Parallel And Sequential Data Structures and Algorithms

Introduction to 15-210

Learning Objectives

- Understand **what kind of course this is** (and how it differs from previous offerings)
- Understand **how we will write algorithms** (imperative *and* functional)
- Understand **how we reason about algorithms** (abstraction, cost, interfaces)

Course Logistics and Goals

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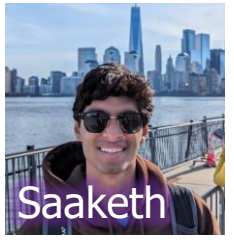
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Course Website and Platforms

- See **the website** for lecture notes, lecture slides, course policies, and resources: https://www.cs.cmu.edu/~15210-s26/
- We will use **Ed** for questions and announcements: https://edstem.org/us/courses/90704/discussion
- Homework will be submitted through **Gradescope**: https://www.gradescope.com/courses/1209244
- For TA office hours, use the Office Hours Queue (**OHQ**): https://210ohq.com/ohq/

Classes

- **Lectures**
  - Monday, Wednesday, 11:00am – 12:20pm (Rashid)
  - Some optional lectures on Friday 11:00am - 12:20pm (Rashid)
  - Note, Friday lecture slot is also used for quizzes and midterms!
- **Recitations**
  - Tuesday, various times, see your schedule on SIO

Assessment

- **Lab Homeworks (Programming and Written)**
  - 11 labs, worth 25% in total
  - Released Tuesdays, due the following Monday
  - "Bucket" system: You only need 90% score to get full points!
  - No lab out on weeks that contain an exam
- **Quizzes**
  - Friday **6th Feb** (Week 4), **13th March** (Week 8), **17th April** (Week 13)
  - Worth 15% in total (6% for your top two, 3% for your lowest)
- **Exams**
  - Midterms on Friday **20th Feb** (Week 6) and **27th March** (Week 10)
  - Midterms are worth 15% each
  - Final exam worth 25%
- **Recitation Participation**
  - Worth 5%

Course Changes

Key Idea: Focus on **algorithm design first**, rather than implementations in a specific programming language

- **New lectures, lecture notes, lecture slides**
- Same topics as before, just different presentation
- No longer *required* to write algorithms in SML
  - On exams and quizzes, you can write algorithms in any language, including pseudocode, as long as it's clear
  - For programming labs, you may choose **Parallel SML or C++**

C++ in 15-210

- All labs can be completed in either **Parallel SML or C++**
- **Same ideas, different syntax:** C++ can be used in a *functional, parallel style,* mirroring the SML track.

Note: **You do not need <u>any</u> advanced or systems-level C++.**

- No memory management
- No pointers
- No OOP concepts (inheritance/overriding/virtual functions)
- No concurrency/atomics/threading
- No move semantics/perfect forwarding/resource management

Imperative vs Functional

Functions

- In functional programming, functions usually correspond to mathematical functions: a map from inputs to outputs

```
fun f(n : int) -> int:
  if (n <= 1): return 1
  else: return f(n - 1) + f(n - 2)
```

- Imperative functions can mutate or depend on shared (external) mutable state

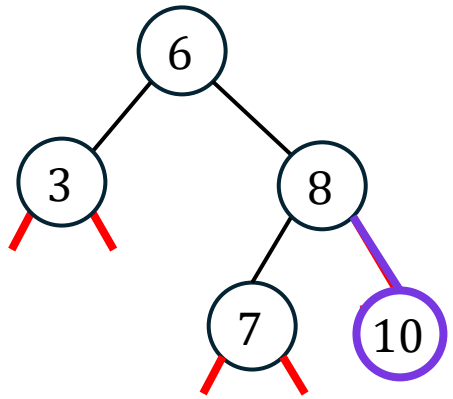
```
x : int = 0
y : int = 0

fun f(n : int) -> int:
  x ← 23
  if (n <= 1): return 1;
  else: return f(n - 1) + y * 3;
```

Definition (Pure function): A pure function is a function that always returns the same output given the same input and has no side effects.

Imperative Data Structures

```
// source: 15-122
tree* bst_insert(tree* T, entry e) {
  if (T == NULL) return leaf(e);
  int cmp = key_compare(entry_key(e), entry_key(T->data));
  if (cmp == 0) T->data = e;
  else if (cmp < 0) T->left = bst_insert(T->left, e);
  else T->right = bst_insert(T->right, e);
  return T;
}
```

- Insertion into an imperative-style (mutable) data structure **modifies** the existing data structure

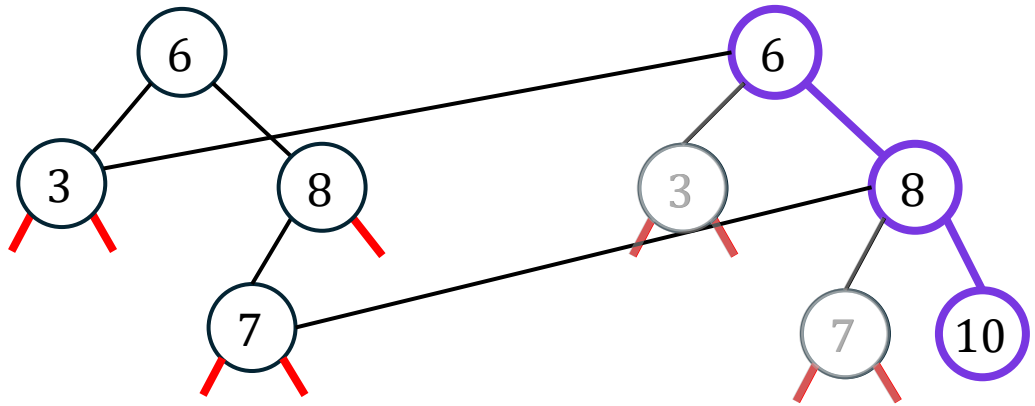


Example: `bst_insert(T, 10)`

- The right child of the 8-node is ***mutated*** to point to a new node containing 10

Functional Data Structures

```
(* source: 15-150 *)
fun insert (k, v) Empty =
    Node (Empty, (k, v), Empty)
  | insert (k, v) (Node (L, (k', v'), R)) =
      case Key.compare (k, k') of
        EQUAL   => Node (L, (k, v), R)
      | LESS    => Node (insert (k, v) L, (k', v'), R)
      | GREATER => Node (L, (k', v'), insert (k, v) R)
```



- Insertion into a functional data structure returns a **new data structure**
- The existing data structure remains unmodified

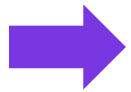
Example: `insert(10, _)`

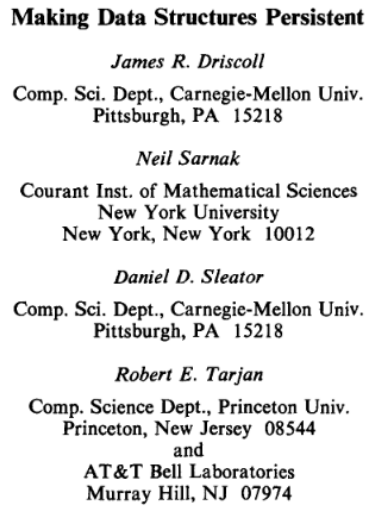
- New nodes are created for each node along the path
- Old nodes not on the insertion path are **reused**

Persistence

- Functional data structures provide a useful property for free:

Definition (Persistent data structure):
A persistent data structure is a data structure that preserves the old versions of itself when it is updated.





Making Data Structures Persistent

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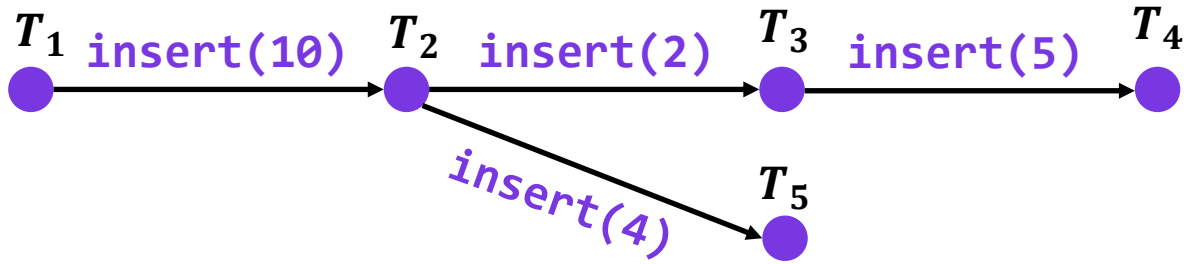
Persistence is like having version-control history of your data structure!



Abstraction: Interfaces and Abstract Data Types

Interfaces and ADTs

Definition (Interface): An interface specifies a **collection of operations** together with their intended behavior. It does not describe how those operations are implemented.

Definition (ADT): An **abstract data type** (ADT) is an interface that describes a data structure. It specifies the operations that can be performed, while leaving the underlying representation unspecified.

- An **implementation** provides algorithms for the operations in the interface
- A **data structure** is an implementation of an abstract data type, consisting of both a representation together with algorithms for the operations.

Why Interfaces?

- **Modularity:** Can improve or replace a data structure with a different one that implements the same interface and the code that uses it still works without needing changes
- **Reasoning:** Correctness proofs and runtime analysis need only reason about the interface and its guarantees, independent of the low-level implementation details (assuming the data structure is correct)
- **Reuse:** The same generic algorithm can be applied to multiple different types if they support the correct interface

The Sequence ADT

Definition (Sequence): A sequence of length n over elements of type T is an ordered collection of values that can be viewed as a mapping from the indices

$$\{0,1,\ldots,n-1\} \to T$$

Interface (Sequence): A `sequence<T>` (with value type T) supports

- **`nth(S : sequence<T>, i : int) -> T`:**
  returns the i^{th} element of the sequence S
- **`length(S : sequence<T>) -> int`:**
  return the length of the sequence S
- **`subseq(S : sequence<T>, i : int, k : int) -> sequence<T>`:**
  returns a view of the subsequence of S starting at index i with length k

Array Sequences

- Assume that the sequences we construct are **`ArraySequence<T>`**, a contiguous fixed-size array, which supports $O(1)$ time operations
- This is the type we will assume is returned by the **`tabulate`** primitive, which constructs a sequence from a function

```
tabulate : (f : (int -> T), n : int) -> ArraySequence<T>
```

- `tabulate(f, n)` returns a sequence of length n where $S[i] = f(i)$, i.e.,

$$[f(0), f(1), \ldots, f(n-1)].$$

- We may also use Python-like syntax in our pseudocode, e.g., we may write

```
parallel [f(i) for i in 0…n-1]
```

Other Example ADTs

Interface (Dictionary): A `dictionary<K,V>` (key type K, value type V) has:

- **`insert(D : dictionary<K,V>, k : K, v : V) -> void`**:
  add the given key, value pair $k{:}\,v$
- **`find(D : dictionary<K,V>, k : K) -> option<(K,V)>`**:
  return the item with the given key k (NONE if it doesn't exist)
- **`delete(D : dictionary<K,V>, k : K) -> void`**:
  delete the item with the given key k

- The most common implementations of dictionaries are:
  - **Hash tables** (common in imperative code)
  - **Binary search trees** (common in functional and imperative code)

Abstraction: Models of Computation and Cost Models

Analyzing Costs of Algorithms

- We make claims like "Insertion sort runs in $O(n^2)$ time" and "Merge Sort runs in $O(n \log n)$ time"

Question: How are we measuring "time"?

- In 15-122, you settled on *"number of execution steps"*
- "Execution steps" include integer arithmetic, conditionals, function calls, reading/writing memory, etc.
- One important caveat: we should not allow *arbitrary-precision arithmetic* in a *single step*

Word RAM Model

Definition (Word RAM model):

- Memory values are ***w*-bits long**,
- Operations on w-bit values, e.g., arithmetic, comparison, branching, reading/writing any memory location, cost $O(1)$,
- w is at least $\log n$ for problem inputs of size n.

- Same in spirit as the *"number of execution steps"* from 15-122
- Only difference is the restriction on the precision to some **fixed number of bits w**

Why Limit Precision?

```
int product = 1;
for (int i = 1; i <= n; i++)
   product *= i;
```

```
product = 1
for i in range(1,n+1):
   product *= i
```

Question: What goes wrong with this C code and Python code?

- In C, the product overflows a 64-bit integer for just $n \geq 21$
- In Python, it takes **more than constant space**
- In summary: don't write algorithms that rely on higher precision arithmetic than the precision of the input

Parallel Cost Models

- The Word RAM model is purely sequential. To analyze parallel algorithms, we will need to extend the model

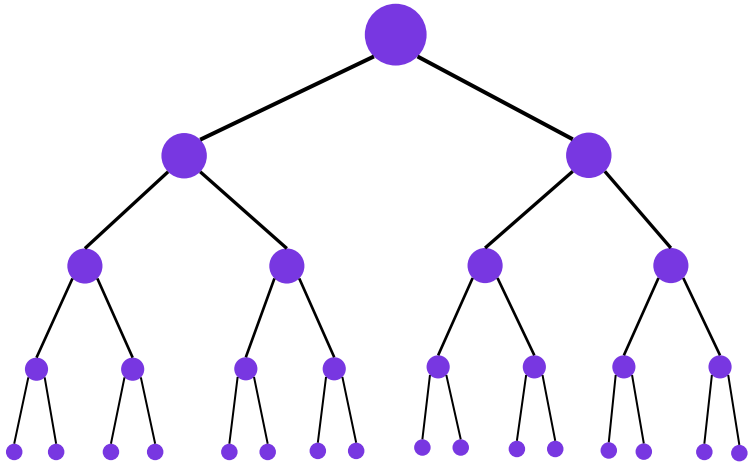
Definition (Fork-Join RAM): Extends the word RAM with a **`fork`** instruction.

- fork creates a fixed number of *child computations* that may execute in parallel.
  - The parent suspends at the fork point and resumes only after all its children have completed; the resumption is called the **join point**
- Child computations may perform forks, allowing parallelism to be nested.

Question: How much abstraction is this compared to real computer?

Nested Parallelism as an Abstraction

- Think of nested parallelism as having ***infinitely many processors!***
- A process can always fork into more, and this can continue recursively (e.g., divide-and-conquer style). No limit.



Remark: Deciding which instructions run on which CPU core is called *scheduling*. Your OS also has a scheduler for running processes.

- Nested parallelism allows algorithms to express parallel structure without specifying *how* computations are assigned to processors

What Do We Measure?

- For sequential algorithms, we just measured one cost *("time")*
- In 15-150, you learned that to describe the cost of a parallel algorithm, we need **two numbers**

Definition (Work): The **work** of a parallel algorithm is sum of the costs of all instructions it executes, across all parallel branches.

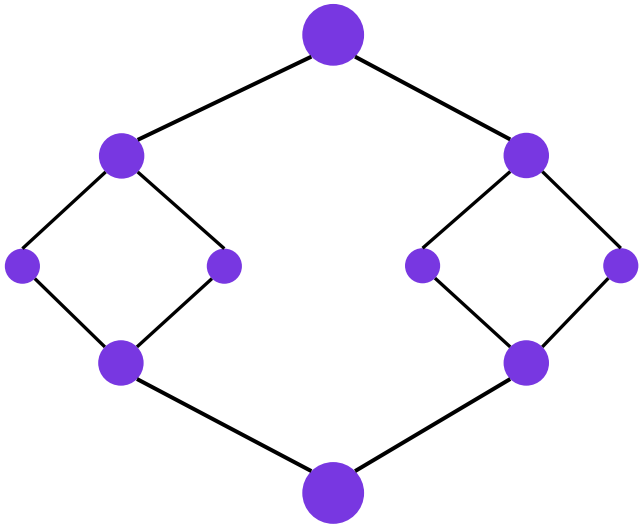
Intuition: Work is the cost of the algorithm on one processor (i.e. no parallelism)

Definition (Span): The **span** of a parallel algorithm is the cost of the longest chain of dependent computations

Intuition: Span is the cost of the algorithm with infinitely many processors!

Example: Parallel Sum (Reduce+)

```
fun sum(S : sequence<int>) -> int:
  match length(S) with:
    case 0: return 0       // Empty sequence
    case 1: return S[0]    // Singleton sequence
    case _:
      L, R = split_mid(S) // Helper function
      Lsum, Rsum = parallel (sum(L), sum(R))
      return Lsum + Rsum
```



Analysis via recurrence relation:

$W_{\text{sum}}(n) = 2W_{\text{sum}}(n/2) + O(1)$ which solves to $O(n)$
$S_{\text{sum}}(n) = S_{\text{sum}}(n/2) + O(1)$ which solves to $O(\log n)$

Less formal analysis: The recursion has depth $O(\log n)$ and does constant work per node, so the span is $O(\log n)$.

Why "infinitely many processors"?

Question: most CPUs have neither one nor infinitely many processors… so why are these quantities useful?

- **Unsatisfying answer**: Span is a natural measurement for nested parallel algorithms since they are described as if there are infinite processors
- **Better answer:** It turns out that by measuring the work and span, we can derive the cost of the algorithm for any number of processors!

Theorem (Brent's Theorem): An algorithm with work W and span S can be scheduled on a P-processor machine in $O\big(\max(W/P, S)\big)$ time

*Brent's Theorem essentially proves that nested parallelism is a **good abstraction**!*

Summary

- You can do your homework in either **Parallel SML or C++**!
- Functional data structures differ from imperative ones by avoiding **mutation** and as a result are naturally **persistent** which can have useful algorithmic applications
- Interfaces and ADTs are useful for specifying operations without committing to a concrete implementation
- Our chosen model for parallel algorithms is **nested fork-join parallelism**, analyzed by their **work** and **span**