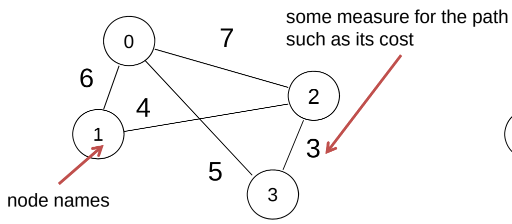
UNIT 7B

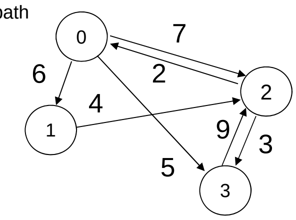
Data Representation: Compression

Last Lecture

- Binary Trees
  - Binary search trees, max-heaps
- Graphs

Undirected and Directed Graphs

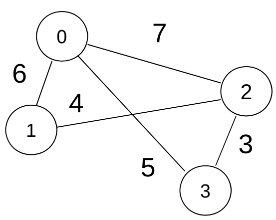




from \ to	0	1	2	3
0	0	6	7	5
1	6	0	4	∞
2	7	4	0	3
3	5	∞	3	0

from \ to	0	1	2	3
0	0	6	7	5
1	∞	0	4	∞
2	2	∞	0	3
3	∞	∞	9	0

Graphs in Ruby

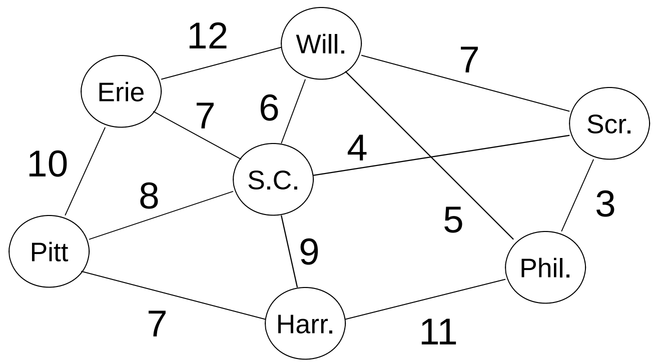


from \ to	0	1	2	3
0	0	6	7	5
1	6	0	4	∞
2	7	4	0	3
3	5	∞	3	0

```
inf = 1.0/0.0

graph =
[ [ 0, 6, 7, 5 ],
  [ 6, 0, 4, inf ],
  [ 7, 4, 0, 3],
  [ 5, inf, 3, 0] ]
```

Original Graph



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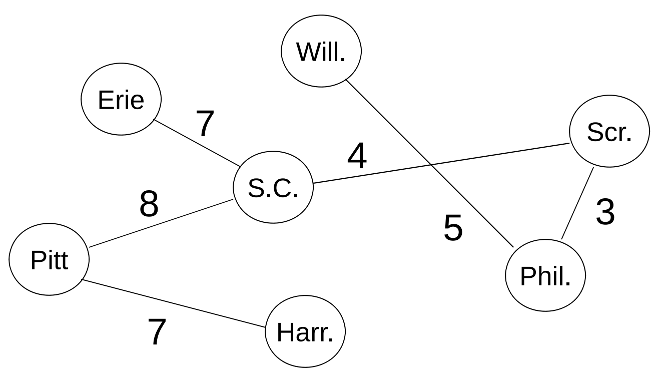
A Minimal Spanning Tree

The minimum total cost to connect all vertices using edges from the original graph without using cycles. (minimum total cost = 34)



For example, what would be the minimum cost for laying cables such that all cities are connected?

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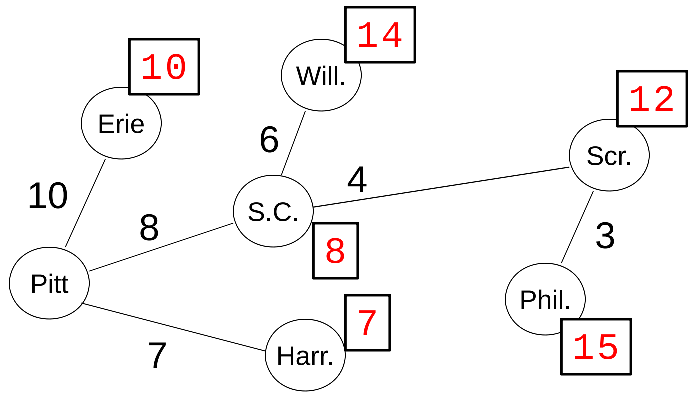
Shortest Paths from Pittsburgh



The total costs of the shortest path from Pittsburgh to every other location using only edges from the original graph.

