that satisfies sentence1 must also satisfy sentence2; False otherwise
  - "If I know 1 holds, then I know 2 holds"

- Satisfy
  - Does model satisfy sentence?
  - Input: model, sentence
  - Output: True if this sentence is true in this model; False otherwise
  - "Does this particular state of the world work?”

(Boolean) Satisfiability Problem (SAT)

- Satisfiable
  - Is sentence satisfiable?
  - Input: sentence
  - Output: True if at least one model satisfies sentence
  - "Is it possible to make this sentence true?"

- SAT problem is the problem of determining the satisfiability of a sentence
  - SAT is a typical problem for logical agents
  - SAT is the first problem proved to be NP-complete
  - If satisfiable, we often want to know what that model is

SAT and Entailment

- A sentence is *satisfiable* if it is true in at least one world
- Suppose we have a hyper-efficient SAT solver; how can we use it to test entailment?
    - Suppose $\alpha \models \beta$
    - Then $\alpha \Rightarrow \beta$ is true in all worlds
    - Hence $\neg(\alpha \Rightarrow \beta)$ is false in all worlds
    - Hence $\alpha \wedge \neg\beta$ is false in all worlds, i.e., unsatisfiable

- More generally, to prove a sentence is valid (i.e., true in all models), introduce the negated claim and test for unsatisfiability; also known as *reductio ad absurdum (reduction to absurdity)*

SAT and CSPs

- SAT problems are essentially CSPs with Boolean variables
  - Can apply backtracking based algorithms
  - Can apply local search algorithms

- Naïve way to solve SAT: Truth table enumeration
- Efficient SAT solvers operate on ***conjunctive normal form***
  - Often based on backtracking and local search

Propositional Logical Vocab: Recap

- Literal
  - Atomic sentence: True, False, Symbol, ¬Symbol

- Clause
  - Disjunction of literals: $A \vee B \vee \neg C$

- Definite clause
  - Disjunction of literals, *exactly one* is positive
  - $\neg A \vee B \vee \neg C$

- Horn clause
  - Disjunction of literals, *at most one* is positive
  - All definite clauses are Horn clauses

Conjunctive Normal Form (CNF)

- Every sentence can be expressed as a conjunction of clauses
- Each clause is a disjunction of literals
- Each literal is a symbol or a negated symbol
- We can convert a sentence to CNF through a sequence of standard transformations

Conjunctive Normal Form (CNF)

- Original sentence:
  - $A \Rightarrow (B \Leftrightarrow C)$
- Biconditional Elimination: Replace biconditional by two implications
  - $A \Rightarrow ((B \Rightarrow C) \wedge (C \Rightarrow B))$
- Implication Elimination: Replace $\alpha \Rightarrow \beta$ by $\neg\alpha \vee \beta$
  - $\neg A \vee ((\neg B \vee C) \wedge (\neg C \vee B))$
- Distribution: Distribute v over $\wedge$, i.e., replace $\alpha \vee (\beta \wedge \gamma)$ by $(\alpha \vee \beta) \wedge (\alpha \vee \gamma)$
  - $(\neg A \vee \neg B \vee C) \wedge (\neg A \vee \neg C \vee B)$

Conjunctive Normal Form (CNF)

- Original sentence:
  - $(\neg(A \vee B) \vee C) \wedge (\neg C \wedge A)$
- De Morgan's Law: Replace $\neg(\alpha \vee \beta)$ by $\neg\alpha \wedge \neg\beta$, and $\neg(\alpha \wedge \beta)$ by $\neg\alpha \vee \neg\beta$
  - $((\neg A \wedge \neg B) \vee C) \wedge (\neg C \wedge A)$
- Distribution: Distribute v over $\wedge$, i.e., replace $\alpha \vee (\beta \wedge \gamma)$ by $(\alpha \vee \beta) \wedge (\alpha \vee \gamma)$
  - $(\neg A \vee C) \wedge (\neg B \vee C) \wedge (\neg C \wedge A)$

Other Logical Equivalences

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) \quad \text{De Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) \quad \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

DPLL Algorithm

- DPLL (Davis-Putnam-Logemann-Loveland) is the core of modern SAT solvers
- Essentially a backtracking search over models with several tricks:
  - *Early termination*: stop if
    - all clauses are satisfied; e.g., $(A \vee B) \wedge (A \vee \neg C)$ is satisfied by {A=true}

    SAT solver can stop with partial models; no need to assign all variables (can assign arbitrarily if a complete model is needed).

    - any clause is falsified; e.g., $(A \vee B) \wedge (A \vee \neg C)$ is satisfied by {A=false, B=false}

    Stop when a conflict is found. Similar to backtracking algorithm for general CSPs.

DPLL Algorithm

- DPLL (Davis-Putnam-Logemann-Loveland) is the core of modern SAT solvers
- Essentially a backtracking search over models with several tricks:
  - *Early termination*
  - *Pure symbols*: if all occurrences of a symbol in as-yet-unsatisfied clauses have the same sign, then give the symbol that value
    - E.g., A is pure and positive in $(A \lor B) \land (A \lor \neg C) \land (C \lor \neg B)$ so set it to true

A = false

Claim: If a sentence has a model to satisfy it, then it has a model in which the pure symbols are assigned values that make their literals true. Why?

W.l.o.g., assume symbol A shows up in all clauses as A. Assume there is a model satisfies the sentence with A=false. Then construct a new model with A=true and everything else the same. Since there are no opposite sign literals, making A=true that could make any clause be false.

DPLL Algorithm

- DPLL (Davis-Putnam-Logemann-Loveland) is the core of modern SAT solvers
- Essentially a backtracking search over models with several tricks:
  - *Early termination*
  - *Pure symbols*: if all occurrences of a symbol in as-yet-unsatisfied clauses have the same sign, then give the symbol that value
    - E.g., A is pure and positive in $(A \lor B) \land (A \lor \neg C) \land (C \lor \neg B)$ so set it to true

Note: In determining the purity of a symbol, the algorithm can ignore clauses that are already known to be true in the model constructed so far

DPLL Algorithm

- DPLL (Davis-Putnam-Logemann-Loveland) is the core of modern SAT solvers
- Essentially a backtracking search over models with several tricks:
  - *Early termination*
  - *Pure symbols*
  - *Unit clauses*: A unit clause is a clause in which all literals but one are already assigned false by the model (i.e., left with a single literal that can potentially satisfy the clause). Set the remaining symbol of a unit clause to satisfy it.
    - E.g., if A=false and the sentence (in CNF) has a clause $(A \vee B)$, then set B true

      Similar to Generalized Forward Checking (nFC0) for general CSPs

    - Unit propagation: Assigning values to the symbol in a unit clause can lead to new unit clauses. Iteratively find unit clauses until no more remains.

      Similar to Constraint Propagation for general CSPs

DPLL Algorithm

```
function DPLL(clauses, symbols, model) returns true or false
    if every clause in clauses is true in model then return true
    if some clause in clauses is false in model then return false

    P, value ←FIND-PURE-SYMBOL(symbols, clauses, model)
    if P is non-null then return DPLL(clauses, symbols–P, modelU{P=value})

    P, value ←FIND-UNIT-CLAUSE(clauses, model)
    if P is non-null then return DPLL(clauses, symbols–P, modelU{P=value})

    P ← First(symbols)
    rest ← Rest(symbols)

    return or(DPLL(clauses, rest, modelU{P=true}),
              DPLL(clauses, rest, modelU{P=false}))
```

Early termination

Essentially backtracking

POLL Problem

Is a sentences in CNF with the following clauses satisfiable?

$x_1 \lor x_4$

$x_1 \lor \neg x_3 \lor \neg x_8$

$x_1 \lor x_8 \lor x_{12}$

$x_2 \lor x_{11}$

$\neg x_7 \lor \neg x_3 \lor x_9$

$\neg x_7 \lor x_8 \lor \neg x_9$

$x_7 \lor x_8 \lor \neg x_{10}$

$x_7 \lor x_{10} \lor \neg x_{12}$

A. Yes
B. No

POLL Problem

Is a sentences in CNF with the following clauses satisfiable?

$x_1 \vee x_4$
$x_1 \vee \neg x_3 \vee \neg x_8$
$x_1 \vee x_8 \vee x_{12}$
$x_2 \vee x_{11}$
$\neg x_7 \vee \neg x_3 \vee x_9$
$\neg x_7 \vee x_8 \vee \neg x_9$
$x_7 \vee x_8 \vee \neg x_{10}$
$x_7 \vee x_{10} \vee \neg x_{12}$

→

$x_1 \vee x_4$
$x_1 \vee \neg x_3 \vee \neg x_8$
$x_1 \vee x_8 \vee x_{12}$
$x_2 \vee x_{11}$
$\neg x_7 \vee \neg x_3 \vee x_9$
$\neg x_7 \vee x_8 \vee \neg x_9$
$x_7 \vee x_8 \vee \neg x_{10}$
$x_7 \vee x_{10} \vee \neg x_{12}$

Pure symbol x_1 =true
Pure symbol x_2 =true
Pure symbol x_3 =false
Pure symbol x_4 =true
Pure symbol x_{11} =true

New pure symbol x_8=true
New pure symbol x_7=true
All constraints satisfied

DPLL Algorithm

Clauses:

$\neg a \vee b \vee c$

$a \vee c \vee d$

$a \vee c \vee \neg d$

$a \vee \neg c \vee d$

$a \vee \neg c \vee \neg d$

$\neg b \vee \neg c \vee d$

$\neg a \vee b \vee \neg c$

$\neg a \vee \neg b \vee c$

Assign $a = true$

DPLL Algorithm

Clauses:

$\neg a \vee b \vee c$

$a \vee c \vee d$

$a \vee c \vee \neg d$

$a \vee \neg c \vee d$

$a \vee \neg c \vee \neg d$

$\neg b \vee \neg c \vee d$

$\neg a \vee b \vee \neg c$

$\neg a \vee \neg b \vee c$

Assign $a = true$

Assign $b = true$

DPLL Algorithm

Clauses:

$\neg a \vee b \vee c$

$a \vee c \vee d$

$a \vee c \vee \neg d$

$a \vee \neg c \vee d$

$a \vee \neg c \vee \neg d$

$\neg b \vee \neg c \vee d$

$\neg a \vee b \vee \neg c$

$\neg a \vee \neg b \vee c$

F F

Assign $a = true$

Assign $b = true$

Find unit clause $\neg a \vee \neg b \vee c$, so $c = true$

DPLL Algorithm

Clauses:

