15-281 Artificial Intelligence HW10

INSTRUCTIONS

- **Due: Wednesday, 20 November 2019 at 10:00 PM EDT.** Remember that you have NO slip days for Written Homework, but you may turn it in up to 24 hours late with 50% Penalty.
- **Format:** Submit the answer sheet pdf containing your answers. You should solve the questions on this handout (either through a pdf annotator, or by printing, then scanning). Make sure that your answers (typed or handwritten) are within the dedicated regions for each question/part. If you do not follow this format, we may deduct points.
- **How to submit:** Submit a pdf with your answers on Gradescope. Log in and click on our class 15-281 and click on the submission titled HW10 and upload your pdf containing your answers.
- **Policy:** See the course website for homework policies and Academic Integrity.

Name	
Andrew ID	
Hours to complete?	

For staff use only

Q1	Q2	Q3	Total
/37	/40	/23	/100

Q1. [37 pts] Variable Elimination

Suppose you are given a Bayes net with the same variables and structure as the alarm Bayes net from lecture, with the conditional probability tables given below.

B	$P(B)$
$+b$	0.1
$-b$	0.9

E	$P(E)$
$+e$	0.1
$-e$	0.9

A	B	E	$P(A \mid B, E)$
$+a$	$+b$	$+e$	0.8
$-a$	$+b$	$+e$	0.2
$+a$	$+b$	$-e$	0.6
$-a$	$+b$	$-e$	0.4
$+a$	$-b$	$+e$	0.6
$-a$	$-b$	$+e$	0.4
$+a$	$-b$	$-e$	0.1
$-a$	$-b$	$-e$	0.9

J	A	$P(J \mid A)$
$+j$	$+a$	0.8
$-j$	$+a$	0.2
$+j$	$-a$	0.1
$-j$	$-a$	0.9

M	A	$P(M \mid A)$
$+m$	$+a$	0.6
$-m$	$+a$	0.4
$+m$	$-a$	0.1
$-m$	$-a$	0.9

Apply the variable elimination algorithm to the query $P(B \mid +j,\ +m)$. You will have to eliminate the variables E and A, in that order. For each variable, write:

(i) the variables involved in the resulting factor (e.g., after eliminating X the resulting factor might be $f_1(Y, Z)$, so the variables involved are Y, Z),

(ii) the summation to calculate the factor (e.g., $f_1(Y, Z) = \sum_x P(Y)P(x \mid Y)P(Z \mid x)$), and

(iii) the numeric values in the factor table.

(a) [9 pts] Eliminating E:

(i) Variables:

(ii) Summation:

(iii) Factor table:

(b) [9 pts] Eliminating A:

(i) Variables:

(ii) Summation:

(iii) Factor table:

(c) [8 pts] After eliminating E and A above, you must multiply the remaining factors to produce yet another factor table. Specify the variables associated with the resulting factor, and fill out the values in the corresponding factor table.

(i) Variables:

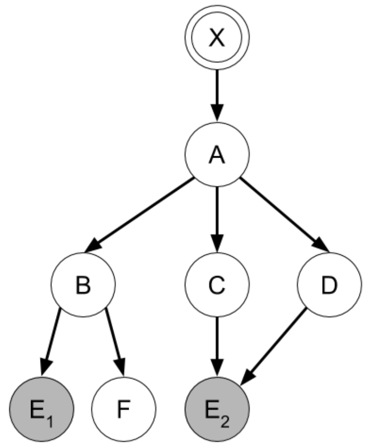
(ii) Factor table:

Now, find the normalizing constant to make this factor the probability distribution we want, $P(B \mid +j, +m)$. Write out the values of this normalized probability table.

(iii) Constant (α or Z):

(iv) Probability table:

Now consider the Bayes net below. Suppose we are trying to compute the query $P(X \mid e_1, e_2)$. Assume all variables are binary.



(d) [6 pts] Suppose we choose to eliminate variables in the order A, C, B, D, F.
Of the factors resulting from summing out over each of these variables, which factor has the most entries in its corresponding table? How many entries are in its table? Assume that we have separate entries for pairs of numbers even if we know sum to one (e.g., we would store both $P(X = +x)$ and $P(X = -x)$).

(i) Factor:	**(ii) # entries:**

(e) [5 pts] An optimal variable elimination ordering is one which minimizes the sum of the sizes of factors generated.

Fill in the table below with an optimal variable elimination ordering. For each variable, include the resulting factor and the number of entries in its table, again assuming that we separately store pairs of numbers which sum to one.