REPRESENTING NUMBERS

Unsigned Integers

- With 8 bits

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

- The minimum value we can represent is 0
- The maximum we value can represent is 255
- The total number of distinct values we can represent is 2^8 = 256

Signed Integers

- Every bit represents a power of 2 except the "left-most" bit, which represents the sign of the number (0 = positive, 1 = negative)
- Example for positive integer (8 bits):

0								
+	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
0	0	1	1	0	1	0	0	
+		2^5	2^4		2^2			
		32 +	16 +		4			= +52

Negative Integers

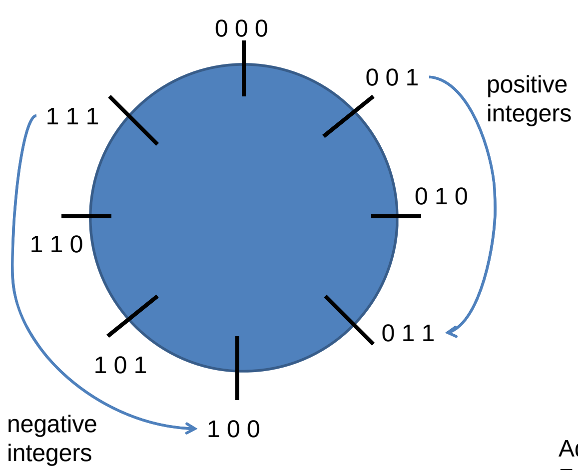
- What about negative numbers?

- We define negative numbers as additive inverse: -x is the number y such that x + y = 0.

- 00110100 is + 52 but 10110100 is not negative

-52 because adding these would not give 0.

Two's complement example



Bit pattern	Decimal value
0 0 0	0
0 0 1	+ 1
0 1 0	+ 2
0 1 1	+ 3
1 0 0	- 4
1 0 1	- 3
1 1 0	- 2
1 1 1	- 1

Adding + n to – n gives 0
For example: 011 + 101 = 000

REPRESENTING TEXT

ASCII Example

- The ASCII code for "M" is 4D hexadecimal.
- Conversion from base 16 to base 2:

hex	binary	hex	binary	hex	binary	hex	binary
0	0000	4	0100	8	1000	C	1100
1	0001	5	0101	9	1001	D	1101
2	0010	6	0110	A	1010	E	1110
3	0011	7	0111	B	1011	F	1111

- 4D (hex) = <u>0100</u> <u>1101</u> (binary) = 77 (decimal)

  (leftmost bit can be used for parity)

Hexadecimal is more convenient to work with

ASCII table

ASCII Code Chart

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2		!	"	#	$	%	&	'	(	)	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	DEL

- 2^7 characters presented in a $2^3 * 2^4$ table.
- Values are represented in hexadecimal (base 16).
- ASCII code for "M" is 4D (hex).

COMPRESSION

Fixed-Width Encoding

- In a fixed-width encoding scheme, each character is given a binary code with the same number of bits.
  - Example:
    Standard ASCII is a fixed width encoding scheme, where each character is encoded with 7 bits.
    This gives us $2^7 = 128$ different codes for characters.

Fixed-Width Encoding

- Given a character set with n characters, what is the minimum number of bits needed for a fixed-width encoding of these characters?
  - Since a fixed width of k bits gives us n unique codes to use for characters, where $n = 2^k$.
  - So given n characters, the number of bits needed is given by $k = \lceil \log_2 n \rceil$. (We use the ceiling function since $\log_2 n$ may not be an integer.)
  - Example: To encode just the alphabet A-Z using a fixed-width encoding, we would need $\lceil \log_2 26 \rceil = 5$ bits:
    e.g. A => 00000, B => 00001, C => 00010, ..., Z => 11001.

Using Fixed-Width Encoding

- If we have a fixed-width encoding scheme using *n* bits for a character set and we want to transmit or store a file with *m* characters, we would need *mn* bits to store the entire file.
- Can we do better?
  - If we assign fewer bits to more frequent characters, and more bits to less frequent characters, then the overall length of the message might be shorter.

Use a method known as Huffman encoding named after David Huffman

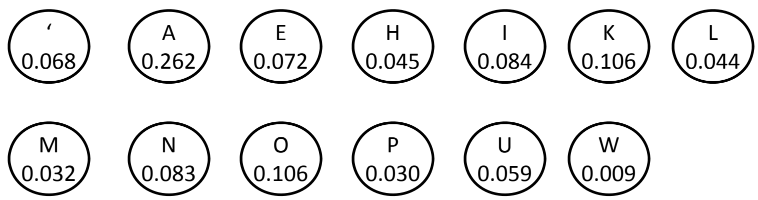
The Hawaiian Alphabet

- The Hawaiian alphabet consists of 13 characters.
  - **'** is the okina which sometimes occurs between vowels (e.g. **KAMA'AINA**)
- The table to the right shows each character along with its relative frequency in Hawaiian words.

'	0.068
A	0.262
E	0.072
H	0.045
I	0.084
K	0.106
L	0.044
M	0.032
N	0.083
O	0.106
P	0.030
U	0.059
W	0.009

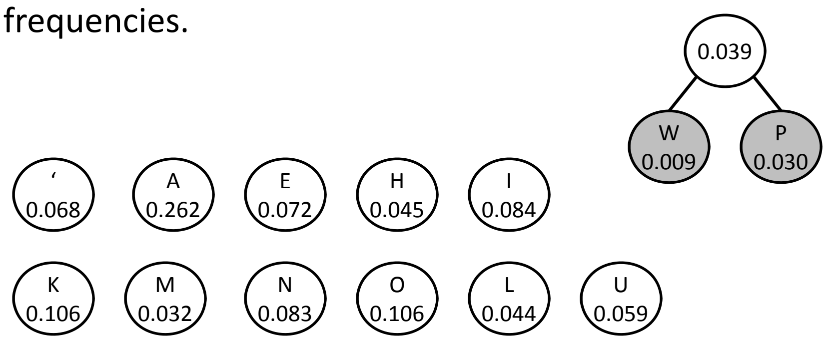
The Huffman Tree

- We use a tree structure to develop the unique binary code for each letter.
- Start with each letter/frequency as its own node:



