Please form groups of 3-4 for today's lecture!

- Warm up: Design an algorithm to determine the winner of three candidates a, b, c given the ranking provided by n individual voters, described by a $3 \times n$ matrix M

function voting(M)

Input: M where $M_{ij} \in \{a, b, c\}$ is the candidate at rank j for voter i
Output: $x \in \{a, b, c\}$ describes the winner

Return x

Example Matrix M

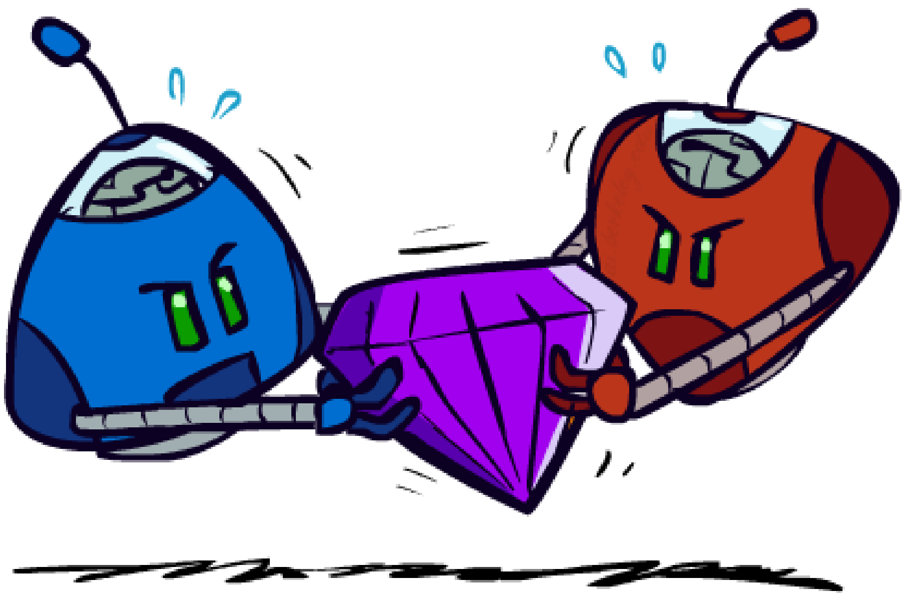
a	c	b	a
b	b	c	b
c	a	a	c

Announcement

- Assignments
  - HW11 (online); ~~Due 11/27 Wed, 10 pm~~ Due 12/2 Mon, 10 pm
  - HW12 (written) will be released soon; ~~Due 12/4 Wed, 10 pm~~
    Due 12/6 Fri, 10 pm
- Piazza post for in-class questions

AI: Representation and Problem Solving

Game Theory: Social Choice



Instructors: Fei Fang & Pat Virtue

Slide credits: CMU AI and http://ai.berkeley.edu

Learning Objectives

- Understand the voting model
- Find the winner under the following voting rules
  - Plurality, Borda count, Plurality with runoff, Single Transferable Vote
- Describe the following concepts, axioms, and properties of voting rules
  - Pairwise election, Condorcet winner
  - Majority consistency, Condorcet consistency, Strategyproof
  - Dictatorial, constant, onto
- Understand the possibility of satisfying multiple properties
- Describe the greedy algorithm for voting rule manipulation

Social Choice Theory

- A mathematical theory that deal with aggregation of individual preferences
- Wide applications in economics, public policy, etc.

Origins in Ancient Greece

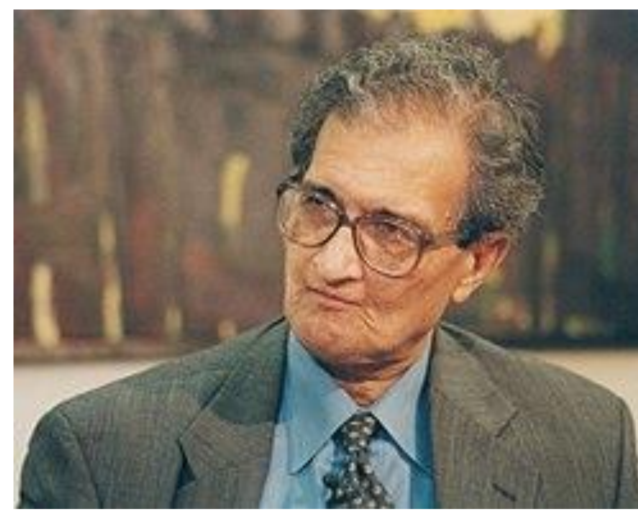
18th century – Formal foundations by Condorcet and Borda

19th Century – Charles Dodgson

20th Century – Winners of Nobel Memorial Prize in Economic Sciences

Kenneth Arrow

Amartya Kumar Sen



Voting Model

- Model
  - Set of voters $N = \{1..n\}$
  - Set of alternatives A $(|A| = m)$
  - Each voter has a ranking over the alternatives
  - Preference profile: collection of all voters' rankings

Voter ID	1	2	3	4
Ranking	a	c	b	a
	b	b	c	b
	c	a	a	c

Voting Rules

- Voting rule: function that maps preference profiles to alternatives that specifies the winner of the election

function voting(M)

Input: M where $M_{ij} \in$ {a, b, c} is the candidate at rank j for voter i
Output: $x \in$ {a, b, c} describes the winner

Return x

Example Matrix M

a	c	b	a
b	b	c	b
c	a	a	c

Voting Rules

- Plurality (used in almost all political elections)
  - Each voter give one point to top alternative
  - Alternative with most points win

Who's the winner? a

Voter ID	1	2	3	4
Ranking	a	c	b	a
	b	b	c	b
	c	a	a	c

Voting Rules

- Borda count (used for national election in Slovenia)
  - Each voter awards $m - k$ points to alternative ranked k^{th}
  - Alternative with most points win

Who's the winner? b

Voter ID	1	2	3	4
Ranking	a	c	b	a
	b	b	c	b
	c	a	a	c

Voting Rules

- Borda count (used for national election in Slovenia)
  - Each voter awards $m-k$ points to alternative ranked k^{th}
  - Alternative with most points win

Who's the winner? b

Voter ID	1	2	3	4	$m-k$
Ranking	a	c	b	a	2
	b	b	c	b	1
	c	a	a	c	0

a: 2+0+0+2=4; b: 1+1+2+1=5; c: 0+2+1+0=3

Pairwise Election

Alternative x beats y in pairwise election if majority of voters prefer x to y

Who beats who in pairwise election? b beats c

Voter ID	1	2	3	4
Ranking	a	c	b	a
	b	b	c	b
	c	a	a	c

Voting Rules

- Plurality with runoff
  - First round: two alternatives with highest plurality scores survive
  - Second round: pairwise election between the two

x beats y if majority of voters prefer x to y

Who's the winner?

Voter ID	1	2	3	4
Ranking	a	c	b	a
	b	b	c	b
	c	a	a	c

Voting Rules

- Plurality with runoff
  - First round: two alternatives with highest plurality scores survive
  - Second round: pairwise election between the two

x beats y if majority of voters prefer x to y

Who's the winner?

Voter ID	1	2	3	4
Ranking	a	c	b	a
	b	b	c	b
	c	a	a	c

Depends on the tie breaking rule.
If break tie alphabetically:
a and b survive in 1st round
a wins the pairwise election

Voter ID	1	2	3	4
Ranking	a	b	b	a
	b	a	a	b

Voting Rules

- Single Transferable Vote (STV)
  - (used in Ireland, Australia, New Zealand, Maine, San Francisco, Cambridge)
  - $m-1$ rounds: In each round, alternative with least plurality votes is eliminated
  - Alternative left is the winner

Who's the winner?

Voter ID	1	2	3	4
Ranking	a	c	b	a
	b	b	c	b
	c	a	a	c

Voting Rules

- Single Transferable Vote (STV)
  - (used in Ireland, Australia, New Zealand, Maine, San Francisco, Cambridge)
  - $m - 1$ rounds: In each round, alternative with least plurality votes is eliminated
  - Alternative left is the winner

Who's the winner?

Voter ID	1	2	3	4
Ranking	a	c	b	a
	b	b	c	b
	c	a	a	c

Depends on the tie breaking rule. If break tie alphabetically, the order of being eliminated is c, b

Voter ID	1	2	3	4
Ranking	a	b	b	a
	b	a	a	b

Tie Breaking

- Commonly used tie breaking rules include
  - Borda count
  - Having the most votes in the first round
  - …

Let's vote for candies!

- On your own, rank your favorite candies
  - M&Ms
  - Snickers
  - Milky Way
  - Kit Kat
  - Skittles
- Compute the Plurality, Borda, STV winners in your group (you may need to choose a tie-breaking rule)

Representation of Preference Profile

- Identity of voters does not matter
- Only record how many voters has a preference

22 voters	30 voters	42 voter
M&Ms	Milky Way	Kit Kat
Snickers	M&Ms	M&Ms
Milky Way	Kit Kat	Skittles
Kit Kat	Skittles	Snickers
Skittles	Snickers	Milky Way

Social Choice Axioms

- How do we choose among different voting rules? What are the desirable properties?