$\neg a \vee b \vee c$
$a \vee c \vee d$
$a \vee c \vee \neg d$
$a \vee \neg c \vee d$
$a \vee \neg c \vee \neg d$
$\neg b \vee \neg c \vee d$
$\neg a \vee b \vee \neg c$
$\neg a \vee \neg b \vee c$

Assign $a = true$

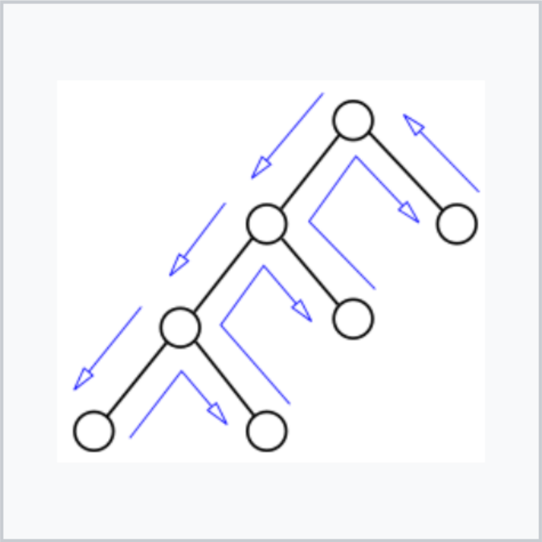
Assign $b = true$

Find unit clause $\neg a \vee \neg b \vee c$, so $c = true$

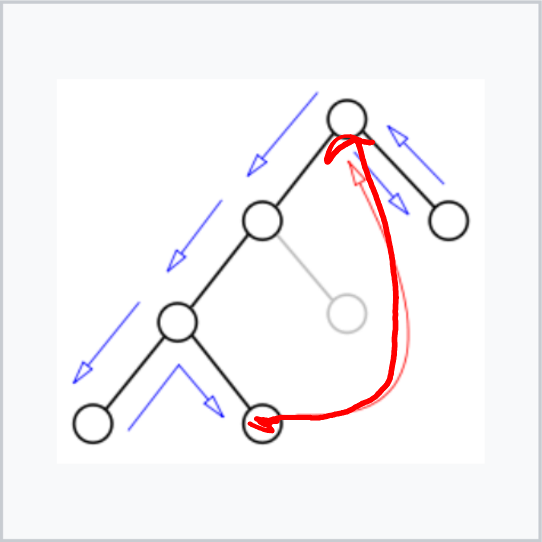
Find unit clause $\neg b \vee \neg c \vee d$, so $d = true$

Backjumping

- Backjumping is a technique in backtracking algorithms
- Go up more than one level in the search tree when backtrack



A search tree visited by regular backtracking



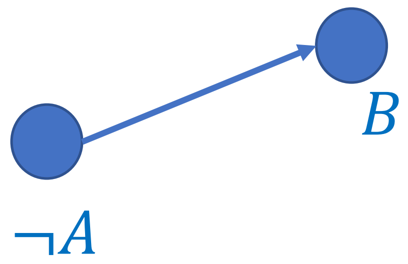
A backjump: the grey node is not visited

https://en.wikipedia.org/wiki/Backjumping

Implication Graph

- A directed graph $G = (V, E)$ composed of vertex set V and directed edge set E. Each vertex in V represents the truth status of a Boolean literal, and each directed edge from vertex u to vertex v represents the implication "If the literal u is true then the literal v is also true".

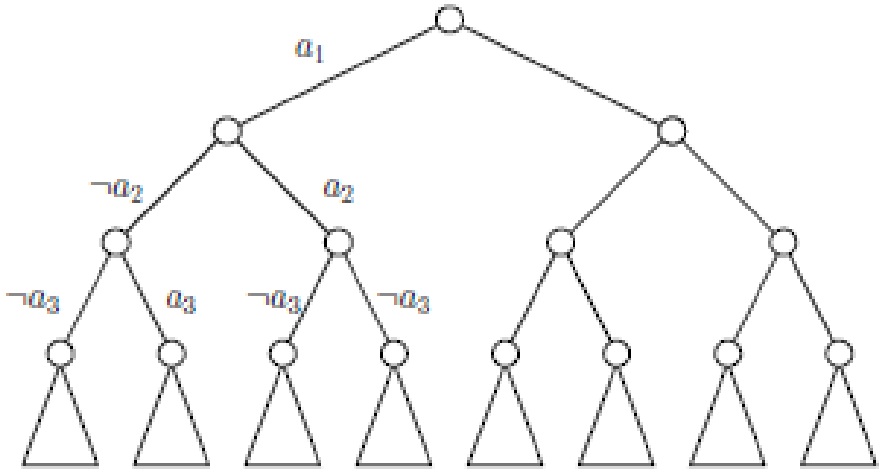
Example: Given a clause (A ∨ B), A=false implies B=true



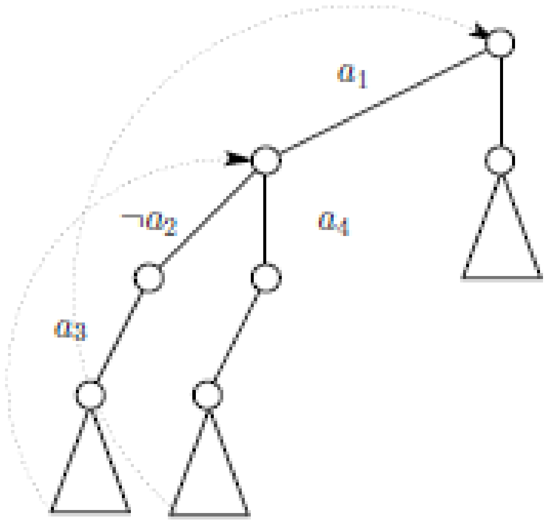
https://en.wikipedia.org/wiki/Implication_graph

Conflict Driven Clause Learning (CDCL)

- Use implication graph
- Use non-chronological backjumping



DPLL



CDCL

Conflict Driven Clause Learning (CDCL)

1. Select a variable and assign True or False
2. Apply unit propagation to build the implication graph
3. If there is any conflict
   a) Find the cut in the implication graph that led to the conflict
   b) Derive a new clause which is the negation of the assignments that led to the conflict
   c) Backjump to the appropriate decision level, where the first-assigned variable involved in the conflict was assigned
4. Otherwise continue from step 1 until all variable values are assigned

https://en.wikipedia.org/wiki/Conflict-driven_clause_learning

Conflict Driven Clause Learning (CDCL)

$x_1 \lor x_4$

$x_1 \lor \neg x_3 \lor \neg x_8$

$x_1 \lor x_8 \lor x_{12}$

$x_2 \lor x_{11}$

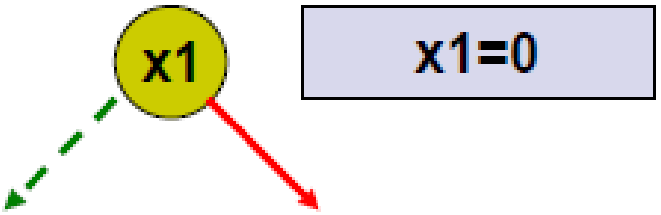
$\neg x_7 \lor \neg x_3 \lor x_9$

$\neg x_7 \lor x_8 \lor \neg x_9$

$x_7 \lor x_8 \lor \neg x_{10}$

$x_7 \lor x_{10} \lor \neg x_{12}$

Step 1



x1=0

Conflict Driven Clause Learning (CDCL)

Step 2

$x_1 \lor x_4$

$x_1 \lor \neg x_3 \lor \neg x_8$

$x_1 \lor x_8 \lor x_{12}$

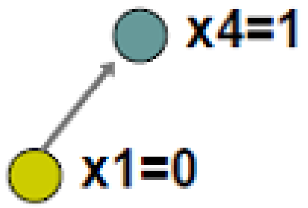
$x_2 \lor x_{11}$

$\neg x_7 \lor \neg x_3 \lor x_9$

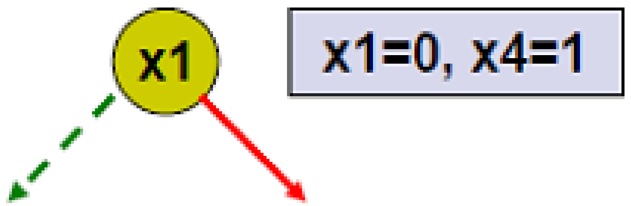
$\neg x_7 \lor x_8 \lor \neg x_9$

$x_7 \lor x_8 \lor \neg x_{10}$

$x_7 \lor x_{10} \lor \neg x_{12}$



Build the implication graph



Conflict Driven Clause Learning (CDCL)

$x_1 \vee x_4$

$x_1 \vee \neg x_3 \vee \neg x_8$

$x_1 \vee x_8 \vee x_{12}$

$x_2 \vee x_{11}$

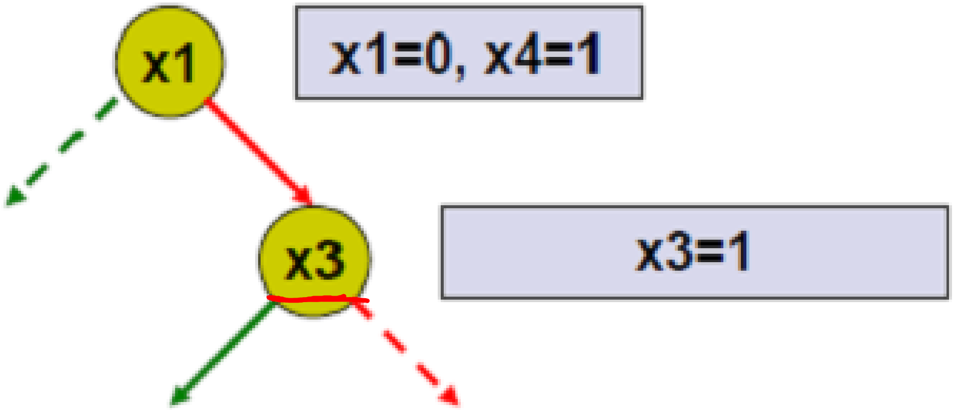
$\neg x_7 \vee \neg x_3 \vee x_9$

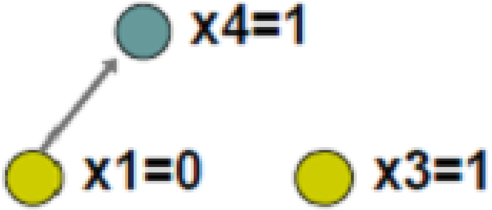
$\neg x_7 \vee x_8 \vee \neg x_9$

$x_7 \vee x_8 \vee \neg x_{10}$

$x_7 \vee x_{10} \vee \neg x_{12}$

Step 3





Conflict Driven Clause Learning (CDCL)

