Algorithmic Thinking: Computing with Lists

So Far in Python

- Data types: int, float, Boolean, string
- Assignments, function definitions
- Control structures: For loops, while loops, conditionals

Last Lecture

- More algorithmic thinking
  - Example: Finding the maximum in a list
- Composite (structured) data type: lists
  - Storing and accessing data in lists
  - Modifying lists
  - Operations on lists
  - Iterating over lists

Any confusion?

- **Print vs Return ------------- -------------**

```
def ??????_(a, b):
    result = a + b
    print (result)
```

```
def ??????(a, b):
    result = a + b
    return (result)
```

- **Between Data Types ------ -------------**

```
"3 + 5" vs  3 + 5          6 * 5   vs  6 * 5.0
```

Representing Lists in Python

We will use a **list** to represent a collection of data values.

```
scores = [78, 93, 80, 68, 100, 94, 85]
colors = [‘red’, ‘green’, ‘blue’]
mixed  = [‘purple’, 100, 90.5]
```

A list is an *ordered* sequence of values and may contain values of any data type.

In Python lists may be heterogeneous (may contain items of different data types).

Some List Operations

- **Indexing** (think of subscripts in a sequence)
- **Length** (number of items contained in the list)
- **Slicing**
- **Membership** check
- **Concatenation**
- …

Some List Operations

```
>>> names = [ "Al", "Jane", "Jill", "Mark" ]

>>> len(names)
4


>>> Al in names
Error  ... name 'Al' is not defined


>>> "Al" in names
True


>>> names + names
['Al','Jane','Jill','Mark','Al','Jane','Jill','Mark']
```

Accessing List Elements


names

"Al" "Jane" "Jill" "Mark"

0 1 2 3

list elements

indices

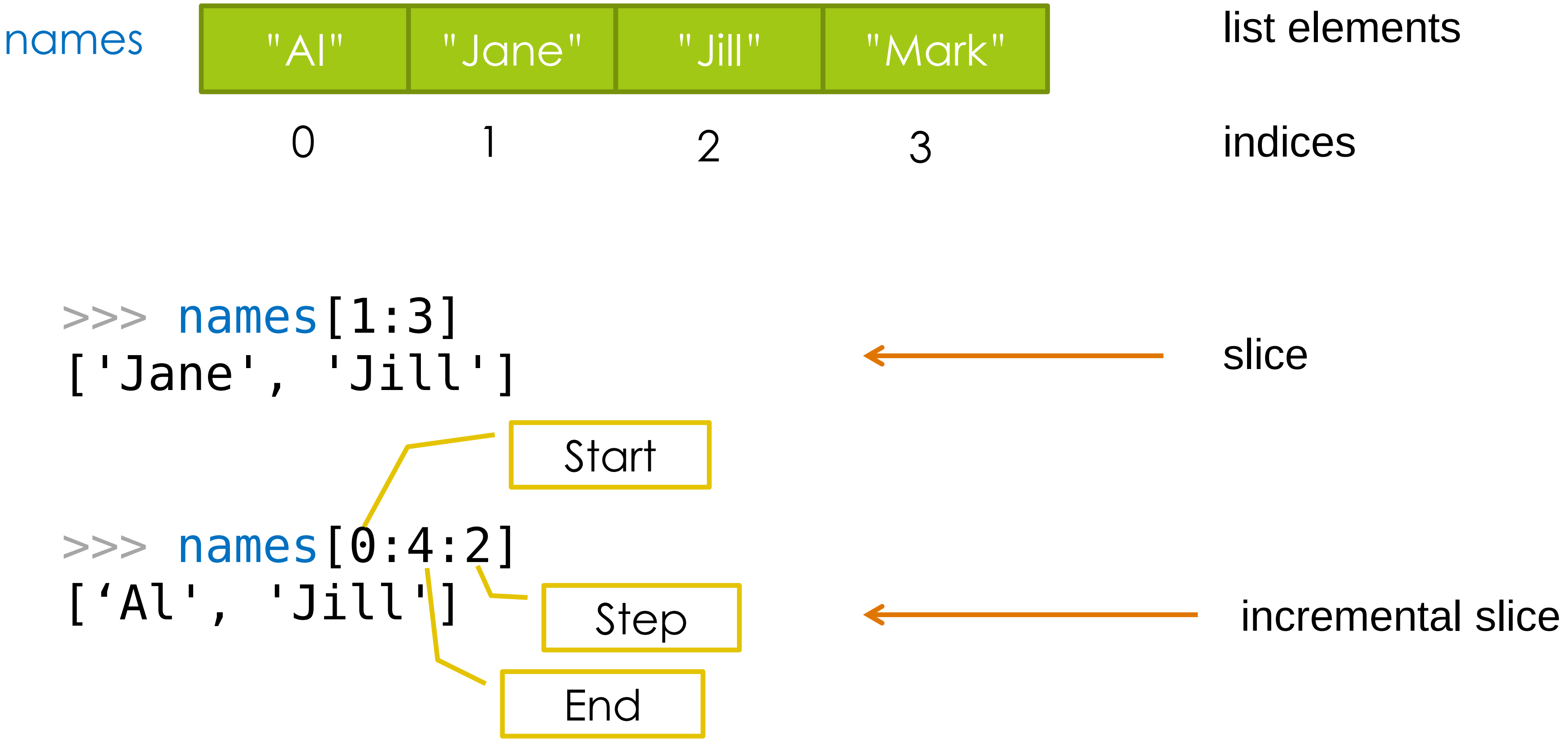
```
>>> names[0]
'Al'
```

```
>>> names[4]
Traceback (most recent call last):
  File "<pyshell#8>", line 1, in <module>
    names[4]
IndexError: list index out of range
```

```
>>> names[3]
'Mark'
```

```
>>> names[len(names)-1]
'Mark'
```

