

RECAP

FLU VIRUS SIMULATION

Example: Flu Virus Simulation

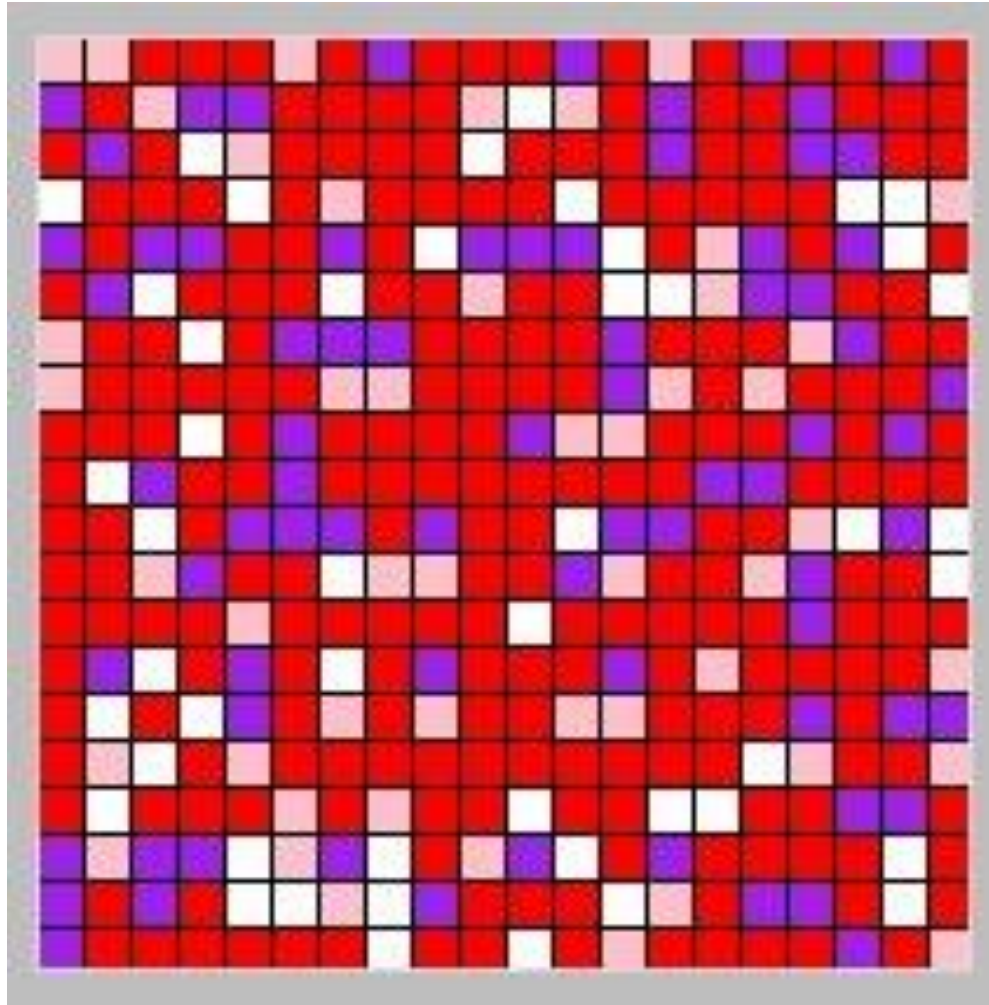
- Goal: Develop a simple simulation that shows graphically how disease spreads through a population.