$x_1 \vee x_4$

$x_1 \vee \neg x_3 \vee \neg x_8$

$x_1 \vee x_8 \vee x_{12}$

$x_2 \vee x_{11}$

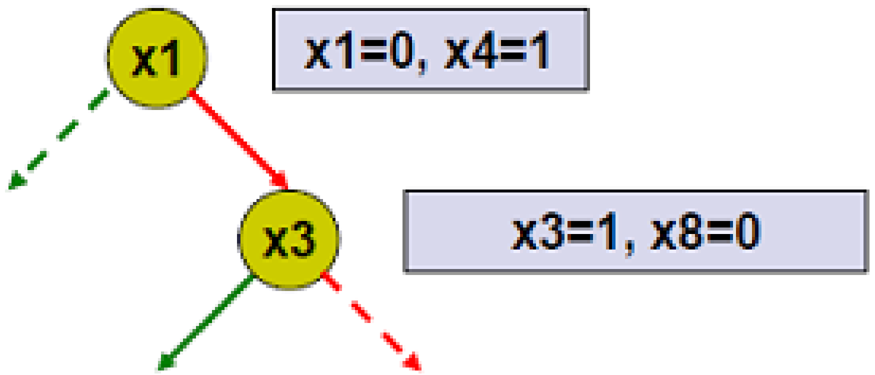
$\neg x_7 \vee \neg x_3 \vee x_9$

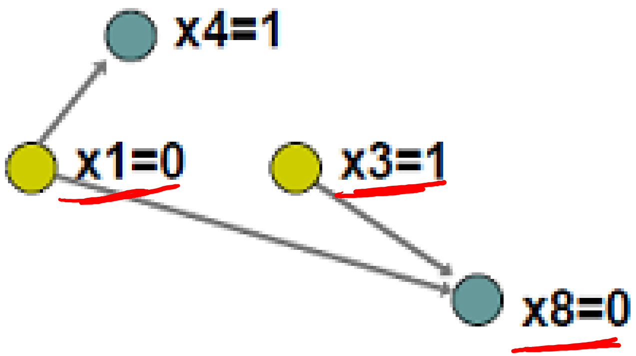
$\neg x_7 \vee x_8 \vee \neg x_9$

$x_7 \vee x_8 \vee \neg x_{10}$

$x_7 \vee x_{10} \vee \neg x_{12}$

Step 4





Conflict Driven Clause Learning (CDCL)

$x_1 \lor x_4$

$x_1 \lor \neg x_3 \lor \neg x_8$

$x_1 \lor x_8 \lor x_{12}$

$x_2 \lor x_{11}$

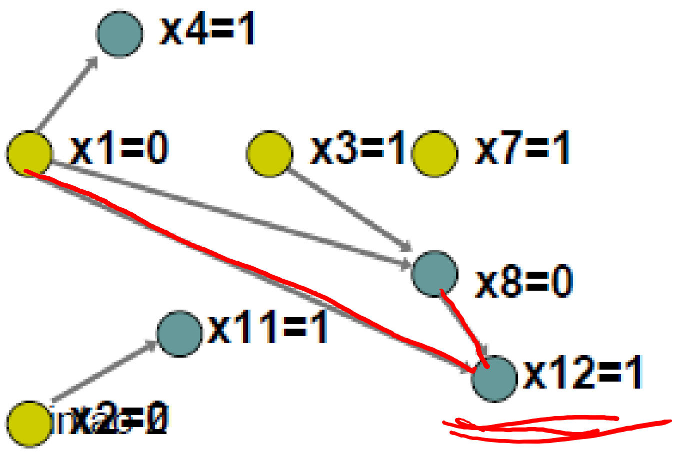
$\neg x_7 \lor \neg x_3 \lor x_9$

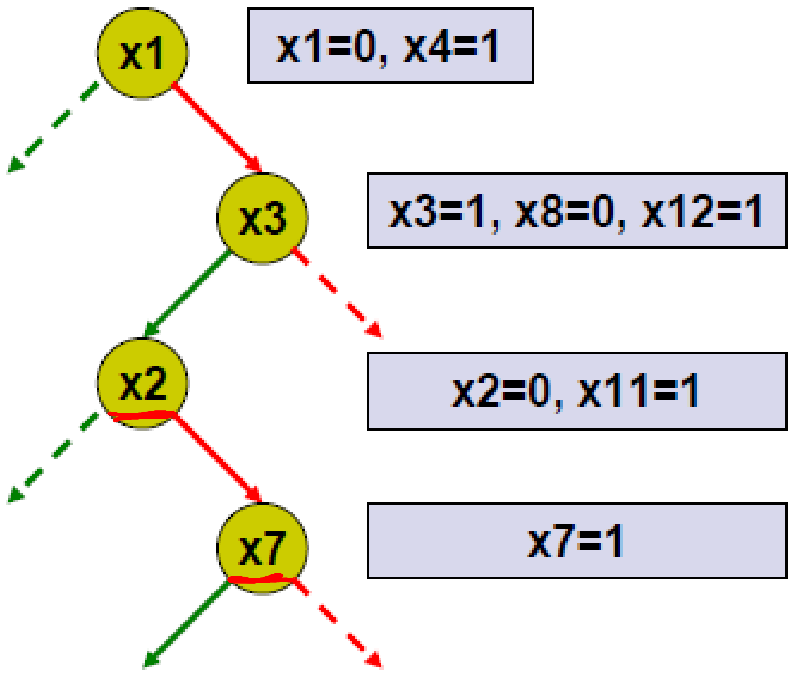
$\neg x_7 \lor x_8 \lor \neg x_9$

$x_7 \lor x_8 \lor \neg x_{10}$

$x_7 \lor x_{10} \lor \neg x_{12}$

Step 9





Conflict Driven Clause Learning (CDCL)

$x_1 \vee x_4$

$x_1 \vee \neg x_3 \vee \neg x_8$

$x_1 \vee x_8 \vee x_{12}$

$x_2 \vee x_{11}$

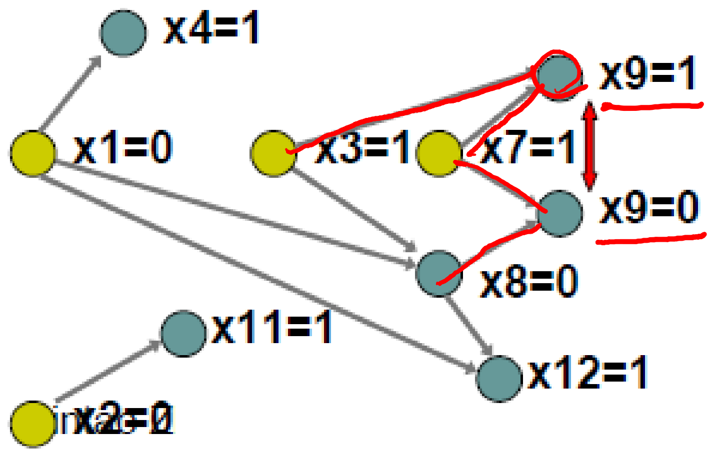
$\neg x_7 \vee \neg x_3 \vee x_9$

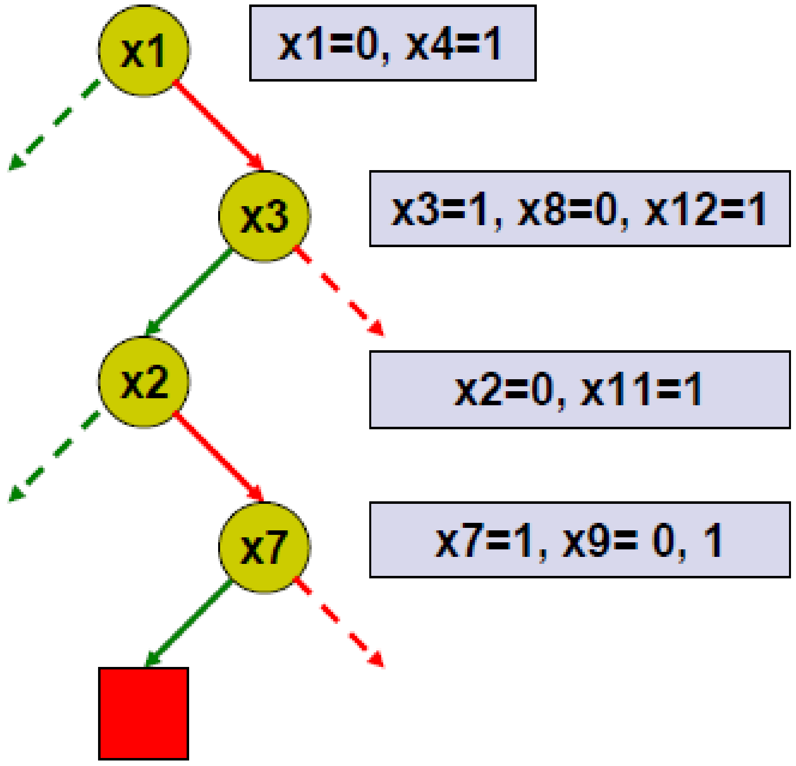
$\neg x_7 \vee x_8 \vee \neg x_9$

$x_7 \vee x_8 \vee \neg x_{10}$

$x_7 \vee x_{10} \vee \neg x_{12}$

Step 10





There is a conflict!