Slicing Lists



names

"Al"	"Jane"	"Jill"	"Mark"
0	1	2	3

list elements

indices

```
>>> names[1:3]
['Jane', 'Jill']
```

slice

```
>>> names[0:4:2]
[‘Al', 'Jill']
```

incremental slice

Slicing Lists

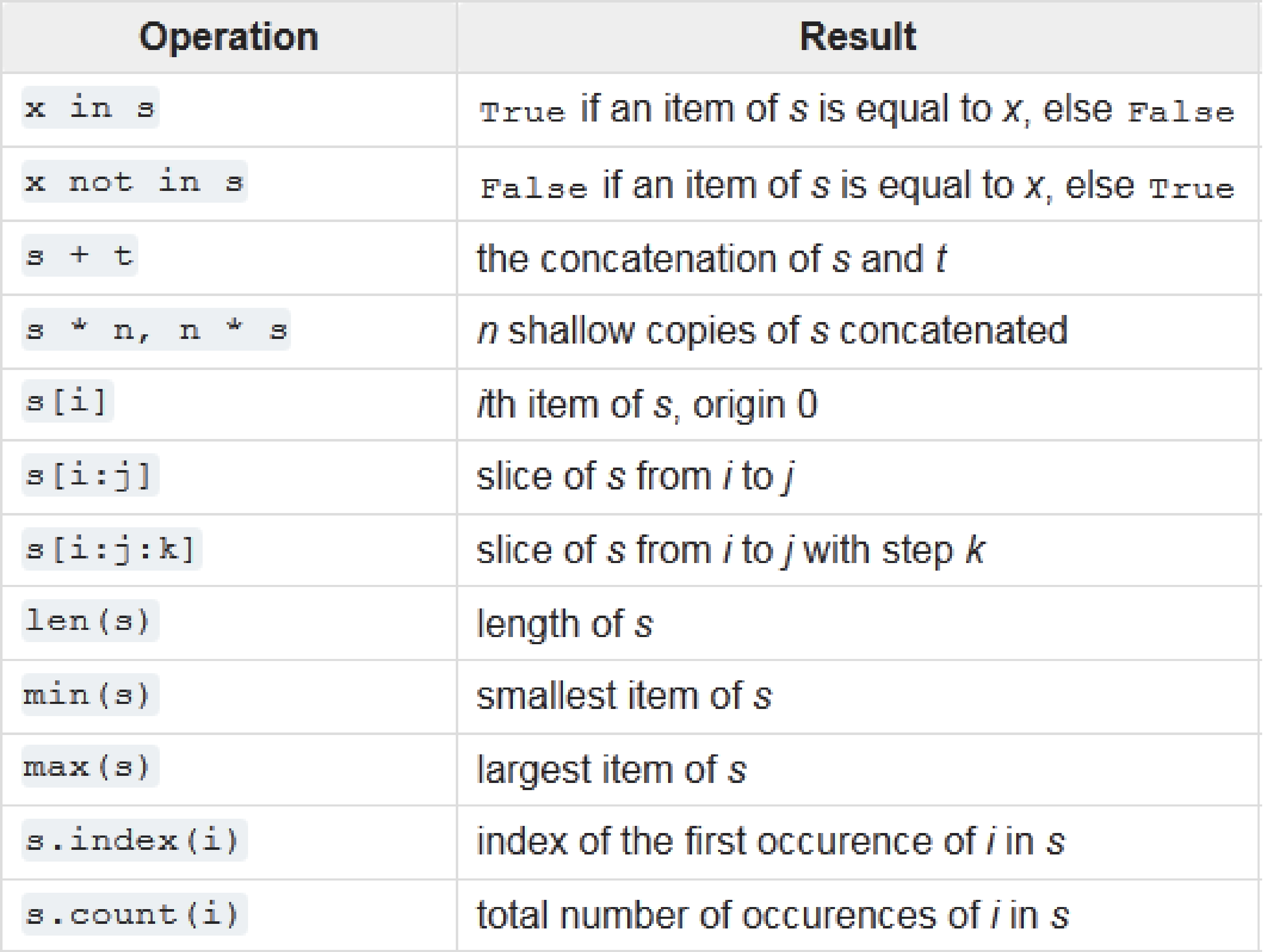

names
"Al" "Jane" "Jill" "Mark"
0 1 2 3
list elements
indices

```
 names, names[0:4],
 names[0,4,1]         ['Al', 'Jane‘, 'Jill', 'Mark']

>>> names[1:3]     ['Jane', 'Jill']
>>> names[1:4]     ['Jane', 'Jill', 'Mark']

>>> names[0:4:2]   [‘Al', 'Jill']
>>> names[:3]      ['Al', 'Jane', 'Jill']

>>> names[:2]      ['Al', 'Jane']
>>> names[2:]      ['Jill', 'Mark']
```



Operation	Result
`x in s`	`True` if an item of *s* is equal to *x*, else `False`
`x not in s`	`False` if an item of *s* is equal to *x*, else `True`
`s + t`	the concatenation of *s* and *t*
`s * n, n * s`	*n* shallow copies of *s* concatenated
`s[i]`	*i*th item of *s*, origin 0
`s[i:j]`	slice of *s* from *i* to *j*
`s[i:j:k]`	slice of *s* from *i* to *j* with step *k*
`len(s)`	length of *s*
`min(s)`	smallest item of *s*
`max(s)`	largest item of *s*
`s.index(i)`	index of the first occurence of *i* in *s*
`s.count(i)`	total number of occurences of *i* in *s*

source: docs.python.org

Modifying Lists

```
>>> names = ['Al', 'Jane', 'Jill', 'Mark']
>>> names[1] = "Kate"
>>> names
['Al', 'Kate', 'Jill', 'Mark']
```

