

Recursive Function Call Tracing

Supplement to Recursion lecture

Tracing the Function Calls

Start with the original function call.

```
def addCards(cards):
    if cards == [ ]:
        return 0
    else:
        smallerProblem = cards[1:]
        smallerResult = addCards(smallerProblem)
        return cards[0] + smallerResult
```

addCards([5, 2, 7, 3])

Call 1

addCards([5, 2, 7, 3])

Tracing the Function Calls

We go to the recursive case and set up a local variable `smallerProblem`. Then call `addCards` again on that variable, putting another function call.

```
def addCards(cards):
    if cards == [ ]:
        return 0
    else:
        smallerProblem = cards[1:]
        smallerResult = addCards(smallerProblem)
        return cards[0] + smallerResult
```

```
addCards([5, 2, 7, 3])
```

Call 2 `smallerResult = addCards([2, 7, 3])`

Call 1 `addCards([5, 2, 7, 3])`



Tracing the Function Calls

When we run through `addCards` a second time, there's a **new local state**. `cards` is now `[2, 7, 3]`. `smallerProblem` is now `[7, 3]`.

```
def addCards(cards):
    if cards == []:
        return 0
    else:
        smallerProblem = cards[1:]
        smallerResult = addCards(smallerProblem)
        return cards[0] + smallerResult
```

```
addCards([5, 2, 7, 3])
```

Call 2

```
smallerResult = addCards([2, 7, 3])
```



Call 1

```
addCards([5, 2, 7, 3])
```

Tracing the Function Calls

Call `addCards` again, this time on `[7, 3]`. Note that the function call tracing helps us keep track of **all** previous calls.

```
def addCards(cards):
    if cards == [ ]:
        return 0
    else:
        smallerProblem = cards[1:]
        smallerResult = addCards(smallerProblem)
        return cards[0] + smallerResult
```

```
addCards([5, 2, 7, 3])
```

Call 3 `smallerResult = addCards([7, 3])`

Call 2 `smallerResult = addCards([2, 7, 3])`

Call 1 `addCards([5, 2, 7, 3])`



Tracing the Function Calls

Now we run the function with `cards` set to `[7, 3]`.

`smallerProblem` becomes `[3]`; we call the function again.

```
def addCards(cards):
    if cards == [ ]:
        return 0
    else:
        smallerProblem = cards[1:]
        smallerResult = addCards(smallerProblem)
        return cards[0] + smallerResult
```

`addCards([5, 2, 7, 3])`

Call 4

`smallerResult = addCards([3])`

Call 3

`smallerResult = addCards([7, 3])`

Call 2

`smallerResult = addCards([2, 7, 3])`

Call 1

`addCards([5, 2, 7, 3])`

Tracing the Function Calls

Run the function with `cards` set to [3].

`smallerProblem` becomes []; run the function again.

```
def addCards(cards):
    if cards == []:
        return 0
    else:
        smallerProblem = cards[1:]
        smallerResult = addCards(smallerProblem)
        return cards[0] + smallerResult
```

addCards([5, 2, 7, 3])

Call 5

smallerResult = addCards([])

Call 4

smallerResult = addCards([3])

Call 3

smallerResult = addCards([7, 3])

Call 2

smallerResult = addCards([2, 7, 3])

Call 1

addCards([5, 2, 7, 3])

Tracing the Function Calls

Now we finally reach the base case. `addCards([])` returns `0` immediately, so `0` takes the place of the function call on the bottom level of the function calls (in Call #5).

```
def addCards(cards):
    if cards == [ ]:
        return 0
    else:
        smallerProblem = cards[1:]
        smallerResult = addCards(smallerProblem)
        return cards[0] + smallerResult
```

`addCards([5, 2, 7, 3])`

Call 5

`smallerResult = 0`

Call 4

`smallerResult = addCards([3])`

Call 3

`smallerResult = addCards([7, 3])`

Call 2

`smallerResult = addCards([2, 7, 3])`

Call 1

`addCards([5, 2, 7, 3])`

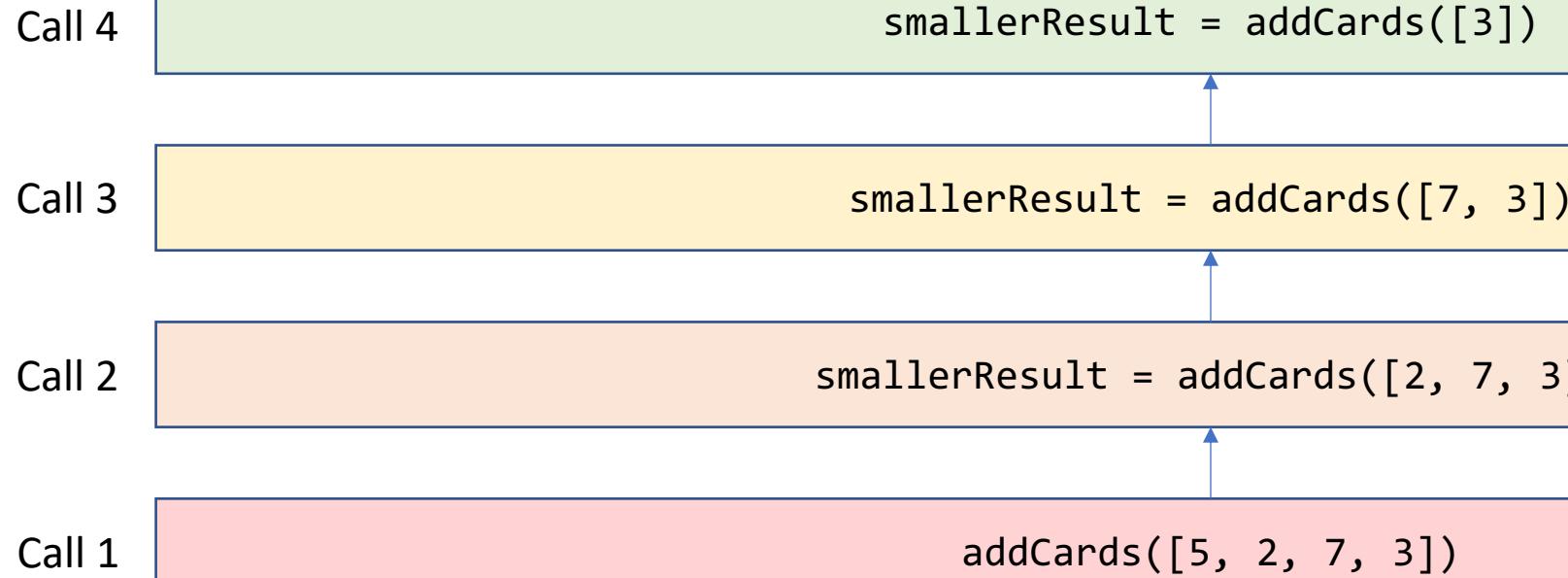


Tracing the Function Calls

How does Python know what to do next? It remembers the **local state** of each level of the function calls! On the green level, it knows that `cards` is `[3]` and `smallerResult` is now `0`.

```
def addCards(cards):
    if cards == [ ]:
        return 0
    else:
        smallerProblem = cards[1:]
        smallerResult = addCards(smallerProblem)
        return cards[0] + smallerResult
```

```
addCards([5, 2, 7, 3])
```



Tracing the Function Calls

Add $3 + 0$ to get 3 ; this can be sent back as the returned value to the previous level of the function calls (in Call #3).

```
def addCards(cards):
    if cards == [ ]:
        return 0
    else:
        smallerProblem = cards[1:]
        smallerResult = addCards(smallerProblem)
        return cards[0] + smallerResult
```

```
addCards([5, 2, 7, 3])
```

Call 4

```
smallerResult = 3
```

Call 3

```
smallerResult = addCards([7, 3])
```

Call 2

```
smallerResult = addCards([2, 7, 3])
```

Call 1

```
addCards([5, 2, 7, 3])
```



Tracing the Function Calls

At this level, `cards` is [7, 3] and `smallerResult` is 3. 7 + 3 gives us a returned value of 10.

```
def addCards(cards):
    if cards == [ ]:
        return 0
    else:
        smallerProblem = cards[1:]
        smallerResult = addCards(smallerProblem)
        return cards[0] + smallerResult
```

`addCards([5, 2, 7, 3])`

Call 3

`smallerResult = 10`

Call 2

`smallerResult = addCards([2, 7, 3])`

Call 1

`addCards([5, 2, 7, 3])`



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Tracing the Function Calls

Now `cards` is `[2, 7, 3]` and `smallerResult` is `10`. Add `2 + 10` to get a returned value of `12`.

```
def addCards(cards):
    if cards == [ ]:
        return 0
    else:
        smallerProblem = cards[1:]
        smallerResult = addCards(smallerProblem)
        return cards[0] + smallerResult
```

`addCards([5, 2, 7, 3])`

Call 2

`smallerResult = 12`

Call 1

`addCards([5, 2, 7, 3])`



Tracing the Function Calls

We've finally reached the original call. `cards` is `[5, 2, 7, 3]`, and `smallerResult` is `12`. Add `5 + 12` to get `17`, which is returned at the top level as the final result of the original function call.

```
def addCards(cards):
    if cards == [ ]:
        return 0
    else:
        smallerProblem = cards[1:]
        smallerResult = addCards(smallerProblem)
        return cards[0] + smallerResult
```

```
addCards([5, 2, 7, 3])
```