Conflict Driven Clause Learning (CDCL)

$x_1 \vee x_4$

$x_1 \vee \neg x_3 \vee \neg x_8$

$x_1 \vee x_8 \vee x_{12}$

$x_2 \vee x_{11}$

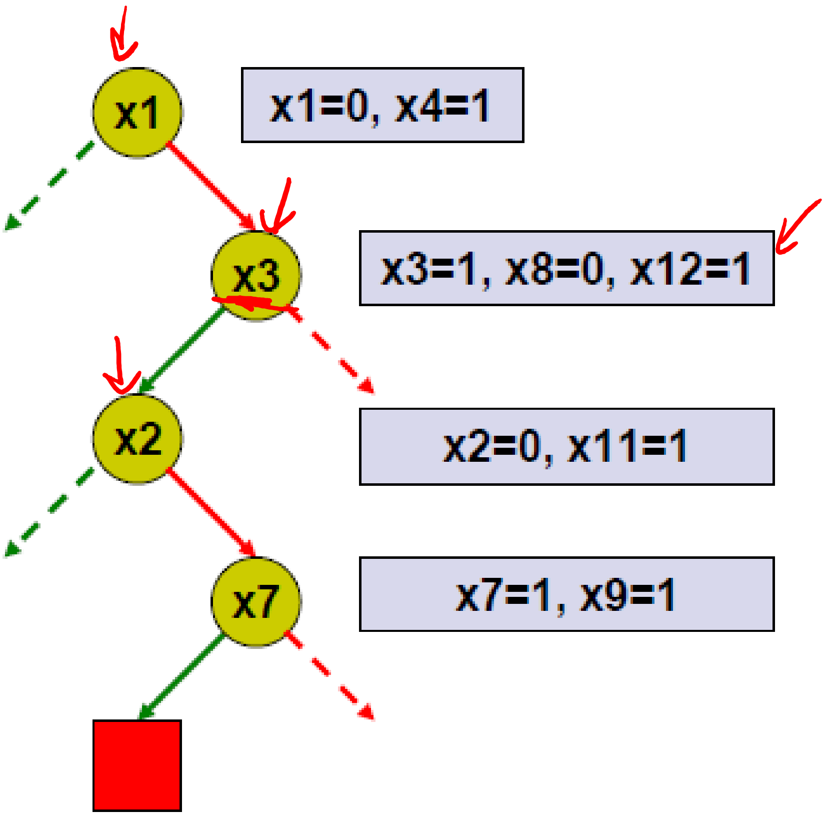
$\neg x_7 \vee \neg x_3 \vee x_9$

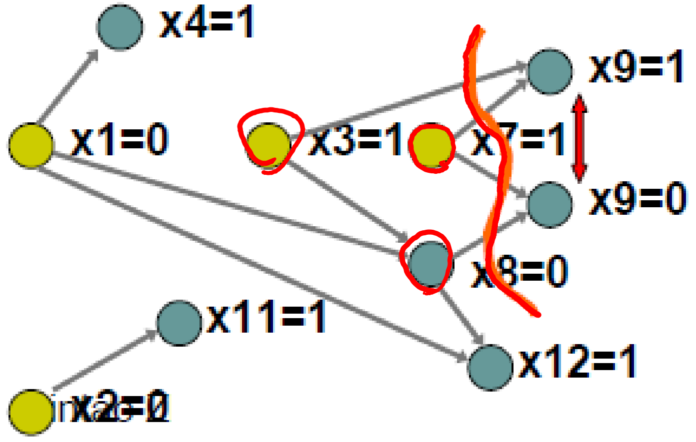
$\neg x_7 \vee x_8 \vee \neg x_9$

$x_7 \vee x_8 \vee \neg x_{10}$

$x_7 \vee x_{10} \vee \neg x_{12}$

Step 11





Find the cut and its corresponding literals: $x_3, x_7, \neg x_8$

Derive a new clause $\neg x_3 \vee \neg x_7 \vee x_8$. Why?

If $x_3 \wedge x_7 \wedge \neg x_8$, then there will be a conflict

Conflict Driven Clause Learning (CDCL)

$x_1 \vee x_4$

$x_1 \vee \neg x_3 \vee \neg x_8$

$x_1 \vee x_8 \vee x_{12}$

$x_2 \vee x_{11}$

$\neg x_7 \vee \neg x_3 \vee x_9$

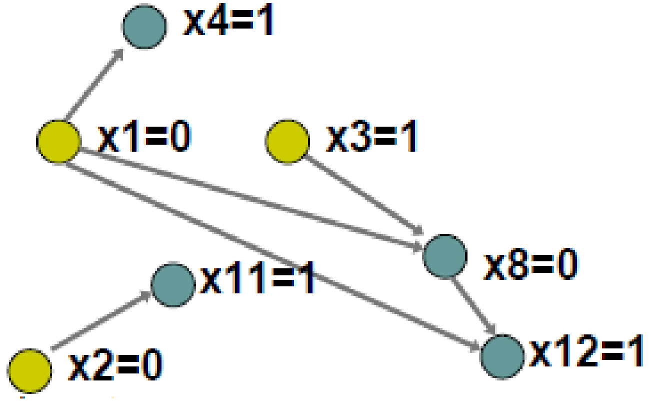
$\neg x_7 \vee x_8 \vee \neg x_9$

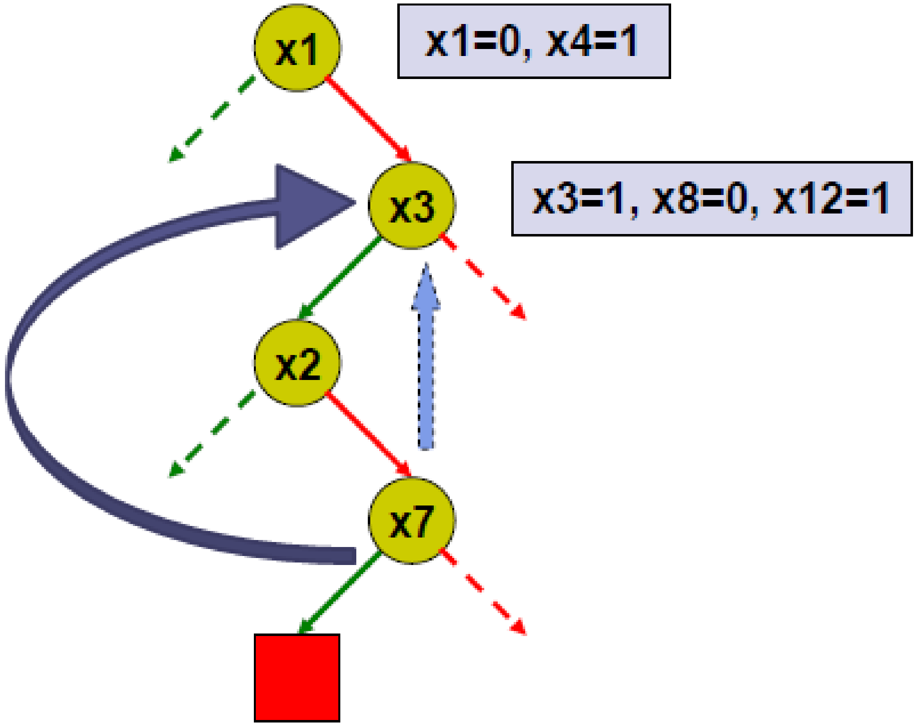
$x_7 \vee x_8 \vee \neg x_{10}$

$x_7 \vee x_{10} \vee \neg x_{12}$

$\neg x_3 \vee \neg x_7 \vee x_8$

Step 14





Backjump to the level where the first-assigned variable involved in the conflict was assigned.

Conflict Driven Clause Learning (CDCL)

$x_1 \vee x_4$

$x_1 \vee \neg x_3 \vee \neg x_8$

$x_1 \vee x_8 \vee x_{12}$

$x_2 \vee x_{11}$

$\neg x_7 \vee \neg x_3 \vee x_9$

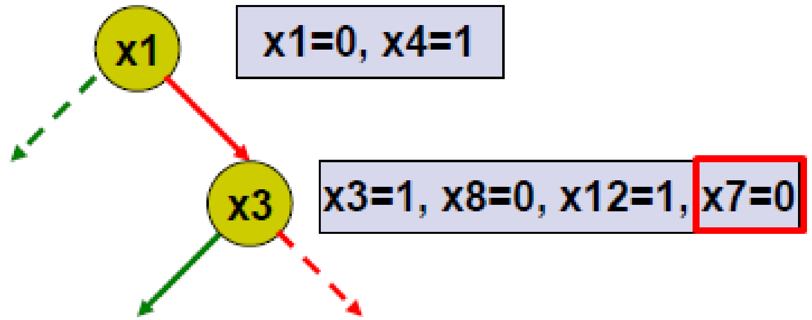
$\neg x_7 \vee x_8 \vee \neg x_9$

$x_7 \vee x_8 \vee \neg x_{10}$

$x_7 \vee x_{10} \vee \neg x_{12}$

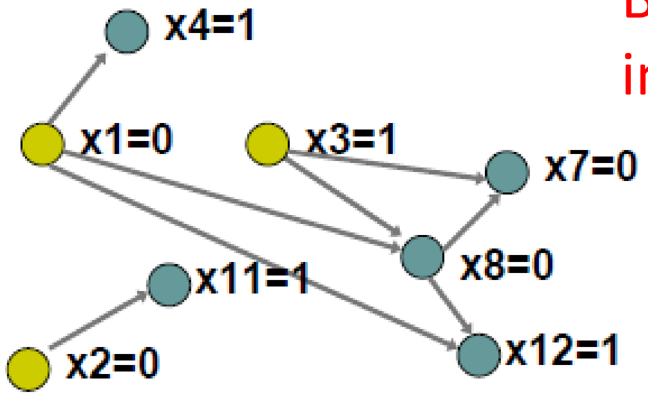
$\neg x_3 \vee \neg x_7 \vee x_8$

Step 15



Continue Unit propagation.