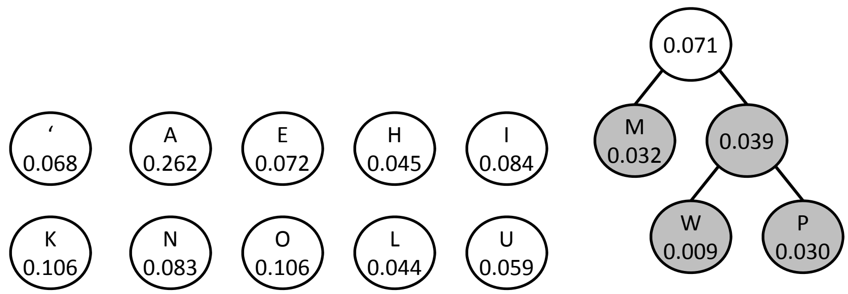
The Huffman Tree

- Combine lowest two frequency nodes into a tree with a new parent with the sum of their frequencies.



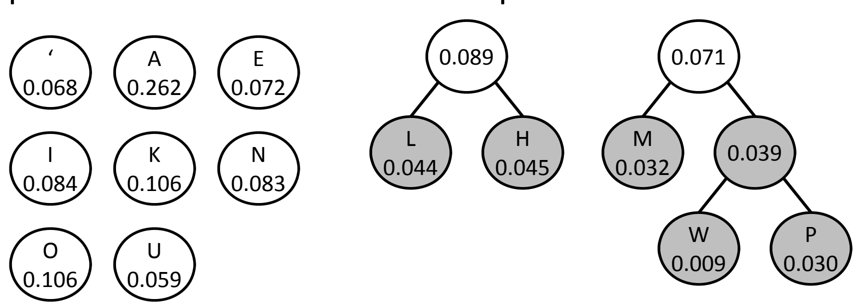
The Huffman Tree

- Combine lowest two frequency nodes (including the new node we just created) into a tree with a new parent with the sum of their frequencies.



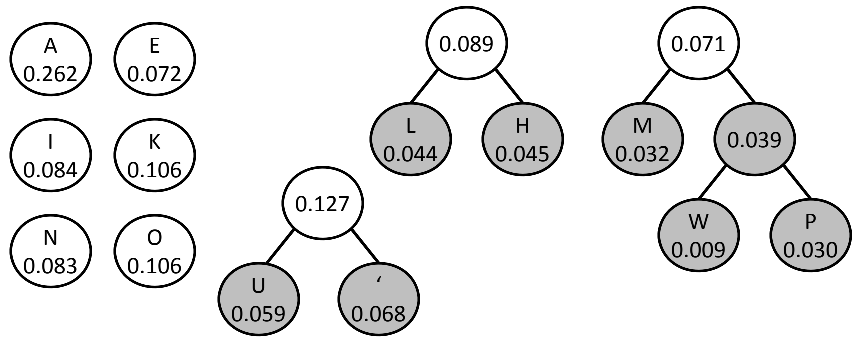
The Huffman Tree

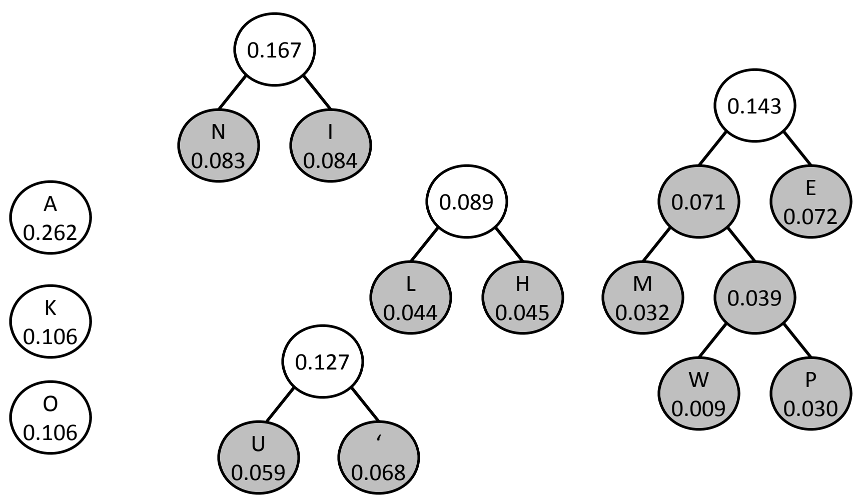
- Combine lowest two frequency nodes (including the new node we just created) into a tree with a new parent with the sum of their frequencies.