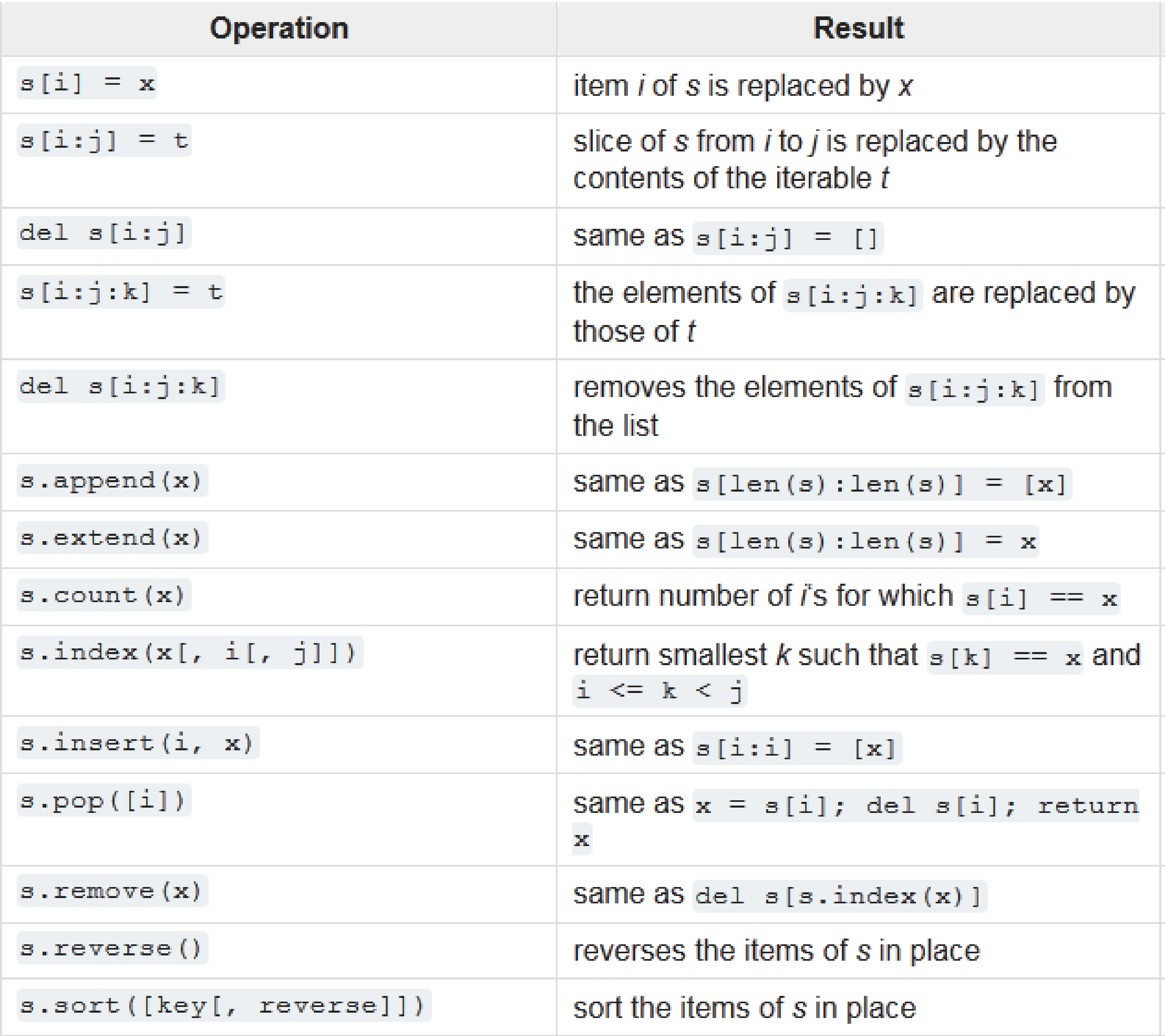
```
>>> a = [1, 2, 3]
>>> a[0:0] = [-2, -1, 0]
>>> a
[-2, -1, 0, 1, 2, 3]

>>> a = [1, 2, 3]
>>> a[0:1] = [-2, -1, 0]
>>> a
[-2, -1, 0, 2, 3]
```

```
>>> names[1:3] = [ "Me", "You" ]
>>> names
['Al', 'Me', 'You', 'Mark']
```

```
>>> names[1:3] = [ "AA", "BB", "CC", "DD" ]
['Al', 'AA', 'BB', 'CC', 'DD', 'Mark']
```

The list grew in length, we could make it shrink as well.