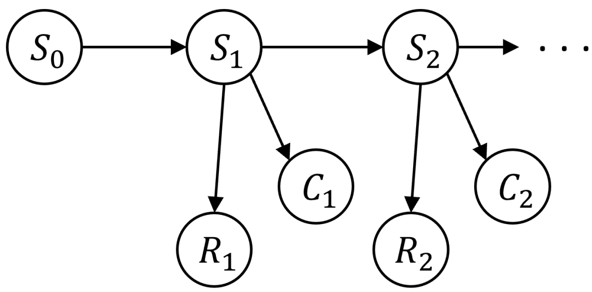
Variable	Factor	# Entries

Q2. [40 pts] Dynamic Bayes Net and Hidden Markov Model

A professor wants to know if students are getting enough sleep. Each day, the professor observes whether the students sleep in class, and whether they have red eyes. Let S_t be the random variable of the student having enough sleep, R_t be the random variable for the student having red eyes, and C_t be the random variable of the student sleeping in class on day t. The professor has the following theory:

- The prior probability of getting enough sleep at time t, with no observations, is 0.6
- The probability of getting enough sleep on night t is 0.9 given that the student got enough sleep the previous night, and 0.2 if not
- The probability of having red eyes is 0.1 if the student got enough sleep, and 0.7 if not
- The probability of sleeping in class is 0.2 if the student got enough sleep, and 0.4 if not

The following represents this information with a Dynamic Bayesian network along with the probability tables for the model.



S_0	$P(S_0)$
$+s$	0.6
$-s$	0.4

S_{t+1}	S_t	$P(S_{t+1} \mid S_t)$
$+s_{t_1}$	$+s_t$	0.9
$-s_{t_1}$	$+s_t$	0.1
$+s_{t_1}$	$-s_t$	0.2
$-s_{t_1}$	$-s_t$	0.8

R_t	S_t	$P(R_t \mid S_t)$
$+r$	$+s$	0.1
$-r$	$+s$	0.9
$+r$	$-s$	0.7
$-r$	$-s$	0.3

C_t	S_t	$P(C_t \mid S_t)$
$+c$	$+s$	0.2
$-c$	$+s$	0.8
$+c$	$-s$	0.4
$-c$	$-s$	0.6

(a) [30 pts] Using the DBN above and these evidence values

$-r_1, -c_1$ = not red eyes, not sleeping in class

$+r_2, -c_2$ = red eyes, not sleeping in class

$+r_3, +c_3$ = red eyes, sleeping in class

perform the following computations:

(i) [21 pts] State estimation: Compute $P(S_t \mid r_{1:t}, c_{1:t})$ for each of $t = 1, 2, 3$

$P(S_1 \mid -r_1, -c_1)$

$P(S_2 \mid r_{1:2}, c_{1:2})$

$P(S_3 \mid r_{1:3}, c_{1:3})$

(ii) [9 pts] Smoothing: Compute $P(S_t \mid r_{1:3}, c_{1:3})$ for $t = 2, 3$. (Hint: AIMA pg. 574)

We can build upon part (i) above, starting with $P(S_3 \mid r_{1:3}, c_{1:3})$.

$P(S_2 \mid r_{1:3}, c_{1:3})$

(b) [10 pts]

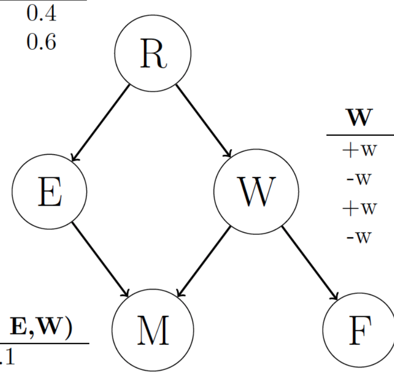
At every time step t for t = 1 to n, you observe a tuple (r_t, c_t) telling you whether the student had red eyes and whether they were sleeping in class. Given these observations and $P(S_k \mid r_{1:k}, c_{1:k})$, find an expression for $P(S_k \mid r_{1:n}, c_{1:n})$, where $0 \leq k \leq n$. You may only use the probability tables in the DBN and $P(S_k \mid r_{1:k}, c_{1:k})$.

$P(S_k \mid r_{1:n}, c_{1:n})$

Q3. [23 pts] Sampling

Consider the following Bayes Net and corresponding probability tables.

R	P(R)
+r	0.4
-r	0.6



E	R	P(E | R)
+e	+r	0.3
-e	+r	0.7
+e	-r	0.6
-e	-r	0.4

W	R	P(W | R)
+w	+r	0.9
-w	+r	0.1
+w	-r	0.2
-w	-r	0.8

M	E	W	P(M | E,W)
+m	+e	+w	0.1
-m	+e	+w	0.9
+m	+e	-w	0.45
-m	+e	-w	0.55
+m	-e	+w	0.35
-m	-e	+w	0.65
+m	-e	-w	0.9
-m	-e	-w	0.1

F	W	P(F | W)
+f	+w	0.15
-f	+w	0.85
+f	-w	0.75
-f	-w	0.25

Consider the case where we are sampling to approximate the query $P(R \mid +f, +m)$.

(a) [15 pts] Fill in the following table with the probabilities of *drawing* each respective sample given that we are using each of the following sampling techniques. *Hint*: $P(+f, +m) = 0.2682$.

Method	$< +r, +e, -w, +m, +f >$	$< +r, -e, +w, -m, +f >$
Prior sampling		
Rejection sampling		
Likelihood weighting		

(b) [8 pts] We are going to use Gibbs sampling to estimate the probability of getting the sample $< +r, +e, -w, +m, +f >$. We will start from the sample $< -r, -e, -w, +m, +f >$ and resample E first then R. What is the probability of drawing sample $< +r, +e, -w, +m, +f >$?

Answer: