Search Algorithms II

15-110 – Monday 03/14

Announcements

- HW4 Partial due **Thursday noon**, HW4 Full due **Monday noon**
  - When working on the coding questions, **start from the class examples** and **read the question prompts thoroughly**
  - Midsemester Feedback Forms are also due **Monday** [optional]
- Check3/Hw3 revision deadline **Thursday at noon**
- Quiz3 on **Friday**
  - Review session: **NSH 3305 from 6 PM on Wed, March 16th**
  - Solutions tomorrow on website; all other material already on website

Announcements

- Final exams dates are released!
  - 15-110's exam: **Monday, May 2, 2022 8:30am-11:30am**
  - **DO NOT BOOK TRAVEL BEFORE THE EXAM**. We do not let students take the exam early.

Learning Objectives

- Identify whether a tree is a **binary search tree**
- Search for values in BSTs using **binary search**
- Analyze the **efficiency** of binary search on a **balanced** vs. **unbalanced** BST
- Search for paths in **graphs** using **breadth-first search** and **depth-first search**
- Analyze the **efficiency** of BFS and DFS on a graph

Binary Search Trees

Revisiting Search Algorithms

Recall the first lecture on Search Algorithms, when we discussed linear and binary search.

We've applied these algorithms to lists; can we apply them to other data structures too? Let's investigate how to search a **tree**.

Linear Search on a Tree

In linear search, we step through each element in a list until we either find the target item or run out of items to look at.

To visit all nodes in a tree, check if each node is the target, then check whether the target is in one of the node's child subtrees. If we find the target in **either** subtree, we should return `True`.

We also have two base cases: one for when we reach an empty tree, and one for when we find the target. In both cases, we know what to return right away.

```
def search(t, target):
  if t == None:
    return False
  elif t["contents"] == target:
    return True
  else:
    return search(t["left"], target) or \
           search(t["right"], target)
```

Binary Search on a Tree

How would we apply Binary Search to a tree?

First, recall that for binary search to work, the input list must be **sorted**. We'll also need to find a way to "split" the tree similarly to how we split the list in binary search.

Discuss: how could we "sort" and "split" a tree?

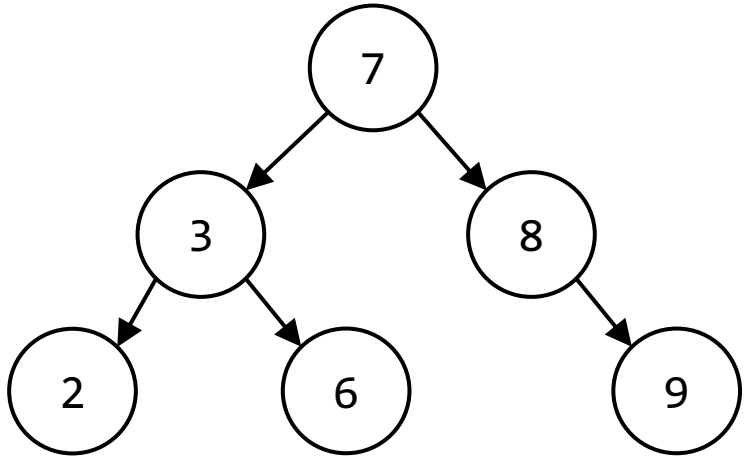
Binary Search Trees (BSTs) are "sorted"

We'll define a new kind of tree, a **Binary Search Tree,** as a binary tree that follows these constraints:

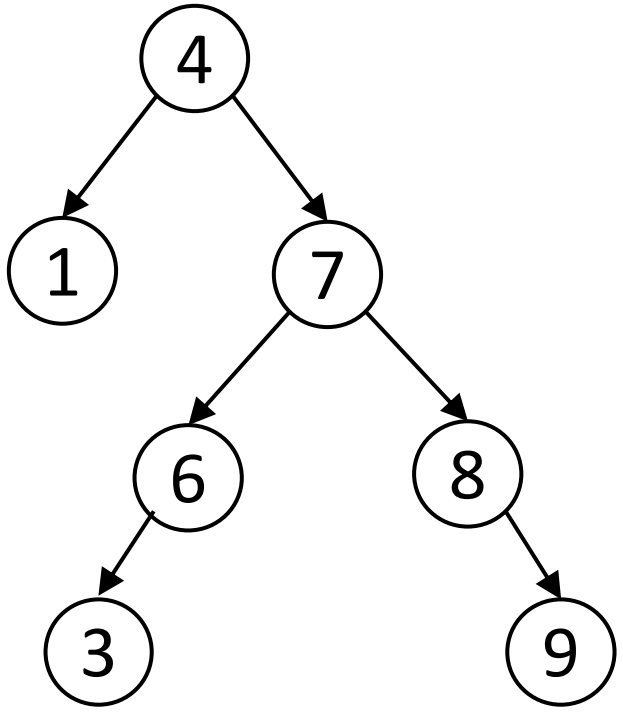
For every node n in a tree which has a value v:

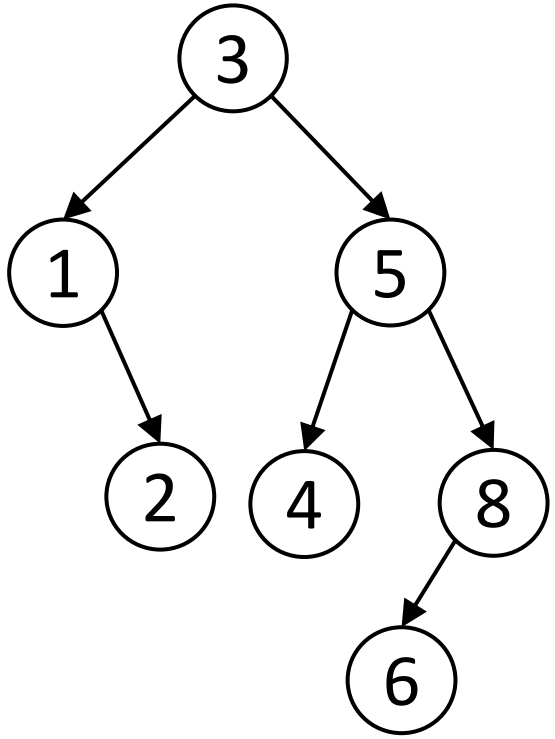
- Each **left** child (and all its children, etc.) must be strictly **less** than v
- Each **right** child (and all its children, etc.) must be strictly **greater** than v

Note: the left and right subtrees are BSTs! BST constraints are **recursive**.



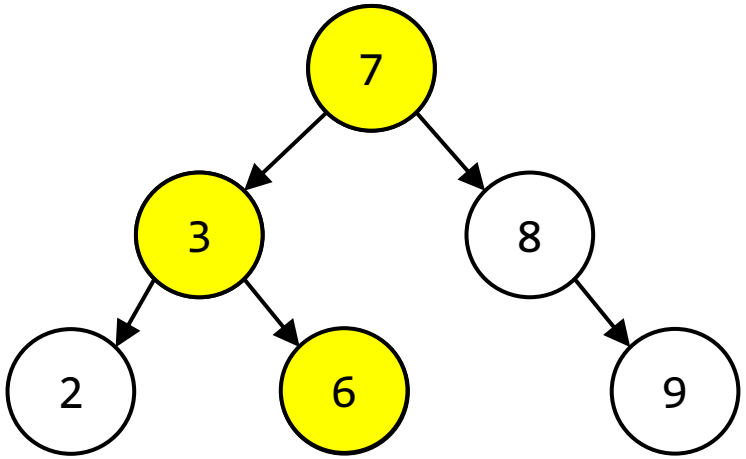
Example: Is this a BST?





Binary Search Trees Can Use Binary Search

When we want to search for the value 5 in the tree to the left, we start at the root node, 7. Because all nodes less than 7 must be in the left child tree and 5 is less than 7, **we only need to search the left child tree.**



Then, when we compare 5 to 3, we know that all values greater than 3 (but less than 7) must be in the right child of 3. 5 is greater than 3, so we **only need to search the right child**.

We 'split' the tree by only looking at one of the node's two children.

BST Search in Python

We would write binary search for a BST as follows:

```
def search(t, target):
    if t == None:
        return False
    elif t["contents"] == target:
        return True
    elif target < t["contents"]:
        return search(t["left"], target)
    else:
        return search(t["right"], target)
```

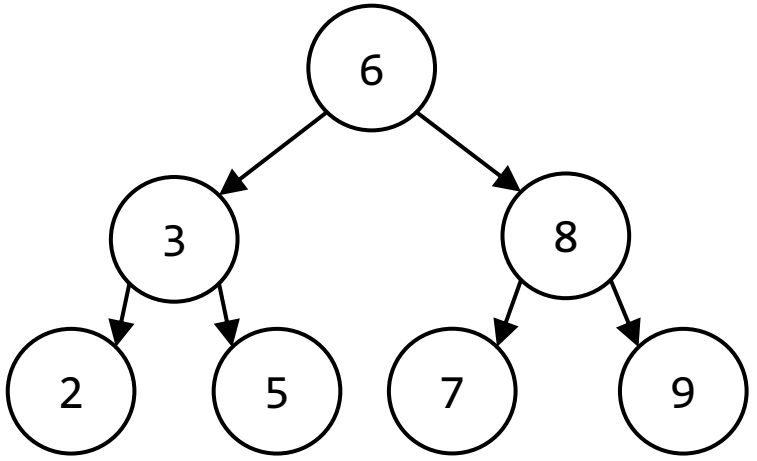
Note that we do just **one** recursive call, either on the left subtree or on the right subtree.

BST Search Runtime – Balanced Trees

Do we get the same O(log n) runtime for BST binary search that we did for list binary search? It depends on the tree.

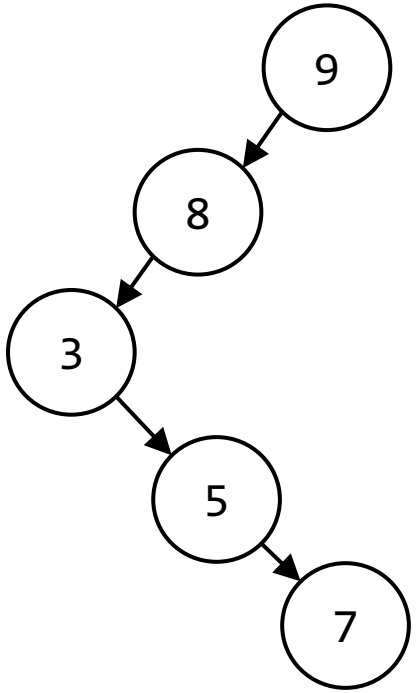
A tree is **balanced** if for every node in the tree, the node's left and right subtrees are approximately the same size. This results in a tree that **minimizes** the number of recursive levels.

Every time you take a search step in a balanced tree, you cut the number of nodes to be searched in half. This means that the algorithm will indeed take O(log n) time.



BST Search Runtime – Unbalanced Trees

A tree is considered **unbalanced** if at least one node has significantly different sizes in its left and right children. For example, consider the tree on the right.



This is a valid BST, but it is still difficult to search! You must visit every single node to determine a number like 6 isn't in the tree. In the worst case, this can still take **O(n)** time.

When we put data into BSTs, we usually strive to make them balanced to avoid these edge cases. For efficiency purposes, assume our BSTs are balanced and the worst case is O(log n).

Benefits of BSTs

At first glance, BSTs may seem less useful than sorted lists. However, they have a few added perks!

BSTs make it much easier to **add new data** to a dataset. In a sorted list, you would need to slide a bunch of values over to make room for a new value; in a BST, you can just run a search for this new value. When you reach a leaf, add a node with the new value.

This is very helpful for systems like hospital priority queues, where patients frequently need to be moved around the queue based on changing circumstances.

In general, try to choose a data structure that matches the task you need to solve.

Breadth-First Search and Depth-First Search

Searching a Graph for a Node

Determining whether a node exists in a graph is really easy given our dictionary structure: just check whether the node exists in the graph's keys.

```
def search(g, node):
    return node in g
```

Finding a node is too easy. Let's ask a more interesting question.

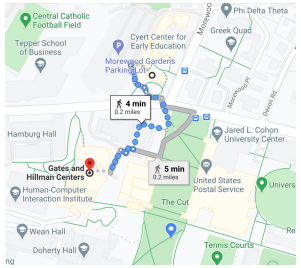
Searching a Graph for a Path

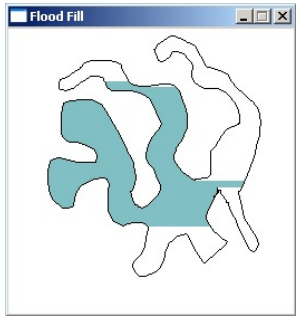
We now know about several data types that **connect** the data points in the structure (trees and graphs). We can search these structures to see if they contain a specific point of data, but it may be more interesting to see whether there is a **connection** between two nodes in the structure.

In other words: can we find a **path** that leads from a start node to a target node in the tree/graph?

This is useful in several contexts, including:

- finding walking/driving routes through a city
- contact tracing to identify people who are at risk
- flood fill in a paint application

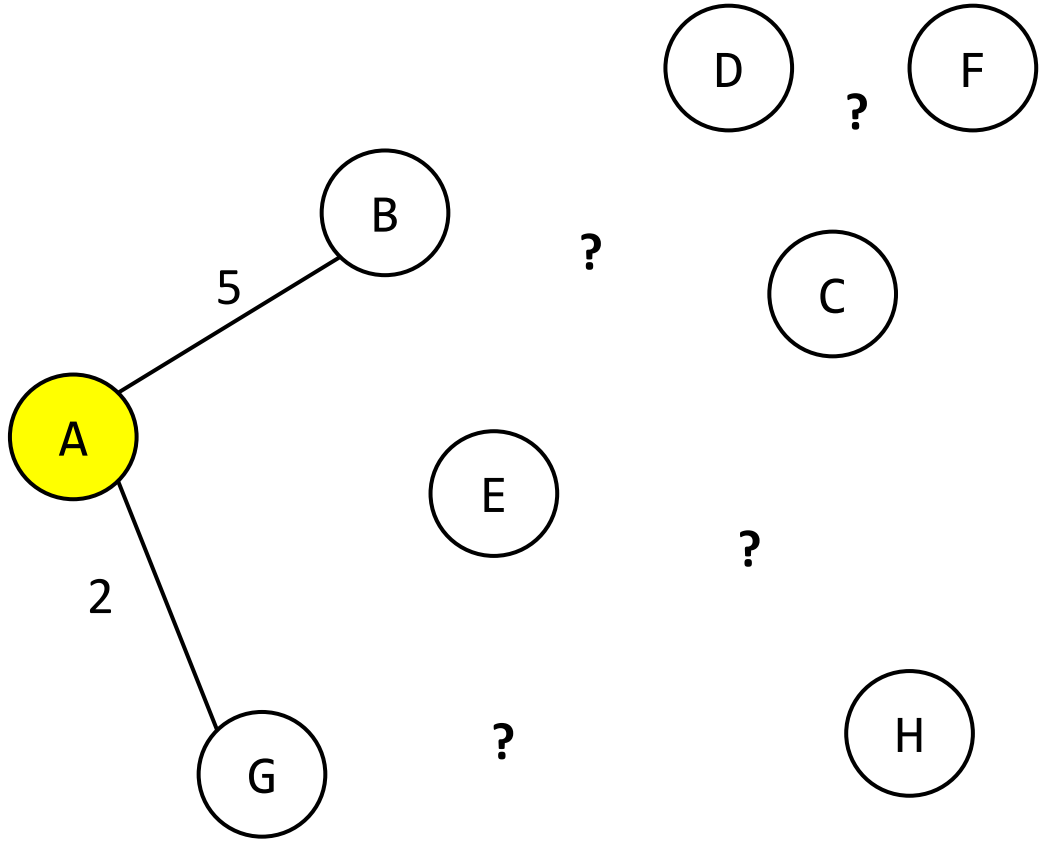




How to Search for Paths?

Recall that the computer can't 'see' graphs the way we do; for each node in the graph, it only sees the neighbor nodes that are directly connected. We need to run a systematic search to determine if two nodes are connected or not.

We'll need to start at the start node and repeatedly follow the edges to find all the other nodes it's connected to. Let's discuss two common approaches for deciding which order to visit the connected nodes in.



Two Search Algorithms: BFS and DFS

In **Breadth-First Search (BFS)**, we slowly move outwards in the graph/tree from the start node. We visit all the neighbors of start, then visit all the neighbors of the already visited nodes, etc., until we've checked all the nodes that were connected to the start node of the graph/tree.

BFS is useful for finding targets that should be nearby (like directions to a location within a 5 mile radius).

In **Depth-First Search (DFS)**, we go all the way down one potential path, then backtrack and try other possible paths. So we choose one neighbor, then choose one of its neighbors, etc., until there are no unvisited neighbors left, then backtrack.

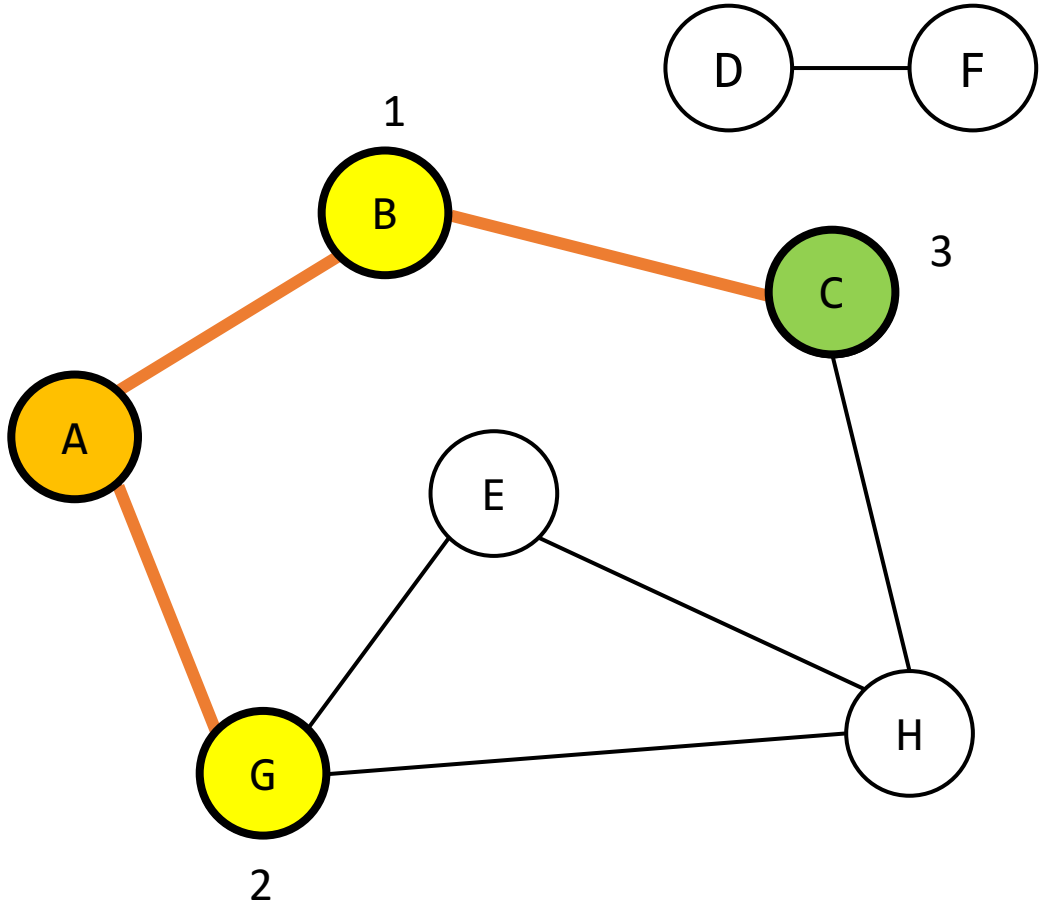
DFS is useful for finding targets that are far away in a specific direction (like a route that goes straight across the United States east-to-west).

Breadth-First Search Example

Let's consider Breadth-First Search on our example graph, starting from A and searching for C.

A has two neighbors, B and G. We can visit B and then G, or G and then B.

Once both have been visited, we visit B and G's neighbors – C, E, and H. (A is a neighbor as well, but we don't visit it because it's been visited before.) As soon as we reach C, we've found the node, and we're done- a path exists!



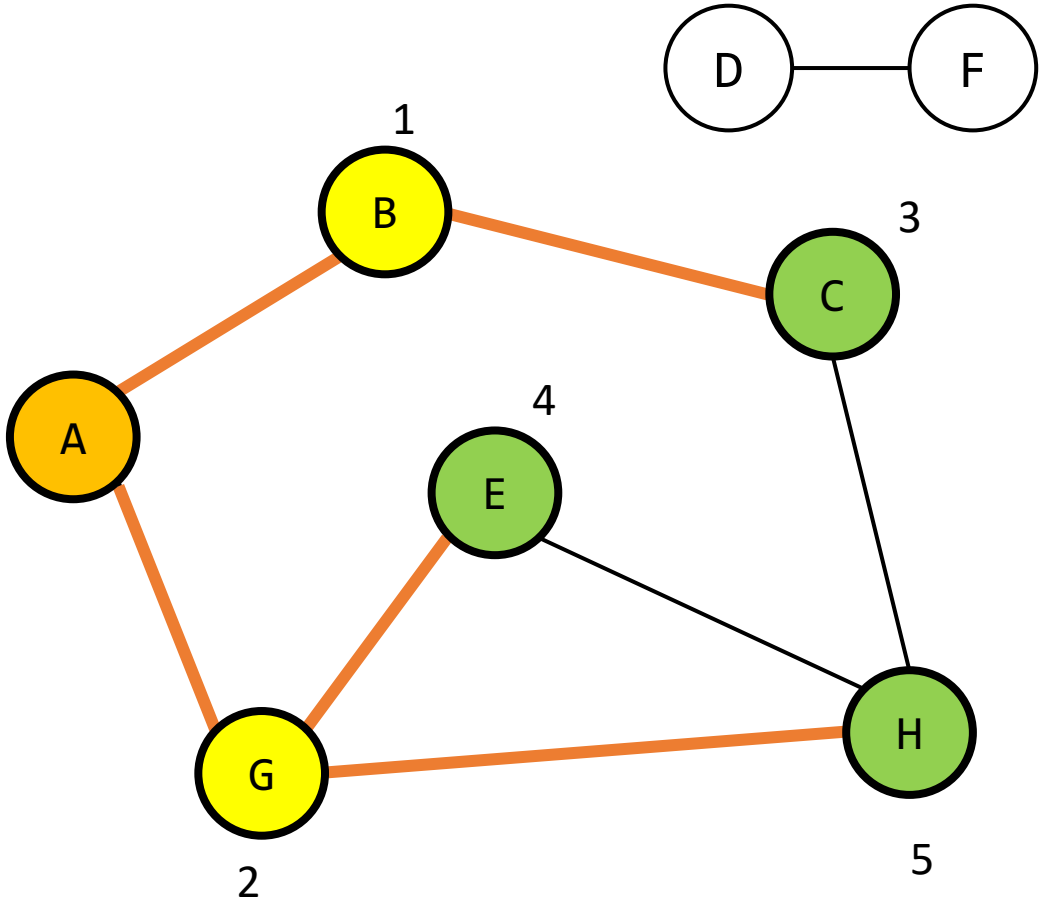
Breadth-First Search Example

Now let's run Breadth-First Search starting from A and searching for a value not connected to it, D.

A has two neighbors – B and G. As before, we can visit B and then G, or G and then B.

Once both have been visited, we visit B and G's neighbors – C, E, and H. Again, these can be visited in any order (CEH, CHE, ECH, EHC, HEC, HCE). We don't revisit A.

At this point, there are no nodes left that are neighbors of C, E, and H and have not been visited. We conclude there is no path from A to D.

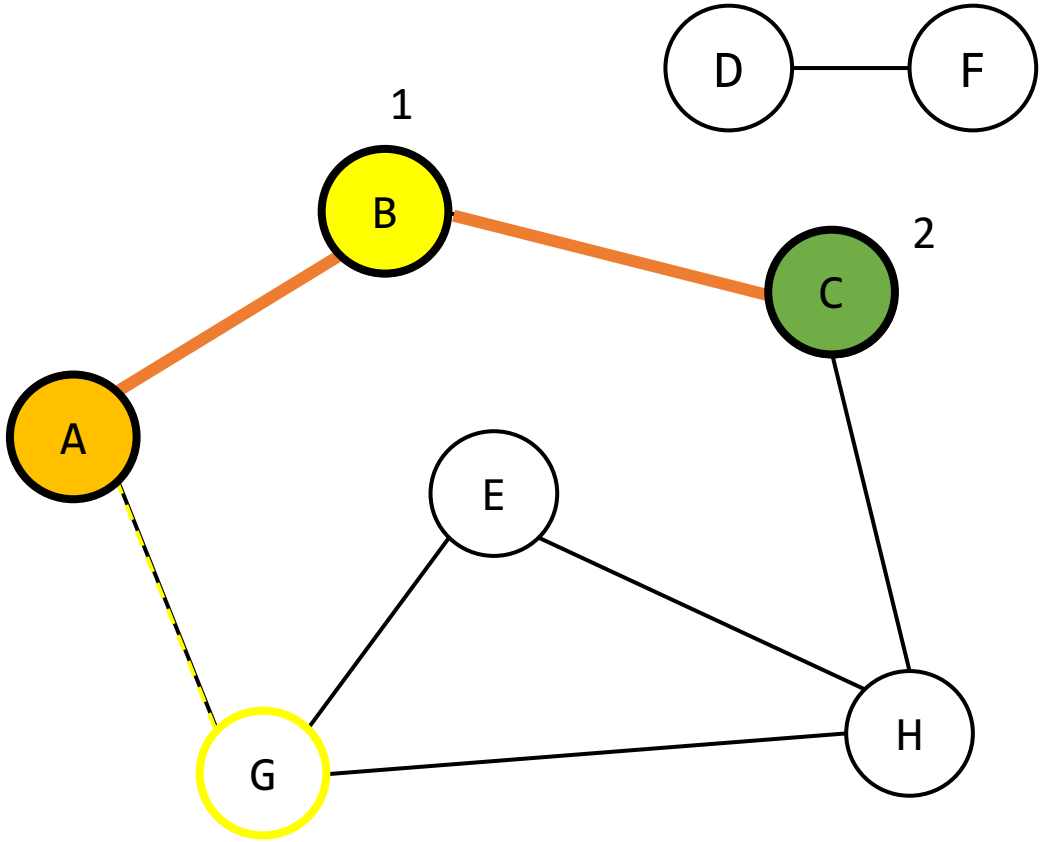


Depth-First Search Example

Now let's search the example graph starting from A with depth-first search, searching for C.

There are two possible starting routes: B or G. Let's choose B. We'll store G as a backup option in case we run into a dead end.

From B, we only have one unvisited neighbor: C. We've found the node we're looking for, so we're done!



Depth-First Search Example

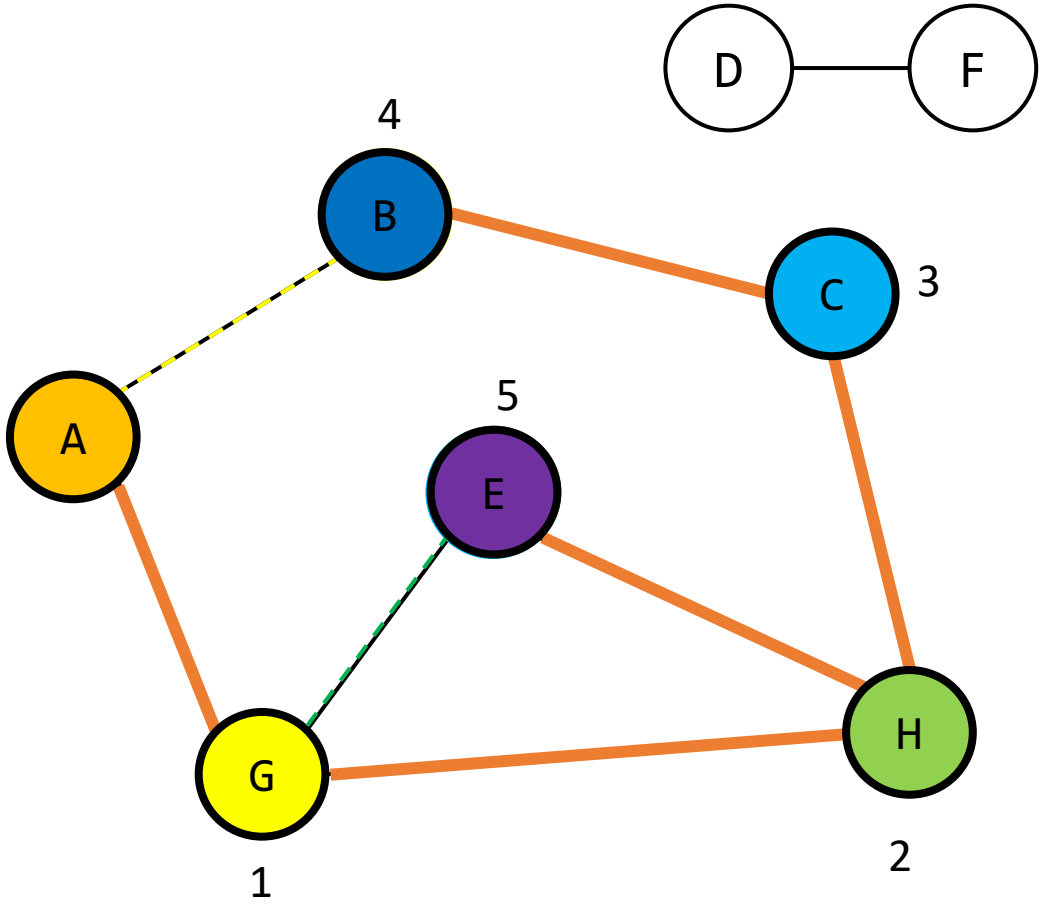
What if we search the example graph starting from A with depth-first search, now looking for D?

There are two possible starting routes: B or G. Choose G, and place B in the **backup list**.

From G, we have two possible routes, E or H; choose H and mark E as backup. Note that A is not a valid choice, as it's already been visited.

From H, we have two more possible routes: E or C (G is not valid). We'll choose C and put E on the backup list again. C's only remaining neighbor is B (H is not valid), so we must visit it.

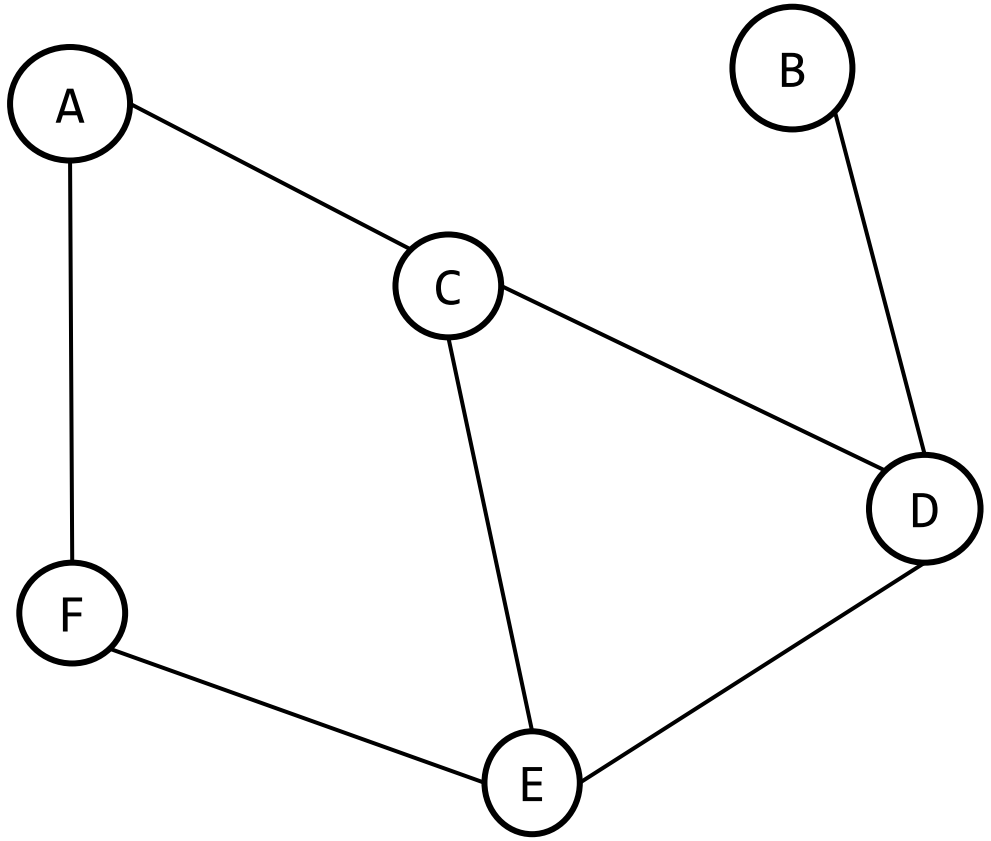
Now B has no unvisited neighbors remaining (A and C are both visited), so we must **backtrack** to the last node that had an unvisited neighbor. If we check our backup list, the only unvisited node remaining is E (which was G and H's neighbor). We visit E, and we're done. There is no path from A to D.



Activity: BFS and DFS Tracing

Given the graph to the right and starting from A while searching for G, what is a valid trace for **Breadth-First Search,** and what is a valid trace for **Depth-First Search**?

Visit neighbors **alphabetically** (while following the search rules) to make things simpler.



Coding BFS and DFS

To code these search algorithms, we'll need to keep track of two pieces of data. One is the **nodes we need to search next**. The other is **the nodes we've already visited**. It's important to keep track of what we've visited so far to avoid cycling back to nodes we've seen before and looping forever!