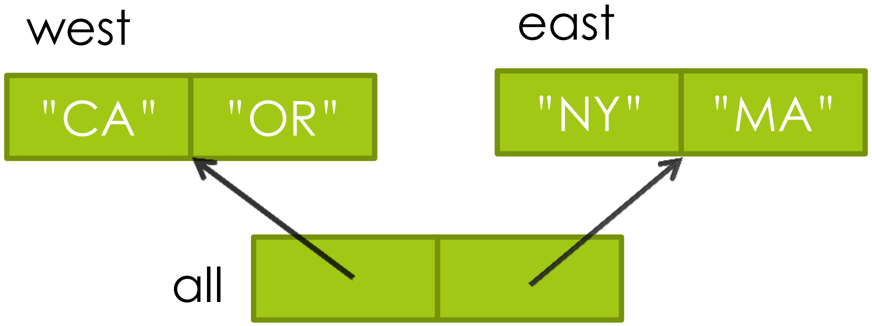


Operation	Result
`s[i] = x`	item *i* of *s* is replaced by *x*
`s[i:j] = t`	slice of *s* from *i* to *j* is replaced by the contents of the iterable *t*
`del s[i:j]`	same as `s[i:j] = []`
`s[i:j:k] = t`	the elements of `s[i:j:k]` are replaced by those of *t*
`del s[i:j:k]`	removes the elements of `s[i:j:k]` from the list
`s.append(x)`	same as `s[len(s):len(s)] = [x]`
`s.extend(x)`	same as `s[len(s):len(s)] = x`
`s.count(x)`	return number of *i*'s for which `s[i] == x`
`s.index(x[, i[, j]])`	return smallest *k* such that `s[k] == x` and `i <= k < j`
`s.insert(i, x)`	same as `s[i:i] = [x]`
`s.pop([i])`	same as `x = s[i]; del s[i]; return x`
`s.remove(x)`	same as `del s[s.index(x)]`
`s.reverse()`	reverses the items of *s* in place
`s.sort([key[, reverse]])`	sort the items of *s* in place

source: docs.python.org

Aliasing

```
>>> west = ["CA", "OR"]
>>> east = ["NY", "MA"]
>>> all = [west, east]
>>> all
[["CA", "OR"],["NY", "MA"]]
```



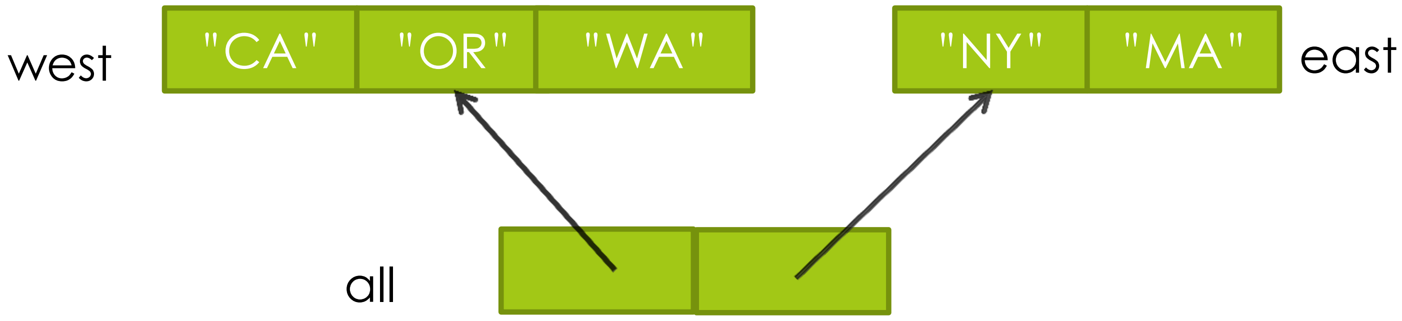
2 paths to the list containing state names in the West Coast.

- One through the variable **west**,
- The other through the variable **all**.

```
>>> west
>>> all[0]
```

This is called **aliasing**.

Mutability Requires Caution



west "CA" "OR" "WA"

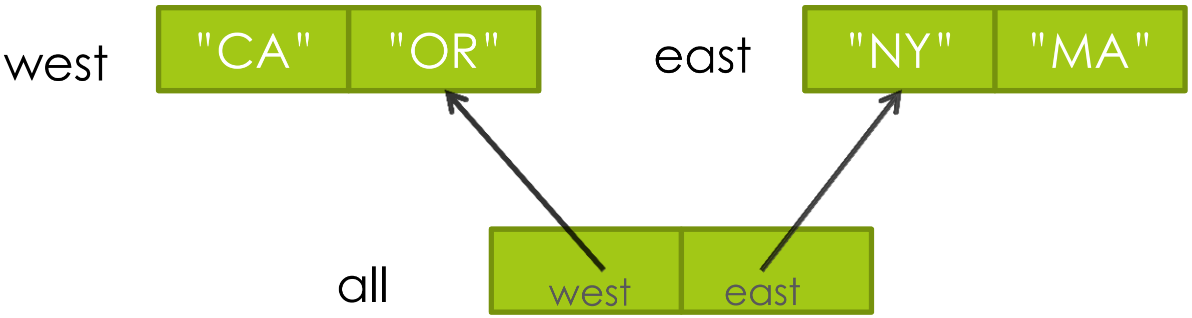
"NY" "MA" east

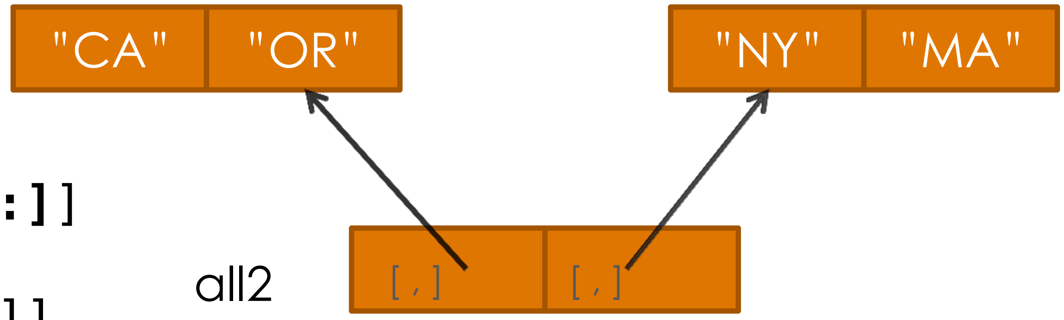
all

```
>>> west = ["CA", "OR"]
>>> east = ["NY", "MA"]
>>> all = [west, east]
>>> west.append("WA")
>>> all
[['CA', 'OR', 'WA'], ['NY', 'MA']]
```

All variables that are bound to the modified object change in value.