Majority consistency

- Majority consistency: Given a voting rule that satisfies Majority Consistency, if a majority of voters rank alternative x first, then x should be the final winner.

Piazza Poll 1

- Which rules are not majority consistent?
  - A: Plurality: Each voter give one point to top alternative
  - B: Borda count: Each voter awards $m - k$ points to alternative ranked k^{th}
  - C: Plurality with runoff: Pairwise election between two alternatives with highest plurality scores
  - D: STV: In each round, alternative with least plurality votes is eliminated
  - E: None
  - F: I don't know

Piazza Poll 1

- Which rules are not majority consistent?
  - A: Plurality
  - B: Borda count
  - C: Plurality with runoff
  - D: STV
  - E: None

4 voters	3 voter	$m-k$
a	c	3
b	b	2
d	d	1
c	a	0

a	b	c	d
4*3+3*0=12	(4+3)*2=14	3*3+4*0=9	(4+3)*1=7

Condorcet Consistency

- Recall: x beats y in a pairwise election if majority of voters prefer x to y
- Condorcet winner is the alternative that beats every other alternative in pairwise election

Does a Condorcet winner always exist?

- Condorcet paradox = cycle in majority preferences

Voter ID	1	2	3
Ranking over alternatives (first row is the most preferred)	a	c	b
	b	a	c
	c	b	a

Condorcet Consistency

- Condorcet consistency: A voting rule satisfies majority consistency should select a Condorcet Winner as the final winner if one exists.

Which of the introduced voting rules (Plurality, Borda count, Plurality with runoff, STV) are Condorcet consistent?

Condorcet Consistency

- Winner under different voting rules in this example

33 voters	16 voters	3 voter	8 voters	18 voters	22 voters
a	b	c	c	d	e
b	d	d	e	e	c
c	c	b	b	c	b
d	e	a	d	b	d
e	a	e	a	a	a

Condorcet Consistency

- Winner under different voting rules in this example
  - Plurality: a; Borda: b; STV: d; Plurality with runoff: e
  - Condorcet winner: c

33 voters	16 voters	3 voter	8 voters	18 voters	22 voters
a	b	c	c	d	e
b	d	d	e	e	c
c	c	b	b	c	b
d	e	a	d	b	d
e	a	e	a	a	a

Strategy-Proofness

- Using Borda Count

Who is the winner?

Voter ID	1	2	3
Ranking over alternatives (first row is the most preferred)	b	b	a
	a	a	b
	c	c	c
	d	d	d

$m-k$
3
2
1
0

Who is the winner now?

Voter ID	1	2	3
Ranking over alternatives (first row is the most preferred)	b	b	a
	a	a	c
	c	c	d
	d	d	b

$m-k$
3
2
1
0

Strategy-Proofness

- Using Borda Count

Who is the winner?

Voter ID	1	2	3	$m-k$
Ranking over alternatives (first row is the most preferred)	b	b	a	3
	a	a	b	2
	c	c	c	1
	d	d	d	0

b: 2*3+1*2=8
a: 2*2+1*3=7

b is the winner

Who is the winner now?

Voter ID	1	2	3	$m-k$
Ranking over alternatives (first row is the most preferred)	b	b	a	3
	a	a	c	2
	c	c	d	1
	d	d	b	0

b: 2*3+1*0=6
a: 2*2+1*3=7

a is the winner

Strategy-Proofness

- A single voter can manipulate the outcome!

Voter ID	1	2	3
Ranking over alternatives (first row is the most preferred)	b	b	a
	a	a	b
	c	c	c
	d	d	d

$m-k$
3
2
1
0

b: 2*3+1*2=8
a: 2*2+1*3=7

b is the winner

Voter ID	1	2	3
Ranking over alternatives (first row is the most preferred)	b	b	a
	a	a	c
	c	c	d
	d	d	b

$m-k$
3
2
1
0

b: 2*3+1*0=6
a: 2*2+1*3=7

a is the winner

Strategy-Proofness

- A voting rule is strategyproof (SP) if a voter can never benefit from lying about his preferences (regardless of what other voters do)
  - Benefit: a more preferred alternative is selected as winner

