UNIT 5A
Recursion: Basics

Recursion

- A "recursive" function is one that calls itself.
- Infinite loop? Not necessarily.

Recursive Definitions

- Every recursive definition includes two parts:
  - Base case (non-recursive)
    A simple case that can be done without solving the same problem again.
  - Recursive case(s)
    One or more cases that are "simpler" versions of the original problem.
    - By "simpler", we sometimes mean "smaller" or "shorter" or "closer to the base case".

Example: Factorial

- $n! = n \times (n-1) \times (n-2) \times \cdots \times 1$

- $2! = 2 \times 1$
- $3! = 3 \times 2 \times 1$
- $4! = 4 \times 3 \times 2 \times 1$
- So $4! = 4 \times 3!$
- And $3! = 3 \times 2!$
- What is the base case? $0! = 1$

How Recursion Works

4! = 4(3!) .... = 4(6) = 24

3! = 3(2!) .... = 3(2) = 6

2! = 2(1!) ..... = 2(1) = 2

1! = 1 (0!) = 1(1) = 1

Base case

make smaller instances of the same problem

build up the result

Factorial in Ruby (Recursive)

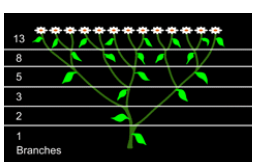
```
def factorial (n)
  if n == 0          % base case
    return 1
  else               % recursive case
    return n * factorial(n-1)
end
```

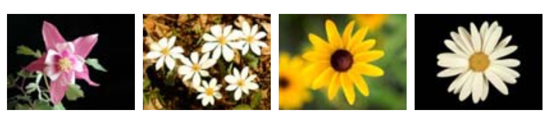
Fibonacci Numbers

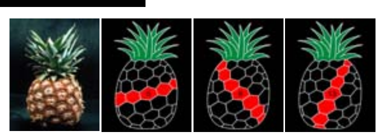
- A sequence of numbers such that each number is the sum of the previous two numbers in the sequence, starting the sequence with 0 and 1.
- 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, etc.

Fibonacci Numbers in Nature

- 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, etc.
- Number of branches on a tree.
- Number of petals on a flower.
- Number of spirals on a pineapple.







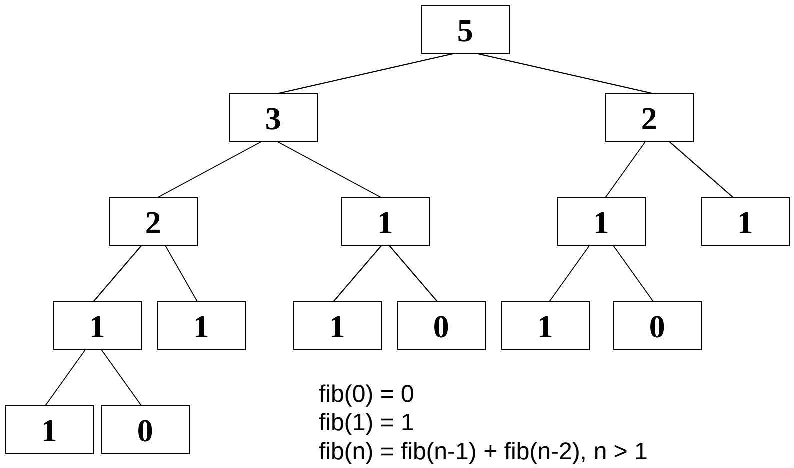
Recursive Definition

- Let fib(n) = the nth Fibonacci number, n ≥ 0
  - fib(0) = 0 (base case)
  - fib(1) = 1 (base case)
  - fib(n) = fib(n-1) + fib(n-2), n > 1

Recursive Fibonacci in Ruby

```
def fib(n)
    if n == 0 or n == 1
        return n
    else
        return fib(n-1) + fib(n-2)
end
```

Recursive Definition



fib(0) = 0
fib(1) = 1
fib(n) = fib(n-1) + fib(n-2), n > 1

Recursive vs. Iterative Solutions

- For every recursive function, there is an equivalent iterative solution.
- For every iterative function, there is an equivalent recursive solution.
- But some problems are easier to solve one way than the other way.

Factorial Function (Iterative)

```
def factorial (n)
    result = 1
    for i in 1..n do
        result = result * i
    end
    return result
end
```


Iterative Fibonacci

```
def fib(n)
    x = 0
    next_x = 1
    for i in 1..n do
        x, next_x = next_x, x+next_x
    end
    return x
end
```

Much faster than the recursive version. Why?