Creating Copies





```
>>> west = ["CA", "OR"]
>>> east = ["NY", "MA"]
>>> all2 = [west[:], east[:]]
>>> all2
[["CA", "OR"], ["NY", "MA"]]
```

Creates a shallow copy.
If list items were mutable objects,
as opposed to strings as we have here,
we would have needed something more.
Don't worry about it now.

No matter how I modify west,
all1 will not see it.

What Happens in the Memory?

```
>>> west = ["CA", "OR"]
>>> east = ["NY", "MA"]
>>> all  = [west, east]
~~>>> all2 = [west[:], east[:]]~~  this is more like
>>> all2 = [["CA", "OR"], ["NY", "MA"]]


>>> print(id(all), all)
48231728 [["CA", "OR"], ["NY", "MA"]]

>>> print(id(all2), all2)
48221880 [["CA", "OR"], ["NY", "MA"]]
```

Iterating over Lists

```
def print_colors(colors):
    for index in range(0, len(colors)):
        print(colors[index])
```

```
>>> print_colors(["red", "blue", "green"])
red
blue
green
```

Alternative Version

```
def print_colors(colors):
    for c in colors:
        print(c)
```

Compare with previous version

```
def print_colors(colors):
    for index in range( 0, len(colors) ):
        print( colors[index] )
```

Python binds c to the first item in colors,
then execute the statement in the loop body,
binds c to the next item in the list colors etc.

Finding the max using Python

```
def findmax(lst):
   max_so_far = lst[0]
      for i in range(1,len(lst)):
         if lst[i] > max_so_far:
            max_so_far = lst[i]
   return max_so_far
```

set 1st item as the **maximum found**

Check all **following** items

if you find a bigger value
update the maximum

After checking all values
return the maximum found

Alternative Version

```
def findmax(lst):
    max_so_far = lst[0]
    for item in lst:
      if item > max_so_far:
          max_so_far = item
    return max_so_far
```

initialize the maximum

"For each item in the list..."

if it is bigger then maximum

keep it as the new maximum

return the maximum after checkin all

Summary

- The list data type (ordered and dynamic collections of data)
    - Creating lists
    - Accessing elements
    - Modifying lists
- Iterating over lists

A 2000 year old algorithm (procedure) for generating a table of prime numbers.

2, 3, 5, 7, 11, 13, 17, 23, 29, 31, …

What Is a “Sieve” or “Sifter”?

Separates stuff you want from stuff you don’t:

We want to separate prime numbers.

Prime Numbers

- An integer is "**prime**" if it is not divisible by any smaller integers except 1.
- 10 is **not** prime because 10 = 2 × 5
- 11 **is** prime
- 12 is **not** prime because 12 = 2 × 6 = 2 × 2 × 3
- 13 **is** prime
- 15 is **not** prime because 15 = 3 × 5

Testing Divisibility in Python

- x is “divisible by” y if the remainder is 0 when we divide x by y

- 15 is divisible by 3 and 5, but not by 2:

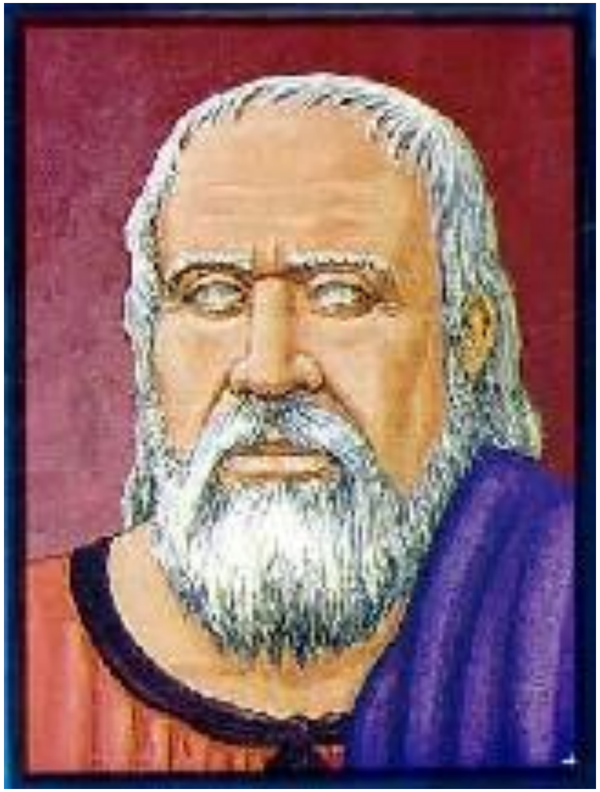
```
>>> 15 % 3
0
>>> 15 % 5
0
>>> 15 % 2
1
```

The Sieve of Eratosthenes

Start with a table of integers from 2 to N.

Cross **out all** the entries that are divisible by the primes known so far.

The first value remaining is the *next* prime.



Finding Primes Between 2 and 50

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

2 is the first prime

Finding Primes Between 2 and 50

	2	**3**	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Filter out everything divisible by 2.

Now we see that 3 is the next prime.

Finding Primes Between 2 and 50

2 **3** 4 **5** 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50

Filter out everything divisible by 3.