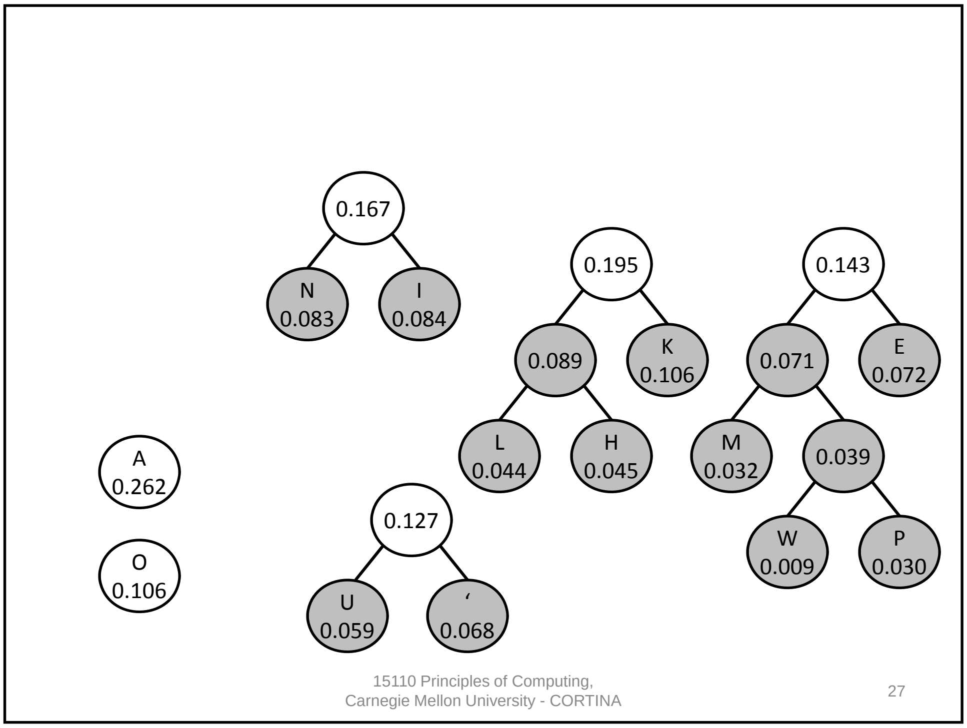


The Huffman Tree

- Combine lowest two frequency nodes (including the new node we just created) into a tree with a new parent with the sum of their frequencies...