Recursive sum of a list

```
def sumlist(list)
    n = list.length
    if n == 0 then
      return 0
    else
      return list[0] + sumlist(list[1..n-1])
    end
end
```

Base case:
The sum of an empty list is 0.

"tail" of list

Recursive case:
The sum of a list is the first element + the sum of the rest of the list.

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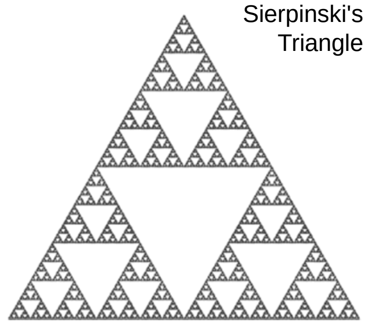
Geometric Recursion (Fractals)

- A recursive operation performed on successively smaller regions.

http://fusionanomaly.net/recursion.jpg



Sierpinski's Triangle

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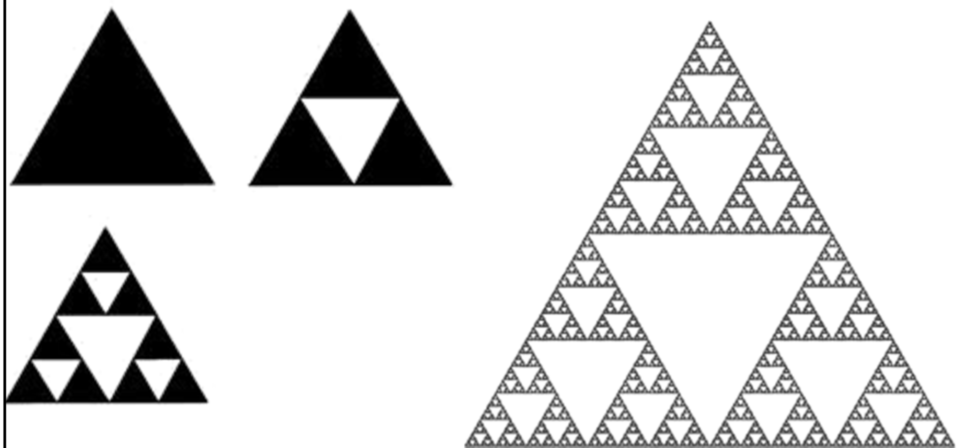
Sierpinski's Triangle



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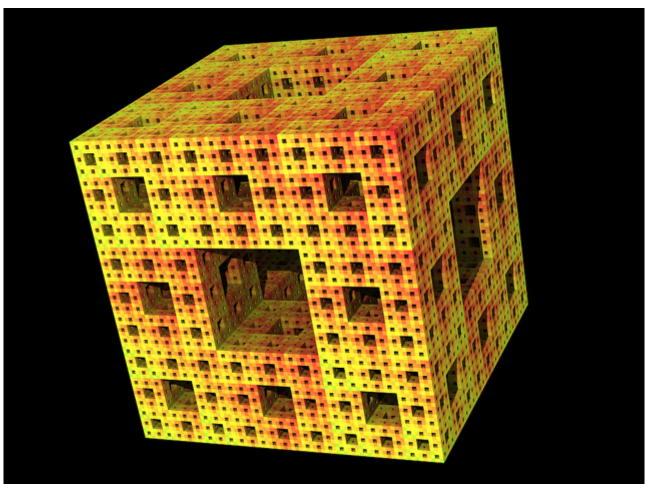
Sierpinski's Carpet



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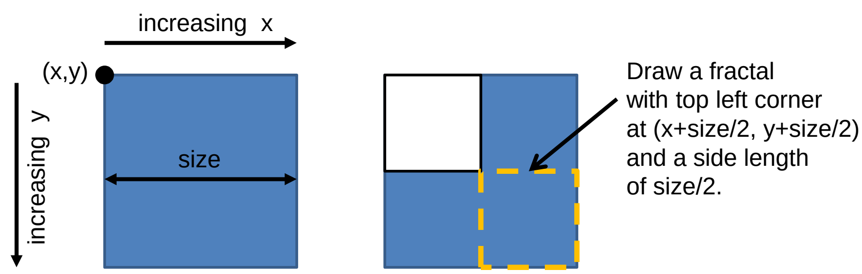
Simple Fractal

To draw a fractal with top-left corner (x,y) and a side length of size:

- Draw a white square with top-left corner (x,y) and a side length of size/2.
- Draw another fractal with top-left corner (x+size/2, y+size/2) and a side length of size/2. [recursive step]



