15-110: Principles of Computing, Spring 2018

Problem Set 7 (PS7)

Due: Friday, March 23 by 2:30PM via Gradescope Hand-in

HANDIN INSTRUCTIONS

Download a copy of this PDF file. You have two ways to fill in your answers:

1. **Just edit (preferred) - Use any PDF editor (e.g., Preview on Mac, iAnnotate on mobile, Acrobat Pro on pretty much anything) to typeset your answers in the given spaces. You can even draw pictures or take a picture of a drawing and import it in the correct place in the document. That's it. (Acrobat Pro is available on all cluster machines.)**

2. **Print and Scan - Alternatively, print this file, write your answers neatly by hand, and then scan it into a PDF file. This is labor-intensive and must be done by the deadline.**

Once you have prepared your submission, submit it on Gradescope. A link to Gradescope is provided in our Canvas course portal.

Fill in your answers ONLY in the spaces provided. Any answers entered outside of the spaces provided may not be graded. Do not add additional pages. We will only score answers in the given answer spaces provided. If we cannot read your answer or it contains ambiguous information, you will not receive credit for that answer.

Be sure to enter your full name below along with your section letter (A, B, C, etc.) and your Andrew ID. Submit your work on Gradescope by 2:30PM on the Friday given above.

REMINDER: Sharing your answers with another student who is completing the assignment, even in another semester, is a violation of the academic integrity policies of this course. Please keep these answers to yourself.

Name (First Last) _____

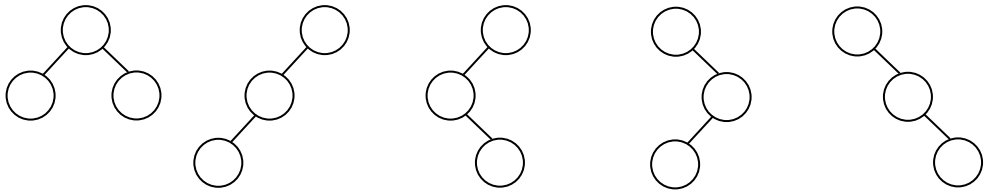
Section _____ Andrew ID _____

1. (1 pt) This problem deals with a binary tree known as a binary search tree.

   (a) Insert the following integers into a binary search tree one at a time in the order given and draw the final result. (You may take a picture of your final drawing and copy and paste it into the square below if you wish.)

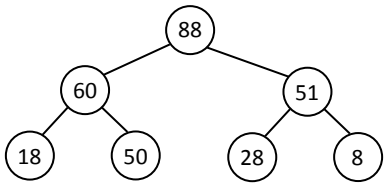
   42 39 10 23 79 65 91 88

   (b) Suppose you insert three data values into an initially empty binary search tree. You could get any one of the following five tree shapes:



   How many tree shapes can you get if you insert four data values into an initially empty binary search tree? (The example above will help you compute part of your solution.)

2. (1.5 pts) This problem deals with a binary tree known as a max-heap. Consider the max-heap `h` shown below.



(a) Show how the max-heap `h` would be stored as a list in Python.

```
h =
```

(b) Using the original max-heap shown above, draw a picture of the max-heap after the value 65 is inserted into the data structure using the algorithm discussed in class.