0.167
N
0.083
I
0.084
0.195
0.089
K
0.106
L
0.044
H
0.045
0.143
0.071
E
0.072
M
0.032
0.039
W
0.009
P
0.030
A
0.262
O
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0.127
U
0.059
‘
0.068
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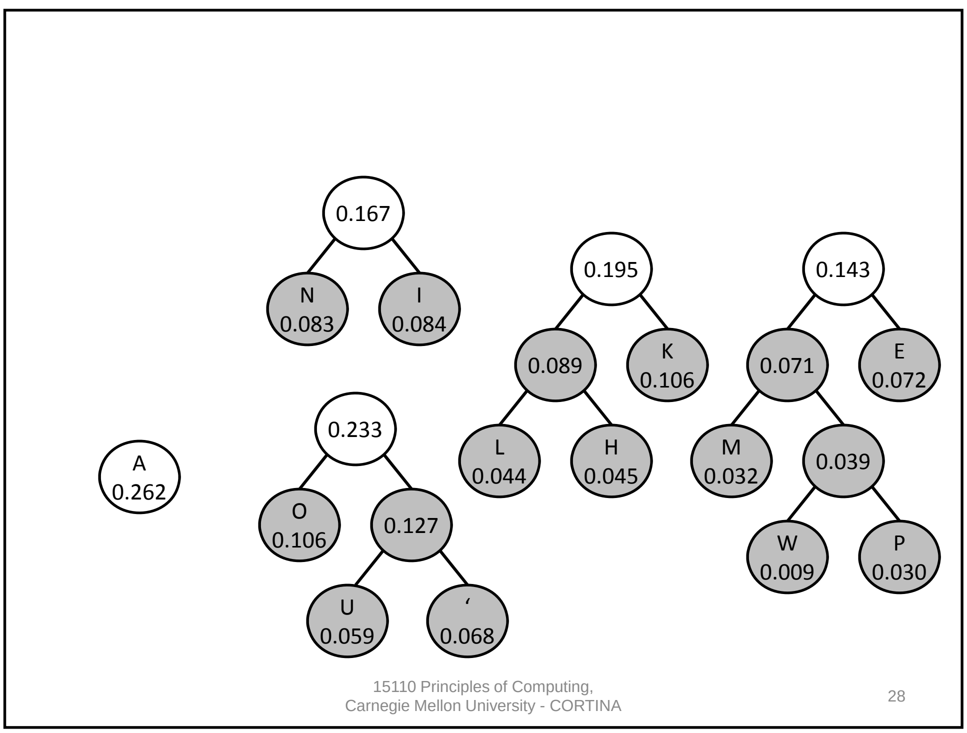
0.167
N
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0.195
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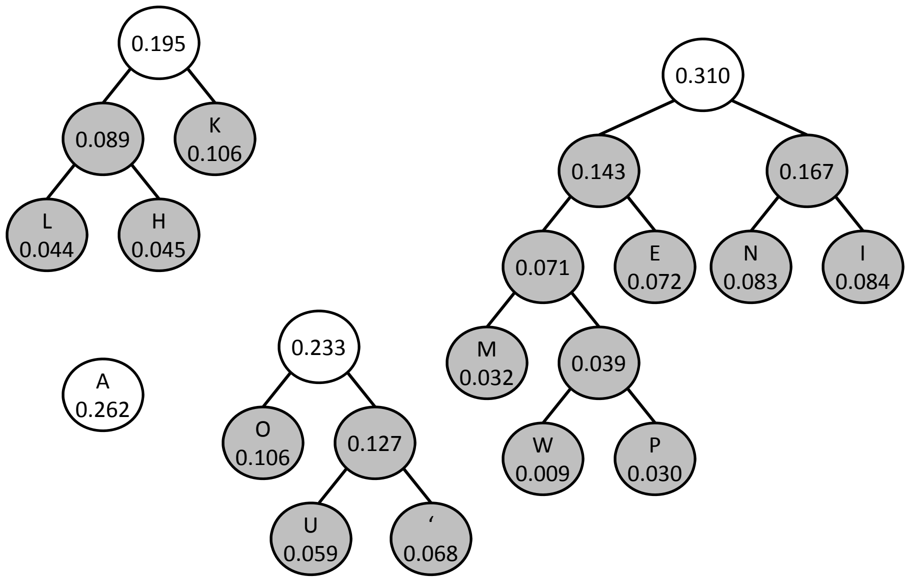