Now we see that 5 is the next prime.

Finding Primes Between 2 and 50

	2	**3**	4	**5**	6	**7**	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Filter out everything divisible by 5.

Now we see that 7 is the next prime.

Finding Primes Between 2 and 50

	2	**3**	4	**5**	6	**7**	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Filter out everything divisible by 7.

Now we see that 11 is the next prime.

Finding Primes Between 2 and 50

2 **3** 4 **5** 6 **7** 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50

Since 11 x 11 > 50, all remaining numbers must be primes. Why?

An Algorithm for Sieve of Eratosthenes

Input: A number *n*:

1. Create a list ***numlist*** with every integer from 2 to n, in order. (Assume *n* > 1.)

2. Create an empty list ***primes***.

3. For each element in ***numlist***

   a. If element is not marked, copy it to the end of ***primes***.

   b. Mark every number that is a multiple of the most recently discovered prime number.

Output: The list of all prime numbers less than or equal to *n*

Automating the Sieve

numlist

2 3 4 5
6 7 8 9
10 11 12 13
…

primes

Use *two* lists: candidates, and confirmed primes.

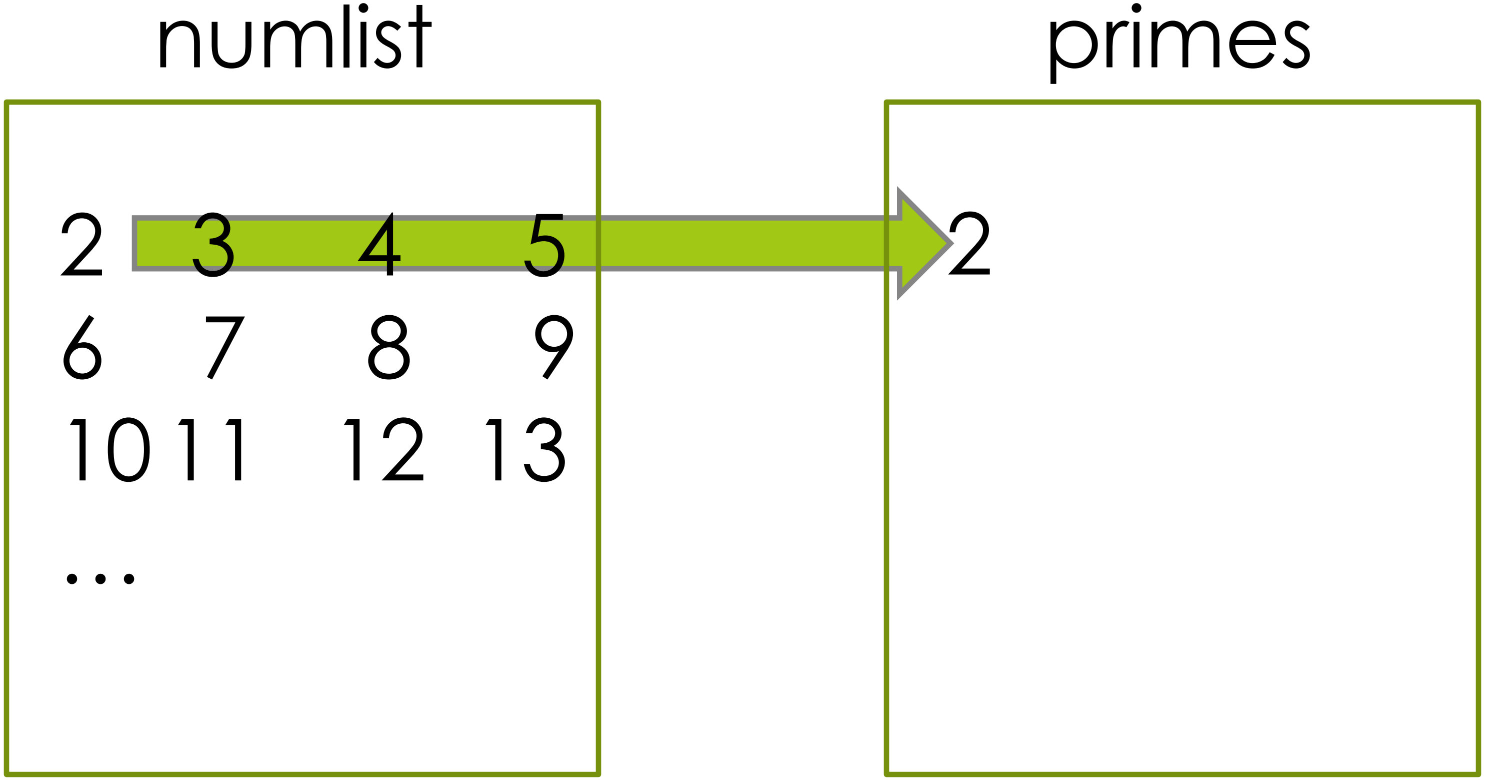
Steps 1 and 2

numlist

2 3 4 5
6 7 8 9
10 11 12 13
…

primes

Step 3a



numlist

2 3 4 5
6 7 8 9
10 11 12 13
…

primes

2

Append the current number in numlist to the end of primes.