Modeling the Spread of Flu Virus

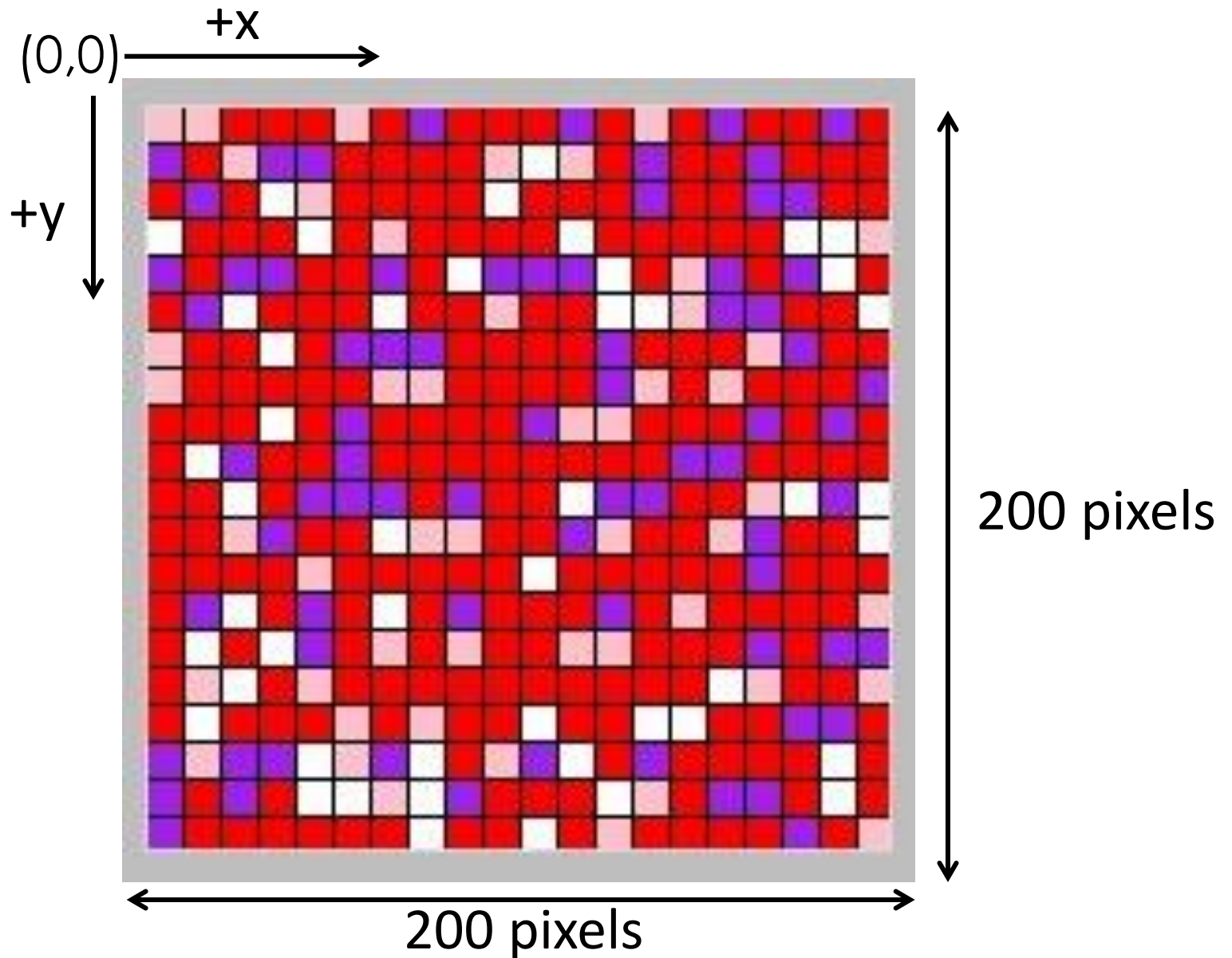
- Every person is either healthy, infected, contagious or immune. We assume that “infected” means infected but not contagious.
- Each day, a healthy person comes in contact with 4 random people. If any of those random people is contagious, then the healthy person becomes infected.
- It takes one day for the infected person to become contagious.
- After a person has been contagious for 4 days, then the person is non-contagious and cannot spread the virus nor can the person get the virus again due to immunity.