We'll use a **while loop** to iterate over the nodes we need to search and update the list as we go. Each iteration will check the next node on the to-search list to see if it's the end node we're looking for.

If we find the end node, we'll return `True` right away (a path is possible; making that path is out of scope for this class). If we're on a different node, we'll add all of its unvisited neighbors to the to-visit list. **How we add the nodes changes based on whether we implement BFS or DFS**.

Breadth-First Search Code

Note that in the BFS code, we add neighbors of each node we visit to the **end** of the to-visit list. This prioritizes neighbors that are connected earlier in the graph/tree.

```
def breadthFirstSearch(g, start, target):
    # Set up two lists for visited nodes and to-visit nodes
    visited = [ ]
    nextNodes = [ start ]

    # Repeat while there are nodes to visit
    while len(nextNodes) > 0:
        next = nextNodes[0]

        if next == target: # If it's what we're looking for- we're done!
            return True
        else: # Otherwise, add unvisited neighbors
            for node in g[next]:
                if node not in visited and node not in nextNodes: # Not seen before
                    nextNodes = nextNodes + [ node ] # Add to the BACK of the list

        nextNodes.remove(next)
        visited.append(next) # We've visited the node now
    return False # When no nodes left- we're done!
```

Depth-First Search Code

In the DFS code, we add neighbors of each node we visit to the **front** of the to-visit list. This prioritizes neighbors that are connected deeper inside the graph/tree. Otherwise, the algorithm is the same.

```
def depthFirstSearch(g, start, target):
    # Set up two lists for visited nodes and to-visit nodes
    visited = [ ]
    nextNodes = [ start ]

    # Repeat while there are nodes to visit
    while len(nextNodes) > 0:
        next = nextNodes[0]

        if next == target: # If it's what we're looking for- we're done!
            return True
        else: # Otherwise, add unvisited neighbors
            for node in g[next]:
                if node in nextNodes: # seen before, but higher-priority now
                    nextNodes.remove(node)
                if node not in visited and node not in nextNodes: # Not seen before
                    nextNodes = [ node ] + nextNodes # Add to the FRONT of the list
        nextNodes.remove(next)
        visited.append(next) # We've visited the node now
    return False # When no nodes left- we're done!
```

BFS / DFS Efficiency

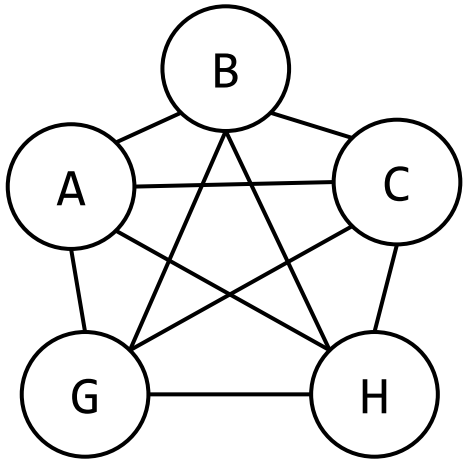
If we only consider the worst case scenario and measure efficiency based on the number of nodes visited, BFS and DFS look the same.

The worst case is when there is no path between the start and target node, but **every other** node in the graph is connected to the start. In either algorithm we would need to visit every single node to determine that there is no path. The overall efficiency is **O(n)**.

... or is it?

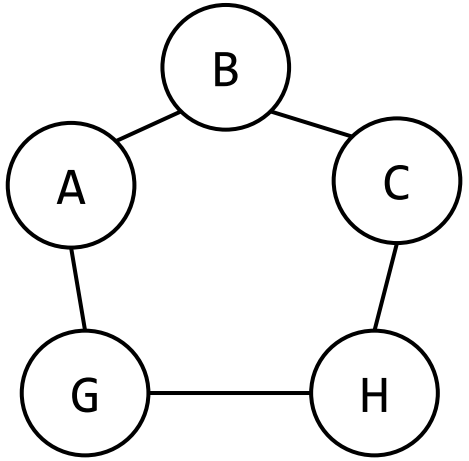
Same Nodes, Different Edges

It's possible for us to visit a node more than once in BFS or DFS if it appears **multiple times** in neighbor lists. For example, consider the two graphs to the right. Each has the same number of nodes.



The upper graph is **heavily connected** – every node is connected to every other node. For each node we visit we must verify that **every** other node in the graph is already in the visited or to-visit list.

In the lower graph we only need to deal with **two** neighbors per node. There are fewer repeats.



Edges Affect Efficiency

The efficiency of BFS/DFS is affected by the **number of edges in the graph**, just like how BST search was affected by how balanced the tree's edges were! When it comes to determining efficiency, **edges matter**.

As with BSTs, we'll simplify things for this class. We'll assume that the number of edges each node has is **constant**. Then the runtime for BFS / DFS can still be **O(n)**.

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- Identify whether a tree is a **binary search tree**
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- Search for values in **graphs** using **breadth-first search** and **depth-first search**
- Analyze the **efficiency** of BFS and DFS on a graph

Feedback: https://bit.ly/110-s22-feedback