Step 3b

numlist

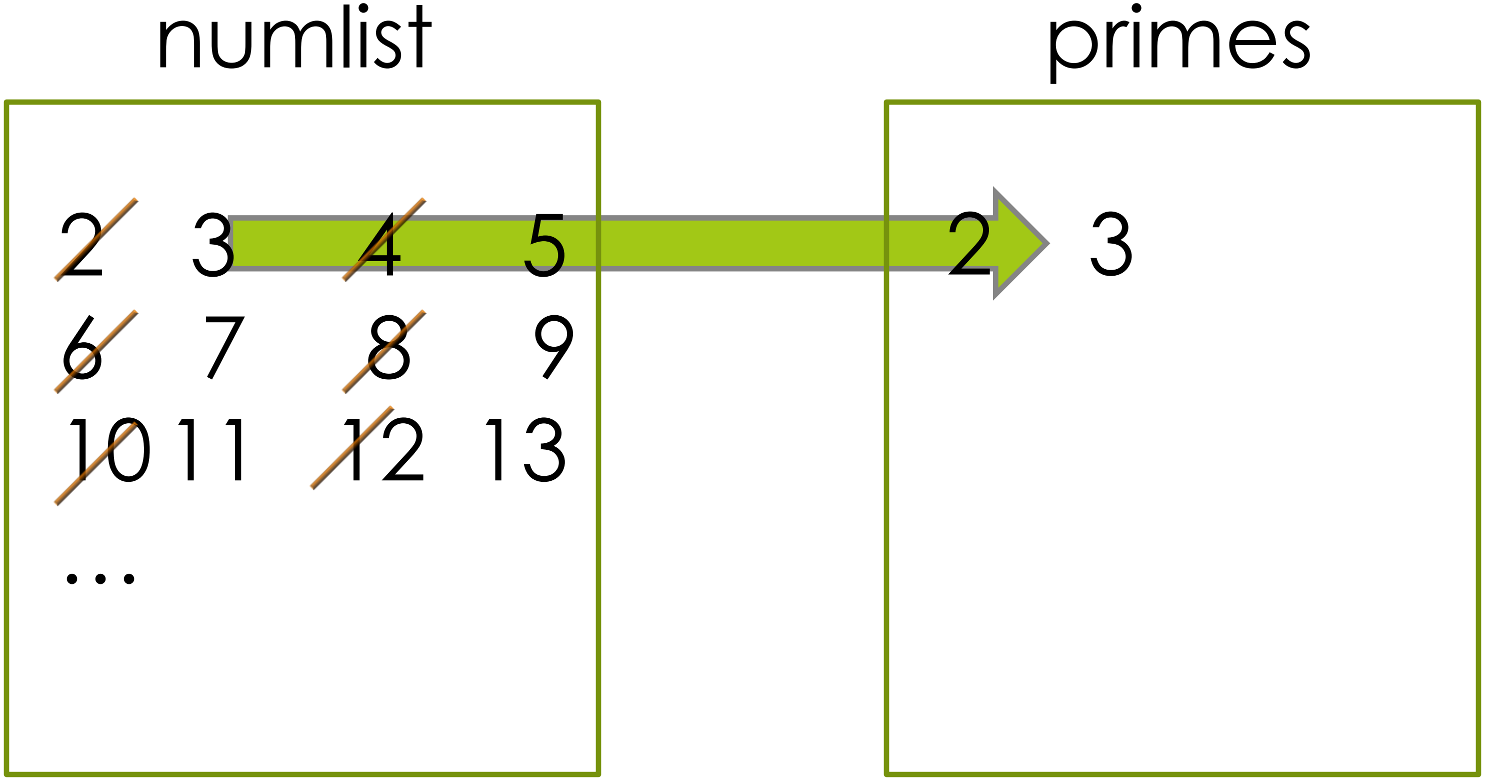
~~2~~ 3 ~~4~~ 5
~~6~~ 7 ~~8~~ 9
~~10~~ 11 ~~12~~ 13
…

primes

2

Cross out all the multiples of the last number in primes.

Iterations



numlist

~~2~~ 3 ~~4~~ 5
~~6~~ 7 ~~8~~ 9
~~10~~ 11 ~~12~~ 13
…

primes

2 3

Append the current number in numlist to the end of primes.