Backtrack to the level where the "problem" is instead of merely trying to fix the "symptom"



Conflict Driven Clause Learning (CDCL)

1. Select a variable and assign True or False
2. Apply unit propagation to build the implication graph
3. If there is any conflict
   a) Find the cut in the implication graph that led to the conflict
   b) Derive a new clause which is the negation of the assignments that led to the conflict
   c) Backjump to the appropriate decision level, where the first-assigned variable involved in the conflict was assigned
4. Otherwise continue from step 1 until all variable values are assigned

Similar ideas can be applied to general CSPs

https://en.wikipedia.org/wiki/Conflict-driven_clause_learning

Local Search Algorithms for SAT

iterative improvement

- WALK-SAT
  - Randomly choose an unsatisfied clause
  - With probability p, flip a randomly selected symbol in the clause → random walk
  - Otherwise, flip a symbol in the clause that maximizes the # of satisfied clauses → best neighbor

WALKSAT

function WALKSAT(*clauses*, *p*, *max_flips*) **returns** a model or *failure*

inputs: *clauses*, a set of clauses

p, the probability of choosing to do a random walk, typically around 0.5

max_flips, number of flips allowed before giving up

model ← a random assignment of *true/false* to the symbols in *clauses*

for i = 1 **to** *max_flips* **do**

  if *model* satisfies *clauses* **then return** *model*

  clause ←a randomly selected clause from *clauses* that is *false* in *model*

  with probability *p* flip the value in *model* of
      a randomly selected symbol from *clause*

  else flip whichever symbol in *clause* maximizes the # of satisfied clauses

return *failure*

Local Search Algorithms for SAT

- WALK-SAT
  - Randomly choose an unsatisfied clause
  - With probability p, flip a randomly selected symbol in the clause
  - Otherwise, flip a symbol in the clause that maximizes the # of satisfied clauses

- GSAT [Selman, Levesque, Mitchell AAAI-92]
  - Similar to hill climbing but with random restarts and allows for downhill/sideway moves if no better moves available

GSAT

function GSAT(*sentence*, *max_restarts*, *max_climbs*) **returns** a model or *failure*

for i = 1 **to** *max_restarts* **do**

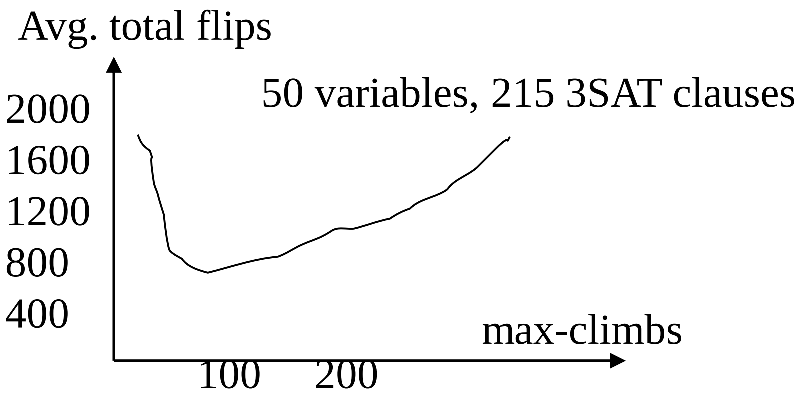
model ← a random assignment of *true/false* to the symbols in *clauses*

for j = 1 **to** *max_climbs* **do**

if *model* satisfies *sentence* **then return** *model*

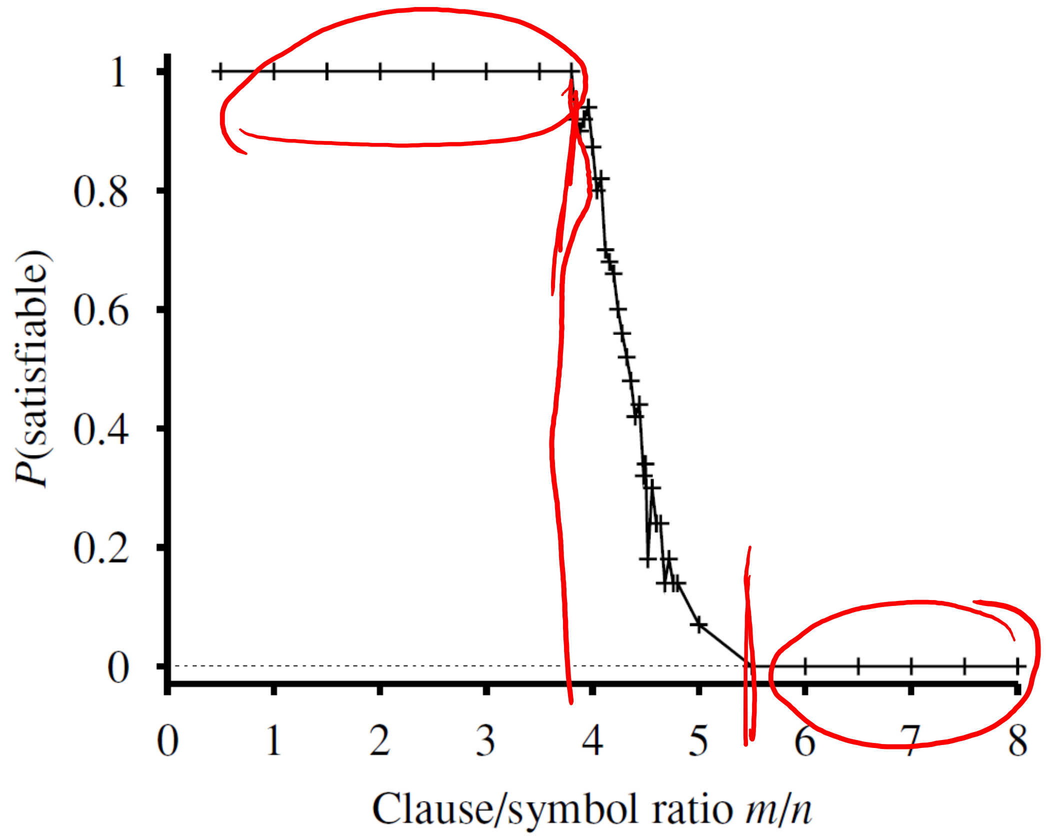
model ← randomly choose one of the best successors

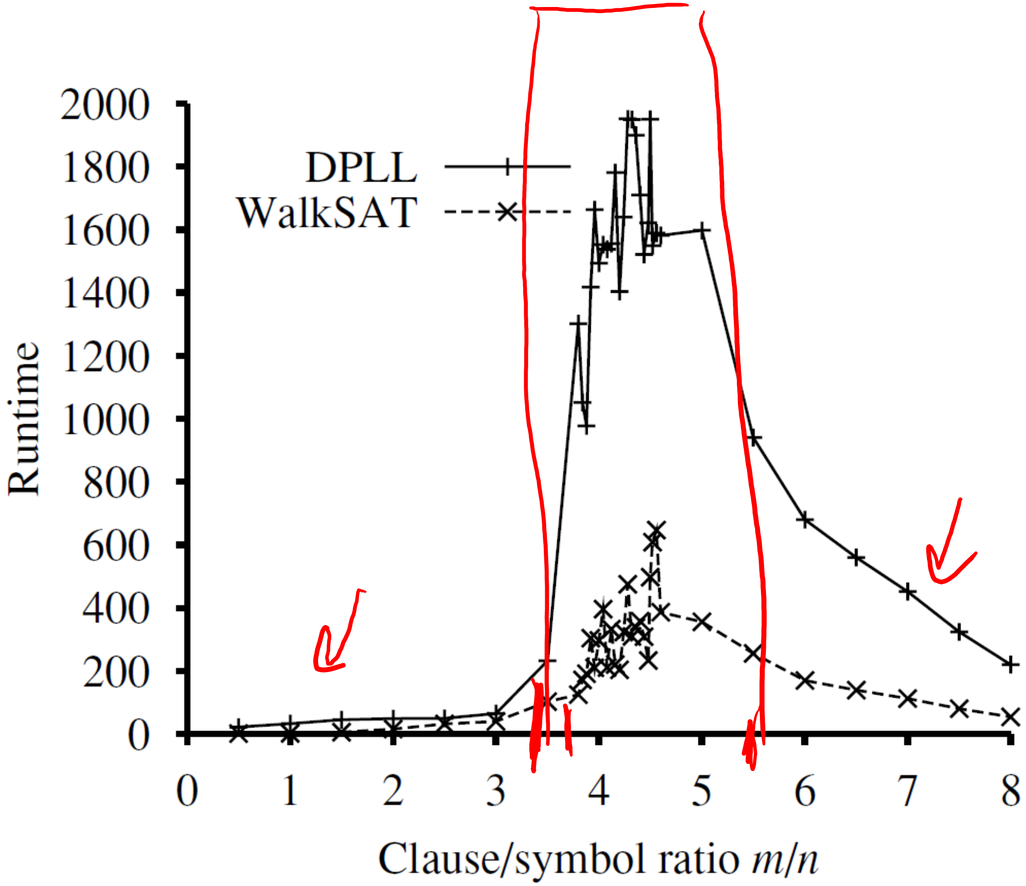
return *failure*



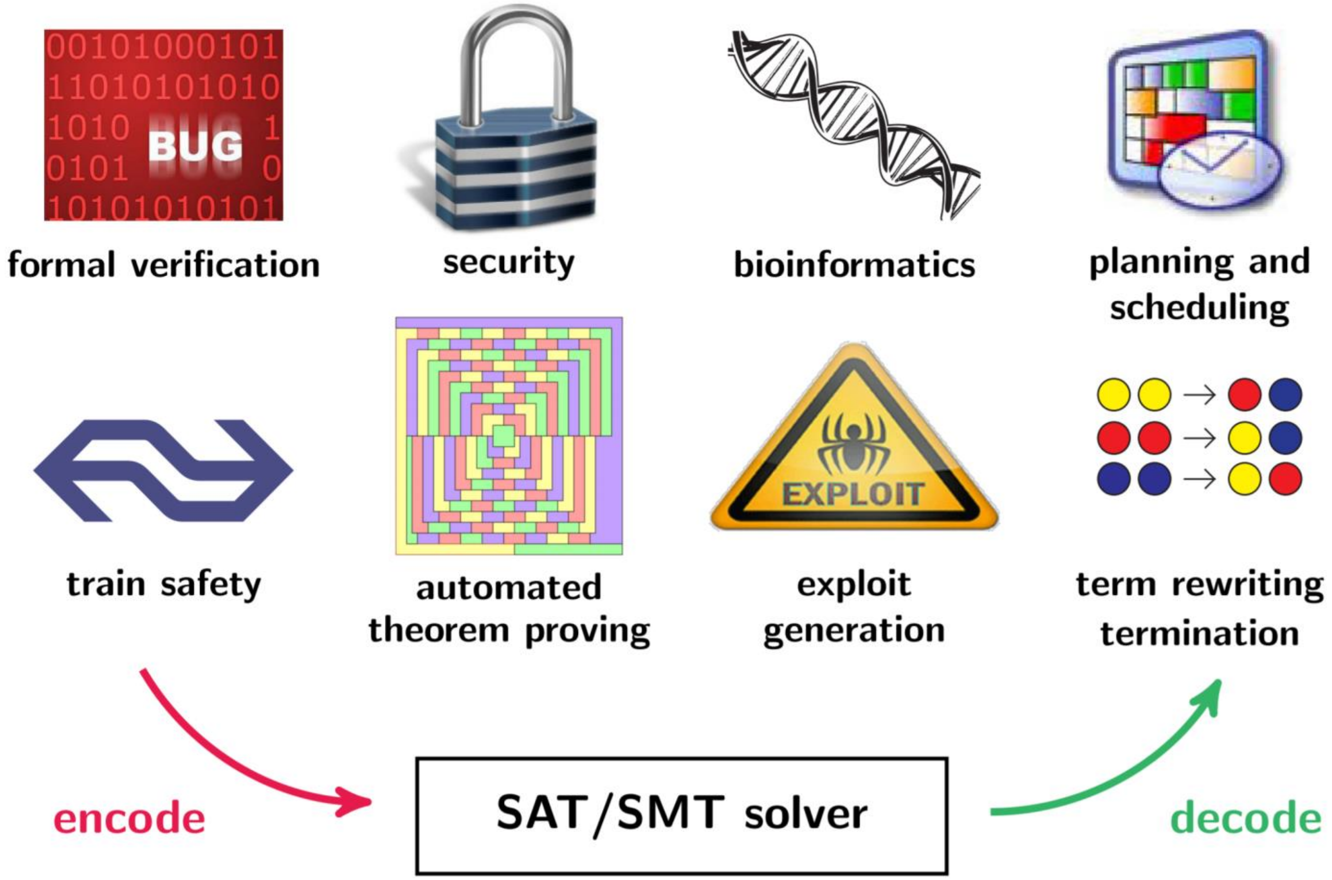
Greediness is not essential as long as climbs and sideways moves are preferred over downward moves.

