Recitation 12

Graph Contraction and MSTs

12.1 Announcements

- *SegmentLab* has been released, and is due Friday, November 17. It’s worth 135 points.
- *Midterm 2* is on Wednesday, November 8.
### 12.2 Contraction

In the textbook, we presented an algorithm for counting the number of connected components in a graph:

**Algorithm 12.1. (Algorithm 17.22 in the textbook.)**

1. \( \text{countComponents} (V, E) = \)
2. \( \text{if } |E| = 0 \text{ then } |V| \text{ else} \)
3. \( \text{let} \)
4. \( (V', P) = \text{starPartition} (V, E) \)
5. \( E' = \{(P[u], P[v]) : (u, v) \in E \mid P[u] \neq P[v]\} \)
6. \( \text{in} \)
7. \( \text{countComponents} (V', E') \)
8. \( \text{end} \)

with \( \text{starPartition} \) implemented as follows:

**Algorithm 12.2. (Algorithm 17.15 in the textbook.)**

1. \( \text{starPartition} (V, E) = \)
2. \( \text{let} \)
3. \( TH = \{(u, v) \in E \mid \neg \text{heads}(u) \land \text{heads}(v)\} \)
4. \( P = \bigcup_{(u, v) \in TH} \{u \mapsto v\} \)
5. \( V' = V \setminus \text{domain}(P) \)
6. \( P' = \{u \mapsto u : u \in V'\} \)
7. \( \text{in} \)
8. \( (V', P' \cup P) \)
9. \( \text{end} \)

Now, suppose we implemented star partitioning for enumerated graphs as follows:

\[ \text{val enumStarPartition} : \text{(int * int) Seq.t} \times \text{int} \rightarrow \text{int Seq.t} \]

Specifically, given a graph represented as a sequence of edges \( E \) where every vertex is labeled \( 0 \leq v < n \), \( \text{(enumStarPartition} (E, n)) \) returns a mapping \( P \) where \( P[v] \) is the super-vertex containing \( v \). (If \( v \) was a star center or was unable to contract, then \( P[v] = v \).)

**Task 12.3.** Implement a function \( \text{enumCountComponents} \) which counts the number of components of an enumerated graph. It should take in a graph represented as \( (E, n) \) and use \( \text{enumStarPartition} \) internally.
12.2.1 Cost Bounds

**Task 12.4.** Recall that a forest is a collection of trees. What are the work and span of \( \text{enumCountComponents} \) when applied to a forest? Assume that \( (\text{enumStarPartition} \ (E, n)) \) requires \( O(n + |E|) \) work and \( O(\log n) \) span.