Iterations

numlist

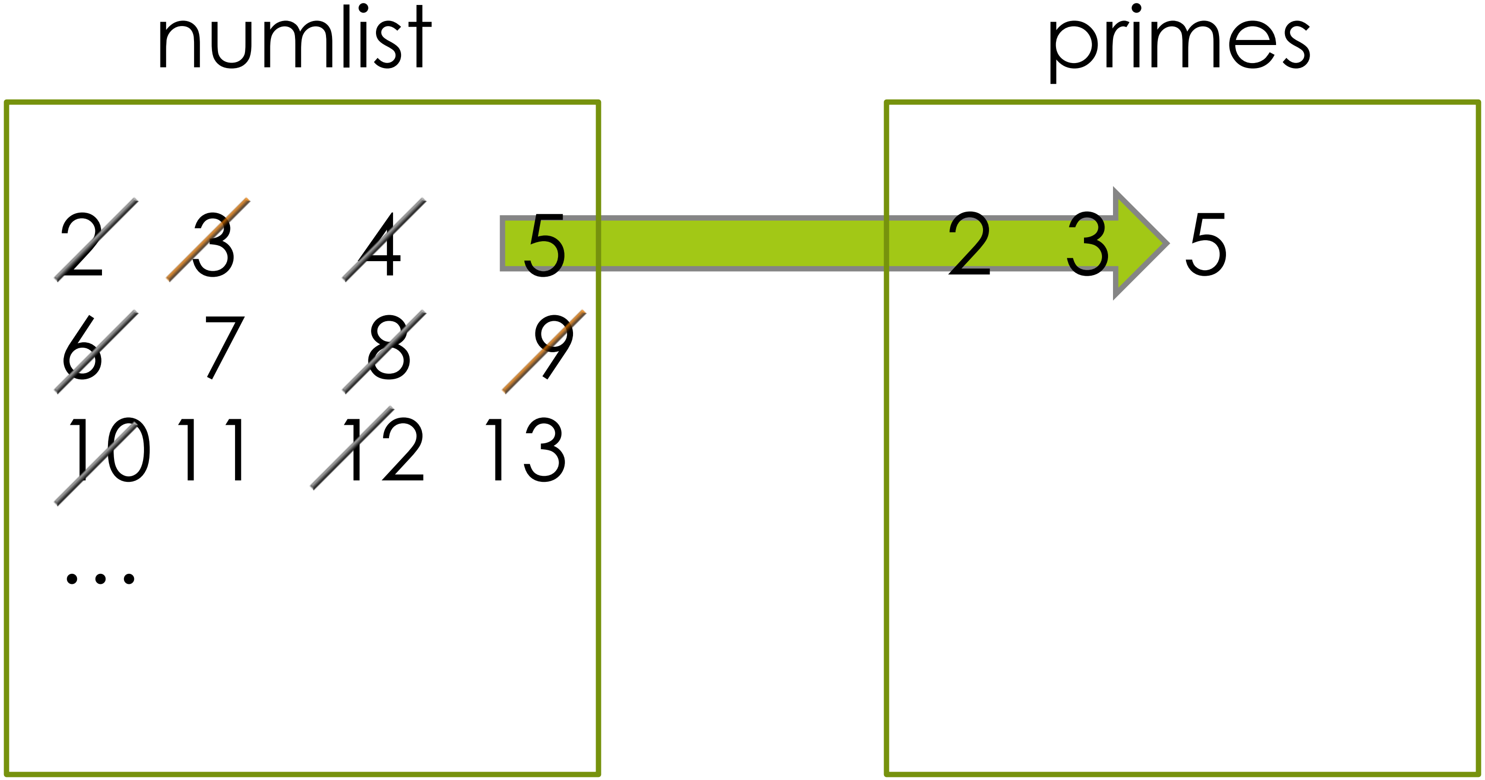
~~2~~ ~~3~~ ~~4~~ 5
~~6~~ 7 ~~8~~ ~~9~~
~~10~~ 11 ~~12~~ 13
…

primes

2 3

Cross out all the multiples of the last number in primes.

Iterations



numlist

~~2~~ ~~3~~ ~~4~~ 5
~~6~~ 7 ~~8~~ ~~9~~
~~10~~ 11 ~~12~~ 13
…

primes

2 3 5

Append the current number in numlist to the end of primes.

Iterations

numlist

~~2~~ ~~3~~ ~~4~~ ~~5~~
~~6~~ 7 ~~8~~ ~~9~~
~~10~~ 11 ~~12~~ 13
…

primes

2 3 5

Cross out all the multiples of the last number in primes.

An Algorithm for Sieve of Eratosthenes

Input: A number *n*:

1. Create a list ***numlist*** with every integer from 2 to n, in order. (Assume *n* > 1.)

2. Create an empty list ***primes***.

3. For each element in ***numlist***

   a. If element is not marked, copy it to the end of ***primes***.

   b. Mark every number that is a multiple of the most recently discovered prime number.

Output: The list of all prime numbers less than or equal to *n*

Implementation Decisions

- How to implement *numlist* and *primes?*
  - *For numlist* we will use a list in which crossed out elements are marked with the special value None. For example,

```
[None, 3, None, 5, None, 7, None]
```

- Use a helper function for step 3.b. We will call it sift.

Relational Operators

- If we want to compare two integers to determine their relationship, we can use these relational operators:

<	less than	<=	less than or equal to
>	greater than	>=	greater than or equal to
==	equal to	!=	not equal to

- We can also write compound expressions using the Boolean operators and and or.

```
x >= 1 and x <= 1
```

Sifting: Removing Multiples of a Number

```
def sift(lst, k):
    # marks multiples of k with None
    i = 0
    while i < len(lst):
        if (lst[i]!=None) and lst[i]%k == 0:
            lst[i] = None
        i = i + 1
    return lst
```

Filters out the multiples of the number k from list by marking them with the special value None (greyed out ones).

Sifting: Removing Multiples of a Number (Alternative version)

```
def sift2(lst,k):
    i = 0
    while i < len(lst):
        if lst[i] % k == 0:
            lst.remove(lst[i])
        else:
            i = i + 1
    return lst
```

Filters out the multiples of the number k from list by modifying the list. Be careful in handling indices.

A Working Sieve

```
def sieve(n):
    numlist = list(range(2,n+1))
    primes = []
    for i in range(0, len(numlist)):
        if numlist[i] != None:
            primes.append(numlist[i])
            sift(numlist, numlist[i])
    return primes
```

We could have used `primes[len(primes)-1]` instead.

Helper function that we defined before

Observation for a Better Sieve

We stopped at 11 because all the remaining entries must be prime since 11 × 11 > 50.

	2	**3**	4	**5**	6	**7**	8	9	10
11	12	**13**	14	15	16	**17**	18	**19**	20
21	22	**23**	24	25	26	27	28	**29**	30
31	32	33	34	35	36	**37**	38	39	40
41	42	**43**	44	45	46	**47**	48	49	50

A Better Sieve

```
def sieve(n):
    numlist = list(range(2, n + 1))
    primes = []
    i = 1
    while i <= math.sqrt(n):
       if numlist[i] != None:
          primes.append(numlist[i])
          sift(numlist, numlist[i])
          i = i + 1
    return primes + numlist
```

Algorithm-Inspired Sculpture

The Sieve of Eratosthenes, 1999 sculpture by Mark di Suvero. Displayed at Stanford University.