Phase Transition of SAT

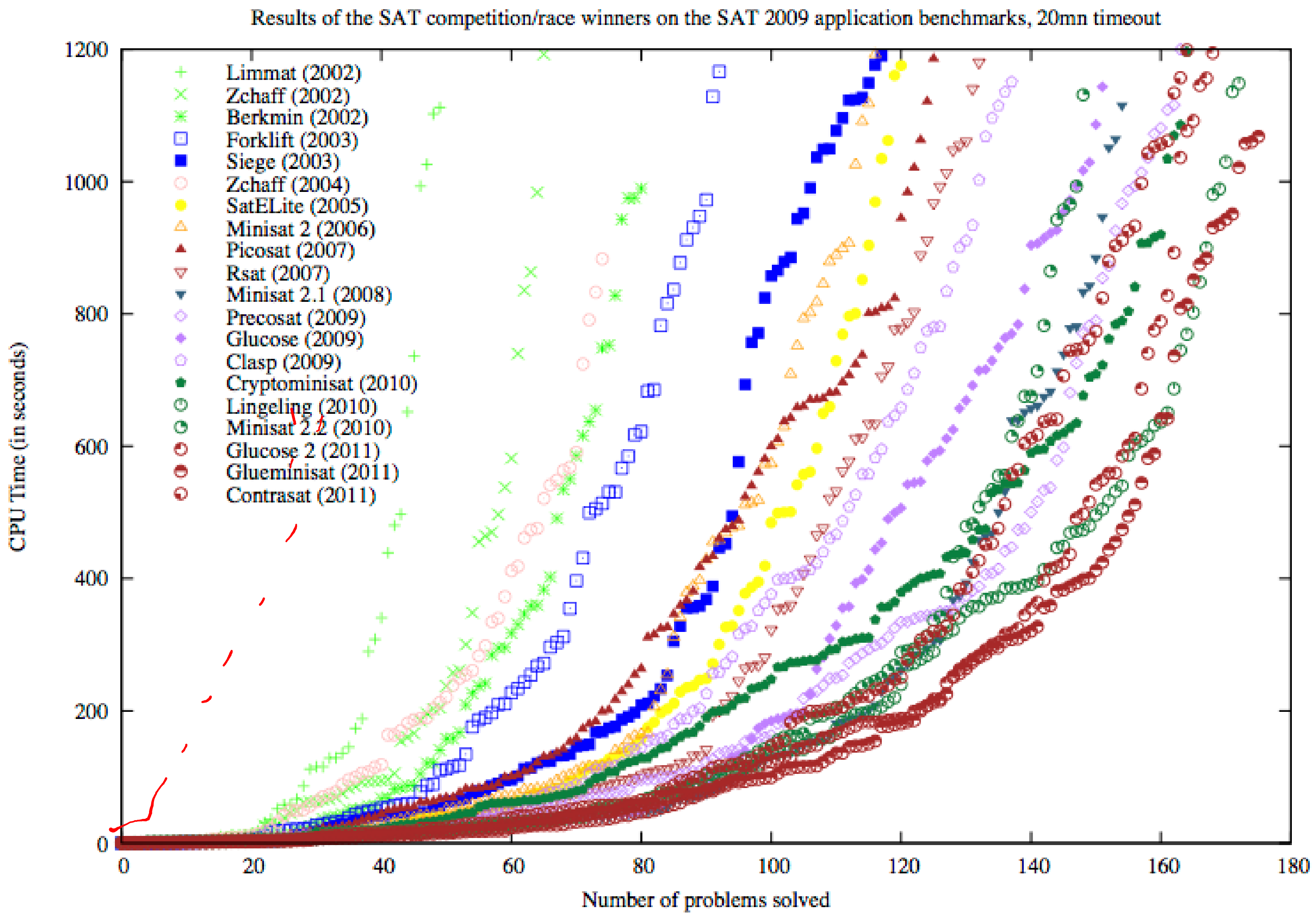




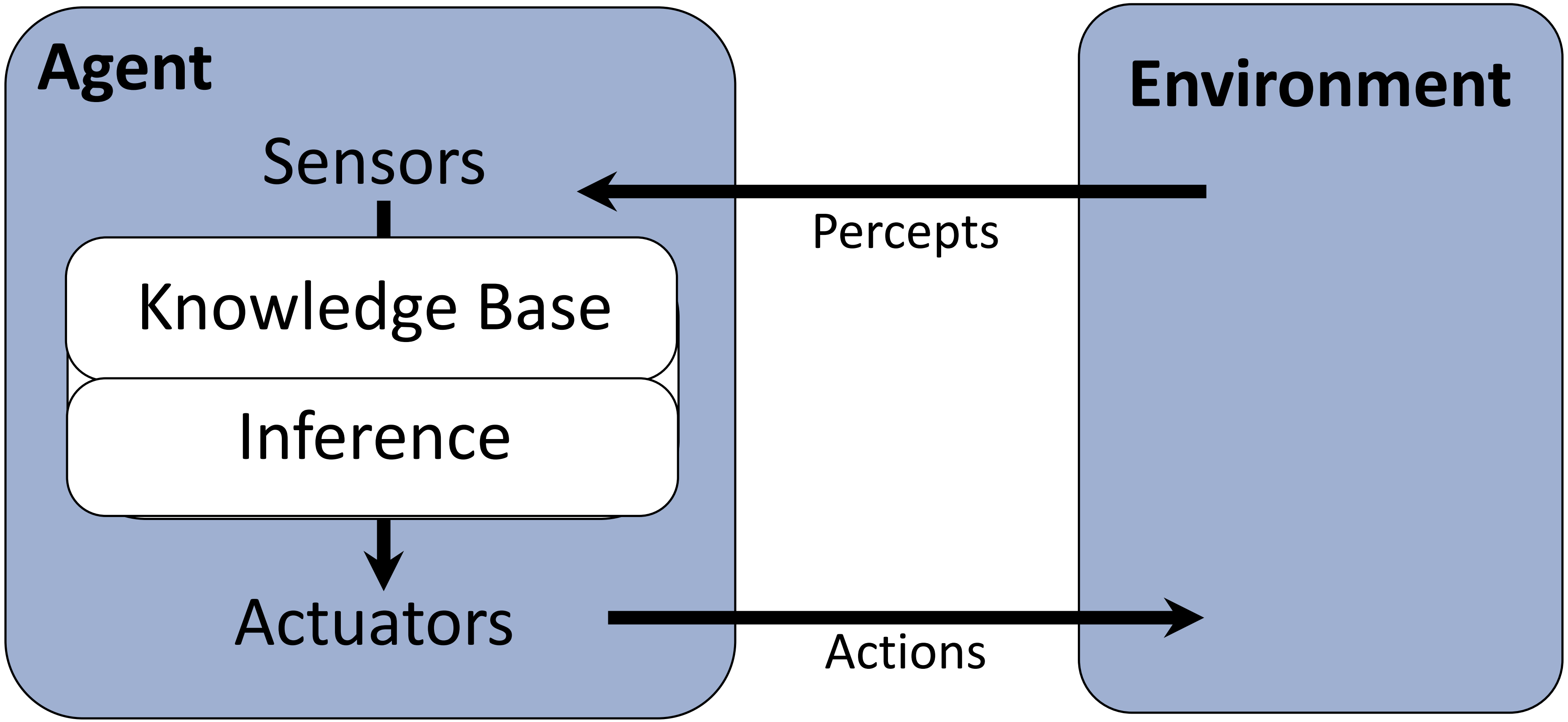
SAT Applications



Evolution of SAT Solvers

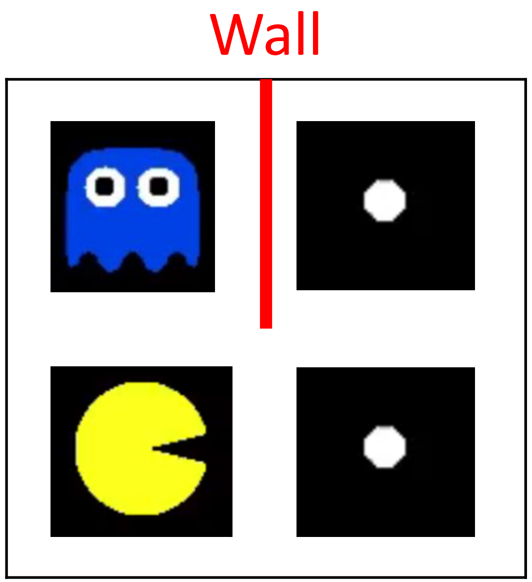


Agent based on Propositional Logic



Planning as Satisfiability (SATPlan)

- Given a hyper-efficient SAT solver, can we use it to make plans for an agent so that it is guaranteed to achieve certain goals?
- For fully observable, deterministic case: Yes, planning problem is solvable iff there is some satisfying assignment for actions etc. (No sensor needed due to full observability; KB does not grow)

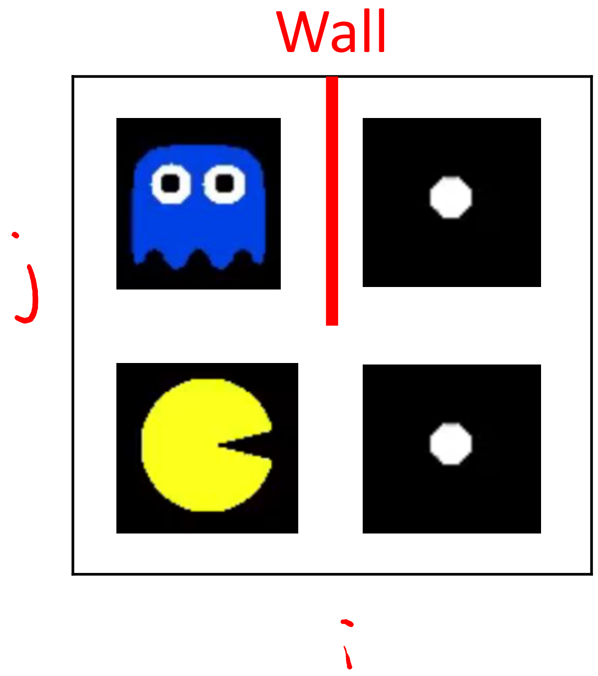


How can Pacman eat all food given that the ghost will move South, then E, then N, then stop there?

Convert it to SAT

Planning as Satisfiability (SATPlan)

How can Pacman eat all food given that the ghost will move South, then E, then N, then stop there?



Use symbols to represent the problem, including aspects of the world that do not change over time (called "atemporal variables"), e.g., $Wall_{ij}^{E}$, and aspects that change over time (called as "fluent", or "state variables"), e.g., location L_{ij}^{t} and action $N^t, S^t, E^t, W^t, \forall t = 1, 2, \ldots, T$

1. Set up KB: Write down all the sentences in KB
2. Solve SAT: Find a model that satisfy all these sentences

What should be the value of T?

Recall Iterative Deepening. Gradually increase T if a small value returns no solution

Planning as Satisfiability (SATPlan)

T_{max}: Max length of planning horizon

function SATPLAN($init$, $transition$, $goal$, $T_{\max}$) **returns** solution or failure
  inputs: $init$, $transition$, $goal$, constitute a description of the problem
    $T_{\max}$, an upper limit for plan length

  for $T = 0$ **to** $T_{\max}$ **do** — T is the length of planning horizon. Gradually increase.
    $cnf \leftarrow$ TRANSLATE-TO-SAT($init$, $transition$, $goal$, T) — Set up the KB
    $model \leftarrow$ SAT-SOLVER(cnf) — Run SAT solver
    if $model$ is not null **then**
      return EXTRACT-SOLUTION($model$)
  return $failure$

Planning as Satisfiability (SATPlan)

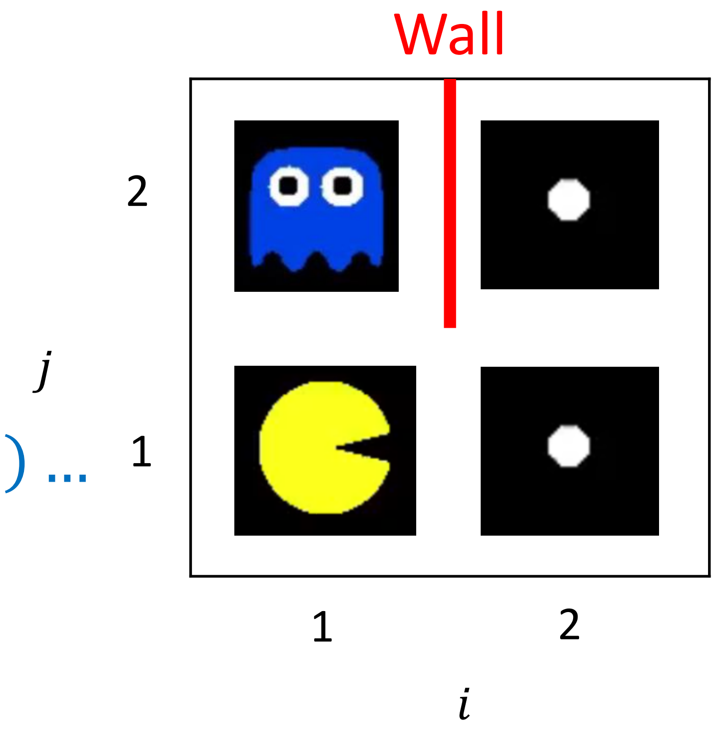
- How to set up the KB? KB often includes sentences describing
  - Initial state

e.g., L_{11}^{0}, $Ghost_{12}^{0}$, $\neg Wall_{12}^{S}$, $Wall_{12}^{E}$, ...

  - Domain constraints

e.g., Pacman cannot be at two locations at the same time

$\neg(L_{11}^{1} \wedge L_{12}^{1}) \wedge \neg(L_{11}^{1} \wedge L_{21}^{1}) \wedge \neg(L_{11}^{1} \wedge L_{212}^{1}) \wedge \neg(L_{12}^{1} \wedge L_{21}^{1})$ ...



Planning as Satisfiability (SATPlan)

- How to set up the KB? KB often includes sentences describing

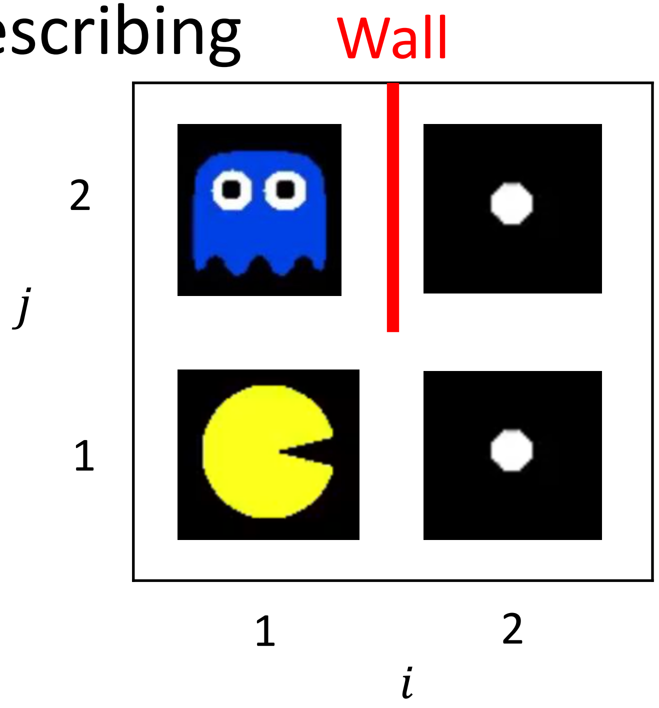
  - Transition model sentences up to time T

  Write down how each *fluent* at each time gets its value based on successor-state axiom:

$$F^{t+1} \Leftrightarrow ActionCausesF^t \vee (F^t \wedge \neg ActionCausesNotF^t)$$

e.g., If "Stop" action is allowed, for L_{12}^1, Pacman was at an adjacent square at time 0 and moved to (1,2) or was at (1,2) and nothing causes to change its location

$$L_{12}^1 \Leftrightarrow \left(\left(L_{11}^0 \wedge N^0 \wedge \neg Wall_{12}^S \wedge \cdots \right) \vee \cdots \right)$$
$$\vee \left(L_{12}^0 \wedge \neg \left(\left(S^0 \wedge \neg Wall_{12}^S \right) \vee \cdots \right) \right)$$



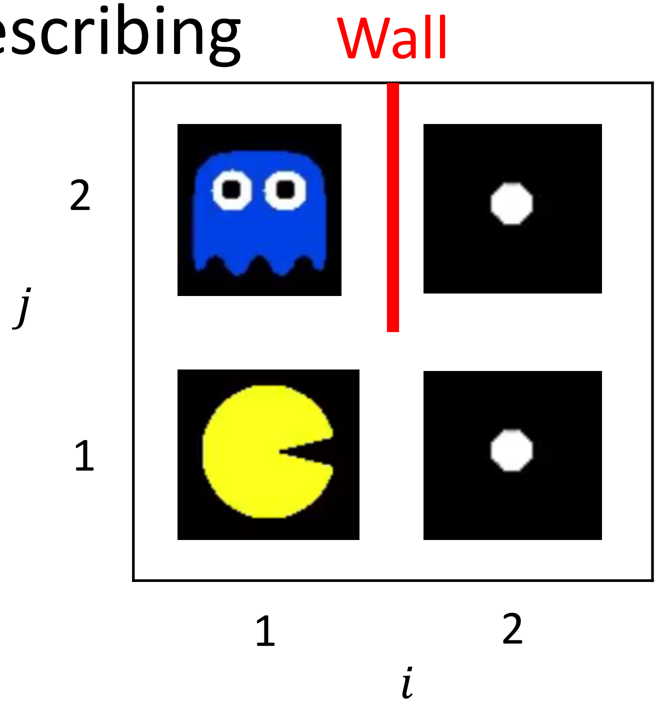
Planning as Satisfiability (SATPlan)

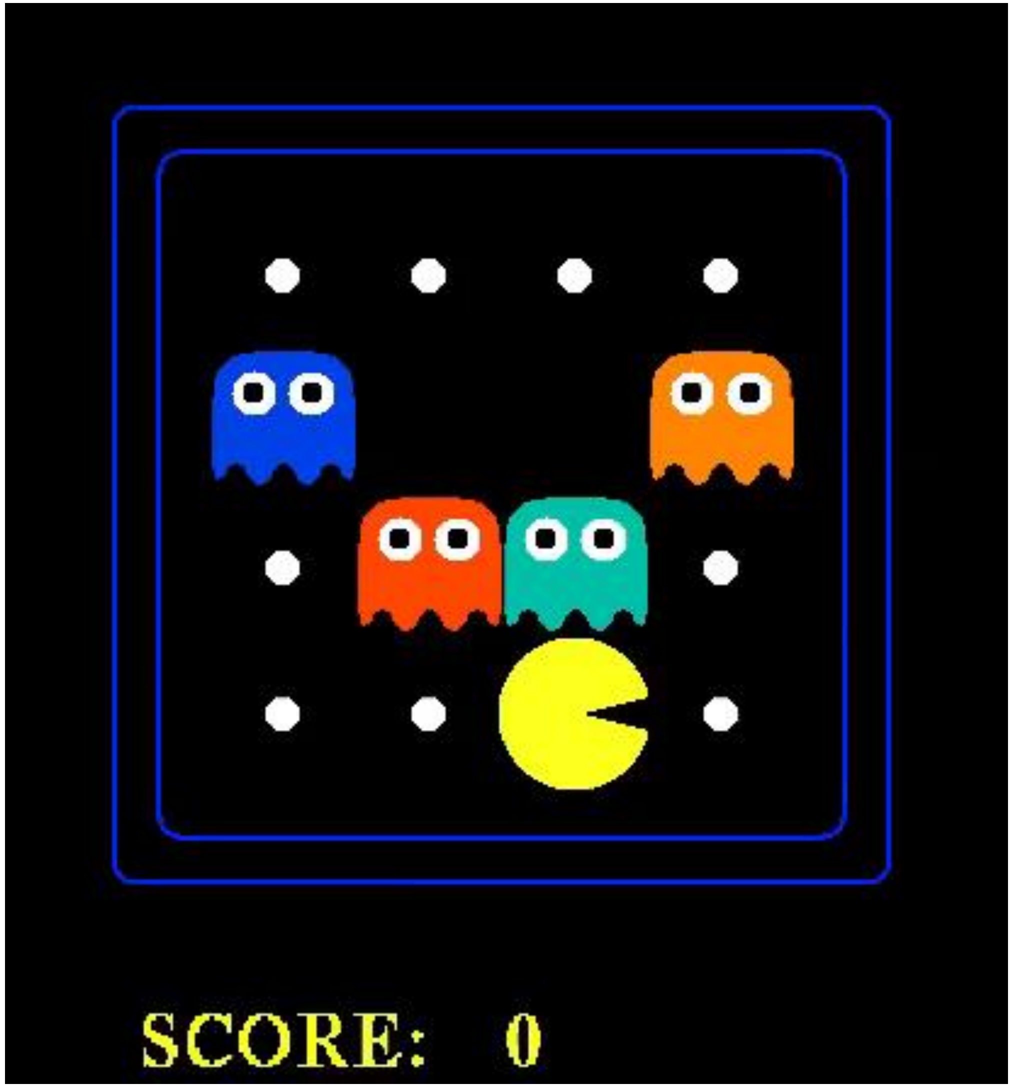
- How to set up the KB? KB often includes sentences describing

- Goal is achieved at time T

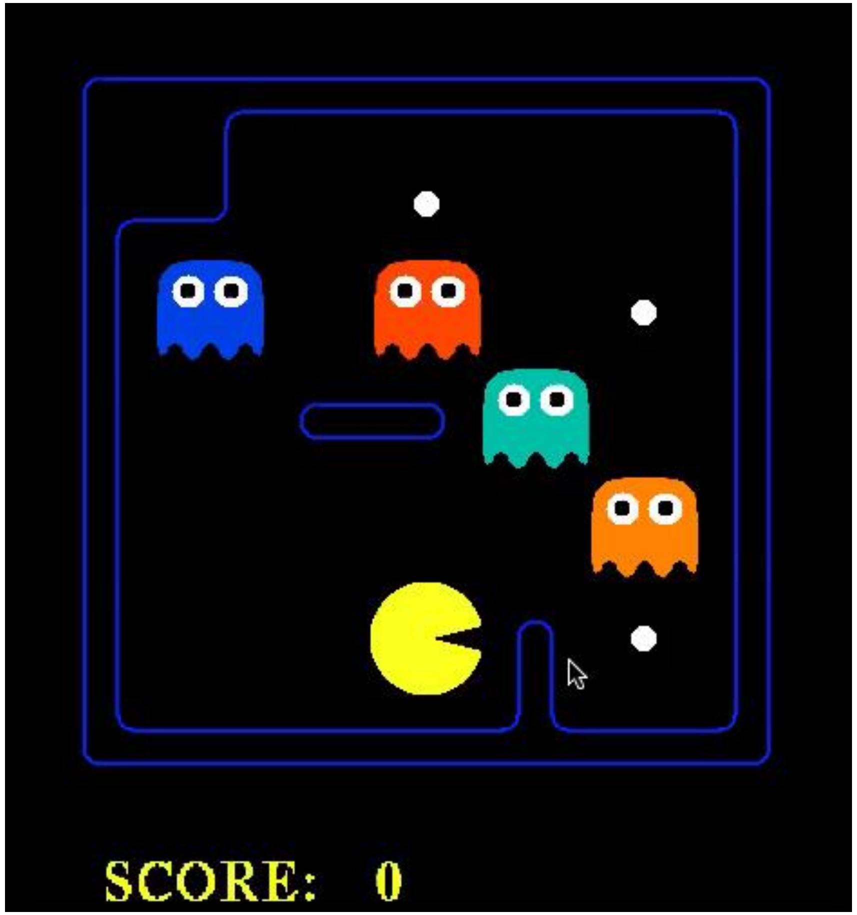
    e.g., no food left at T

    $\neg Food_{11}^{T} \land \neg Food_{12}^{T} \land \neg Food_{21}^{T} \land \neg Food_{22}^{T}$



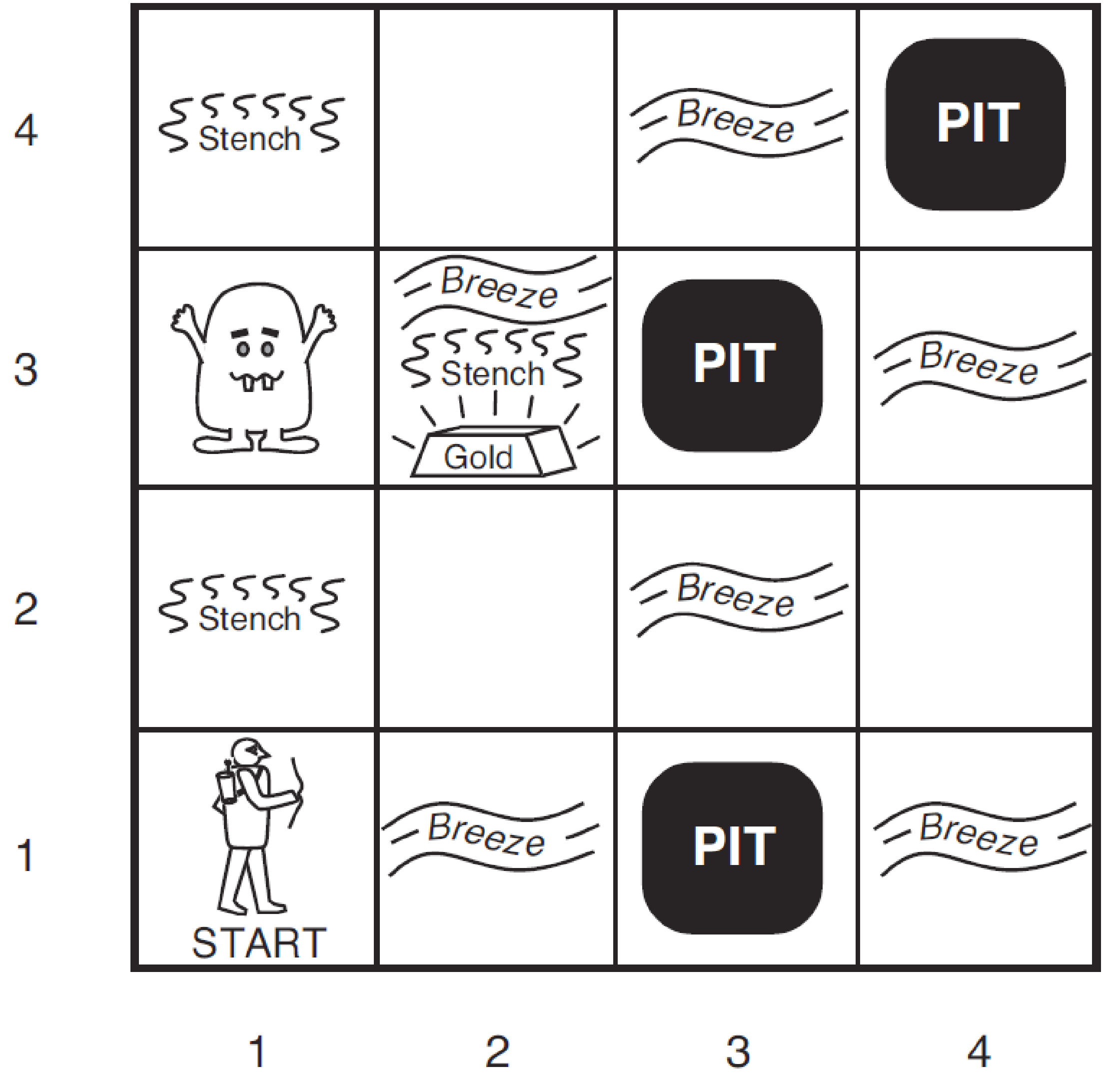

SCORE: 0


SCORE: 0


SCORE: 0

Wumpus World

- The world is not fully observable from the beginning
- KB consists of
  - Facts
  - Rules
  - Percept and Actions
- Keep adding sentences to the KB with new percepts and actions
- At any time step, we can Ask the KB about the current state, e.g., whether a square is safe



B_{ij} = breeze felt; S_{ij} = stench smelt
P_{ij} = pit here; W_{ij} = wumpus here; G = gold

Hybrid Agent

- Plan actions by combining search and logical inference
- Maintain and update a KB as well as a current plan
- Construct a plan based on a decreasing priority of goals
- In Wumpus world
  - Ask KB to work out which squares are safe and which have yet to be visited
  - If there is glitter, construct a plan to grad the gold and go back safely
  - If there is no current plan, use A* search to plan a route that only goes through safe squares to the closest unvisited safe square
  - If no such safe squares to explore, ask questions to determine whether to shoot at one of the possible wumpus locations

Summary

- Many problems can be reduced to SAT
- Efficient SAT solvers operates on CNF and uses ideas in solving CSPs such as backtracking and local search
- Can frame a planning problem as a satisfiability problem

Learning Objectives

- Describe the definition of (Boolean) Satisfiability Problem (SAT)
- Describe the definition of Conjunctive Normal Form (CNF)
- Describe the following algorithms for solving SAT
  - DPLL, CDCL, WalkSAT, GSAT
- Determine whether a sentence is satisfiable
- Describe Successor-State Axiom
- Describe and implement SATPlan (Planning as Satisfiability)
- (Hybrid Agent)