0.195
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K 0.106
L 0.044
H 0.045
A 0.262
0.233
O 0.106
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0.310
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W 0.009
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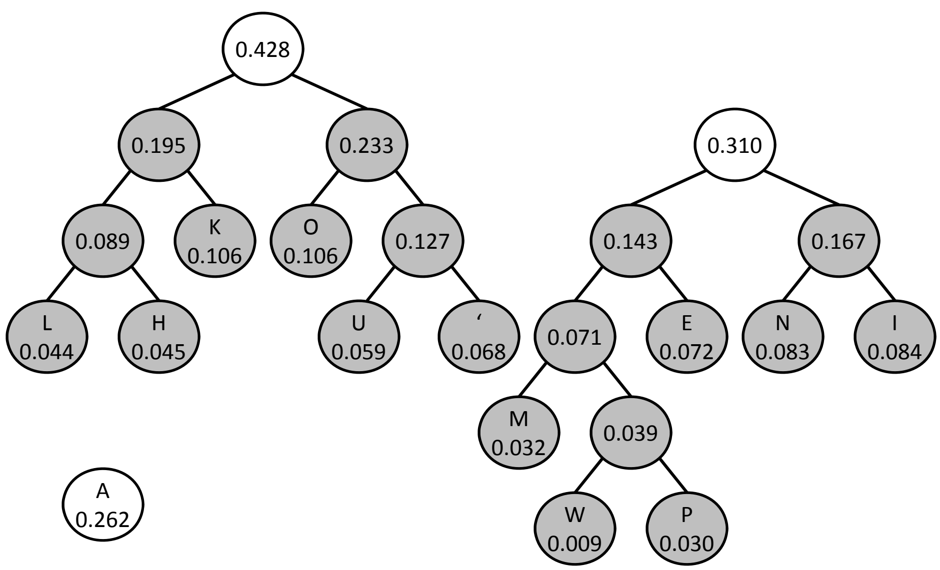
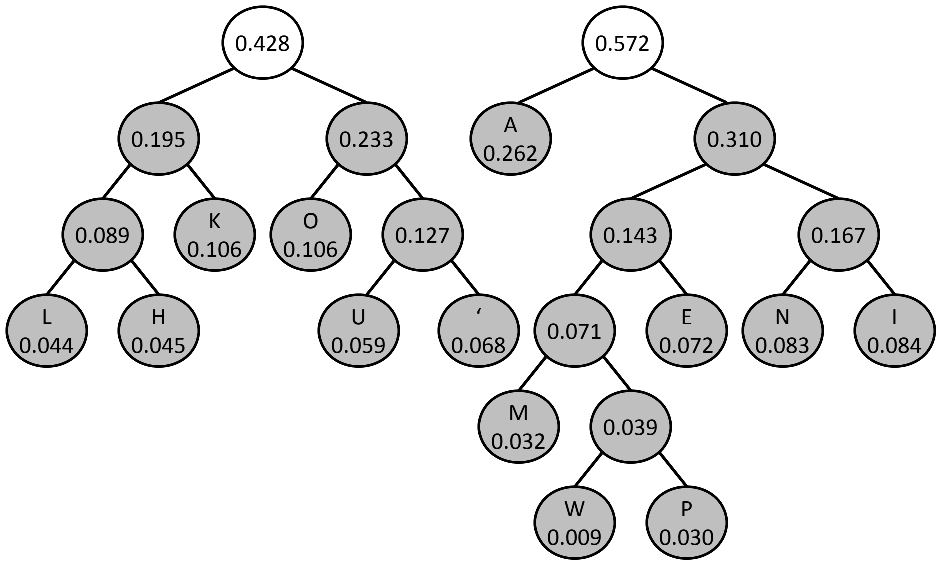
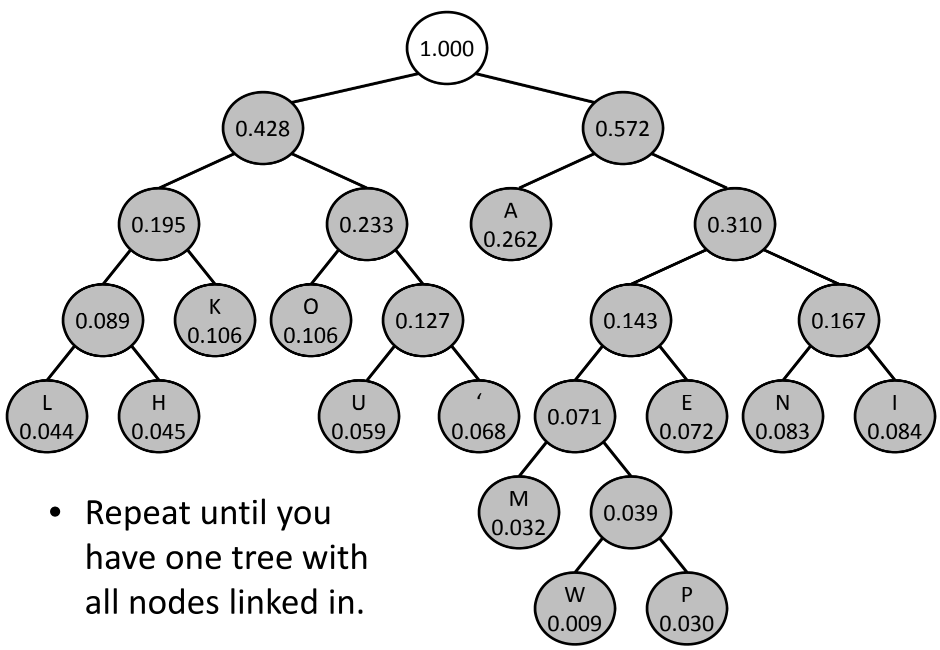
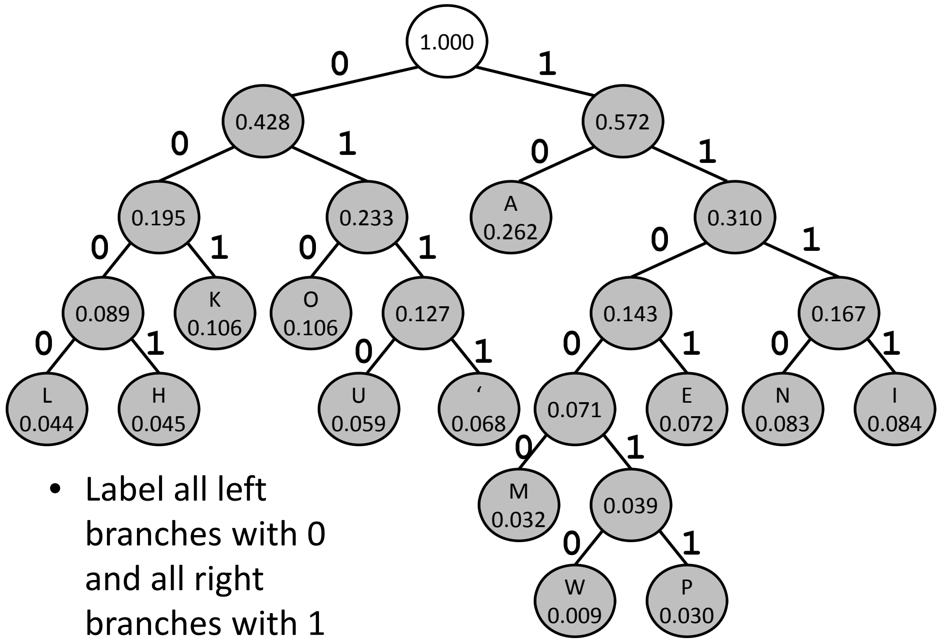