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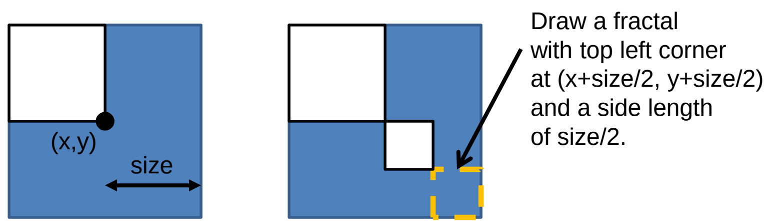

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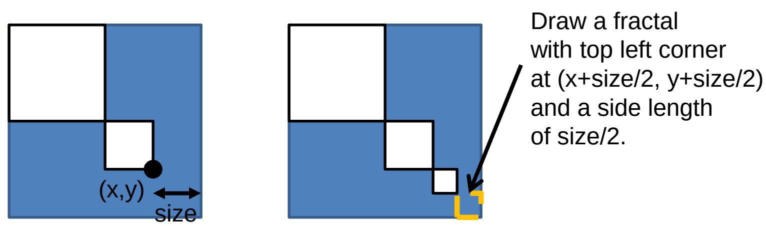
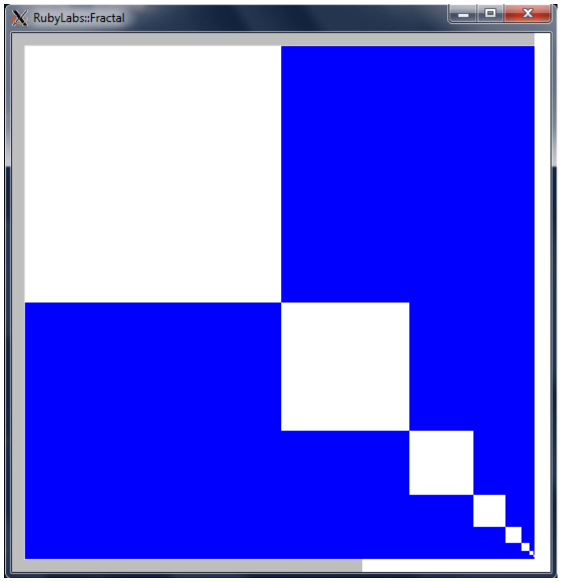


Simple Fractal in Ruby

(not all code shown)

```
def fractal(x, y, size)
    return if size < 2          # base case
    draw_square(x, y, size/2)
    fractal(x+size/2, y+size/2, size/2)
end

def draw_fractal()
    # initial top-left (x,y) and size
    fractal(0, 0, 512)
end
```



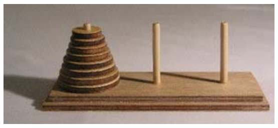
Next Lecture

- Binary search: Apply the technique of recursion in doing search
- Analyze its time complexity

Famous Puzzle of "Towers of Hanoi"

EXTRA RECURSION EXAMPLE

Towers of Hanoi



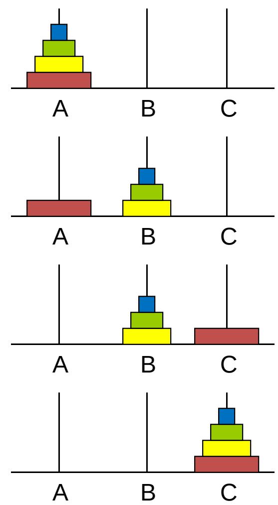
Towers of Hanoi with 8 discs.

- A puzzle invented by French mathematician Edouard Lucas in 1883.
- At a temple far away, priests were led to a courtyard with three pegs and 64 discs stacked on one peg in size order.
  - Priests are only allowed to move one disc at a time from one peg to another.
  - Priests may not put a larger disc on top of a smaller disc at any time.
- The goal of the priests was to move all 64 discs from the leftmost peg to the rightmost peg.
- According to the story, the world would end when the priests finished their work.

Towers of Hanoi

Problem: Move n discs
from peg A to peg C using peg B.

1. Move n-1 discs from peg A to peg B using peg C. (recursive step)
2. Move 1 disc from peg A to peg C.
3. Move n-1 discs from peg B to C using peg A. (recursive step)



Towers of Hanoi in Ruby

```
def towers(n, from_peg, to_peg, using_peg)
  if n >= 1 then
    towers(n-1, from_peg, using_peg, to_peg)
    puts "Move disc from " + from_peg
         + " to " + to_peg
    towers(n-1, using_peg, to_peg, from_peg)
  end
end
```

In irb: `towers(4, "A", "C", "B")`

How many moves do the priests need to move 64 discs?