Displaying the Population

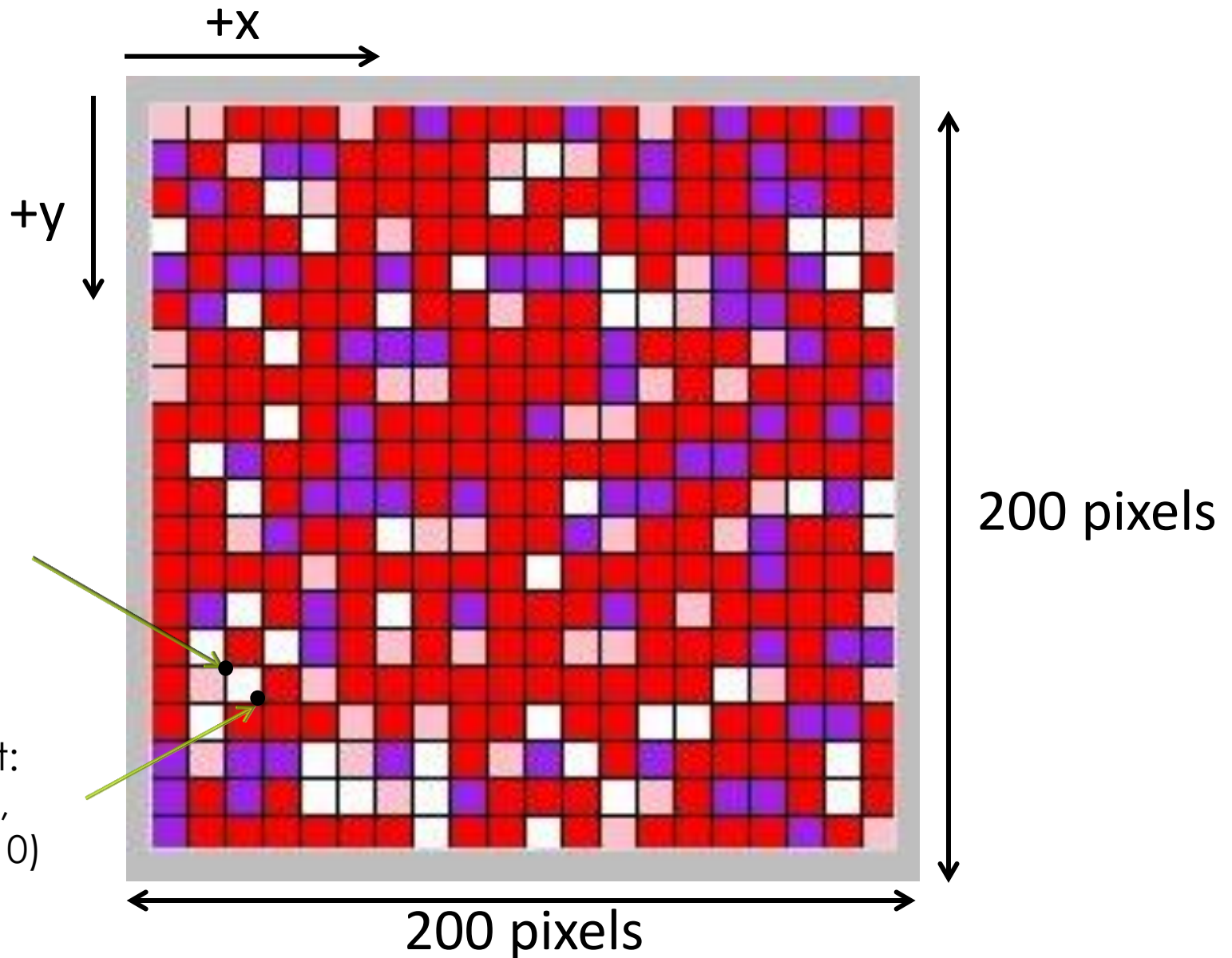
Color of each cell indicates the health state of the person corresponding to that cell