0.428
0.195
0.233
0.089
K 0.106
O 0.106
0.127
L 0.044
H 0.045
U 0.059
‘ 0.068
0.310
0.143
0.167
0.071
E 0.072
N 0.083
I 0.084
M 0.032
0.039
W 0.009
P 0.030
A 0.262

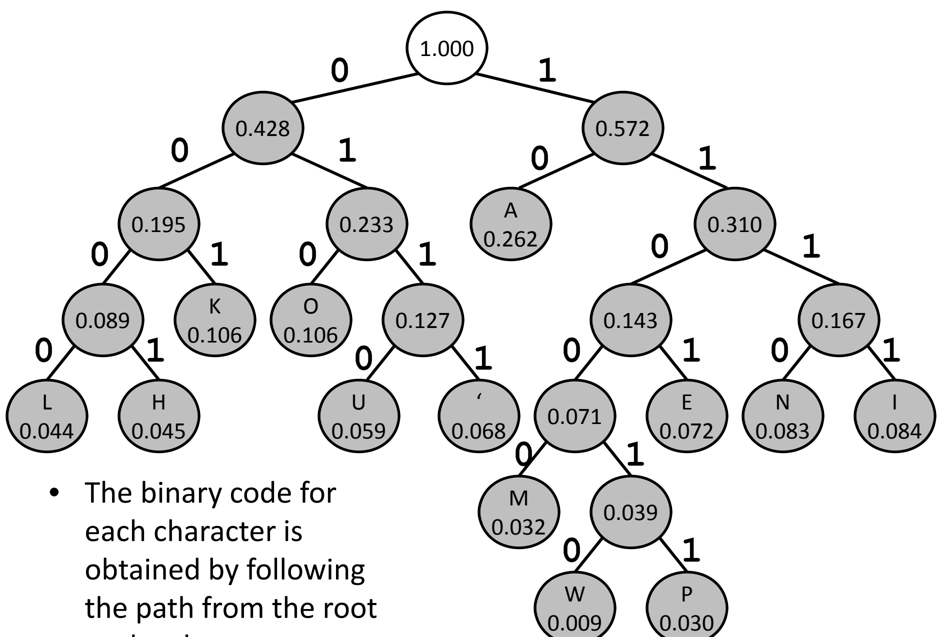




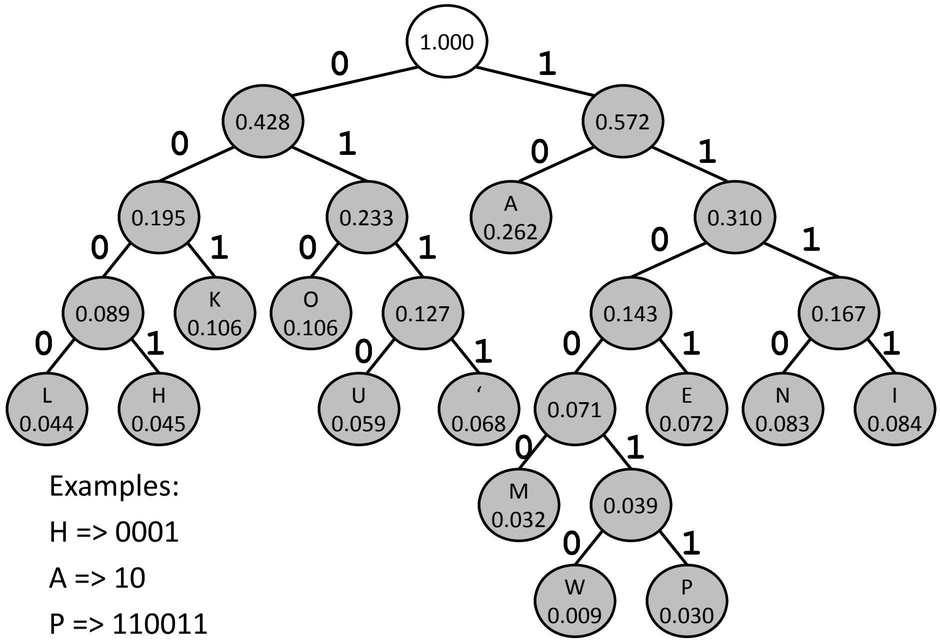
- Repeat until you have one tree with all nodes linked in.



- Label all left branches with 0 and all right branches with 1



- The binary code for each character is obtained by following the path from the root to the character.



Examples:

H => 0001

A => 10

P => 110011

Fixed Width vs. Huffman Coding

'	0000	'	0111
A	0001	A	10
E	0010	E	1101
H	0011	H	0001
I	0100	I	1111
K	0101	K	001
L	0110	L	0000
M	0111	M	11000
N	1000	N	1110
O	1001	O	010
P	1010	P	110011
U	1011	U	0110
W	1100	W	110010

ALOHA

Fixed Width:

0001 0110 1001 0011 0001

20 bits

Huffman Code:

10 0000 010 0001 10

15 bits

Variable Length Codes

- In a fixed-width code, the boundaries between letters are fixed in advance:

  0001 0110 1001 0011 0001

- With a variable-length code, the boundaries are determined by the letters themselves.
  - No letter's code can be a prefix of another letter.
  - Example: since A is "10", no other letter's code can begin with "10". All the remaining codes begin with "00", "01", or "11".

Programming the Huffman Tree

- Let's write Ruby code to produce a Huffman encoding of an alphabet.
- At each step we need to find the two nodes with the lowest frequency scores.
- This will be easy if nodes are kept in a list that is sorted by score value.
- Solution: use a **priority queue**.