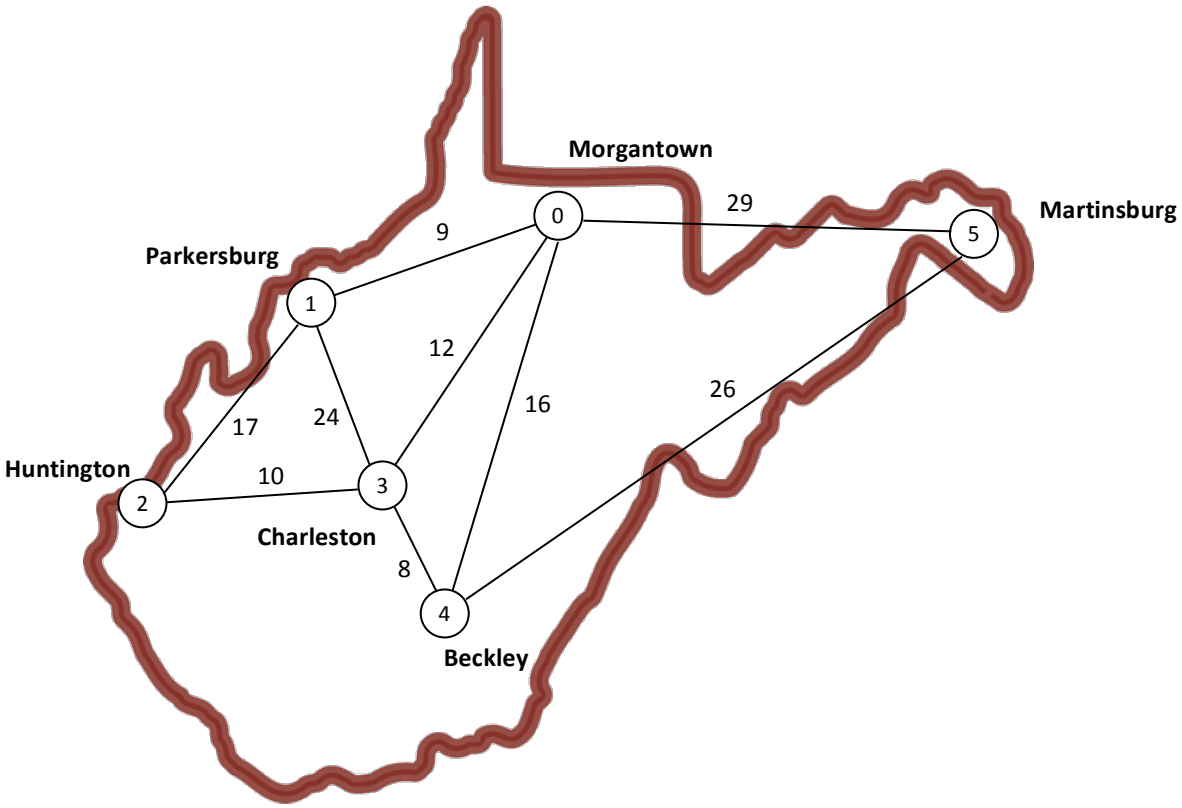
(c) Using the original max-heap shown above, draw a picture of the max-heap after the value 88 is removed from the data structure using the algorithm discussed in class.

(d) If a max-heap had 1024 values in it, what is the maximum number of swaps needed to insert a new value? (HINT: the max-heap is a balanced tree structure.)

3. (1.5 pts) A new bus company WV-Bus has consulted with an analyst to determine the cost in millions of dollars per year to run buses between a set of designated towns in West Virginia:

```
towns = [ "Morgantown", "Parkersburg", "Huntington",
          "Charleston", "Beckley", "Martinsburg" ]
```

The analyst creates a picture of a `graph` based on the `towns` list above to represent the operating costs for possible direct service between the towns in millions of dollars per year:



(a) Show how the graph above is represented in Python:

```
f = float("inf")

graph = [ [ ___, ___, ___, ___, ___, ___ ],
          [ ___, ___, ___, ___, ___, ___ ],
          [ ___, ___, ___, ___, ___, ___ ],
          [ ___, ___, ___, ___, ___, ___ ],
          [ ___, ___, ___, ___, ___, ___ ],
          [ ___, ___, ___, ___, ___, ___ ]  ]
```

(b) WV-Bus has limited funds, so the company wants to start with a minimal bus network. They want to include only those direct bus routes so that all towns are connected (either directly or indirectly through bus transfers) and the total operating cost is as low as possible.

One algorithm we can use to solve this problem is to sort the routes by cost (lowest to highest). For each route in sorted order, include it in your solution if the two towns were not connected already in your solution, directly or indirectly.

Using this algorithm, which direct bus routes (town to town) are needed to create the minimal operating network for this company so that all towns are connected and the overall operating cost of the bus network is as low as possible? List them in the order that you select them in the answer box below, along with their costs. Also, indicate how much the minimal network will cost to operate in millions of dollars per year.

Direct town-to-town routes needed to build minimal operating network:
(You may not need all lines below)

Route	Cost (in $M/year)
_____ to _____	_____
_____ to _____	_____
_____ to _____	_____
_____ to _____	_____
_____ to _____	_____
_____ to _____	_____
_____ to _____	_____
_____ to _____	_____

Total cost for minimal service connecting all 6 towns: $_____M/year

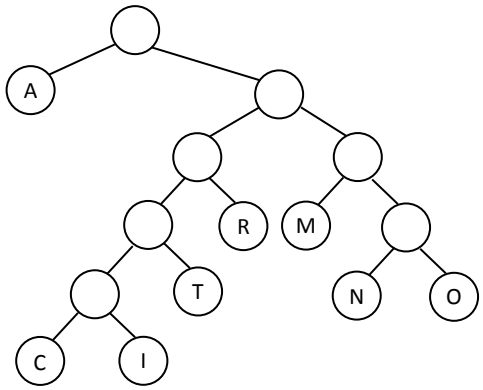
4. (1.5 pts) Consider the 8-bit value 11010111.

(a) What is its value in decimal if it is interpreted as an unsigned integer? Show your work for full credit.