Graphics

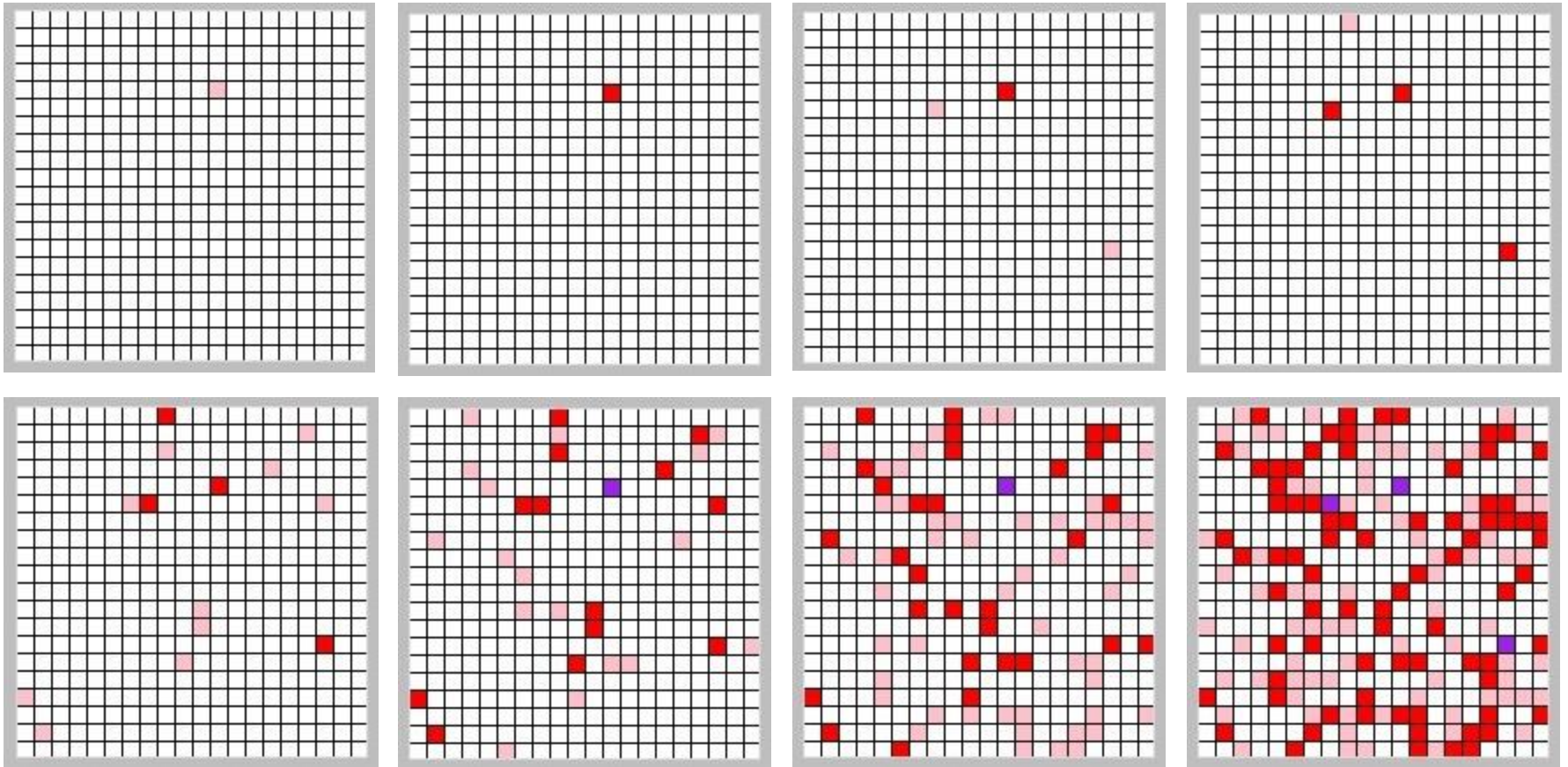


Coordinates of each cell



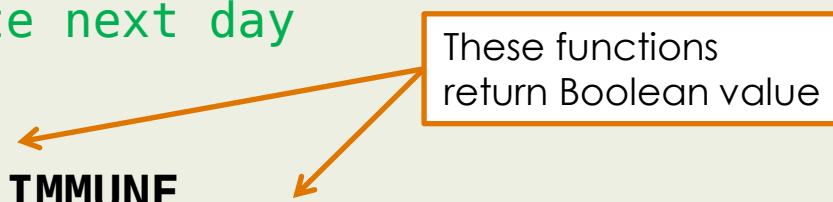
Graphical Simulation

Simulation captures the evolution of the health state of the population over time. It evolves in discrete steps: change occurs instantaneously as a new day begins.