Priority Queues

NOTE: For this unit, you will need RubyLabs set up and you will need to include BitLab (see p. 167)

- A priority queue (PQ) is like an array that is sorted.
```
pq = PriorityQueue.new
=> []
```
- To add element into the priority queue in its correct position, we use the **<<** operator:
```
pq << "peach"
pq << "apple"
pq << "banana"
=> ["apple", "banana", "peach"]
```


Priority Queues (cont'd)

- To remove the first element from the priority queue, we will use the **`shift`** method:
```
fruit1 = pq.shift
=> "apple"
pq
=> ["banana", "peach"]
fruit2 = pq.shift
=> "banana"
pq
=> ["peach"]
```

Tree Nodes

- We can store all of the node data into a 2-dimensional array:

```
table = [ ["'", 0.068], ["A", 0.262],
 ["E", 0.072], ["H", 0.045], ["I", 0.084],
 ["K", 0.106], ["L", 0.044], ["M", 0.032],
 ["N", 0.083], ["O", 0.106], ["P", 0.030],
 ["U", 0.059], ["W", 0.009] ]
```

- A tree node consists of two values, the character and its frequency. Making one of the tree nodes:

```
char = table[2].first     # "E"
freq = table[2].last      # 0.072
node = Node.new(char, freq)
```


Building a PQ of Single Nodes

```
def make_pq(table)
   pq = PriorityQueue.new
   for item in table do
      char = item.first
      freq = item.last
      node = Node.new(char, freq)
      pq << node
   end
   return pq
end
```

Remember: each item in the table is a 2-element array with a character and a frequency.

Building our Priority Queue

```
pq = make_pq(table)
=> [( W: 0.009 ), ( P: 0.030 ),
    ( M: 0.032 ), ( L: 0.044 ),
    ( H: 0.045 ), ( U: 0.059 ),
    ( ': 0.068 ), ( E: 0.072 ),
    ( N: 0.083 ), ( I: 0.084 ),
    ( K: 0.106 ), ( O: 0.106 ),
    ( A: 0.262 )]
```

This is our priority queue showing the 13 nodes in sorted order based on frequency.

Building a Huffman Tree

(Slightly different than book version fig 7.9)

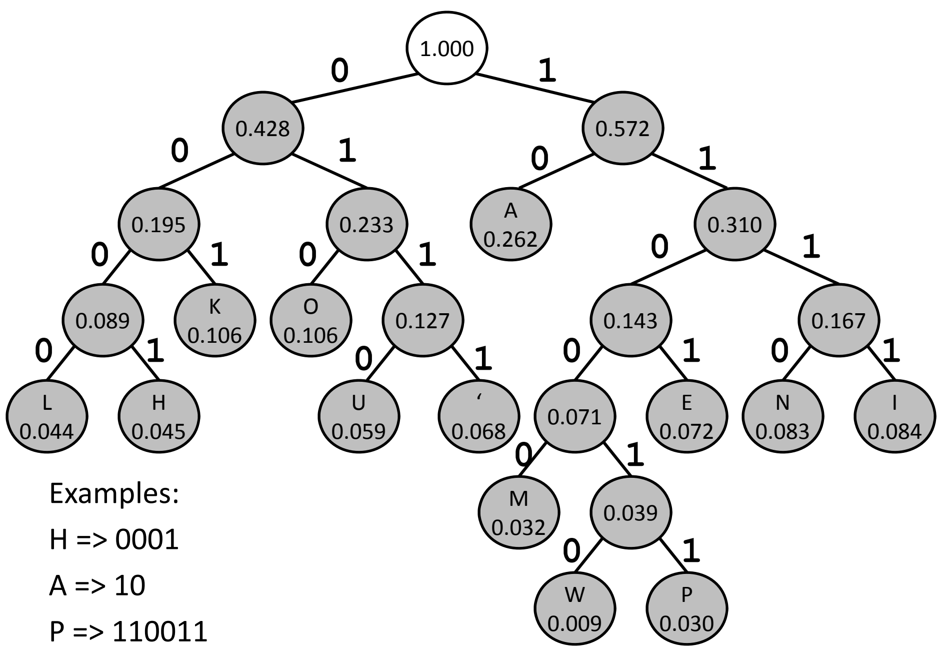
```
def build_tree(pq)
   while pq.length > 1
       node1 = pq.shift
       node2 = pq.shift
       pq << Node.combine(node1, node2)
   end
   return pq.first
end
```

Creates a new node with node1 as its left child and node2 as its right child

Building our Huffman Tree

```
tree = build_tree(pq)
=> ( 1.000 ( 0.428 ( 0.195 ( 0.089
  ( L: 0.044 ) ( H: 0.045 ) ) ( K: 0.106 ) )
  ( 0.233 ( O: 0.106 ) ( 0.127 ( U: 0.059 )
  ( ': 0.068 ) ) ) ) ( 0.572 ( A: 0.262 )
  ( 0.310 ( 0.143 ( 0.071 ( M: 0.032 )
  ( 0.039 ( W: 0.009 ) ( P: 0.030 ) ) )
  ( E: 0.072 ) ) ( 0.167 ( N: 0.083 )
  ( I: 0.084 ) ) ) ) )
```

This is just our Huffman tree expressed using recursively nested parenthetical components:
`( root ( left ) ( right ) )`



Examples:

H => 0001

A => 10

P => 110011

Assigning Codes, Encoding & Decoding

```
ht = assign_codes(tree)

ht["W"]
=> 110010
ht["A"]
=> 10

msg = encode("ALOHA", tree)
=> 100000010000110
decode(msg, tree)
=> "ALOHA"
```

from BitLab
takes a Huffman tree and returns a hash table that maps each letter to its binary code

Note the [ ] syntax.
This returns the code associated with the character from the hash table.

from BitLab
encode and decode functions

Next Lecture

- Representing images and sound