(b) What is its value in decimal if it is interpreted as a signed integer? Show your work for full credit.

(c) Write this 8-bit value in hexadecimal.

5. (1 pt) Consider the following Huffman tree that encodes an alphabet of 8 letters based on the frequency of each letter in a set of electronic documents:



In the Huffman coding tree above, left branches are represented as zeroes (0) and right branches as ones (1).

(a) Which letter occurs most frequently in the documents? How can you tell?

(b) Decode the following "word" using the Huffman tree above:

100001111101100110001111100

(c) Is Huffman coding a *lossy* or *lossless* technique?

6. (1 pt) This question deals with image pixels and audio samples.

(a) The RGB color for Simpsons Yellow is given in hexadecimal as `FFD90F`. Express the red, green and blue components of this color in decimal. Show your work for full credit.

Red:

Green:

Blue:

(b) A tuning fork emits a pure tone (i.e. single frequency) once the higher overtones die out. This analog tone is used to help us tune instruments or prepare to sing a specific note. Suppose we want to digitally record a tuning fork that vibrates at 1760 Hz (cycles/second). At what minimum rate (samples/second) should we sample this sound so that we can reconstruct the same audio frequency later when we go from digital back to analog?

If we use 16-bit sampling (2 bytes per sample), and we sample in stereo at the rate above for 20 seconds, how big will our file be? Express your answer in kilobytes. Show your work.

7. (1 pt) As discussed in class, we can use the notion of parity to detect when an error occurs after transmission of data. Universal Product Codes (UPCs) also use this idea, often called a check digit. A UPC barcode is made of 11 digits followed by a check digit as shown at right.

Let's store a UPC without the check digit as a list in Python. For example, the UPC at right, without the check digit, would be stored in a list as:
`upc = [ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 ]`

The following function in Python computes and returns the check digit for an 11-digit UPC:

```
def compute_checkdigit(upc):
    x = 0
    for i in range(0,11,2):
        x = x + upc[i]
    y = 0
    for j in range(1,10,2):
        y = y + upc[j]
    z = (3*x + y) % 10
    if z == 0:
        return z
    else:
        return 10 - z
```

(a) Using the algorithm above, determine the check digit needed to complete the following UPC barcode: `16384581930_`. Show the computation for `x`, `y` and `z` to justify your answer.

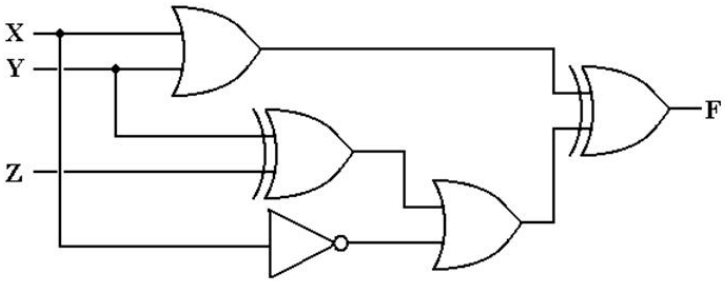
```
x = _____ + _____ + _____ + _____ + _____ + _____ = _______

y = _____ + _____ + _____ + _____ + _____ = _______

z = (3*x+y) % 10 = _________ % 10 = _______    Return: ________
```

(b) We are given the following 12-digit UPC barcode (including check digit): `086871296613`
Which of the following statements is true about this 12-digit barcode?

(A) The barcode is correct.
(B) The barcode has an incorrect digit, and we cannot correct it.
(C) The barcode has an incorrect digit, but we can correct it.
(D) We have no way of knowing whether the bar code is correct or incorrect.

ANSWER :

8. (1.5 pts) Consider the following digital circuit with 3 inputs:



(courtesy of Edward Bosworth)

(a) Complete the truth table to the right that describes the output of the circuit.

X	Y	Z	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

(b) A Boolean expression for the circuit above is:

$$F = (X \vee Y) \oplus ((Y \oplus Z) \vee \neg X)$$

Using your truth table, write an equivalent Boolean expression for the circuit above that does not use the exclusive-or operator. Your expression should be of the form: F = _____ v _____ v _____ … where each sub-expression corresponds to a row of the truth table.

F = _____

(c) Bernie is writing a Python program to display the text of an amendment to the Constitution of the United States given the amendment's number as an integer. There are 27 ratified amendments to date (numbered 1 through 27). If the number is invalid, he wants to print an error message. He writes the following Python statement:

```
if not((number >= 1) and (number < 28)):
     print("ERROR: Invalid number of a ratified amendment.")
```

Rewrite the statement using DeMorgan's Law to simplify it:

```
if (number ____ 1) ______ (number ____ 28):
     print("ERROR: Invalid number of a ratified amendment.")
```