Do not lie: b is the winner

Voter ID	1	2	3
Ranking	b	b	a
	a	a	b
	c	c	c
	d	d	d

Lie: a is the winner

Voter ID	1	2	3
Ranking	b	b	a
	a	a	c
	c	c	d
	d	d	b

If a voter's preference is a>b>c, c will be selected w/o lying, and b will be selected w/ lying, then the voter still benefits

Piazza Poll 2

- Which of the introduced voting rules are strategyproof?
  - A: Plurality: Each voter give one point to top alternative
  - B: Borda count: Each voter awards $m - k$ points to alternative ranked k^{th}
  - C: Plurality with runoff: Pairwise election between two alternatives with highest plurality scores
  - D: STV: In each round, alternative with least plurality votes is eliminated
  - E: None
  - F: I don't know

Piazza Poll 2

- Which of the introduced voting rules are strategyproof?
  - A: Plurality
  - B: Borda count
  - C: Plurality with runoff
  - D: STV
  - E: None

Previous example already showed that Borda count is not.
My true preference is a>b>c, but b>c>a for 50 other voters and c>b>a for the remaining 50 voters. Assume the tie-breaking rule will make c the winner if I report truthfully. Then I would report b>c>a to make b the winner instead of c under Plurality, Plurality with runoff, and STV. So none.

Greedy Algorithm for f −Manipulation

- Given voting rule f and preference profile of $n-1$ voters, how can the last voter report preference to let a specific alternative y *uniquely* win (no tie breaking)?

Greedy algorithm for f −Manipulation

Rank y in the first place
While there are unranked alternatives
    If $\exists x$ that can be placed in the next spot without preventing y from winning
        place this alternative in the next spot
    else
        return false
return true (with final ranking)

Correctness proved (Bartholdi et al., 1989)

Greedy Algorithm for f −Manipulation

- Example with Borda count voting rule

Voter ID	1	2	3
Ranking over alternatives (first row is the most preferred)	b	b	a
	a	a	
	c	c	
	d	d	

Greedy Algorithm for f −Manipulation

- Example with Borda count voting rule

Voter ID	1	2	3	$m-k$
Ranking over alternatives (first row is the most preferred)	b	b	a	3
	a	a	b	2
	c	c		1
	d	d		0

b: 2*3+1*2=8
a: 2*2+1*3=7

Cannot put b here

c: 2*1+1*2=4
a: 2*2+1*3=7
c can be placed second

Voter ID	1	2	3	$m-k$
Ranking over alternatives (first row is the most preferred)	b	b	a	3
	a	a	c	2
	c	c	d	1
	d	d	b	0

b: 2*3+1*1=7
b cannot be placed third

d: 2*0+1*1=1
d can be placed third

Other Properties

- A voting rule is dictatorial if there is a voter who always gets his most preferred alternative

- A voting rule is constant if the same alternative is always chosen (regardless of the stated preferences)

- A voting rule is onto if any alternative can win, for some set of stated preferences

  Which of the introduced voting rules (Plurality, Borda count, Plurality with runoff, STV) are dictatorial, constant or onto?

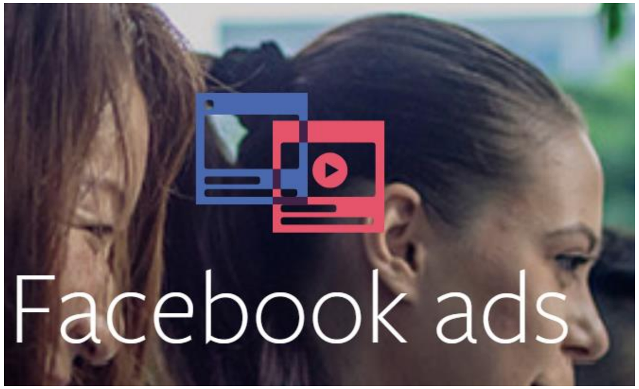
Results in Social Choice Theory

- Constant functions and dictatorships are SP Why?

- Theorem (Gibbard-Satterthwaite): If $m \geq 3$, then any voting rule that is SP and onto is dictatorial
  - Any voting rule that is onto and nondictatorial is manipulable
  - It is impossible to have a voting rule that is strategyproof, onto, and nondictatorial

(Optional) Mechanism Design Overview and Second Price Auction

Google Ads



Let's have an auction!

- 1 box of milk chocolate to be purchased
- Each participant submit a bid (a number ≥ 0)
- I will give the chocolate to the one with the highest bid
- The person who get the chocolate needs to pay a price that equals the second highest bid provided by any participant (in dollars)
- How much will you bid?