Updating the matrix

```
def simNextDay(data):
    nextDayData = []          # create new matrix and initialize
    for i in range(20):
        nextDayData.append([0] * 20)
    for i in range(20):      # create next day
        for j in range(20):
            if immune(data, i, j):
                nextDayData[i][j] = IMMUNE
            elif infected(data, i, j) or contagious(data, i, j):
                nextDayData[i][j] = data[i][j] + 1
            elif healthy(data, i, j):
                nextDayData[i][j] = meetPeople(data, i, j)
    return newMatrix
```



These functions return Boolean value

```
def meetPeople(currMatrix, row, col):
    for counter in range(4): # repeat 4 times
        if contagious(currMatrix, randrange(20), randrange(20)):
            return INFECTED
    return currMatrix[row][col]
```


What if Our Model Changes?

- If a healthy person contacts a contagious person, she gets sick 40% of the time.

```
def meetPeople(currMatrix, row, col):  
    for counter in range(4): # repeat 4 times  
        if contagious(currMatrix, randrange(20), randrange(20))  
            and randrange(100) < 40 :  
            return INFECTED  
    return currMatrix[row][col]
```

What if Our Model Changes? (cont'd)

- The current model does not capture neighbor relationship. The adjacency of 2 cells does not indicate that they are neighbors.
- What if we used to grid to capture neighbor relationship and assumed that a healthy person gets infected if they have at least one contagious neighbor.

Neighbors

```
cell = matrix[i][j]  
north = matrix[i-1][j]
```

NO!

```
if i == 0:  
    north = None  
else:  
    north = matrix[i-1][j]
```

YES!

Continuous Simulation

N-Body Problem

Continuous-Time Simulations

- Often used to model physical phenomena involving forces acting on objects.
- Is “time” really continuous?
 - Philosophical question. No one knows.
 - Just pretend it is.
- Is simulated time continuous?
 - No. It's divided into discrete time steps.
 - But they can be as small as we like.

N-Body Problem

- **Newton's theory:** Planets and other bodies move according to the gravitational effects of the objects around them
 - **N-body problem:** Predicting the individual motions of a group of objects interacting with each other gravitationally
 - **With just two bodies**, we can write a **simple formula** to calculate their positions at any future time, given their starting positions.
 - *But **with 3 or more bodies**, no formula exists for this, because the system is highly nonlinear, and potentially chaotic.*
- Our only recourse is **simulation**.

N-Body Simulation

- Using simulation to predict future locations of bodies

Astronomers use simulations to predict locations of satellites, plan space travel, track dangerous asteroids etc.

Main idea of the simulation:

- Start with the current location and heading of each planet. Then repeatedly
 - Determine where the planets would be a short time later if they move according to a straight line
 - Calculate adjustments to headings

Simulating Gravitational Attraction

Newton's law of universal gravitation:

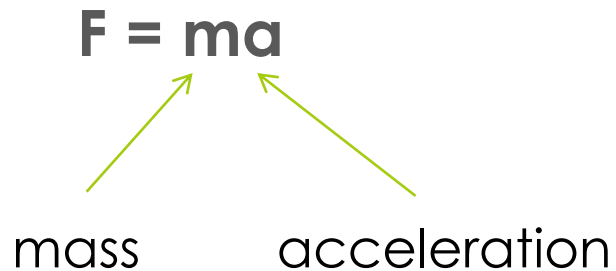
$$\mathbf{F} = \mathbf{G} \cdot m_1 \cdot m_2 / d^2$$

where G = gravitational constant,
 m_1 and m_2 are the masses, and
 d is the distance between them.

Force and Acceleration

Newton's second law:

if some external force is applied to a body then
the body accelerates (its velocity changes)

$$\mathbf{F} = m\mathbf{a}$$


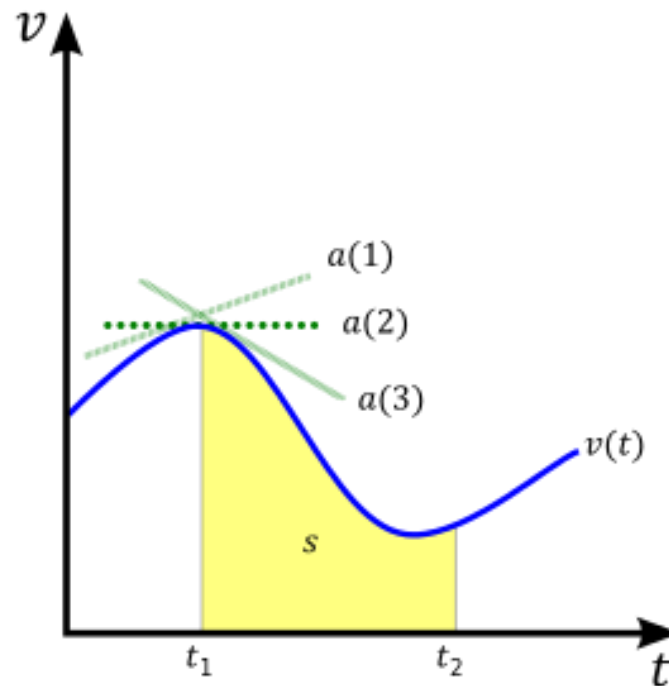
mass acceleration

The diagram illustrates the equation $\mathbf{F} = m\mathbf{a}$. Below the equation, the word "mass" is positioned under the "m" and "acceleration" is positioned under the "a". Two green arrows point upwards from "mass" and "acceleration" towards the "ma" part of the equation, indicating that mass and acceleration are the components of the product on the right-hand side.

Moving A Single Body

- ▣ Calculate the **force** and **acceleration** influencing the body **at a given time**
- ▣ **Suppose that acceleration is constant** for a given interval of time and calculate the **velocity** and **distance** moved

Velocity versus Time graph



a lines represent the values for acceleration at different points along the curve and the yellow area under the curve represents displacement **s**

Integrating Acceleration

- When an object accelerates, its velocity $v(t)$ changes. How can we model this?
- Divide time into tiny steps Δt .
- Re-calculate the velocity at each time step.
$$v(t + \Delta t) = v(t) + a(t) \cdot \Delta t$$
- Smaller Δt brings greater accuracy. Why?

Velocity Is Rate of Change of Position

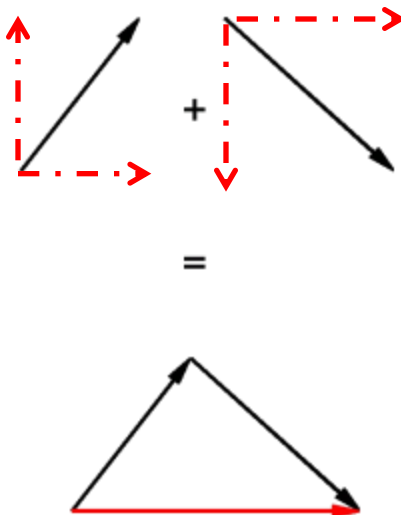
- If an object has non-zero velocity, its position is changing.
- We can use the same integration trick to update the body's position based on velocity.

$$x(t + \Delta t) = x(t) + v(t) \cdot \Delta t$$

Force Vectors

- We can use vectors to keep track of positions, velocities, and accelerations: (x, y, z) coordinates
- Forces are additive and vector addition performs ordinary addition on each component:

$$(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1+x_2, y_1+y_2, z_1+z_2)$$



The vectors in this example has 0 for the z coordinate.

The north and south vectors cancel out each other

The east vectors add up

Force Action on a Single Body

- Calculate all the force vectors influencing the body
- Add the vectors together to determine the cumulative force

Moving Multiple Bodies

- At each time step move each body by calculating the force vectors in each direction
- Suppose we are given a method $\text{interaction}(a,b)$ that calculates the gravitational force between the bodies a and b
- We need to compute all pairwise interactions.
- How many force calculations are there?
 - For the first body interactions with each of the remaining $N-1$ bodies, for the second one interactions with each of the remaining $N-2$ bodies because we already took into account its interaction with the first one etc.
 - $N-1 + N-2 + \dots + 1 = N \times (N-1)/2 \Rightarrow O(N^2)$

Simulation At Extreme Scales

- Cosmologists use simulations to study the formation of galaxies (clusters of stars), and even clusters of galaxies.
- At the other extreme, physicists simulate individual atoms and molecules, e.g., to model chemical reactions.

Next Time

■ Internet