Mechanism Design

- Mechanism specifies what actions the agent can take, and what is the outcome after the agents take actions
- Given a mechanism, the interaction among agents can be seen as a n-player game
- Mechanism design: Choose a mechanism that can will cause rational agents to behave in a desired way, i.e., the solution or equilibrium of the induced game satisfy properties or optimize certain goals

Bayesian Game

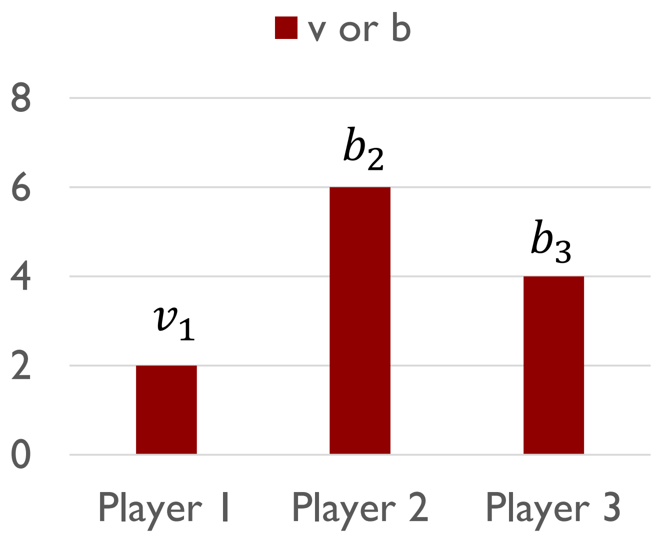
- A player's utility function depends on his "type"
- In chocolate auction, a person on diet may have different valuation of the chocolate from a person who loves chocolate and is not on diet, leading to different utility functions
- Each participant knows his own type but only knows a prior distribution of other players' type
- Keep in mind that once the auction mechanism is specified, it is a game among participating agents

Truthfulness

- A mechanism is truthful if each agent i's equilibrium strategy is to report his true valuation (or utility function)
- Just a different name of strategyproofness in the context of mechanism design

Second Price Auction

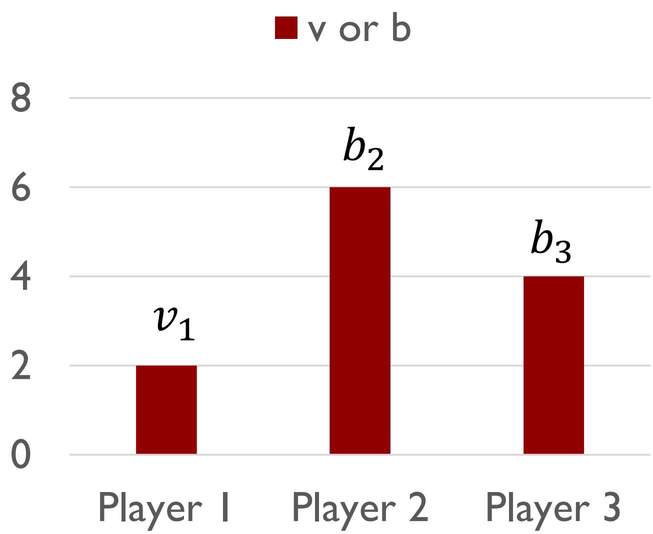
- Every participant submitting a bid that equals their true valuation is a (Weakly) Dominant Strategy Equilibrium



If $v_1 < b_2$	If $v_1 \geq b_2$
If $b_1 < b_2$	If $b_1 < b_2$
If $b_1 > b_2$	If $b_1 \geq b_2$
If $b_1 = v_1$ $(< b_2)$	If $b_1 = v_1$ $(\geq b_2)$

Second Price Auction

- Every participant submitting a bid that equals their true valuation is a (Weakly) Dominant Strategy Equilibrium



If $v_1 < b_2$		**If $v_1 \geq b_2$**	
If $b_1 < b_2$	$u_1 = 0$	If $b_1 < b_2$	$u_1 = 0$
If $b_1 > b_2$	$u_1 = v_1 - b_2$ (< 0)	If $b_1 \geq b_2$	$u_1 = v_1 - b_2$ (> 0)
If $b_1 = v_1$ $(< b_2)$	$u_1 = 0$ (optimal)	If $b_1 = v_1$ $(\geq b_2)$	$u_1 = v_1 - b_2$ (optimal)